GATE MINING FORMULAE

Matrices & Det.

{Cayley-Hamilton Theorem: Pg - 90} A^x, x index

→ $I^{-1} = I$ → **Idempotent**: $A^2 = A$ → **Involutory**: $A^2 = I$ → **Nilpotent**: $A^x = 0$

→ Trace = Σ diag. elements. → $tr(\lambda A) = \lambda\, tr(A)$ → $tr(A+B) = tr(A) + tr(B)$

Types of Matrices

1) Idempotent $A^2 = A$
2) Involutory $A^2 = I$
3) Nilpotent $A^x = 0$ x - index

1) $[ABC]^T = C^T B^T A^T$

1) Conjugate = $\bar{A}$ $[i \to -i]$
2) Transp Conj $A^\theta = (\bar{A})^T$

$[ABC]^\theta = C^\theta B^\theta A^\theta$

1) Symmetric $A^T = A$
2) Skew Sym $A^T = -A$
3) Orthogonal $A^T = A^{-1}$

1) Hermitian $A^\theta = A$
2) Skew Herm $A^\theta = -A$
3) Unitary $A^\theta = A^{-1}$

cof $= (-1)^{i+j} |M_{ij}|$ minor

1) $\text{Adj}\, A = [\text{cof}(A)]^T$
2) $A^{-1} = \frac{1}{|A|} \text{adj}(A)$
3) $(ABC)^{-1} = C^{-1} B^{-1} A^{-1}$

E.V. ⇒ $\lambda^2 - 5\lambda + 1 = 0$

⇒ $A^2 - 5A + I = 0$ | λ: eigen val. of matrix A

Rank

1) Non-zero determinant of order n_{max}

$n \to$ rank

2) Row reduced echelon form

Diagonal ke niche elements = 0

→ rank = non-zero rows (no. of)

Eigen Values

→ $|A - \lambda I| = 0$

$\lambda \to$ eigen values

→ $(A - \lambda) X = 0$

$(A - \lambda I) X = 0$

$X \to$ eigen vectors

{Cayley-Hamilton eq^n}

Homogeneous Linear Equations

$a + b + c + d = 0$

$\ldots = 0$

$\ldots = 0$

⇒ System is always consistent

$X = [0, 0, 0 \ldots] \Rightarrow$ sol^n

⇒ Consistent unique sol^n ⇒ rank[A] = no. of variables ($r = n$)

⇒ Cons ∞ sol^n ⇒ $r < n$

- $n - r$ linearly indepent sol^n
- But total infinite sol^n

Non-Homogenous LE

$A_{11}x_1 + A_{12}x_2 + A_{13}x_3 = a$

$A_{21}x_1 \quad\quad A_{23}x_3 = b$

$A_{31}x_1 \quad\quad A_{33}x_3 = c$

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$A \quad\quad X = B$$

→ Augmented matrix = $[A|B] = \begin{bmatrix} A & A & A & a \\ A & A & A & b \\ A & A & A & c \end{bmatrix}$

① Inconsistency

$r(A) \neq r(A|B)$

② Unique sol^n

$r(A) = r(A|B) = n$

③ ∞ sol^n

$r(A) = r(A|B) < n$

$n \to$ no. of variables

Limits

1) $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2$

2) $(1-x)^{-1} = 1 + x + x^2 + x^3 \dots$

1) $a^x = 1 + x\log a + \frac{x^2}{2!}(\log a)^2 + \frac{x^3}{3!}(\ln a)^3$

2) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

1) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$

2) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$

3) $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5$

1) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$ $[|x|<1]$

2) $\ln(1-x) = -\left[x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4}\right]$

1) $\sin^{-1}x = x + \frac{x^3}{6} + \frac{3x^5}{40}$

2) $\tan^{-1}x = x - \frac{x^3}{5} + \frac{x^5}{5}$

1) $\lim_{x\to 0} \frac{\sin x}{x} = \frac{\tan x}{x} = 1$

2) $\lim_{x\to 0} (1+x)^{1/x} = e$

3) $\lim_{x\to 0} (1+nx)^{1/x} = e^n$

4) $\lim_{x\to \infty} \left(1 + \frac{1}{x}\right)^x = e$

5) $\lim_{x\to \infty} \left(1 + \frac{a}{x}\right)^x = e^a$

1) $0.\infty = \frac{0}{1/\infty}$ L-hos.

2) $0^0, 1^\infty, \infty^0 \Rightarrow$ Take log + do step-1

→ Continous at $[a,b]$ iff
i) cont. from right at a
ii) cont from left at b
iii) cont in (a,b)

same cond^n in different

→ Progressive derivative $= Rf'(x_0)$, $f'(x_0^+) = \lim_{h\to 0} \frac{f(x_0+h) - f(x)}{h}$

→ Regressive derivative $= Lf'(x_0)$ or $f'(x_0^-)$ $= \lim_{h\to 0} \frac{f(x-h) - f(x)}{-h}$

Rolles' Theorem : $f'(c) = 0$

$f(x) \to$ continous $[a,b]$ $c \in (a,b)$

$f'(x)$ exists @ (a,b)

Langrage Mean Value Theorem

3 same cond^n $f'(c) = \frac{f(b) - f(a)}{b-a}$

$\boxed{\text{Derivative}}$

- $\log_a x = \frac{1}{x}\ln a$
- $a^x = a^x \ln a$
- $\tan x = \sec^2 x$
- $\sec x = \sec x \tan x$
- $\operatorname{cosec} x = -\operatorname{cosec} x \cot x$
- $\cot x = -\operatorname{cosec}^2 x$
- $\sinh x = \cosh x$
- $\cosh x = \sinh x$
- $\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}$
- $|x| = \frac{x}{|x|}$
- $\cot^{-1}x = \frac{1}{1+x^2}$
- $\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$
- $\operatorname{cosec}^{-1}x = -\frac{1}{x\sqrt{x^2-1}}$
- $\tan^{-1}x = 1/1+x^2$
- $\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$

$y = f(x)^{g(x)} \Rightarrow \log y = g(x)\log(f(x))$

Maxima + minima

i) $f^{2n-1}(x) = 0$ $\Rightarrow f^{2n}(x) > 0 +$ minima

$\Rightarrow f^{2n}(x) < 0 \Rightarrow$ maxima

$\boxed{\text{Taylor Series}}$: $f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + \frac{h^3}{3!}f'''(a)$

$\boxed{\text{Fourier Series}}$: $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cdot \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right]$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x) \cdot \cos\left(\frac{n\pi x}{L}\right) dx$$

$2L$ = period of f^n

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x) \cdot \sin\left(\frac{n\pi x}{L}\right) dx$$

$\boxed{\text{Convergence}}$:. GP : $|r|<1 \Rightarrow$ convergent ; $r<-1 \Rightarrow$ oscillating

- $\lim_{n\to\infty} \frac{f(n)}{g(n)} = +K \Rightarrow f(n)$ & $g(n)$ are of same nature

- $\frac{1}{n^p} \Rightarrow$ convergent $p>1$ divergent $p<1$

$\boxed{\text{Partial Derivative}}$: $\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)$

AGP $\rightarrow \Sigma(AP).(GP) = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

Sample variance $= \sqrt{\dfrac{\Sigma(x_i-\mu)^2}{N-1}}$

Prob. & Stats.

std normal distri. $N(0,1)$

$z = \dfrac{x-\mu}{\sigma}$ mean $= 0 = E(x)$, var $= 1 = V(x)$

→ MODE = 3 Median − 2 mean

→ $\sigma^2 = \dfrac{\Sigma(x_i-\bar{x})^2}{n}$; S.D. $= \sigma$ [population variance]

$\sigma^2 = \dfrac{1}{n}\Sigma x_i^2 - \mu^2 = \dfrac{n\Sigma x_i^2 - (\Sigma x_i)^2}{n^2}$

$\Rightarrow \sigma = +\sqrt{\dfrac{1}{n}\Sigma(x_i-\mu)^2} = \sqrt{\dfrac{\Sigma x_i^2}{n} - \mu^2}$

→ Coeff of var $= \dfrac{SD}{mean} = \dfrac{\sigma}{\mu}$

Pdf → prob density f^n ⇒ continuous variable

prob mass f^n pmf ⇒ discrete variable

Discrete Distribution		Continuous Distr.
$\Sigma Px = 1$		$\int_{-\infty}^{\infty} f(x)dx = 1$
$E(x) = \Sigma x P(x)$	Mean	$E(x) = \int_{-\infty}^{\infty} x f(x)dx$
$V(x) = E(x^2) - (E(x))^2$ $= \Sigma x^2 P(x) - [\Sigma x P(x)]^2$	Variance	$V(x) = E(x^2) - [E(x)]^2$ $= \int_{-\infty}^{\infty} x^2 f(x)dx - \left[\int_{-\infty}^{\infty} x f(x)dx\right]^2$
	mean $= E(x)$ var $= V(x)$ $= E(x^2) - (E(x))^2$	

