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Pedestrian–bridge dynamic interaction, including human participation

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ABSTRACT

The pedestrian–bridge dynamic interaction problem based on bipedal walking model and damped compliant legs is presented in this work. A time-variant damper is modeled at a given walking speed. A control force is applied by the pedestrian to compensate for energy dissipated with the system damping in walking and to regulate the walking performance of the pedestrian. The effects of stiffness, damping of the leg and the landing angle of attack are investigated in the numerical studies. Simulation results show that the dynamic interaction will increase with a larger vibration level of structure. More external energy must be input to maintain steady walking and uniform dynamic behavior of the pedestrian in the process. The simple bipedal walking model could well describe the human–structure dynamic interaction.

1. Introduction

Human–structure interaction (HSI) is an important but relatively new topic in designing slender structures occupied and dynamically excited by humans [1]. Human–structure interaction with stationary (standing or sitting) human body and activities on site (jumping, bobbing) has been well studied recently [1–7]. Human–structure synchronization in both the vertical direction and lateral direction during walking was often regarded as the human–structure interaction by many researchers. If the amplitude of vibration of structure exceeds a critical value, the pedestrians are affected by the motion and tend to adjust their walking frequency and phase, synchronizing with the motion of the structure. Various mathematical models of human-induced dynamic walking loads have been proposed to include this interaction in the dynamic analysis of the structure [8–14]. In all of these models, the crowd was only an imposed load rather than a dynamical system. This ignores the possibility of interaction that can occur due to the 'mechanical' properties of the human body and the structure [7].

The first attempt to model the crowd and the structure as parts of a complex dynamical system in the human–structure interaction is referred to Refs. [15–17]. However, this model only considered the inertial force induced by the crowd. The contribution of human to structural stiffness and damping was not included. Moreover, this model cannot explicitly model the dynamics of the human body. The different influencing parameters associated with the walking force, such as the stiffness, damping, length and the angle of attack of human leg, do not have contributions in these models. Problems like how the pedestrian influences the dynamic properties of footbridge, such as the natural frequency and damping,

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Nomenclature

$a(t)$	coefficient of damping
c_{leg}	leg damping
c_l	damping of the leading leg
c_t	damping of the trailing leg
c_s	damping of the beam
$\mathbf{C}$	damping matrix
d_i	ith step length
E_0	initial energy input of human body
$E(t)$	energy of human body at time 't'
f_i	ith natural frequencies of the HSI system
$F(t)$	force vector
$F_{\text{ctrl}}(t)$	horizontal control force
k_{leg}	leg stiffness
$\mathbf{K}$	stiffness matrix
l_0	rest length of leg
$L_t(0)$	length of the trailing leg at the start of double support phase
$L_t(t)$	length of the trailing leg at the time step 't'
L_l	length of the leading leg
L_t	length of the trailing leg
m	mass per unit length of the beam
m_h	lumped mass
$\mathbf{M}$	mass matrix
Q_i	ith generalized forcing function
T	kinetic energy
u	horizontal displacements of COM
$\mathbf{U}$	displacement vector
$\dot{\mathbf{U}}$	velocity vector
$\ddot{\mathbf{U}}$	acceleration vector
v_l	axial velocity of the leading leg
v_t	axial velocity of the trailing leg
V	potential energy
δW	variation of virtual work
x_0	initial position of pedestrian
x_l	time dependent position of the leading foot
x_t	time dependent position of the trailing foot
y	displacement of the beam
$y\|_N$	vertical displacement of beam at the points of contact of the leading leg in the Nth step cycle
$y\|_{N-1}$	vertical displacement of beam at the points of contact of the trailing leg in the Nth step cycle
y''	curvature of the beam
$Y_i(t)$	modal coordinates of the bridge
z	vertical displacements of COM
$\Delta u(t)$	horizontal displacement increment of COM
$\phi_i(x)$	vibration modes
λ	eigenvalue
ξ_i	damping ratios of the HSI system corresponding to f_i

cannot be studied using these force models. Recently, a simple human body model (ISO 1981) [18] with two vertical degrees-of-freedom (DOFs) was adopted considering human–structure dynamic interaction [19]. However, this model can only represent the vertical displacement of the center of mass (COM) rather than any physical motion of the leg. The leg stiffness corresponding to vertical displacement of the COM is not the real leg stiffness. Moreover, the model cannot reproduce the dynamics of standing observed in the double support phase with two legs. Hence, how structural vibration influences the forces induced by pedestrian and how pedestrian influences the dynamic properties of a footbridge, such as the natural frequency and damping, is not well understood.

