IIT(BHU) VARANASI

STUDENTS' NOTES

UNDERGROUND MINE ENVIRONMENT

FIFTH SEMESTER

TOPIC-FLUID

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Underground Mine Environment

Gases are compressible fluids due to large intermolecular spaces and weak intermolecular forces.

Difference between gas & a liquid :-

(i) Intermolecular Forces & spaces
(ii) Diffusion.
(iii) Compressibility

VOLUME FLOW, MASS FLOW & CONTINUITY EQ^n :

M : mass flow, $kg\,s^{-1}$ Q : Volume flow m^3/s

So $\rho = \frac{m/s}{v/s} = \frac{M}{Q} \Rightarrow \boxed{M = Q\rho}$

In a duct with no inlets & outlets & in steady state mass flow is constant.

$\therefore$ $M = \rho Q$ = constant.

This is simplest form of Continuity Equation.

Now, $Q = u \times A$ u: velocity A: cross section.

So $M = \rho u A$ = const

The density of air can be assumed constant

$\therefore$ $\boxed{uA = \text{constant}}$

FLUID PRESSURE : Two conclusion made out of bombardment of large no. of particles on a boundary : (i) force exerted by static fluid is always normal to surface.

(ii) In a static fluid pressure is same

PRESSURE HEAD: $P = h\rho g$

If ρ is known and assuming g a constant then P can be expressed in terms of h, head of liquid. Used in manometers.

This can also be used for gases and air. It should be kept in mind that air density may vary with height. So a mean value can be taken.

Standard atm. pressure: One standard atm. pressure is that which will support a column of mercury of height 0.760 m and where $g = 9.8066\, m s^{-2}$.

So One s.a.p $= h\rho g$

$$= 0.76 \times 13.5951 \times 10^3 \times 9.8066$$

$$= 101.324 \text{ KPa}$$

Gauge Pressure: It is the difference between the pressure inside a system ~~and~~ and external atmospheric pressure

~~Gauge pressure~~ ∴ Absolute Pressure $= P_{atm} + $ Gauge pressure

If P_{Abs} is less than P_{atm} then (-) ve ~~PG~~ P_{gauge} is called suction pressure or vaccum.

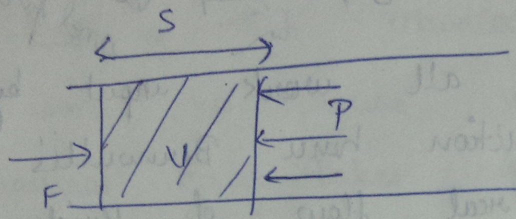
* Head of one fluid can be converted into other using: $\rho_1 g h_1 = \rho_2 g h_2 \Rightarrow \boxed{h_2 = \frac{\rho_1}{\rho_2} h_1}$

FLUIDS IN MOTION: For an ideal fluid stream flowing in a duct there are 3 types of energies associated with it

(1) KINETIC ENERGY : $\frac{1}{2}mu^2$

(2) Potential Energy : mgZ

(3) <u>Flow Work / Pressure Energy</u> :



When a plug of fluid tends to insert inside a pipe, there is a resistance due to pressure already inside the pipe. We must exert a force F to overcome this pressure. So

Work done $= FS = \underbrace{PAS}_{\text{volume}} = PV$

Now $\rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho}$

$\therefore$ Flow work $= \frac{mP}{\rho}$

$\therefore$ Total mechanical Energy = KE + PE + Flow work

$$= \frac{1}{2}mu^2 + mgZ + \frac{mP}{\rho}$$

This must remain constant throughout the duct.