Properties

→ $E(ax_1+b) = aE(x_1) + b$

→ $V(ax_1+b) = a^2 V(x_1)$

• $cov(x,y) = E(XY) - E(X)E(Y)$

→ $E(ax_1 + bx_2) = aE(x_1) + bE(x_2)$

→ $V(ax_1 + bx_2) = a^2 V(x_1) + b^2 V(x_2) + 2ab\, cov(x_1, x_2)$

If x_1, x_2 are independent $cov(x_1,x_2) = 0$

→ $E(XY) = E(X)E(Y)$

Binomial

mean $= np$

var $= npq$

Poisson

$P(X=x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$

mean $= \lambda = np$

var $= \lambda = np$

mean $= E(x) = \int x f(x)dx$

var $= E(x^2) - [E(x)]^2$

$E(x^2) = \int x^2 f(x)dx$

pdf $= f(x)$

prob. dist. $f^n = F(x)$

$f(x) = \dfrac{dF}{dx}$

Exponential: $f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x \leq 0 \end{cases}$

mean $= \dfrac{1}{\lambda}$

var $= 1/\lambda^2$

Normal

$f(x) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

mean $= \mu$

var $= \sigma^2$

ODE

$\rightarrow \frac{dy}{dx} + Py = Q$

Integrating factor (IF) $= e^{\int P dx}$

sol$^n \Rightarrow y(IF) = \int Q(IF)dx + c$

$\rightarrow f(x) \Rightarrow xy = c$

Orthogonal Traj.

$x\frac{dy}{dx} + y \cdot 1 = 0$

$\frac{dy}{dx} \rightarrow \frac{-dx}{dy}$

Order

Bisection	= 1
Regula Falsi	= 1
Secant	= 1.62
Newton-Raphson	= 2

Numerical Method

Trapezoid Rule $\Rightarrow \int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 \cdots + y_{n-1}) + y_n\right]$

Simpson's 1/3rd $\Rightarrow \int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[y_0 + y_n + 4(y_1 + y_3 \cdots) + 2(y_2 + y_4 \cdots\right]$ → most accurate

Simpson's 3/8th $\Rightarrow \int_{x_0}^{x_n} y\,dx = \frac{3h}{8}\left[y_0 + y_n + 3(y_1 + y_2 + y_4 + y_5 + y_7 \cdots) + 2(y_3 + y_6 + y_9)\right]$

Vector Calculus

$\rightarrow$ curl grad $f = 0$; $\rightarrow$ div curl $f = 0$

$\rightarrow$ div grad $f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

$\rightarrow \nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$

$\rightarrow$ Gradient = Grad(f) $= \nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k$

$\rightarrow$ Divergence = (Grad.) $\cdot$ $(i + j + k)$

$\rightarrow$ Curl $= \nabla \times V = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ V_1 & V_2 & V_3 \end{vmatrix}$

$(f = V_1 i + V_2 j + V_3 k)$

$\rightarrow a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$

$\rightarrow$ area of Δ formed by $\vec{a}, \vec{b}, \vec{c}$ $= \frac{1}{2}\left|(\bar{a} - \bar{b}) \times (\bar{a} - \bar{c})\right|$

$\rightarrow$ Unit vector $\perp$ to $\bar{a}$ & $\bar{b}$ $= \frac{a \times b}{|a \times b|} = \frac{a \times b}{|a||b|\sin\theta}$

Line Integral

Vectors & Angles

f are cos

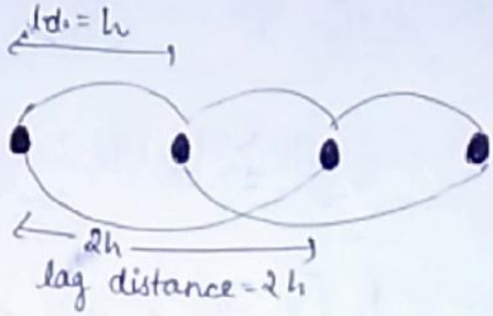
Semi- Variogram

① $\bar{G} = \dfrac{\sum g_i}{n}$ ② $V = \dfrac{1}{n} \sum (g_i - \bar{g})^2$

X_i = tonnage of ore at i
G_i = grade of ore at i

① $\gamma(h) = \dfrac{\text{variance}}{2} = \dfrac{\sigma^2(h)}{2}$

$= \dfrac{1}{2}\left[\dfrac{1}{n(h)} \sum (g_i - g_j)^2\right]$

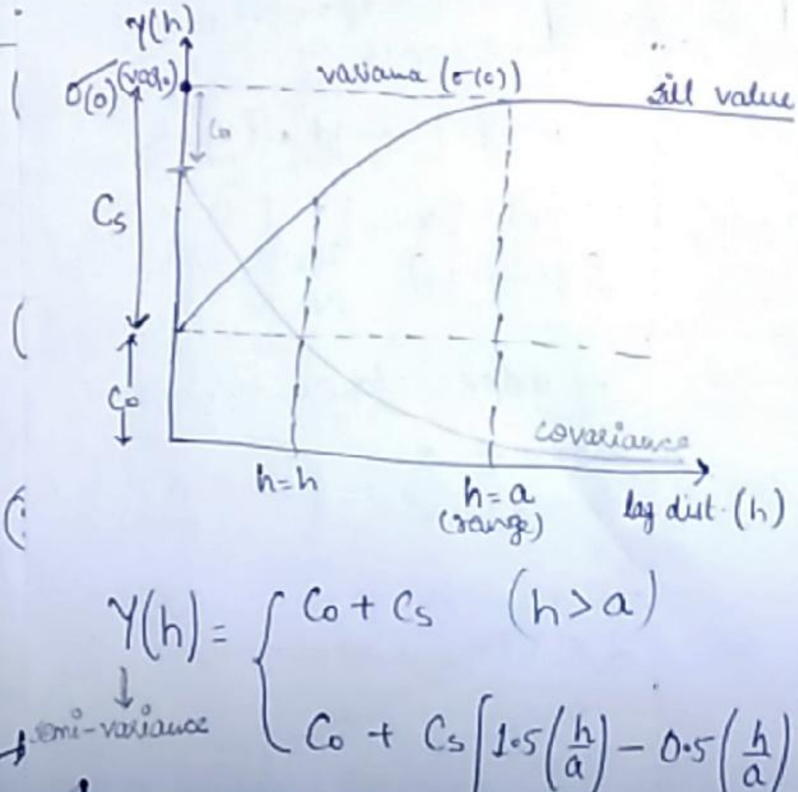


② $\gamma(2h) = \dfrac{\text{st var}}{2} = \dfrac{S^2(2h)}{2}$

$= \dfrac{1}{2}\left[\dfrac{1}{n(2h)} \sum (g_i - g_j)^2\right]$

g_i, g_j = grade of holes
$\gamma(h)$ = Geostatistical variance or semi- variance

Semi- Variogram (Spherical model)



variance

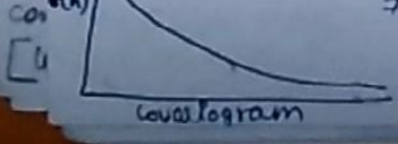
C_s = partial sill
$C_s + C_0$ = sill value $\gamma(\infty)$
C_0 = nugget effect

$C(h)$ = Co-variance at h
$\text{cov}(h) = \left(\text{sill value}\right) - \gamma(h)$

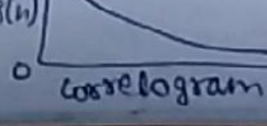
$a \rightarrow$ range variance $= C(0)$
$C(0) = \text{cov}(0) = (C_0 + C_s) - (C_0) = C_s$
$\sigma(0)$ = sill value - nugget

$$\gamma(h) = \begin{cases} C_0 + C_s & (h > a) \\ C_0 + C_s\left[1.5\left(\dfrac{h}{a}\right) - 0.5\left(\dfrac{h}{a}\right)^3\right] & (h \le a) \end{cases}$$

