IIT(BHU) VARANASI

STUDENTS' NOTES

TOPIC-RELIABILITY

www.MINEPORTAL.in

Date 5/10/15
Page

SYSTEM SAFETY AND FAULT TREE ANALYSIS

Safety can be broadly defined as the avoidance of conditions that can cause injury, loss of life or severe damage to equipment and possibly the surrounding envt. Therefore, the focus here is on failures that may create safety hazards.

Reliability ⟶ operational hazards
Safety ⟶ safety hazards

The objective is to determine the design how these failures likely to occur, to estimate their probability of occurance and to ~~take~~ take corrective action. Often safety related failures of modes have a low probability of occurance, and therefore difficult to estimate.

Reliability testing at the system ~~way~~ level, may fail to generate an unsafe condition. Additionally because of design safety features, we backup or redundancy. A system safety failure is usually caused by a combination of events.

For eg. a combination of equipment failure; human error and an alarm failure may be neccessary before a boiler begins to overheat causing pressure buildup.

FAULT TREE ANALYSIS

Fault tree ⟶ logical representation of the systems

Fault tree is most common technique used for schematic ~~relationship~~ representation of system. It uses a causal relationship b/w various events and use a logic relationship b/w various events

FT starts at a possible system failure mode and then works backwards to identify events (fault events) which may contribute to a failure event; these events are enumerated in logical sequence i.e. event statements through logical connections (logic gates).

Such an analysis produce a tree like structure having basic events at its extremities. The basic events are those for which failure data is available or which cannot be further ~~dear~~ dissected into more further fundamental events.

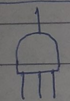
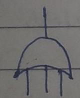
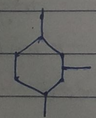
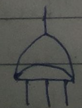
The basic events may be ~~denoted~~ sometimes distinguished as initiating or triggering or enabling events. An initiating event is the first failure event in an event sequence. While an enabling event i.e. warning system disabled for maintenance, ~~bo~~ will cause a higher level of failure when accompanied by a lower level of failure. Thus a fault tree is a boolean logic diagram comprised principally of AND ~~Gates~~ & OR logic gates.

Date / /
Page

The output event of the AND gate occurs if the input events occur simultaneously and from an ~~ ~~ OR gate if any of the input events occurs.

It will be observed that a fault free component an AND gate represents a parallel system, ~~which all must~~ all component must fail for the system to fail. Hence the system has degree of redundancy as the system will operate event if a component fails. An OR gate represents a series system, in which all components must operate for the system to operate.

GATE SYMBOLS

Gate Symbol	Gate Name	Causal relation
	AND	Output event occurs if all input events occur simultaneously
	OR	. . . any one of the input event occurs
	Inhibit	input produces output when conditional event occurs
	Priority AND	output event occurs if all input events occur in the order from L to R
	Exclusive OR	Output event occurs if one, but not both, of the input events occur

07/10/15

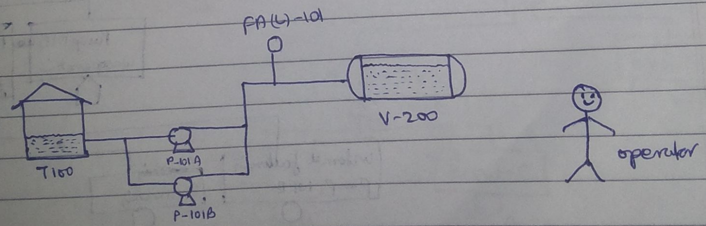
	m out of n Gate (Voting Gate)	output occurs if m out of n input events occur

EVENT SYMBOLS

02/10/15

Event Symbols	Meaning of symbols
circle	Basic event with sufficient data
diamond	underdeveloped event
rectangle	event represented by a gate
oval	Conditional event used with inhibit gate
house	house event, either occurring or not occurring
triangle	transfer symbols

Q1)