Thus, Bernoulli's Eqn:

$$\frac{1}{2}mu^2 + mgZ + \frac{mP}{\rho} = \text{const}$$

$$\therefore m\left\{\frac{u^2}{2} + gZ + \frac{P}{\rho}\right\} = \text{const}$$

For station ① $\longrightarrow$ ②

$$m\left\{\frac{u_1^2}{2} + gZ_1 + \frac{P_1}{\rho}\right\} = m\left\{\frac{u_2^2}{2} + gZ_2 + \frac{P_2}{\rho}\right\}$$

$$\boxed{\frac{u_1^2 - u_2^2}{2} + g(Z_1 - Z_2) + \frac{P_1 - P_2}{\rho} = 0}$$

interrelation between vel, elevation and pressure

VISCOSITY: In mine ventilation all work input by fans is used against air friction hence Bernoulli's Eq^n must be amended for real flow of fluids.

Let us consider two layers of fluid & contact area A.

$\mu_{air} = (17 + 0.045t) \times 10^{-6}$

$\mu_{water} = \left(\frac{64.72}{t + 31.766} - 0.2455\right) \times 10^{-3}$

$\longrightarrow u + du$

$\longleftarrow u$

Faster one tends to drag the slower one with it & latter acts as a brake on former.

$\therefore$ Shear stress, $\tau = \frac{F}{A}$

Now, Issac Newton stated shear stress $\propto \frac{du}{dy}$ (velocity gradient)

$\therefore \quad \tau = \mu \frac{du}{dy}$ μ: coefficient of dynamic viscosity or dynamic viscosity.

$\mu \downarrow$ temp. dependant

Now, $\nu(nu) = \frac{\mu}{\rho \text{ (density)}} \longrightarrow$ Kinematic viscosity

Causes of viscosity : (i) intermolecular forces (dominant in liquid)
(ii) movement of molecules from one layer to other.

<u>Effect of temp</u> :- Liquids :- Here interm. forces are major cause. On $\uparrow$ temp, forces weaken hence viscosity decreases.

Gases :- Here reason is transfer of inertia from one to other layer. On ↑ Temp. KE of molecules ↑ ∴ viscosity increases.

Dynamic eq. is achieved b/w higher & lower velocity layer. ~~But~~ & thus maintaining net energy levels. But since no process is real ∴ loss of mechanical E. takes place. This results in observable pressure drop also called frictional pressure drop.

∴ Rewriting B.E.: $\quad$ energy from mech $\rightarrow$ heat

$$\frac{u_1^2}{2} + gZ_1 + \frac{P_1}{\rho} = \frac{u_2^2}{2} + gZ_2 + \frac{P_2}{\rho} + F_{12}$$

~~Q. $\rho_{mean} = 1.2\ kgm^{-3}$ $t = 18°C$~~

FLOW $\rightarrow$ (i) Laminar

FLOW $\rightarrow$ (ii) Turbulent

(i) Also known as streamline flow. Fluid flows in form of parallel layers.

(ii) Fluid undergoes irregular fluctuations or mixing.

Reynold's Number: $Re = \frac{\rho u d}{\mu}$

$d \rightarrow$ hydraulic mean diameter

$u \rightarrow$ mean velocity

$\mu \rightarrow$ coefficient of viscosity.

Re < 2000 - laminar (viscous forces prevail)

Re b/w 2500 - 3000 → turbulent

Q. $\rho = 1.2\ kgm^{-3}$ $\quad d = 5m \quad Q = 200 m^3/s \quad t = 18°C.$

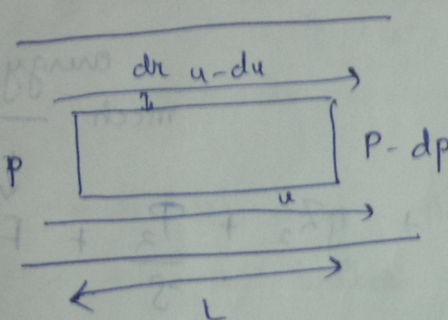
$Re = \frac{\rho u d}{\ }$ $\quad \mu_{air} = 17.81 \times 10^{-6}\ Nsm^{-2}$