In biomechanics, human is treated as systems of actuators and dampers. It has been shown that a bipedal model consisting of a lump mass supported by spring limbs with damper can be tuned to simulate periodic human walking behavior [20–22]. Previous studies have shown that active energy input in a feedback or feed-forward manner was required to compensate for energy loss to achieve continuous step cycle behavior [22]. However, no such feedback control was included in current bipedal walking model. Moreover, the damper of the bipedal model is assumed to be time-invariant linear component at a given walking speed. This assumption is unreasonable because if the damping remains constant, the ground reaction force generated by the pedestrian will not be zero when the leg touches down at the start of each step cycle.

The principal aim of this paper is to study the dynamic response of a footbridge under a pedestrian by considering the effect of interaction between the human and structure. The analysis is presented for a bipedal walking model with damped compliant legs and the footbridge is modeled as a simply supported Euler–Bernoulli beam with uniform cross-section. The pedestrian and structural systems are integrated for studying the problem. A time-variant damper will be modeled as a function of a given walking speed. A control force in a feedback manner is applied by the pedestrian to compensate for energy dissipated with the system damping in walking and to regulate the walking performance of the pedestrian. The effects of stiffness and damping of the leg and the landing angle of attack of legs are investigated in the numerical studies.

2. Human–structure dynamic interaction system

A pedestrian is simulated as a bipedal walking model with two DOFs as shown in Fig. 1. The human body is modeled as a lumped mass m_h at the center of mass (COM) and the two legs are described as two massless, linear springs of equal rest length l_0 with stiffness k_{leg} and a time-variant damper neglecting the roller feet. A passive spring provides a compliant mechanism to absorb collision impacts and to generate push-off impulses, whereas the damper restrained excessive motion of COM [22]. Both the springs and dampers act independently and they influence the model dynamics only during standing when the spring and damper forces oppose the gravitational force of the human body.

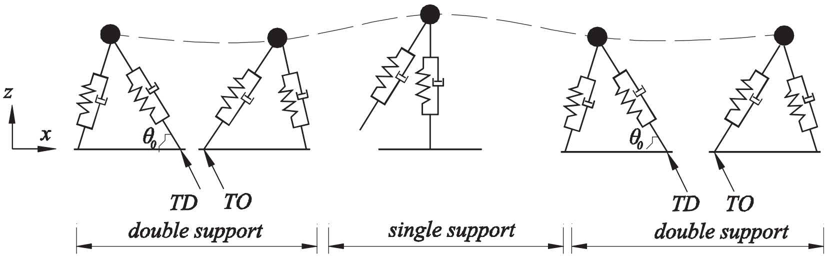

Fig. 1. Schematic of the biomechanical walking model (θ_0 is the angle of attack).

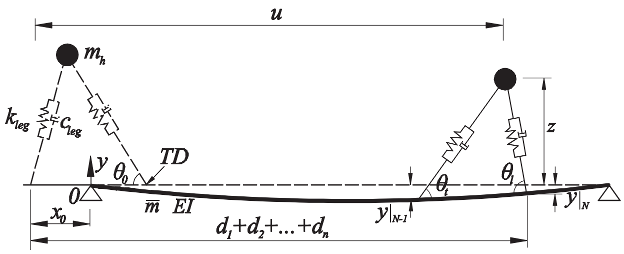

Fig. 2. Mathematical model for the dynamic analysis of the footbridge.