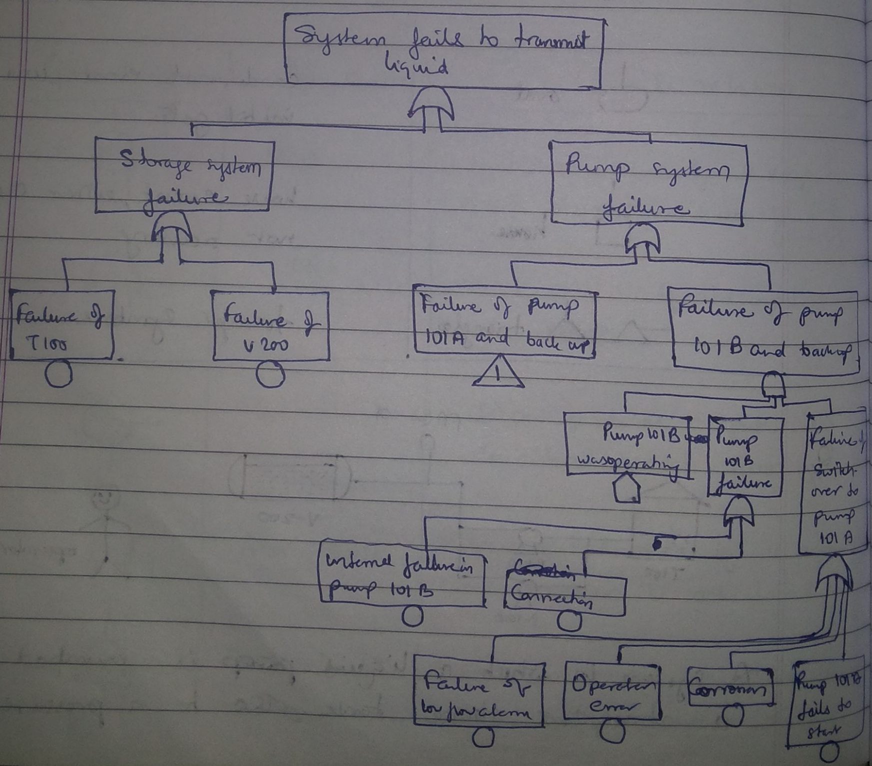


A system in which a liquid ~~poop~~ is pumped from a atmospheric storage tank ~~.....~~ to a pressure vessel V-200

Date / /
Page

Two pumps are available for service P 101 A (steam turbine driven) and P101 B (electric motor driven). Either pump has sufficient capacity to handle the full pumping load by itself. At any given time one pump is operating and the other is on standby. If the working pump fails, the operator has five minutes to switch to the standby pump. The operator is aware of a no-flow condition by a ~~[illegible]~~ no-flow alarm. The system is said to have failed, if the following statement is satisfied.

- System fails to transmit liquid from T-100 to V-100 for an interrupted period of 5 min or more due to failure of equipment or due to operating error in the section.



www.MINEPORTAL.in Mine Portal App Available www.fb.com/mineportal.in

Date / /
Page

$\triangle$; // triangle on 1 continues

Quantification of fault Tree (FT)

- Basic event – lowest events in the FT
- Intermediate event
- Top event

OR Gate – $p_g = 1 - \prod_{i=1}^{n} (1 - p_i)$

p_g → probability at the gate

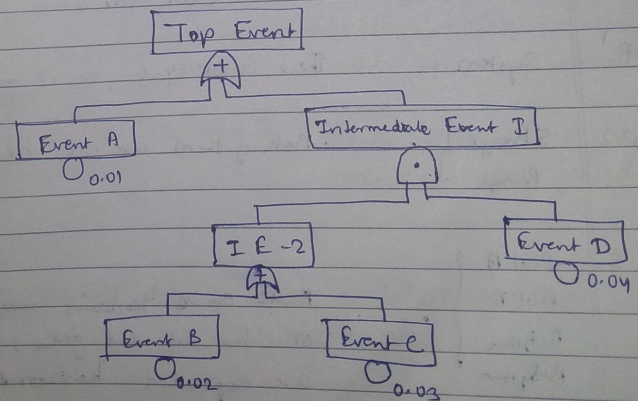
p_i → failure prob. of the i th event

AND Gate – $p_g = \prod_{i=1}^{n} p_i$

Voting Gate – $p_g = \frac{n!}{(n-r)!\, r!} p^r (1-p)^{n-r}$

p → prob. of event occurring

Eg:



Date / / Page

Method Cut Sets

A cut set is a collection of basic events such that, if all the basic events were to occur, the top event would occur.

A minimum cut set is a minimum ~~or~~ and sufficient combination of ~~for~~ functional elements, whose failure leads to system failure.