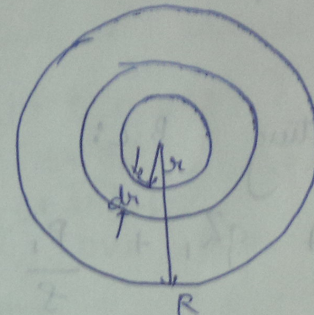
$Av = Q \Rightarrow 200 = \pi \frac{(5)^2}{4} \times v$

$v = \frac{\overset{8}{\cancel{200}}}{\frac{\cancel{25}\pi}{4}} = \frac{8 \times 4}{\pi} = \frac{32}{\pi} = 10.18\ ms^{-1}$

So $Re = \frac{1.2 \times 10.18 \times 5}{17.81 \times 10^{-6}} = 3.43 \times 10^{6} \longrightarrow$ turbulent.

FRICTIONAL LOSSES IN LAMINAR FLOW :





Propogation of fluid is due to pressure difference at two ends. Force $= \pi r^2 \times p$

-viscous force, $\tau = -\mu \frac{du}{dr}$ (due to u-du)

For steady flow ~~stress stress~~ $\tau \times A$ = force due to pressure diff

$\therefore \cancel{\pi} r^{\cancel{2}} p = -\mu \frac{du}{dr} \times 2\cancel{\pi}\cancel{r} L$

$\therefore$ ~~dr~~ ~~$\mu\pi r^2 p\, dr$~~

$\Rightarrow \frac{du}{dr} = -\frac{p}{L} \frac{r}{2\mu}$ --> const

-- constant

$\therefore du = -\frac{p}{L} \frac{r}{2\mu} dr$

$\Rightarrow u = -\frac{p}{L} \frac{r^2}{4\mu} + C$

when ~~Br~~ $r = R$ $u = 0$

$\therefore \quad C = \dfrac{PR^2}{4\mu L}$

So $\quad u = \dfrac{P}{4\mu L}(R^2 - r^2)\ ms^{-1}$

General exp for velocity.

u is max when $r \to 0$.

$\therefore \quad u_{max} = \dfrac{PR^2}{4\mu L} \longrightarrow$ max velocity

Mean velocity $\quad u_{mean} = \dfrac{Q}{A}\ ms^{-1}$

we know: $\quad Q = AV$

$dQ = u \times dA = u \times 2\pi r\, dr$

~~$Q = u\,\pi r^2 + C$~~

$dQ = \dfrac{P}{2\cancel{4}\mu L}(R^2 - r^2) \times \cancel{2}\pi r\, dr$

$dQ = \dfrac{P\pi}{2\mu L}(R^2 r - r^3)$

$Q = \dfrac{P\pi}{2\mu L}\left[\left(\dfrac{R^2 r^2}{2} - \dfrac{r^4}{4}\right)\right]_0^R$

$\boxed{Q = \dfrac{P\pi R^4}{8\mu L}} \longrightarrow$ poiseuille's eq^n.

$$Q = \frac{P}{L}\frac{\pi R^4}{8\mu} \Rightarrow P = \frac{Q\, 8\mu L}{\pi R^4} \Rightarrow P = R_L Q$$

$$\therefore \boxed{R_L = \frac{8\mu L}{\pi R^4} \text{ (Resistance of Laminar flow)}}$$

Now,

$$u_m = \frac{Q}{A} = \frac{P}{L}\frac{\cancel{\pi R^4}\, R^2}{8\mu \times \cancel{\pi R^2}} = \frac{PR^2}{8\mu L}$$

$$\boxed{P = \frac{8\mu u_m L}{R^2}}$$

Frictional pressure drop

Frictional pressure drop

Now using Bernoulli's eqn:

$$\frac{u_1^2 - u_2^2}{2} + g(Z_1 - Z_2) + \frac{(P_1 - P_2)}{\rho} = F_{12}$$

$P_1 - P_2 \to P$

$$\frac{u_1^2 - u_2^2}{2} + g(Z_1 - Z_2) + \frac{P}{\rho} = F_{12}.$$

For incompressible fluid along a level of pipe :-

$u_1 = u_2 = u_m, \quad Z_1 = Z_2$

$$\therefore \frac{P}{\rho} = F_{12} \Rightarrow \boxed{F_{12} = \frac{8\mu u_m L}{R^2 \rho}}$$