A complete step, defined as the interval between 'heel strike' of progressive footfalls, is divided into two periods: single support phase and double support phase. For example, the double support phase begins with 'touch down' (TD) of leading leg and ends with 'touch off' (TO) of trailing leg as shown in Fig. 1, where the single support phase begins. Then, the trailing leg is repositioned ahead of the body's COM at a given angle of attack and becomes the leading leg for the next step. When the trailing leg hits the ground, the single support phase is completed.

2.1. Equation of motion of human–structure interaction system

Consider a footbridge subjected to a pedestrian, as shown in Fig. 2. The footbridge is modeled as a simply supported Euler–Bernoulli beam with uniform cross-section. The pedestrian is assumed to remain in contact with the bridge surface at all times.

The equations of motion governing the vibration of the bridge and pedestrian are derived from the Lagrangian equation. The kinetic energy T and potential energy V of human–structure interaction system in double support phase can be obtained as

$$T=\frac{1}{2}m_h\dot{z}^2+\frac{1}{2}m_h\dot{u}^2+\frac{1}{2}\int_0^L m\dot{y}(x,t)^2\mathrm{d}x \tag{1a}$$

$$V=\frac{1}{2}k_{\mathrm{leg}}(L_l-l_0)^2+\frac{1}{2}k_{\mathrm{leg}}(L_t-l_0)^2+m_h gz+\frac{1}{2}\int_0^L EI\left(y''(x,t)\right)^2\mathrm{d}x \tag{1b}$$

The motion of the COM is described by the generalized coordinate $\{z,u\}$, where z and u denote the vertical and horizontal displacements of the COM respectively. y is the displacement of the beam; E and m are Young's modulus of material and the mass per unit length of the beam; g is the acceleration due to gravity; L_l and L_t are the length of two legs, respectively. The subscripts 'l' and 't' indicate the leading and trailing legs, respectively. $y''(x,t)$ is the curvature of beam.

By separation of variables, the vertical displacement of the beam can be expressed as:

$$y(x,t)=\sum_{i=1}^{\infty}Y_i(t)\phi_i(x)=\mathbf{\Phi}(x)\mathbf{Y}(t) \tag{2}$$

where $\{\phi_i(x),\ i=1,2,\ldots,n\}$ are the assumed vibration modes that satisfy the boundary conditions and $\{Y_i(t),\ i=1,2,\ldots,n\}$ are the modal coordinates of the bridge.

The length of the leading leg and trailing leg are respectively

$$L_l=\sqrt{\left(\sum_{i=1}^{N}d_i-u\right)^2+\left(z-y|_N\right)^2} \tag{3a}$$

$$L_t=\sqrt{\left(u-\sum_{i=1}^{N-1}d_i\right)^2+\left(z-y|_{N-1}\right)^2} \tag{3b}$$

where d_i, $y|_N$ and $y|_{N-1}$ are the ith step length and the vertical displacement of beam at the points of contact of the leading and the trailing legs in the Nth step cycle, respectively.

Substituting Eq. (2) and (3) into Eq. (1) gives

$$T=\frac{1}{2}m_h\dot{z}^2+\frac{1}{2}m_h\dot{u}^2+\frac{1}{2}\int_0^L m\left(\sum_{i=1}^{\infty}\dot{Y}_i(t)\phi_i(x)\right)^2\mathrm{d}x \tag{4a}$$

$$V=\frac{1}{2}k_{\mathrm{leg}}\left(\sqrt{\left(\sum_{i=1}^{N}d_i-u\right)^2+\left(z-y|_N\right)^2}-l_0\right)^2+\frac{1}{2}k_{\mathrm{leg}}\left(\sqrt{\left(u-\sum_{i=1}^{N-1}d_i\right)^2+\left(z-y|_{N-1}\right)^2}-l_0\right)^2+m_hgz$$
$$+\frac{1}{2}\int_0^L EI\sum_{i=1}^{\infty}{Y_i}^2(t)\left(\frac{\mathrm{d}^2(\phi_i(x))}{\mathrm{d}x^2}\right)^2\mathrm{d}x \tag{4b}$$