Algorithm for quantification by minimum cut set :-

Step I Determine the cut sets
Step II Eliminate repeat events within a cut set
Step III Eliminate repeat cut sets
Step IV Eliminate "super cut sets"
Step V Calculate the top event probabilities

Developing minimum cut sets :-

Step 1 System ------ There is OR Gate, so it expands vertically

Step 2 Storage, Pumps ----- Both of them are also OR gates and expanded vertically

Step 3 P101 A, P101 B, A-System, B-System ----- Basic events remains unchanged, the other two are and gate, therefore they are expanded horizontally

Supply $F100^*$
$V\text{-}200^*$
A operator A fails
B operator

———— × ————

14/10/15

Analysis of failure and repair data

Sources
- operational & field data
- generated data

Censoring : unit is removed before failure occurs

Singly censored data
- censored on left
- censored on right
  - Type I : units are tested for a certain length of time
  - Type II : tested until a no. of components have failed

Multiply censored data : Testing units removed from test at different time

Emperical methods

- non parametric.
- distribution free.
- ~~objective~~ :- derive expression for failure distribution, reliability fn and hazard rate fn.

Ungrouped complete data

$t_1, t_2, t_3, \dots t_n$ where $t_i \le t_{i+1}$

n - ordered failure time

$$\hat{R}(t_i) = \frac{n-i}{n} = 1 - \frac{i}{n}$$

$$\hat{F}(t_i) = 1 - \hat{R}(t_i) = \frac{i}{n}$$

however, a better estimate is

$$\hat{F}(t_i) = \frac{i}{n+1}$$

$$\hat{R}(t_i) = 1 - \frac{i}{n+1} = \frac{n+1-i}{n+1}$$

plotting positions :

- ~~based on mean~~
- Mean plotting position (based on mean)
- Median plotting position (based on median)

$$\hat{F}(t_i) = \frac{i-0.3}{n+0.4} \rightarrow \text{Benard's approximation}$$

$$\hat{f}(t_i) = \frac{\hat{R}(t_{i+1}) - \hat{R}(t_i)}{t_{i+1} - t_i}$$

$$= \frac{1}{(t_{i+1} - t_i)(n+1)}$$

$$\hat{\lambda}(t_i) = \frac{1}{(t_{i+1} - t_i)(n+1-i)} \quad \text{for } t_i < t < t_{i+1}$$

$$\widehat{MTTF} = \sum_{i=1}^{n} \frac{t_i}{n}$$

$$S^2 = \sum_{i=1}^{n} \frac{(t_i - \widehat{MTTF})^2}{n-1}$$

$$S^2 = \frac{\sum_{i=1}^{n} t_i^2 - n(\widehat{MTTF})^2}{n-1}$$

$100(1-\alpha)$ · percent $\longrightarrow$ confidence interval for underlying MTTF

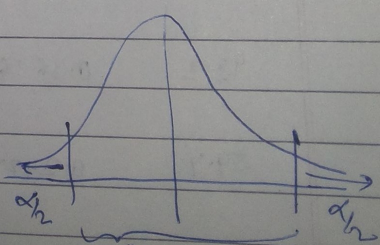
$$MTTF \pm t_{\alpha/2,\, n-1} \frac{S}{\sqrt{n}}$$

confidence $\Rightarrow 100(1-\alpha)$

for $\alpha = 5\%$

$100(1-0.05)$

$= 95\%$ confidence MTTF



"two-tailed estimation"

Date / /
Page

Q. Given the following 10 failure times in hours, estimate $R(t)$, $F(t)$, $f(t)$ and $\lambda(t)$. Also calculate a 90% confidence interval for the MTTF of these ungrouped data.