↓
semi-variance



$\Rightarrow \dfrac{\sigma(h)}{\sigma(0)} \Rightarrow$

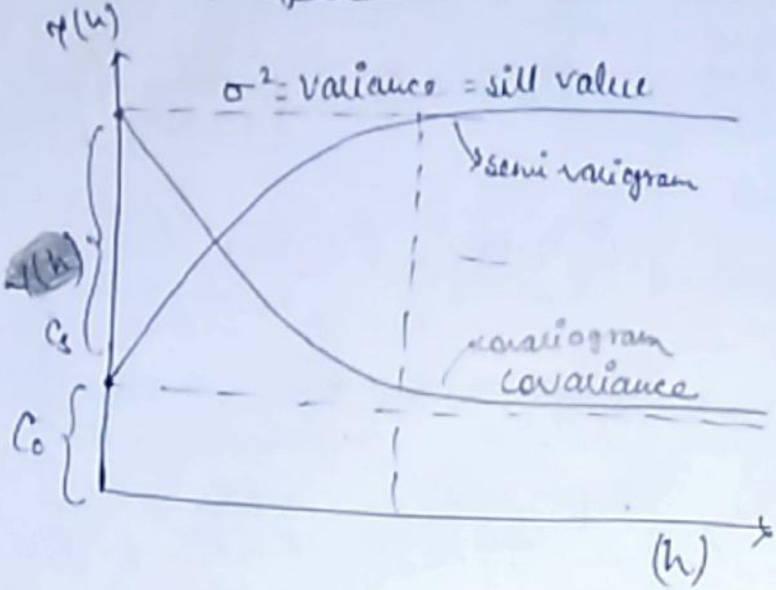


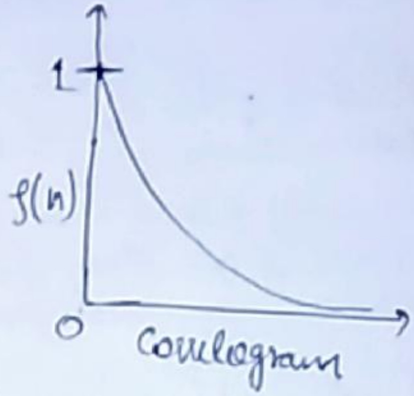
range = a = the h at which $\gamma(h) = 95\% \text{ (sill)}$
$\sigma(h) = \sigma(0) - \gamma(h)$
$\rho(h) = \sigma(h)/\sigma(0)$
$\rho(h) = 1 - \gamma(h)/\sigma(0)$

Semi-Variogram

C_s = partial sill

$$\boxed{\rho(h) = \frac{cov(x)}{\sigma^2}}$$





$\boxed{C(0) = cov(0) = \sigma^2 = \text{sill}}$

~~$C(x_i, x_j) = C($~~ $\rho = \dfrac{cov(x_i, x_j)}{\sigma(x_i) \cdot \sigma(x_j)}$

$2\gamma(h) \rightarrow$ variogram
$\gamma(h) \rightarrow$ semi-variogram

---

① Constant distance weighting Technique $\Rightarrow$ $\bar{g} = \dfrac{\Sigma \frac{g_i}{d_i}}{\Sigma \frac{1}{d_i}}$

② Inverse distance weighting tech $\Rightarrow$ $\bar{g} = \dfrac{\Sigma\, g_i/d_i^2}{\Sigma\, 1/d_i^2} = \dfrac{\Sigma \frac{g_i}{d_i^2}}{\Sigma \frac{1}{d_i^2}}$

③ Triangle

$\bar{g} = \dfrac{g_1 t_1 + g_2 t_2 + g_3 t_3}{3}$

* Avg grade of ore mined = $\dfrac{T_O G_O + T_d G_D - T_l G_l}{T_O + T_D + T_l}$

Assay $\Rightarrow$ process of determining metal content. Output = Assay value (grade only) [used for precious metals like gold] less conc^n (ppm)

T_O, G_O = tonnage & grade of ore
$T_d\ G_D$ =   of diluted ore
$T_l\ G_l$ =   of ore loss

Rock Mechanics

① $\varepsilon = \frac{\Delta l}{l}$ — Tensile/compressive strain

② $\gamma \; E = \frac{\sigma}{\varepsilon}$ — Modulus of elasticity

③ Poisson's Ratio : $\nu = \frac{\varepsilon_{lat}}{\varepsilon_{long}} = \frac{\varepsilon_{radial}}{\varepsilon_{axial}}$ $\left(\frac{\Delta D / D}{\Delta L / L}\right)$

④ Young Modulus : $E = \frac{\text{axial stress}}{\text{axial strain}}$

⑤ Shear Modulus $G = \frac{\text{shear stress}}{\text{shear strain}}$

⑥ Bulk Modulus : $K = \frac{\text{hydrostatic pressure}}{\text{volumentric strain}}$

⑦ $E = 2G(1+\nu) = 3K(1-2\nu)$

⑧ $L = \sqrt{\frac{2Rt}{\gamma \, FOS}}$

L = length of rupture
R = modulus of rupture
T = thickness
γ = unit weight

⑨ P-wave velocity $V_P = \sqrt{\frac{\frac{4G}{3} + K}{\rho}}$

⑩ $V_S = \sqrt{\frac{G}{\rho}}$

$\frac{\text{Horiz. stress}}{\text{vertical stress}} = \frac{\sigma_H}{\sigma_V} = \frac{\nu}{1-\nu} = K$

Point Load Strength : $I = \frac{P}{d^2}$

Strength of Pillar $= 7.2 \frac{W^{0.46}}{H^{0.66}}$ (MPa)

$\sigma_{ucs} = 24\, I$ [if diam = 50mm] [L:D = 2:1]

Salman Muno $= 1320 \frac{W^{\alpha}}{H^{\beta}}$ (lb/in²) $\alpha = 0.46$, $\beta = 0.66$

Brazilian Test : $\sigma_t = \frac{2F}{\pi D L}$

σ_t = kg/cm²
F = kg
D, L = cm

$\sigma_t = 0.0675 \frac{P}{d^2}$??

Tributary Area $= (a+b)^2 - (\text{Pillar area})$

Punch shear Test : $\cancel{S = \frac{F}{A}}$ $\sigma = \frac{F}{A} = \frac{F}{\pi d t}$

F = load (kg)
S = shear strength (kg/cm²)
t = thickness of disc
d = dia of punch

$\underset{(\sigma_c)}{UCS} = \frac{2c\cos\phi}{1-\sin\phi}$

c = cohesion
φ = angle of internal friction

[Unconfined compressive strength]

Volumetric Strain $= \varepsilon(1-2\nu)$ ε = strain

$= \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$

$= \frac{(\sigma_1 + \sigma_2 + \sigma_3)(1-2\nu)}{E}$

$\varepsilon_1 \varepsilon_2 \varepsilon_3$ = principal strain
$\sigma_1, \sigma_2, \sigma_3$ = principal stress

$\underset{(UTS)}{\sigma_t} = \frac{2c\cos\phi}{1+\sin\phi}$ — Unconfined Tensile Strength

$C_P = \sigma_c + \frac{\sigma_c}{\sigma_t}(P)$ P = pressure

C = Compressive strength (confining pressure P alternatively)

$$\rightarrow \boxed{\sigma_1 = \sigma_c + \left(\frac{\sigma_c}{\sigma_t}\right)\sigma_3 \quad ; \quad \sigma_1 = \left[\frac{2c\cos\phi}{1-\sin\phi}\right] + \left[\frac{1+\sin\phi}{1-\sin\phi}\right]\sigma_3}$$

σ_1, σ_3 = major & minor principal stress

$1\,g/cm^3 = 9.81\ kN/m^3$

$$\rightarrow L = \frac{B}{2}\left(\frac{100 - RMR}{100}\right)$$

L = length of bolt
B = span (gallery width)

Darcy's Law

$$Q = kA\left(\frac{dH}{dL}\right)$$

Q = flow rate
K = hydraulic conductivity
A = cross-sectional area
or K → coefficient of permeability
dH/dL → hydraulic gradient

$1\ darcy = 9.86 \times 10^{-9}\ cm^2$

$$\text{Immediate roof (Caving height)} = \frac{\text{Mining height}}{\text{Bulking factor} - 1}$$

Bulking factor = swelling factor

$$\text{moisture content} = \frac{W_{water}}{W_{solids}} \times 100$$

Bulk unit weight = W/V

Dry Density = W_s / V_{Total}

$$\text{Void Ratio} = \frac{\text{Vol of VOIDS}}{\text{Vol of SOLIDS}}$$

$$\text{Porosity} = \frac{V_{void}}{V_{Total}}$$

$$\text{void ratio} = \frac{\text{Porosity}}{1 - \text{Porosity}}$$

$$\text{Porosity} = \frac{\text{void R}}{1 + \text{void rat}}$$

$$\text{Degree of Saturation} = \frac{V_{water}}{V_{void}}$$

$$\text{porosity} = \frac{\rho_s - \rho_d}{\rho_s}$$

ρ_d = dry density

$$\text{Specific Gravity} = \frac{(M_2 - M_1)}{(M_2 - M_1) + (M_4 - M_3)}$$

M_1 = mass of empty pycnometer
M_2 = M_{pyc} with dry sample
M_3 = M_{pyc} soil (solid + water)
M_4 = M_{pyc} filled with water

$$\text{moisture content} = \left\{\left(\frac{M_2 - M_1}{M_4 - M_3}\right)\left(\frac{1 - G}{G}\right) - 1\right\} \times 100$$

$$\text{specific gravity} = \frac{w}{w - S}$$

w = weight in air
W = weight when suspended in water

① $\tau = c + \sigma_n \tan\phi$

[mohr-coloumb criteria] $\boxed{\sigma_1 = UCS + \sigma_\phi \sigma_3}$

τ = shear stress
c = cohesion
σ_n = normal stress

② $\sigma_1 = \sigma_3 + \sigma_{ucs}\left[m\frac{\sigma_3}{\sigma_{ucs}} + s\right]^a$ (Hoek Brown) → Intact rock