The virtual work is performed by the damping force of the footbridge and two legs, and the variation of virtual work δW of the human–structure interaction system is given by [23]

$$\delta W=-c_l\nu_l\delta(\Delta L_l)-c_t\nu_t\delta(\Delta L_t)-\int_0^L c_s\dot{y}''\delta y''\mathrm{d}x=Q_1\delta Y_1+Q_2\delta Y_2+\cdots+Q_n\delta Y_n+Q_{n+1}\delta z+Q_{n+2}\delta u \tag{5}$$

where c_l, c_t and c_s are the damping of the two legs and the damping of the beam, respectively; ν_l and ν_t are the axial velocity of the leading and trailing legs; $\delta y''$ is variation of the curvature of the beam. $Q_1, Q_2,\ldots,Q_{n+2}$ are the generalized forcing functions corresponding to the coordinate $\{Y_1, Y_2,\ldots,Y_n, z, u\}$. $\delta(\Delta L_l)$ and $\delta(\Delta L_t)$ are the virtual decompressions of leg. $\{\delta Y_i,\ i=1,2,\ldots,n\}$ are the virtual displacements of beam.

The axial spring velocity can be obtained from the derivative of the decompressions of two legs in Eq. (3)

$$\nu_l=(\dot{z}-\dot{y}|_N)\sin\theta_l-\dot{u}\cos\theta_l \tag{6a}$$

$$\nu_t=(\dot{z}-\dot{y}|_{N-1})\sin\theta_t+\dot{u}\cos\theta_t \tag{6b}$$

where θ_l, θ_t are the leading leg angle and the trailing leg angle, respectively.

We assume the sum of damping of two legs equal to c_{leg} in double support phase. The damping of the leading leg changes from zero at the touch down of the leading leg to c_{leg} at the touch off of the trailing leg in the double support phase while it remains constant at c_{leg} in the single support phase. Therefore, the coefficient of damping in the double support phase can be represented by

$$\alpha(t)=\frac{L_t(t)-L_t(0)}{l_0-L_t(0)} \tag{7}$$

where $L_t(0)$, $L_t(t)$ are the length of the trailing leg at the start of double support phase and the time step 't', respectively.

According to the above assumption, the damping of the leading and trailing legs can be expressed as follows:

$$c_l=\alpha(t)c_{\mathrm{leg}} \tag{8a}$$

$$c_t=(1-\alpha(t))c_{\mathrm{leg}} \tag{8b}$$

Substituting Eqs. (2), (6) and (8) into Eq. (5) gives

$$\delta W=\sum_{i=1}^{\infty}(\alpha(t)c_{\mathrm{leg}}\nu_l\phi_i(N)\sin\theta_l+(1-\alpha(t))c_{\mathrm{leg}}\nu_t\phi_i(N-1)\sin\theta_t)\delta Y_i(t)$$
$$+\sum_{i=1}^{\infty}\left(\int_0^L c_s\left(\sum_{i=1}^{\infty}\dot{Y}_i(t)\frac{\mathrm{d}^2(\phi_i(x))}{\mathrm{d}x^2}\right)\frac{\mathrm{d}^2(\phi_i(x))}{\mathrm{d}x^2}\mathrm{d}x\right)\delta Y_i(t)$$
$$+\left(-\alpha c_{\mathrm{leg}}\nu_l\sin\theta_l-(1-\alpha(t))c_{\mathrm{leg}}\nu_t\sin\theta_t\right)\delta z+\left(\alpha(t)c_{\mathrm{leg}}\nu_l\cos\theta_l-(1-\alpha(t))c_{\mathrm{leg}}\nu_t\cos\theta_t\right)\delta u \tag{9}$$

The Lagrangian equations of motion of the human–structure interaction system are given by [23]

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial\dot{Y}_i}\right)+\frac{\partial V}{\partial Y_i}=Q_i\quad(i=1,2,\ldots,n) \tag{10a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial\dot{z}}\right)+\frac{\partial V}{\partial z}=Q_{n+1} \tag{10b}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial\dot{u}}\right)+\frac{\partial V}{\partial u}=Q_{n+2} \tag{10c}$$