24.5, 18.9, 54.7, 48.2, 20.1, 29.3, 15.4, 33.9, 72.0, 86.1

Sol^n –

$n = 10$

Rank ordering of data in ascending order

15.4, 18.9, 20.1, 24.5, 29.3, 33.9, 48.2, 54.7, 72.0, 86.1

Time	Reliability $R(t)$	Cumm. failure prob. $F(t)$	Probability density $f(t)$	Hazard rate $\lambda(t)$
0.0	1	0	0.005903	0.005903
15.4	0.9090	0.090909	0.025974	0.028571
18.9	0.818181	0.181818	0.075757	0.0925927
20.1	0.727272	0.272727	0.020661	0.0284091
24.5	0.636363	0.363636	0.018939	0.029762
29.3	0.545454	0.454545	0.019763	0.036232
33.9	0.454545	0.545454	0.006357	0.013986
48.2	0.363636	0.636363	0.013986	0.038462
54.7	0.272727	0.727272	0.005255	0.019268
72.0	0.181818	0.818181	0.006447	0.035461
86.1	0.090909	0.909090		

$$\hat{f}(0.0) = \frac{1}{15.4} \times \frac{1}{11}$$

$$\hat{R}(15.4) = \frac{n+1-i}{n+1} = \frac{10+1-1}{10+1}$$

$$\hat{F}(15.4) = 1 - \hat{R}(15.4) = \frac{i}{n+1} = \frac{1}{10+1}$$

$$\widehat{MTTF} = \frac{15.4 + 18.9 + \dots + 86.1}{10} = 40.31$$

$$s^2 = \frac{15.4^2 + 18.9^2 + \dots + 86.1^2 - 10(40.31)^2}{9} = 585.5454$$

$$s = 24.198$$

From table: $t_{0.05,9} = 1.833$

the desired interval,

$$40.31 \pm 1.833 \times \frac{24.198}{\sqrt{10}}$$

$$= 40.31 \pm 14.026$$

$$= [26.284, 54.34]$$

Date 02/11/15

Page

Grouped complete data

$n_1, n_2, \ldots, n_k$ no. of units survived ordered times $t_1, t_2, \ldots, t_k$.

$$\hat{R}(t_i) = \frac{n_i}{n} \qquad i = 1, 2, \ldots, n$$

$$\hat{f}(t_i) = -\frac{\hat{R}(t_{i+1}) - \hat{R}(t_i)}{t_{i+1} - t_i} \qquad \text{for } t_i < t < t_{i+1}$$

$$= \frac{n_i - n_{i+1}}{(t_{i+1} - t_i)\, n}$$

$$\hat{\lambda}(t_i) = \frac{\hat{f}(t_i)}{\hat{R}(t_i)} = \frac{n_i - n_{i+1}}{(t_{i+1} - t_i)\, n_i} \qquad \text{for } t_i < t < t_{i+1}$$

$$\widehat{MTTF} = \sum_{i=0}^{k-1} \bar{t}_i \, \frac{n_i - n_{i+1}}{n} \qquad \text{when } \bar{t}_i = \frac{t_i + t_{i+1}}{2}$$

and $t_0 = 0$, $n_0 = 0$

fraction failed in the interval $i+1$

the sample variance,

$$S^2 = \sum_{i=0}^{k-1} \bar{t}_i^{\,2} \, \frac{n_i - n_{i+1}}{n} - \widehat{MTTF}^{\,2}$$

Q. 70 compressors are observed at 5 months interval with the following no. of failures: 3, 7, 8, 9, 13, 18, 12. Estimate $\hat{R}(t)$, $f(t)$ and $\lambda(t)$. Also determine the sample mean time to failures and sample standard deviation.

upper bound month	nos. failing	nos. surviving	Reliability	failure density	Hazard rate
0	0	70	1.00	0.00857	0.00857
5	3	67	0.95714	0.0200	0.0209
10	7	60	0.85714	0.02286	0.0267
15	8	52	0.74286	0.02571	0.03461
20	9	43	0.61429	0.03714	0.06046
25	13	30	0.42857	0.05143	0.12000
30	18	12	0.17143	0.034286	0.19997
35	12	0	0	—	—

$$\widehat{MTTF} = \frac{2.5\times3 + 7.5\times7 + 12.5\times8 + \dots + 32.5\times12}{70}$$

$= 21.357$

$$s^2 = \frac{2.5^2\times3 + 7.5^2\times7 + 12.5^2\times8 + \dots + 32.5^2\times12}{70} - 21.357^2$$