σ_1, σ_3 = principal stresses, σ_{ucs} = UCS

m = reduced material constant, s, a = Rock mass properties

③ $\sigma_{cs} = \sigma_{ucs}(s)^a$

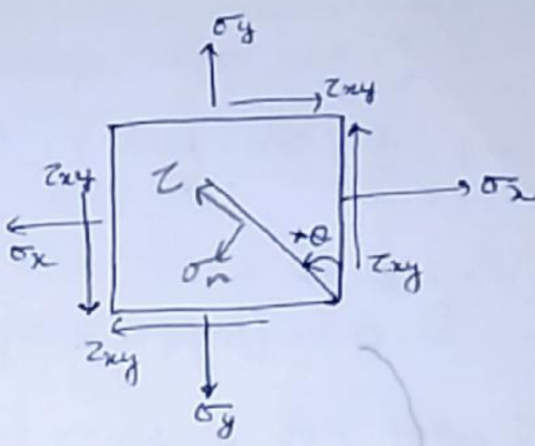
④ $\sigma_t = -\frac{s\sigma_{ucs}}{m}$ } Rock mass

Global rock mass strength →
Intact rock

(Hoek Brown)

Griffith Criterion : $\sigma_1 = \sigma_3 + 4\sigma_T \pm 4\sqrt{\sigma_T\sigma_3 + \sigma_T^2}$

Stresses on Inclined Plane



$\theta \Rightarrow$ anti-clockwise $\Rightarrow +ve$

$\theta \Rightarrow$ clockwise $\Rightarrow -ve$

σ_T = Tensile stress $\Rightarrow -ve$

σ_c = Compression $\Rightarrow +ve$

shear Clockwise $\tau \Rightarrow$

shear anti-clockwise $\tau \Rightarrow$

$\Rightarrow \sigma_r = \sqrt{\sigma_n^2 + \tau^2}$

$\phi = \tan^{-1}(\tau/\sigma_n)$

Principal stress

$\Rightarrow$ [Shear stress = 0 are called principle planes & stress normal to it are principal stresses.]

Major PS $\Rightarrow \sigma_1 = \dfrac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

minor PS $\Rightarrow \sigma_3 = \dfrac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Principal Planes $\Rightarrow \tan 2\theta = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

Normal stress

$$\tau = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Shear/ Tangential stress

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Ela Str Ene

Strains

Bi-axial Stress $[\tau_{xy} = 0]$

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta$$

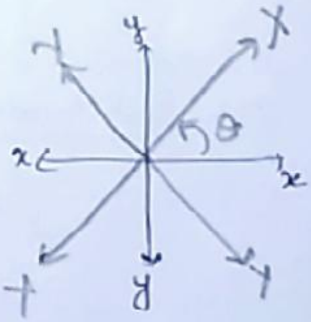
shear/ tangential stress $\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta$

Changing co-ordinates

$$\sigma_{XX} = \left(\frac{\sigma_{xx} + \sigma_{yy}}{2}\right) + \left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)\cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{YY} = \left(\frac{\sigma_{xx} + \sigma_{yy}}{2}\right) - \left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)\cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{XY} = -\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)\sin 2\theta + \tau_{xy} \cos 2\theta$$



Strains

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right]$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu\left(\sigma_{xx} + \sigma_{zz}\right)\right]$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu\left(\sigma_{xx} + \sigma_{yy}\right)\right]$$

Elastic Strain Energy $\Rightarrow U = \frac{P^2 L}{2AE}$

Strain Energy $\boxed{U = \frac{P^2 L}{2AE}}$

$P =$

L = length A = area

E = Young's Modulus

Resilience ; $U = \frac{\sigma^2}{2E} \times \text{volume}$

Mohr Circle

GSI = Geological Strength Index

RQD (Rock quality designation)

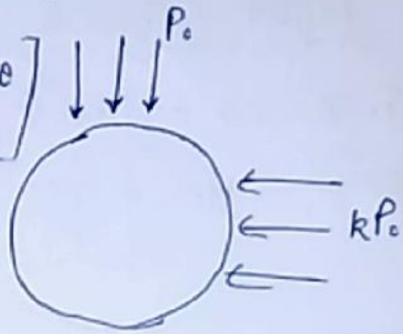
① $RQD = \frac{\Sigma \text{length (cores} > 10\text{cm)}}{\Sigma \text{all lengths}} \times 100$

② $\boxed{RQD = 100 e^{-0.1\lambda}(0.1\lambda + 1)}$ λ = no of fractures per meter

③ $RQD = 115 - 3.3V$ V = no. of discontinuities / unit volume or joints / unit length (volumetric joint count)

Stress Distribution: Around Circular Opening

Radial stress $\sigma_r = \frac{P_0}{2}\left[(1+k)\left(1-\frac{a^2}{r^2}\right) - (1-k)\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta\right]$



$$\sigma_{\theta\theta} = \frac{P}{2}\left[(1+k)\left(1+\frac{a^2}{r^2}\right) + (1-k)\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right]$$

At $r = a$;

$$\sigma_{rr} = 0$$
$$\sigma_{\theta\theta} = P\left[(1+k) + 2(1-k)\cos 2\theta\right]$$
$$\sigma_{r\theta} = 0$$

$\theta = 0$ $r \to$ large ;

$$\sigma_{rr} = KP$$
$$\sigma_{\theta\theta} = P$$
$$\sigma_{r\theta} = 0$$

[Hydrostatic stress field $K = 1$]

Plane failure

1.2 & 1.3 [Metalliferous & Opencast]

Mohr-Coloumb Criteria

① $\tau = c + \sigma \tan\phi$ $\tau_r = \sigma \tan\phi_r$

② $\sigma_1 = UCS + \left[\frac{UCS}{\sigma_T}\right]\sigma_3$

τ = peak shear strength
τ_r = residual shear strength
σ_1, σ_3 = principal stresses
c = cohesive strength
ϕ = friction angle

Hoek-Brown

$\rightarrow \sigma_1 = \sigma_3 + \sigma_{ci}\left(m\frac{\sigma_3}{\sigma_{ci}} + s\right)^a$

$\rightarrow \sigma_3 = 0 \Rightarrow \sigma_1 = UCS$

$\Rightarrow \boxed{UCS = \sigma_{ci} \cdot s^a}$

$\rightarrow \sigma_1 = \sigma_3 = \sigma_3 \Rightarrow$ tensile strength

$\boxed{\sigma_t = \frac{s\sigma_{ci}}{m}}$

σ_{ci} = unconfined comp strength
UCS = Uni-axial comp strength
m_b = reduced value of material constant
s =
a =

D = degree of disturbance
= 0 for undisturbed (no-fracture)
= 1 (for sand like)

Uni-axial Comp S Test

$\sigma_{(CS)} = \frac{F}{A} = \frac{F}{\pi r^2}$ $\Rightarrow$ for $D:L = 1:L$

$\sigma_{UCS} = \frac{\sigma_{ci}}{0.778 + 0.222(D/L)}$

↳ for L:D ≠ 1:1

Point load strength $I = \frac{P}{d^2}$ $UCS = 24I$

Bending Test

$\sigma_t = \frac{3PLn}{bd^3}$

L = length b/w supports (cm)
b = width of specimen
d = depth of specimen
n = dist. from neutral axis to far fibre

Brazilian Test

Tensile strength $\sigma_t = \frac{F}{\pi r t} = \boxed{\frac{2F}{\pi DT}}$

Punch Shear Test

Shear strength $= \frac{P}{A} = \frac{P}{\pi d t}$ [disc shape]

Mining Methods

→ Proximate Analysis

- Moisture (%) $= \frac{W_{loss} \text{ in drying}}{\text{init. } W_{sample}} \times 100\%$

- Ash = weight of residue after complete combustion

- Volatile matter = Gases released on heating.