So Bernoulli's Eqn becomes :-

$$\boxed{\frac{u_1^2 - u_2^2}{2} + g(Z_1 - Z_2) + \frac{P_1 - P_2}{\rho} = \frac{8\mu u_m L}{R^2 \rho}}$$

Q. $d = 0.02m$ $L = 2000m$ $P_{atm} = 100 kPa$

$Q = 1.6 \ lt/min = \frac{1.6}{1000 \times 60} = 2.67 \times 10^{-5} \ m^3 s^{-1}$.

$Re = \frac{\rho u d}{\mu} = 1491$.

$A = \pi (0.01)^2 = 3.14 \times 10^{-4} m^2$.

$R = \frac{8 \mu L}{\pi R^4}$ $\mu_{water} = \left(\frac{64.72}{t + 31.766} - 0.2455 \right) \times 10^{-3}$

$= 1.138 \times 10^{-3} \ Nsm^{-2}$

$R = \frac{8 \times 1.138 \times 10^{-3} \times 2000}{3.14 \times (0.01)^4} \times 10^8 = 580 \times 10^6 \ \frac{Ns}{m^5}$

$u_m = \frac{Q}{A} = \frac{2.67 \times 10^{-5}}{\pi (0.01)^2} = 0.08488 \ ms^{-1}$.

$\therefore$ $F_{12} = \frac{8 \mu u_m L}{R^2 \rho} = 15.461 \ J/kg$.

Frictional Losses In Turbulent Flow :

- CHEZY - DARCY EQ^n: Chezy found that mean velocity of water in open ducts is directly prop. to sq. root of cross sectional area, channel gradient & inversely to wetted perimeter. i.e

$$u \propto \sqrt{\frac{Ah}{\underbrace{per\, L}_{\text{wetted perimeter}}}}$$

$\frac{h}{L}$: hydraulic gradient

$$\therefore \boxed{u = \underbrace{c}_{\text{Chezy coefficient}} \sqrt{\frac{Ah}{per\, L}}} \longrightarrow \text{Chezy equation}$$

~~Fluid Mechanics~~

Shear stress on fluid $= \tau$

Drag force $= \tau$ per L.

Force due to pressure difference $= pA$.

Now, $pA = \tau$ per L

$\tau \propto \frac{\rho u^2}{2} \Rightarrow \tau = f \frac{\rho u^2}{2}$.

$\Rightarrow \cancel{f \frac{\rho u^2}{2} = \tau \text{ per L}} \Rightarrow f \frac{\rho u^2}{2} \text{ per L} = \underbrace{\rho g h}_{p = \rho g h} A$

$\Rightarrow f \frac{\cancel{\rho} u^2}{2} \text{ per L} = \cancel{\rho} g h A$

$u = \sqrt{\frac{2g}{f}} \sqrt{\frac{Ah}{\text{per L}}} \quad \therefore \boxed{C = \sqrt{\frac{2g}{f}}} \rightarrow$ chezy constant

Now, $A = \frac{\pi d^2}{4}$, per $= \pi d$.

So $u = \sqrt{\frac{2g}{f}} \sqrt{\frac{\cancel{\pi} d^{\cancel{2}} h}{4 \cdot \cancel{\pi} \cancel{d} L}} = \sqrt{\frac{2g}{f}} \sqrt{\frac{hd}{2 \cancel{4} L}}$

$\Rightarrow u^2 = \frac{hdg}{2fL} \Rightarrow \boxed{h = \frac{2fLu^2}{gd}}$

↳ CHEZY - DARCY EQ^n.

It can be written as: $\boxed{p = \frac{2\rho f L u^2}{d}}$