$= 76.557$

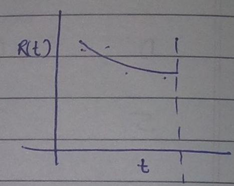
$s = 8.75$

Ungrouped censored data

n units are placed, r failures occur $r \le n$

$$\hat{R}(t_i) = \frac{n+1-i}{n+1} , \quad \hat{f}(t) = \frac{1}{(t_{i+1}-t_i)(n+1)}$$

$$\hat{\lambda}(t_i) = \frac{1}{(t_{i+1}-t_i)(n+1-i)}$$



Methods for estimating reliability of ungrouped censored data :-

1) Product limit estimator
2) Kaplan - Meier form of product limit estimator
3) Rank adjustment method

$$\hat{R}(t_i) = \frac{n+1-i}{n+1}$$

$$\hat{R}(t_{i-1}) = \frac{n+2-i}{n+1}$$

$$\Rightarrow \frac{\hat{R}(t_i)}{\hat{R}(t_{i-1})} = \frac{n+1-i}{n+2-i}$$

$$\Rightarrow \hat{R}(t_i) = \frac{n+1-i}{n+2-i} \hat{R}(t_{i-1})$$

If t_i is a censored time

$$\hat{R}(t_i) = \hat{R}(t_{i-1})$$

Date / /
Page

$$\delta_i = \begin{cases} 1 & \text{— if failure occurs at time } t \\ 0 & \text{— if censoring occurs at time } t_i \end{cases}$$

then, $R(t_i) = \left(\dfrac{n+1-i}{n+2-i}\right)^{\delta_i} \hat{R}(t_{i-1})$

when $\hat{R}(0) = 1$

The following failures & censored times are recorded on 10 turbine vanes in ~~of~~ o/p hours

150, 340+, 560, 800, 1130+, 1720, 2470+, 4210+, 5230, 6890

i	t_i	n_j	$1-\frac{1}{n_j}$	$\hat{R}(t_i+0)$ (reliability following immediately the i^{th} failure)	Standard dev.
1	150	10	9/10	$R(150) = 0.90$	0.09487
2	340+				
3	560	8	7/8	$R(560) = \frac{7}{8} \times 0.90 = 0.7875$	0.134033
4	800	7	6/7	$R(800) = \frac{6}{7} \times 0.7875 = 0.675$	$0.675 \times \sqrt{\frac{1}{90}+\frac{1}{56}+\frac{1}{42}} = 0.1507$
5	1130+				
6	1720	5	4/5	$R(1720) = 0.54$	0.17312
7	2470+				
8	4210+				
9	5230	2	1/2	$R(5230) = 0.27$	0.209625
10	6890	1	0		

$$\hat{R}(t) = \prod \left(1 - \frac{1}{n_j}\right)$$

$$\hat{SD}(t) = \hat{R}(t) \sqrt{\sum_{\{j:t_j<t\}} \frac{1}{n_j(n_j-1)}}$$

Kaplan-Meier form

Date / /
Page

spring summ

A shoe ~~cost~~
The shoe cost is ₹4000/- and retail is ₹6000/- per pair.
If there still remains shoes not sold upto July, they will be put on clearance sale at ₹ 3000/-. It is expected that all remaining shoes can be sold during the sale. The demand range is b/w 350 – 650 pairs.
Determine is optimum quantity of shoe order.

P.M.

02/11/15

Kaplan – Meier form of product limit estimator

t_j = ordered failure times

n_j = no. remaining at risk just prior to the j^{th} failure

$$\hat{R}(t) = \prod_{\{j : t_j \le t\}} \left(1 - \frac{1}{n_j}\right) \quad \text{for } 0 \le t \le t_i \qquad R(0) = 1$$

$$\hat{var}\left[\hat{R}(t)\right] = \hat{R}(t)^2 \sum_{\{j : t_j < t\}} \frac{1}{n_j(n_j - 1)}$$

Rank Adjustment method

$$\hat{F}(t_i) = \frac{i - 0.3}{n + 0.4}$$

$$\hat{R}(t_i) = 1 - \hat{F}(t_i)$$

$$\text{Rank increment} = \frac{(n+1) - i_{t_{i-1}}}{1 + \text{no. of units beyond present censored unit}}$$

n = ~~...~~ total no. of units at risk

$i_{t_{i-1}}$ is the rank order of failure time $i-1$.