→ Strength of Pillar (Salmon - Munro) $= K \frac{W^{\alpha}}{H^{\beta}}$

$K = 1320 \ lb/in^2$
W = width (ft) $\alpha = 0.46$
H = height (ft) $\beta = 0.66$

=

→ Absolute weight strength = $\underline{\underline{AWS}}$ = theoretical energy

Relative $\underline{\underline{RWS}} = \frac{AWS_{exp}}{AWS_{ANFO}} \times 100$

Absolute Bulk strength = $\underline{\underline{ABS}} = (AWS) \times \rho$ energy per unit volume

Relative Bulk strength = $\underline{\underline{RBS}} = \frac{ABS}{ABS_{anfo}} \times 100$

→ Powder Factor = Tonne / kg

→ Detonation Pressure = $0.25 \times VOD^2 \times \rho \Rightarrow \boxed{\rho V^2/4}$

→ V_{max} = Peak particle Velocity

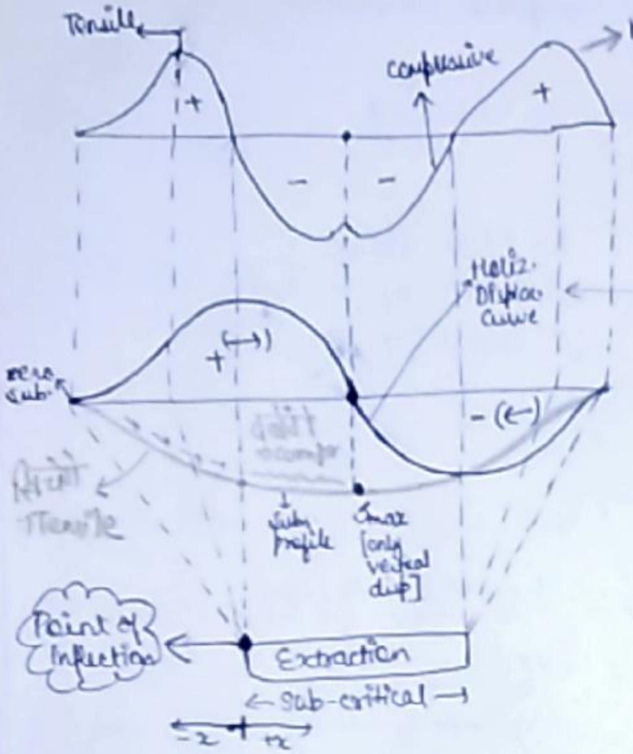
$V = K \left[\frac{D}{\sqrt{Q}}\right]^{E}$ D = distance of measuring transducer
Q = max. charge weight per delay

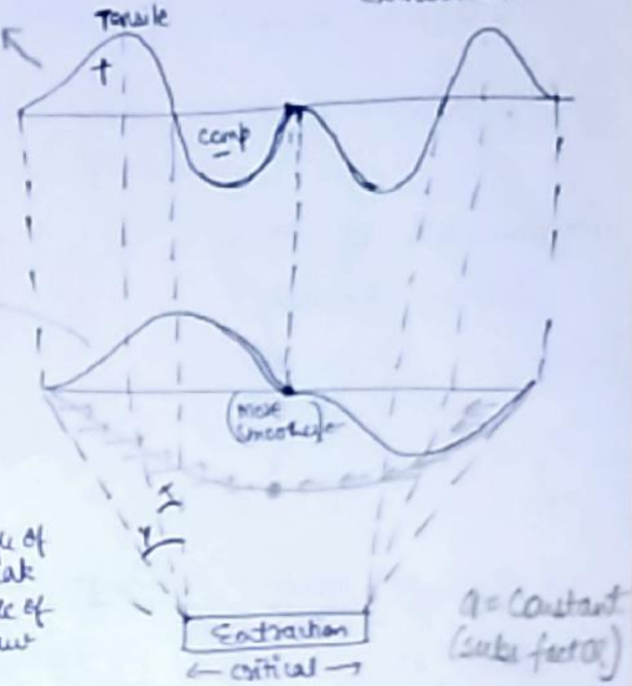
→ Subsidence

→ Subsidence

Horiz. Strain Curve is derivative of Horiz. Displacement curve

- Critical Point – Last point where vertical displacement increases.
- Sub-critical area – Areas where if length of extraction is increased, then max. vertical subs. also ↑es or ↓es.
- Super Critical – Areas where max. vert. subs is constant irrespective of extraction.





α = angle of break

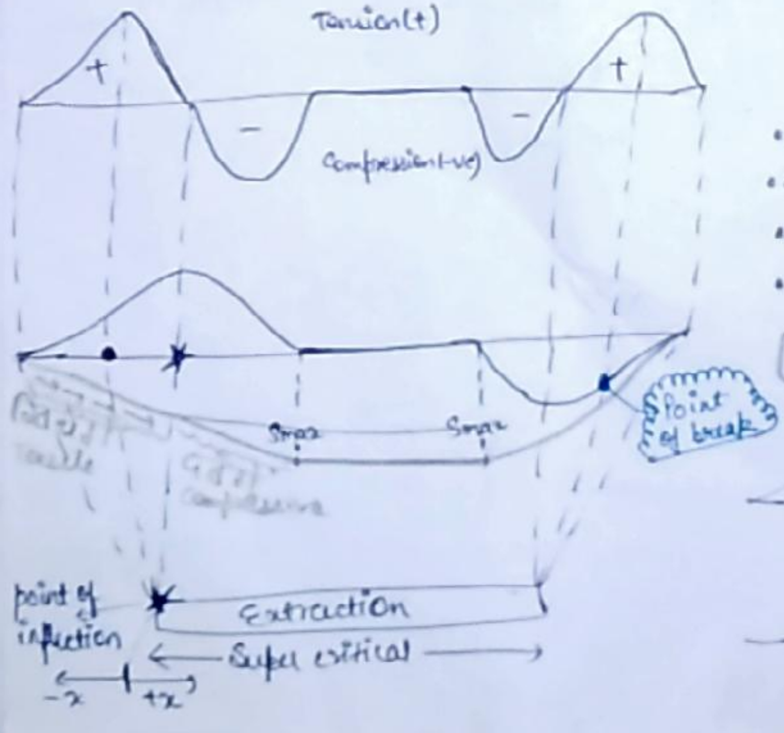
γ = angle of draw

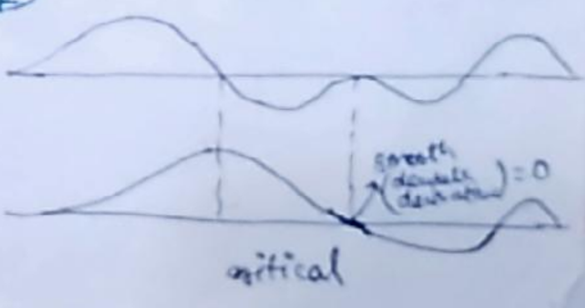
a = Constant (subs factor)

$$\rightarrow \boxed{S(x) = \frac{S_{max}}{2}\left[1 - \tanh\left(\frac{cx}{B}\right)\right]}$$

- $\boxed{S_{max} = a \times h}$; • h = height of panel (extr)
- S(x) = Sub. profile
- S_{max} = max subsid
- C = arbitrary constant (1–4)
- x = distance from inflection point
- B = radius of critical area of excavation

$\boxed{B = D\tan\gamma}$ D = depth of mining, γ = angle of draw





Environment

→ See air quality standards Table*

→ Gravity settler : ~~L, D, H =~~

L, H = length/height of settler
ρ = density of matter ; μ = viscosity
d = diam. of particulate matter ; V = horiz velo.

$$\eta = \frac{LD^2 \rho g}{18 \mu V \cdot H} \Rightarrow \eta = \frac{L}{H} \times \frac{\rho g d^2}{18 \mu V}$$

respirable < 5

→ Electrostatic Precipitator

$$\eta = 1 - e^{[VA/Q]}$$

V = terminal velocity

→ Cyclone seperator

$$\eta = \frac{N \times V_c}{9W} \times \frac{\pi d^2 \rho}{\mu}$$

N = no. of turns
d = dia. of PM
W = width of inlet

V_c = centrifugal velocity

→ Mine Disease + Hazards*

→ Sound Intensity Level : $L_i = 10 \log\left[\frac{I}{I_{ref}}\right]$ $I_{ref} = 10^{-12}\ W/m^2$

→ Sound Power Level : $L_w = 10 \log\left(\frac{W}{W_{ref}}\right)$ $W_{ref} = 10^{-12}\ W$

→ Sound Pressure Level : $SPL = 20 \log\left[\frac{P}{P_0}\right]$ $P_0 = 2 \times 10^{-5}$ µbar or µPa

→ Time interval sounds : $L_i = 10 \log_{10}\left[t_1 * 10^{l_1/10} + t_2 * 10^{l_2/10} \dots\right]$

→ Sound Propagation : Point source $\propto 1/r^2$

$$L_{pres.} = L_{power} - 20 \log(r) - 11$$

$$L_{p_1} - L_{p_2} = 20 \log\left(\frac{r_2}{r_1}\right)$$

Line source $\propto 1/r$

$$L_{p_1} - L_{p_2} = 10 \log\left(\frac{r_2}{r_1}\right)$$

*[Read noise standards]

→ Illumination (lux) $= \frac{P \cos\theta}{r^2}$ → $\boxed{\frac{I_1}{I_2} = \frac{\cos\theta_1}{\cos\theta_2} \times \frac{r_2^2}{r_1^2}}$

• Lumen = light emitted
  Lm = power × 4π
• Lux = lumen/m²

⇒ θ = angle b/w normal + light rays

⇒ 1 fc = 10.764 lux (footcandle)

41.37

1.33

→ **BOD/COD**

- $COD > BOD$
- COD test = 2 hrs
- BOD test = 5 days

- $BOD_t = BOD_{ult}[1 - e^{-kt}]$ $\boxed{BOD_t = BOD_{ult}(1 - e^{-kt})}$

- $BOD_5 = (DO_i - DO_f) \times \dfrac{V_{soln}}{V_{sample}}$

$\dfrac{V_{soln}}{V_{sample}}$ = Dilution Factor $= \dfrac{V_1 + V_2}{V_1}$ V_2 = water