$i_{t_i} = i_{t_{i-1}}$ + rank increment

i	t_i	Rank increment	i_{t_i}	$\hat{R}(t_i) = 1 - \frac{i_{t_i} - 0.3}{n + 0.4}$
1	150	1	1	0.9327
2	340^+			
3	560	$(11-1)/(1+8) = 1.1111$	$1 + 1.1111 = 2.1111$	0.8259
4	800	1.1111 (rank increment remains same)	$2.1111 + 1.1111 = 3.2222$	0.71902
5	1130^+			
6	1720	$(11 - 3.2222)/(1+5) = 1.2963$	$3.2222 + 1.2963 = 4.5185$	0.5944
7	2470^+			
8	4210^+			
9	5230	$(11 - 4.5185)/(1+2) = 2.1605$	$4.5185 + 2.1605 = 6.679$	0.3866
10	6890	2.1605 (remains same)	$6.679 + 2.1605 = 8.8395$	0.1789

Page

Identify failure and repair distribution

~~Empirical~~

Q, Why do we prefer to feed a theoretical distribution over an empirically developed models

1) Empirical models do not provide information beyond the range of data. In reliability engineering the tails of the distribution are of most interest.

For ~~seemingly~~ singly censored data, extrapolation ~~beyond~~ beyond the censored data is possible with a theoretical distribution.

2) We are interested in determining the probabilistic nature of the underlying failure process. A sample is a small subset of a population of failure times. And it is the distribution the sample came from and not the sample itself we want to establish.

3) Often the failure process is a result of some physical phenomena that can't be associated with a particular distribution.

For example, the "Central limit theorem" provides justification for using the ~~modern~~ normal distribution when additive effects are present, or the lognormal distribution when multiplicative effects are present.

Date / /
Page

Identify failure and repair distribution

~~Empirical~~

Q) Why do we prefer to feed a theoretical distribution over an empirically developed models

1) Empirical models do not provide information beyond the range of data. In reliability engineering the tails of the distribution are of most interest.
For ~~seemingly~~ singly censored data, extrapolation beyond the censored data is possible with a theoretical distribution.

2) We are interested in determining the probabilistic nature of the underlying failure process. A sample is a small subset of a population of failure times. And it is the distribution the sample came from and not the sample itself we want to establish.

3) Often the failure process is a result of some physical phenomena that can't be associated with a particular distribution.
For example, the "Central limit theorem" provides justification for using the ~~modern~~ normal distribution when additive effects are present, or the lognormal distribution when multiplicative effects are present.

Date / /
Page

4) Small sample size provide very little information ~~concer~~ concerning the failure process, however if the sample is consistent with a theoretical, then much stronger ~~result~~ result, based on the properties of the

for example, the effect of changes on the MTTF can easily be accomplished.

04/11/15

Identification of candidate distribution
Estimate parameters
Perform goodness of fit list

~~Stat~~ - understanding of the failure process ~~know~~
- knowledge of the characteristic distribution

<u>Statistical analysis of data</u>

Sturge's rule
$K = [1 + 3.3 \log n]$

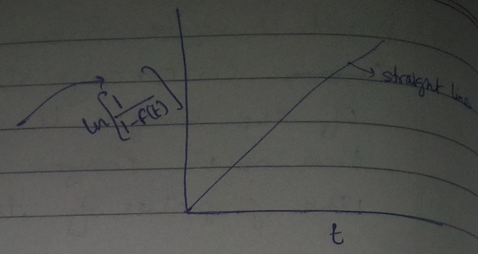
1) Construct a histogram of failure and repair time
2) Compute the descriptive statistics
3) Analyze the empirical failure rate
4) Use prior knowledge of the failure process
5) Use properties of the theoretical distribution
6) Construct a probability plot

- least square method (least of deg...)
- MLE

$F(t) = 1 - e^{-\lambda t}$

$\ln\left(1 - F(t)\right) = -\lambda t$

$\ln\left[\dfrac{1}{1-F(t)}\right] = \lambda t$



$F(MTTF) = 1 - e^{-1} = 0.632$

MLE (Maximum Likelihood Estimator)

$P_r\{X = x\} = f(x) = (1-p)^{x-1} \cdot p$

X = discrete random variable representing the no. of trials neccessary to obtain the first failure

p = probability of const. fails

$x_1, x_2, \dots, x_n$ represents sample of size n from the distribution

$f_{x_1, \dots x_n}(x_1, x_2, \dots, x_n) = f(x_1) f(x_2) \dots f(x_n)$

likelihood fn

$= (1-p)^{x_1 - 1} \cdot p * (1-p)^{x_1 - 1} \cdot p * \dots * (1-p)^{x_n - 1} \cdot p$

$= p^n (1-p)^{\sum_{i=1}^{n} (x_i - 1)}$

$\max_{0 \le p \le 1} g(p) = p^n (1-p)^{\sum_{i=1}^{n} (x_i - 1)}$

$\max \ln g(p) = \ln\left\{\prod_{i=1}^{n} f(x_i)\right\} = n \ln(p) + \sum_{i=1}^{n} (x_i - 1) \ln(1-p)$

Date / /
Page

Solving $\frac{d(\ln(g(p)))}{dp} = 0$

$$\frac{n}{p} + \frac{\sum_{i=1}^{n} x_i - 1}{1-p}(-1) = 0$$

$$\Rightarrow \hat{p} = \frac{n}{\sum_{i=1}^{n} x_i}$$

$$L(\theta_1, \theta_2, \ldots, \theta_k) = \prod_{i=1}^{n} f(t_i \mid \theta_1, \ldots, \theta_k)$$

$$\frac{\partial \ln L(\theta_1, \theta_2 \ldots \theta_n)}{\partial \theta_i} = 0 \quad , \quad i = 1, 2 \ldots k$$

for single censored data

$$L(\theta_1, \theta_2, \ldots, \theta_k) = \prod_{i=1}^{r} f(t_i \mid \theta_1, \theta_2, \ldots, \theta_k) \left[R(t_*)\right]^{n-r}$$

where, r is the no. of failures and n is the no. at risk

$\left[R(t_*)\right]^{n-r}$ probability that censored units do not fail prior to the determination of the

$$\hat{\lambda} = \frac{r}{T}$$

r → no. of failures

T → cumulative test time

Date / /
Page

Goodness-of-fit Test

$H_0 \rightarrow$ null hypothesis
$H_1 \rightarrow$ alternative hypothesis

H_0: The failure times came from the specified distribution
H_1: ——— didn't came from the specified distribution

	H_0 true	H_0 not true
Accept H_0	Correct decision	Type II error
Reject H_0	Type I error	Correct decision

Two type of tests

General test : CHI-SQUARE Goodness-of-fit Test

- applicable to both continuous & descrete data

descrete data $\rightarrow$ Ladge
continuous $\rightarrow$ MLE

χ^2 'ki'

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

K = no. of classes
O_i = observed no. of failures (or repairs) in i^{th} class
$E_i = np_i \rightarrow$ expected no. of failures in the i^{th} class

n = probability of failure occuring in the i^{th} class if H_0 is true

The following 35 failure times (in o/p hours) were obtained from field data over a six month period.

1476, 300, 98, 221, 157, 182, 499, 552, 1563, 36, 246, 442, 20, 796, 31, 47, 438, 279, 247, 210, 284, 553, 767, 1297, 214, 428, 597, 2025, 185, 467, 401, 210, 289, 1024

Sturge's rule,

$$K = \left[1 + 3.3 \log_{10} 35\right] = 6.0954 \approx 6$$

(count + max) 6 → for 6 classes

cell	upper bound	count (E → actual)
1	354	18
2	688	10
3	1022	2
4	1356	2
5	1690	2
6	2026	1

7 (after clubbing) because frequency was less than 5

$$\hat{\lambda} = \frac{1}{\hat{MTTF}} = 0.00206 \quad \left(\frac{1}{485.4}\right)$$

E calculated

$$E_1 = 35P_1 = 35\left[1 - e^{\frac{-354}{485.4}}\right] = 18.121$$

$$E_2 = 35P_2 = 35\left[1 - e^{\frac{-688}{485.4}} - P_1\right] = 8.897$$

$$E_3 = 35P_3 = 35 \quad 1 - P_1 - P_2 \quad = 8.482$$

H_0 : failure times are exponential with $\lambda = 0.00206$

H_1 : ———— not ————

$$\chi^2 = 0.56602$$

$$\alpha = 0.10$$

degree of freedom $= 3-1-1 = 1$

??

Date / /
Page

$$\chi^2_{critical} = \chi^2_{0.10,1} = 2.71 > \chi^2$$

Hence H_0 is accepted.