--x---x---x x---x---x x---x---x---

Surveying

Correction Factors

1. **Incorrect length of chain** $= \dfrac{(L \pm \Delta L)}{L}$

   True length $= \dfrac{(L \pm \Delta L)}{L} \times \left(\text{Measured length}\right)$

2. **Slope correction**

   $CF = \dfrac{H^2}{2L}$ actual length $= \left(\text{Measured length}\right) - \dfrac{H^2}{2L}$

3. **Temperature**

   $C_t = \alpha [T - T_0] L$ True length $= L + \alpha [T - T_0] L = L[1 + \alpha(T - T_0)]$

4. **Pull**

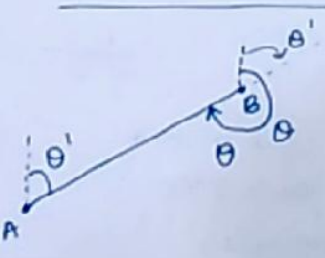
   True length $= L + \dfrac{(P - P_0) \times L}{AE}$

5. **Sag**

   $= L - \dfrac{L(mg)^2}{24 P^2}$ P = applied pull
   m = mass of tape b/w two ends

Back-Bearing

→ Back bearing = θ'



* Magnetic Bearing wrt M-North

Theodolite

→ Magnifying Power = $\frac{\text{Size of object}}{\text{size of image}}$ → $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

→ Linear Error = $\frac{P}{1000}\sqrt{1 + \frac{Ne^2}{12}}$

P = perimeter of traverse
N = no. of sides
e = permissible error per angle [1′]
L = least count (1′ or 20″)

→ Angular error = $L\sqrt{N}$

→ Latitude = North-South   Longtitude = East-West = **Departure**

→ Close Traverse Survey :
→ Sum of Latitudes = $\Sigma L\cos\theta = 0$; Sum of departures = $\Sigma L\sin\theta = 0$
→ Closing error = $\sqrt{(\Sigma L\cos\theta)^2 + (\Sigma L\sin\theta)^2} = \sqrt{(\Sigma \text{latitude})^2 + (\Sigma \text{departure})^2}$
→ Dir^n of closing error = $\tan^{-1}\left[\frac{\Sigma(L\sin\theta)}{\Sigma(L\cos\theta)}\right]$

Area Computation

i) Mid-ordinate Rule :

41.37

Levelling

→ diff b/w 2 levels $= \dfrac{(b_1-a_1)+(b_2-a_2)}{2}$; $a_1, b_1 = 1^{st}$ set of reading

$a_2, b_2 = 2^{nd}$ — " — from opposite end

error : $e = \dfrac{(b_1-a_1)-(b_2-a_2)}{2}$

d = 12742 km

→ Correction for Curvature : $\boxed{-0.0785D^2}$; D = dist b/w Instrum & staff

$= -D^2/d \simeq D^2/2R = D^2/\text{diam.}$

→ Refraction : $+\left(\frac{1}{7}\right)(C_{curv}) = \boxed{+\frac{1}{7} \times 0.0785D^2}$

$= +D^2/7d$

Tacheometric survey.

f = focal length

* i = dist. b/w upper & lower stadia hairs

s = staff intercept $= S_2 - S_1$

S_3 = height of mid-reading of staff

* k = dist. b/w instrument & its glass

Case-I : $D = \left(\frac{f}{i}\right)S + (f+k)$

$RL = H - S_3$

Case-II : Line of sight inclined 'θ'

Staff = vertical

$D = \left(\frac{f}{i}\right)S\cos^2\theta + (f+k)\cos\theta$

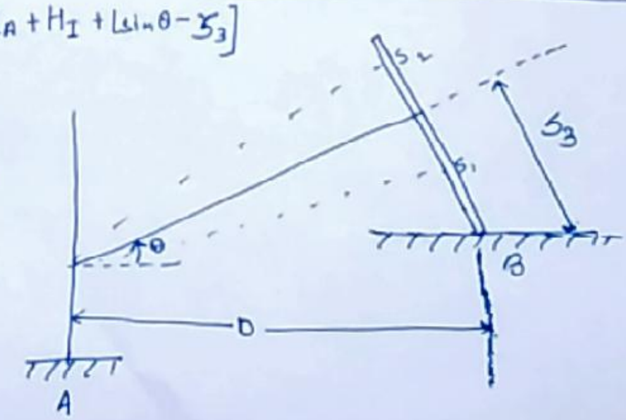
$RL = H + V - S_3 = [RL_A + H_I + L\sin\theta - S_3]$

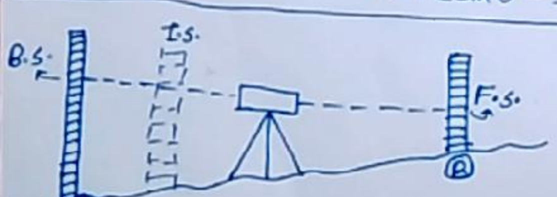
Case-III :

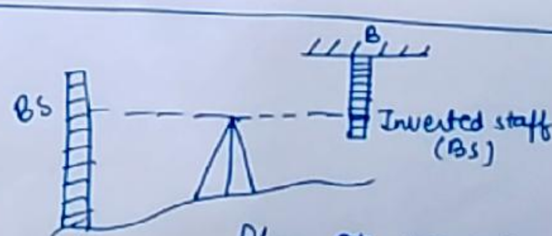
$D = \left[\frac{f}{i}\right]S\cos\theta + (f+k)\cos\theta + S_3\sin\theta$

$RL_B = RL_A + H_I + V - S_3\cos\theta$

$= RL_A + H_I + L\sin\theta - S_3\cos\theta$







$RL_B = HI + BS$

Photogrammetry & GPS : Book

41.37

11.33

Triangulation

→ True Value = free from all errors — {Book Theory 174-82}

→ Most probable value = more likely to be true than other values.

→ True error = Diff. b/w True value & observed value.

→ Residual error = (most prob. value) − (observed value)

→ Indirect observation: {see book Pg-218} [5.9.2.3 & .4]

→ Computed quantities:

1. $x = ka \Rightarrow error_x = k \times error_a$

2. $x = a_1 + a_2 + a_3 + a_4 \Rightarrow error_x = \sqrt{e_{a_1}^2 + e_{a_2}^2 + e_{a_3}^2}$

   $x = a + a + a + a \dots + a \Rightarrow error_x = e_a\sqrt{n}$

3. $x = f(a) \Rightarrow e_x = e_a\left[\frac{dx}{da}\right]$

4. $x = f(a,b) \Rightarrow e_x = \sqrt{\left(e_a\frac{dx}{dx}\right)^2 + \left(e_b\frac{dx}{db}\right)^2}$

$A = \theta_1 \quad B = \theta_2$
$A + B = \theta_3$
$\Rightarrow 2A + B = \theta_1 + \theta_3$
$A + 2B = \theta_2 + \theta_3$

solve & find A & B

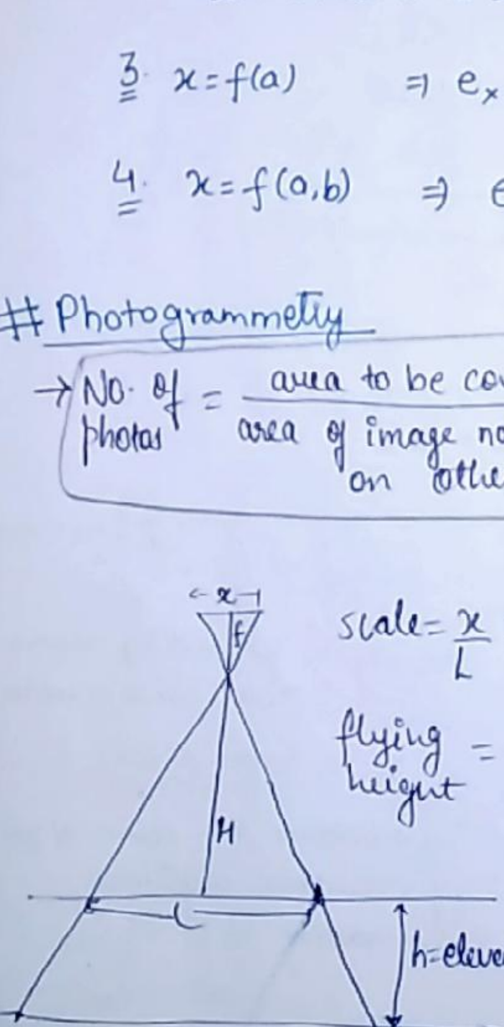
Photogrammetry

→ $\boxed{\text{No. of photos} = \frac{\text{area to be covered}}{\text{area of image not overlapping on other}}}$

→ ~~Intervals b/w exposure~~ (hr)

Lq = distance travelled by plane b/w exposures in km

→ $\boxed{\text{Interval b/w exposure} = \frac{Lq\,(m)}{V\,(m/s)} \quad (s)}$



$\text{scale} = \frac{x}{L} = \frac{f}{H}$

flying height $= H + h$

$A = \theta_1$, weight = 2
$A = \theta_2$, weight = 5
$\Rightarrow (2\times1)A = 2\theta_1$
$\Rightarrow (2\times5)2A = 10\theta_2$
$\rightarrow A = \frac{2\theta_1 + 10\theta_2}{22}$

$A = \theta_1 \Rightarrow 1\times A = 1\times\theta_1$
$2A = \theta_2 \Rightarrow 2\times2A = 2\times\theta_2$
$5A = \theta_3 \Rightarrow 5\times5A = 5\times\theta_3$
$\Rightarrow A = \frac{\theta_1 + 2\theta_2 + 5\theta_3}{1 + 4 + 25}$

~~A = 2θ₁ weight = 2 ⇒ (2×2)A = (2×2)2θ₁~~
~~2A = θ₂ weight = 5 ⇒ 2~~

Mine Ventilation

→ Stinkdamp ⇒ H_2S

Blackdamp ⇒ $CO_2 = 13\%$, $N_2 = 87\%$ $[CO_2 + N_2]$

Firedamp ⇒ $CH_4 + N_2 + CO_2$ + Hydrocarbons

White Damp $= CO$

Afterdamp ⇒ $N_2 + CO_2 + CO$ + Residual air

→ Permissible Conc^n ⇒ $CH_4 = 0.75\%$ (return air) 1.25 (in mine)

respirable < 5μ

$CO_2 = 0.5\%$ $CO < 0.005\%$

$O_2 > 19\%$ N-oxides $< 0.0005\%$

$H_2S < 0.001\%$ $SO_2 < 0.0005\%$

→ Degree of Gassiness

	% of inflammable gas in return air	Gas emmission $[m^3/te]$
I	<0.1	<1
II	>0.1	1-10
III		>10

→ Coward Diagram ⇒ Methane

$$CH_4 + 2O_2 + 8N_2 \longrightarrow CO_2 + 2H_2O + 8N_2$$

Explos. limit → 5.4-14.8%

→ Le-Chatelier equation

$$\text{Limit of explosibility} = \frac{100}{\frac{V_1}{x_1} + \frac{V_2}{x_2} \ldots}\% = \frac{V_1 + V_2 + V_3}{\frac{V_1}{x_1} + \frac{V_2}{x_2} + \frac{V_3}{x_3}} \quad (\text{If } V_1 + V_2 + V_3 \neq 100)$$

V_1, V_2 = % by volume

x_1, x_2 = limits of explos.

→ Safe distance from methane source ⇒ $L = \frac{22r}{\sqrt{f}}$

L → distance from source of gas

f = resistance coefficient

r = radius of duct

Methane Layering no. → $L = \frac{V}{[4.32Q/w]^{1/3}} = \left[\frac{24V^2}{C\sqrt{F}}\right]^{1/3}$

→ Methane layering No.

$$L = \frac{V}{(4.32q/w)^{1/3}}$$

$$L = \left[\frac{24V^2}{C\sqrt{A}}\right]^{1/3}$$

W =
V = velocity of air
q = rate of CH_4 emission
C = methane content [%]
A = cross-sectional area

→ Graham's Ratio = $\boxed{CO/O_2 \text{ deficiency ratio}}$

$$= \frac{CO \text{ produced}}{O_2 \text{ consumed}} \times 100 \; ; = \frac{CO_i - CO_f}{\frac{20.95}{79.02} N_{2f} - O_{2f}} = \frac{CO_i - CO_f}{\frac{21}{79} N_{2f} - O_{2f}} = \frac{CO_i - CO_f}{0.265 N_{2f} - O_{2f}}$$

$$= \frac{CO_i - CO_f}{\frac{21}{79} N_{2f} - O_{2f}} \times 100$$

0.1-0.5%	Normal
1%	spontaneous heating exist
2%	Heating in adv. stage; approaching active fire
>3%	Active Fire

→ Reynold's no = $\boxed{\frac{VD\rho}{\mu}}$

$$= \frac{VD}{\nu}$$

V = velocity D = diameter
μ = viscosity $\frac{\rho}{\mu}$ = kinematic viscosity (ν)
ρ = density

Laminar Flow: $Re = 2000$; $f = 64/Re$

→ $$\Delta P = \frac{\rho f}{8} \times LP \times \frac{V^2}{A} = KS\frac{V^2}{A} = \frac{KS}{A^3}Q^2 = RQ^2$$

f = Darcy resist coeff (coeff of resist.)

k = coeff of friction = $\frac{f\rho}{8}$ (coeff of friction)

S = rubbing surface area = LP

R = resistance = KS/A^3

→ Shock loss = $X P_v$

$$= X \frac{1}{2}\rho V^2$$

P_v = velocity pressure

→ Coeff of Contraction = $\frac{1}{\quad}$

→ Eq. resis ⇒ series = ΣR parallel ⇒ $\frac{1}{\sqrt{R_{eq}}} = \frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}} + \frac{1}{\sqrt{R_3}}$

→ Power = $\Delta P \times Q = RQ^3$ (watt) $= R_1Q_1^3 + R_2Q_2^3 \dots$

→ Area = $1.2 / \sqrt{R}$ [eq. orifice]

→ NPV = $\frac{gDB}{0.287}\left[\frac{T_u - T_D}{T_u \cdot T_D}\right]$ 'Pa' ; $B_d = B_u$ = pressure (kPa)
D = depth

→ Motive column ⇒ $h = \frac{NVP}{\rho_d g} = \left[\frac{T_u - T_D}{T_u}\right] \times D$

→ Fan Laws:

Head $\propto D^2$ $Q \propto D^3$ Power $\propto D^5$

Head $\propto n^2$ $Q \propto n$ $P \propto n^3$

D → diameter   n → speed

→ Relative Humidity

$= 100 - 7(D - W)$ [DBT > 25°]

$= 100 - 8(D - W)$ [20° < D < 25°]

$= 100 - 9(D - W)$ [D < 20°C]

Quantity of Air ⇒ $6 m^3/min$ per person or $2.5 m^3/min$ / tonne output of day

$= \text{Max}\,[6 m^3/min \times$ no. of person , $2.5 m^3/min \times$ daily output]

Respiratory Quotient = $\frac{CO_2 \text{ produced}}{O_2 \text{ consumed}}$

Machinery

→ **Pitch** $= \dfrac{\text{no. of teeth}}{\text{pitch diameter}}$

→ Centre-centre distance $= \dfrac{(\text{Pitch dia})_1 + (\text{Pitch dia})_2}{2}$

→ **Gear Ratio** $= \dfrac{N_{\text{driven gear}}}{N_{\text{drive gear}}}$

[N - teeth number
m - module]

$= \dfrac{(N_1 + N_2)m}{2}$

→ velocity = RPM × circumference

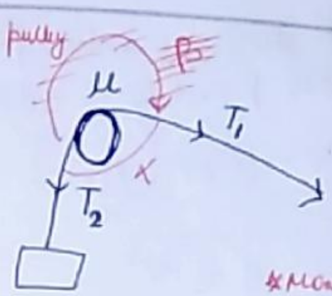
→ $T_1 = T_2 e^{\mu \beta}$

$\boxed{\beta = 180 - \alpha}$ → in rope pulley

β = lap angle (radian)

$\beta = 180 - \alpha$ in rope pulley

μ = coeff of friction



→ Super Elevation $= \dfrac{BV^2}{gR}$ (road) or $\dfrac{dV^2}{gR}$ (railways)

B = width of road
d = distance b/w tracks

* Moment of Inertia

→ $\boxed{\text{mass}_{\text{rope}} = kd^2 \ \{\text{kg/m}\}\ d \to \text{cm}}$

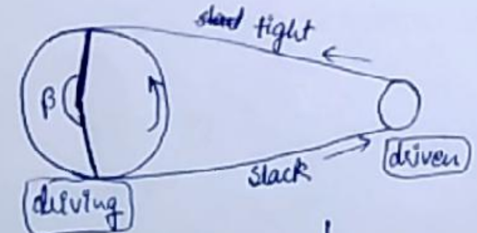
→ Rope Haulage : rope lies on surface. {[or friction = $\mu_{dr} m_r g$] μ_{dr} = b/w rope & drum; m_r = mass of rope}

→ Tractive effort = μW (μ = adhesion coeff W = weight = 'mg' always)

→ $\boxed{\text{Draw-Bar Pull = Tractive effort − Force required}}$

→ **Pump Laws:**

Capacity	Head	BHP
$Q \propto D$	$H \propto D^2$	$P \propto D^3$
$Q \propto N$	$H \propto N^2$	$P \propto N^3$

→ 

!! ⇒ rpm question

$\boxed{T_{\text{tight}} = T_{\text{slack}}\, e^{\mu\beta}}$

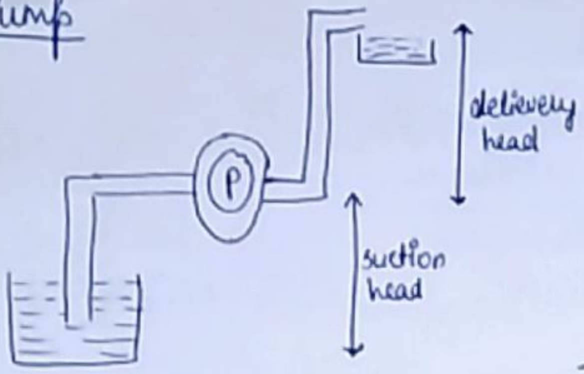
β = lap angle = angle of contact ⇒ (bigger angle **)

→ $\boxed{\text{Power} = F \cdot V = T \cdot \omega}$

$\boxed{1\,m^3 = 10^3 L}$

$\boxed{\text{Power} = \{T_{\text{tight}} - T_{\text{slack}}\} \times V = (T_1 - T_2)\omega r}$

→ Pump



→ Static head = Del. head + Suction head

→ manometric head = static head + head losses

↓

= difference in manometic reading at delievery & suction

→ Pressure head = $P_0/\rho g$

→ $\boxed{\text{velocity head}}$ = $V_0^2/2g$ (vel. loss head)

→ work done by centif. pump [only boundary se outlet]

$$V = (2\pi r) \cdot \frac{n}{60} \qquad n \to rpM$$

$$\boxed{\text{head} = \frac{V^2}{g}}$$

$\boxed{\text{✗ Air pressure} = 103,125 \text{ Pa}}$

→ Centrifugal force = $\frac{mv^2}{r}$

→ $\boxed{\text{Centrifugal tension} = \frac{mv^2}{r}} = \rho v^2$ ρ - kg/m

→ Electric Rotor motor [Power factor/3-Phase]

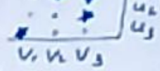
→ Tower mounted friction hoist ⇒ θ =

$$\boxed{\text{→ RMS Torque} = \sqrt{\frac{\Sigma \tau^2 t}{\Sigma t}} = \sqrt{\frac{\tau_1^2 t_1 + \tau_2^2 t_2 + \tau_3^2 t_3}{t_1 + t_2 + t_3}}}$$

System Engineering

☆ Balanced problem ⇒ sum of demands = sum of supply

1. N-W corner method → assign from N-W corner cells.
2. Matrix Minimum / Least cost method → assign from least cost cell.
3. Vogel approx. method : 1] Penalty = (2^{nd} minimum − minimum)
   [penalties are same, then go with one having min. demand/supply].
   2] Select max penalty row/coloumn.
   3] Assign value to least cost of that ~~row~~ row/coloumn.
4. U-V method : 1] Write only allocated cell of VAM.



↓ [$v_3 = 0$ or $u_1 = 0$]

   2] Row/Column having max allocation ⇒ give $u/v = 0$ for that
   3] $u_i + v_j = 0$ find all values of u & v
   4] Now write unallocated cell with resp. cost C_{ij}
   5] Every $C_{ij} - (u+v) \geq 0$; if not select most −ve & assign θ allocation to it.
   6] Subtract & add θ in closed loop, so demand and supply are justified.

☆ Unbalanced : Add dummy row/column & assign costs = 0.

☆ Hungarian assign. method = Row redⁿ ⟶ col redⁿ ⟶ optim. solⁿ (or) ⟶ draw min. lines to cover all zeroes ⟶ from uncovered entries, select min entry (c) ⟶ subtract c from uncovered entries & add c to intersection point of lines.

☆ Project Planning :

$$time_1 = \frac{a+4m+b}{6}$$

$$\sigma_1 = \frac{b-a}{6} \quad var = \sigma^2$$

Total time $= t_1 + t_2 + t_3$

total var $= \sigma_1^2 + \sigma_2^2 + \sigma_3^2$

$\Rightarrow$ S.D. total $= \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2}$

→ min time = time on critical path (max. time path)

→ Crashing will be only on CP activities such that time of critical path does not become lesser than any other path [CAN BE EQUAL].

11:33

* Reliability : $R(t) = e^{-\int_0^t \lambda(t)\,dt}$ $\boxed{\lambda = \text{failure rate}}$

→ if λ = constant (0.15/hours) ⇒ $R(t) = e^{-\lambda t}$

→ $MTBF = \int_0^\infty R(t)\,dt = \frac{1}{\lambda}$ ⇒ $m = \frac{1}{\lambda}$

→ Reliability of series = $R_1 \times R_2 \times R_3$ $m = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3}$

→ Rel. of Parallel = $1 - (1-R_1)(1-R_2)\ldots$ [atleast one should work]

→ k-out-of-n ⇒ Binomial

MTBF : $m = \frac{1}{\lambda} \sum_{j=k}^{n} \frac{1}{j}$

⇒ $R(t) = e^{-\lambda t}$ → exponential dist

λ → constant failure rate (failure/hr)

mean = $1/\lambda$ = MTBF

Variance = $1/\lambda^2$

* Queuing

n = no of units in system [waiting + served]

λ = arrival rate (cust. arriving per unit time)

μ = service rate (customer served per unit time)

W_s = waiting time per customer in system

W_q = waiting time per cust in queue

N_s = expected no of customer in system

N_q = in queue

P_n = prob of n customers in the system

ρ = prop of time server is busy.

(1) $\rho = \frac{\lambda}{\mu}$ Busy Time prop = $\frac{\text{Arrival rate}}{\text{Service rate}}$

(2) $W_s = \frac{1}{\mu - \lambda}$

(3) $W_q = \frac{\lambda}{\mu(\mu - \lambda)}$

(4) $N_s = W_s \lambda = \frac{\lambda}{\mu - \lambda} = \frac{\rho}{1-\rho}$

(5) $N_q = W_q \lambda = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\rho^2}{1-\rho}$

(6) $P_0 = 1 - \lambda/\mu$

(7) $P_{>k} = \left(\frac{\lambda}{\mu}\right)^k$

41.37 7 + 0 = 13 = 11.33

MINE ECONOMICS [Only highlighted ques from book]

- ★ Non-discounting Tech. ⇒ no iderest interest

- ★ $NPV = -X_0 + \frac{X_1}{1+i} + \frac{X_2}{(1+i)^2} \cdots + \frac{X_n}{(1+i)^n}$

- ★ Profitability Index (PI) $= \dfrac{\frac{X_1}{1+i} + \frac{X_2}{(1+i)^2} \cdots + \frac{X_n}{(1+i)^n}}{X_0} = \dfrac{NPV + X_0}{X_0}$

- ★ Rate of Return : $X(1+i)^n = Y\left[\frac{(1+i)^n - 1}{i}\right]$

  X → investment
  Y → profit/year

  $X(1+i)^n$ ⇓ Future worth of x Rs after n years

  $Y\left[\frac{(1+i)^n - 1}{i}\right]$ ⇓ F.W. of all profit combined after n yrs

- ★ Internal Rate of Return (IRR or DCFROR) = i for NPV = 0.

- ★ Payback Period ⇒ $X(1+i)^n = A\left[\frac{(1+i)^n - 1}{i}\right]$

- ★ Interest rate annualy = i ; compounded quarterly

  $FW = P\left[1 + \frac{i}{4}\right]^{4n}$ n → no of years

- ★ Depreciation = $\dfrac{\text{Investment} - \text{salvage value}}{\text{Life of project}}$

- ★ Depletion = Decrease in value of resource base [Pg-377, Q.30]

- ★ Redemption =

Hmishra

41.37

Mineral Dressing [May refer book for Theory Part]

* Primary Crusher
  - → Jaw Crusher
    - → Blake
    - → Dodge
    - → Universal
  - → Gyratory Cr.

Secondary crusher
  - → cone crushers
  - → Gyradisc crusher
  - → Roll crushers

Tertiary/Impact crushers → Hammer mill [Impact]

* Grinding Mills → Tumbling mills
  - → Rod
  - → Ball
  - → Autogenous

  Stirred mills

* Ball Mill ⇒ critical speed of mill =

* <u>Terminal Velocities</u> :

Free settling , $V = \frac{gd^2(\rho_s - \rho_L)}{18\eta}$

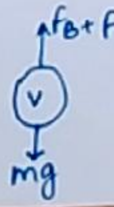
Stoke's law ⇒ drag force is due to viscosity of liquid

Hindered settling, $V = \frac{gd^2(\rho_s - \rho_p)}{18\eta}$

$\boxed{\text{Drag force} = 3\pi d \eta v}$

ρ_p = pulp density

Newton ⇒ Drag force $= 0.055\pi d^2 v^2 \rho_L$



$F_B = \rho_L V g$

F_D = drag

$mg = \rho_s V g$

** sample variance $S^2 = \frac{1}{N-1}\Sigma(x_i - \mu)^2$

populn variance $= \frac{1}{N}\Sigma(x_i - \mu)^2$