Substituting Eqs. (4) and (9) into Eq. (10) results in the dynamic equation of the human–structure interaction system in a matrix form as Eq. (11). This formulation is original but it is placed in the Appendix just for an orderly presentation.

$$\mathbf{M}\ddot{\mathbf{U}}+\mathbf{C}\dot{\mathbf{U}}+\mathbf{K}\mathbf{U}=\mathbf{F}(t) \tag{11}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$, $\mathbf{U}$, $\dot{\mathbf{U}}$, $\ddot{\mathbf{U}}$ and $\mathbf{F}(t)$ are the mass, damping and stiffness matrices, the displacement, velocity, acceleration and force vectors, respectively. For the present case, the mass, damping, stiffness matrices, and the force vector can be expressed as

$$\mathbf{M}=\begin{bmatrix} M_1 & 0 & \dots & 0 & 0 \\ 0 & M_2 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & m_h & 0 \\ 0 & 0 & \dots & 0 & m_h \end{bmatrix}_{(n+2)\times(n+2)}$$

$$\mathbf{C}=\begin{bmatrix} c_{1,1} & c_{1,2} & \dots & c_{1,n+1} & c_{1,n+2} \\ c_{2,1} & c_{2,2} & \dots & c_{2,n+1} & c_{2,n+2} \\ \dots & \dots & \dots & \dots & \dots \\ c_{n+1,1} & c_{n+1,2} & \dots & \alpha(t)c_1+(1-\alpha(t))c_2 & (1-\alpha(t))c_4-\alpha(t)c_3 \\ c_{n+2,1} & c_{n+2,2} & \dots & (1-\alpha(t))c_4-\alpha(t)c_3 & c_{leg}-\alpha(t)c_1-(1-\alpha(t))c_2 \end{bmatrix}_{(n+2)\times(n+2)} \tag{12}$$

$$\mathbf{K}=\begin{bmatrix} \omega_1^2 M_1 & 0 & \dots & -k_{t,v}\phi_1(N-1)-k_{l,v}\phi_1(N) & 0 \\ 0 & \omega_2^2 M_2 & \dots & -k_{t,v}\phi_2(N-1)-k_{l,v}\phi_2(N) & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & k_{t,v}+k_{l,v} & 0 \\ 0 & 0 & \dots & 0 & k_{t,h}-k_{l,h} \end{bmatrix}_{(n+2)\times(n+2)}$$

$$\mathbf{F}(t)=[0,0,\dots,-m_h g,0]^{\mathrm{T}}_{n+2},\quad \mathbf{U}=[Y_1,Y,\dots,Y_n,z,u]^{\mathrm{T}}_{n+2}$$

in which

$$\begin{aligned} c_{i,i}&=2\xi_i\omega_i M_i+(1-\alpha(t))c_2\Phi_{i,i}(N-1)+\alpha(t)c_1\Phi_{i,i}(N)\quad(i=1,2,\dots,n)\\ c_{i,j}&=c_{j,i}=(1-\alpha(t))c_2\Phi_{i,j}(N-1)+\alpha(t)c_1\Phi_{i,j}(N)\quad(i\neq j\le n)\\ c_{i,n+1}&=-\alpha(t)c_1\phi_i(N)-(1-\alpha(t))c_2\phi_i(N-1)\quad(i=1,2,\dots,n)\\ c_{i,n+2}&=\alpha(t)c_3\phi_i(N)-(1-\alpha(t))c_4\phi_i(N-1)\quad(i=1,2,\dots,n)\\ c_{n+1,i}&=-\alpha(t)c_1\phi_i(N)-(1-\alpha(t))c_2\phi_i(N-1)\quad(i=1,2,\dots,n)\\ c_{n+2,i}&=\alpha(t)c_3\phi_i(N)-(1-\alpha(t))c_4\phi_i(N-1)\quad(i=1,2,\dots,n) \end{aligned} \tag{13}$$

$$c_1=c_{\text{leg}}\sin^2\theta_l,\quad c_2=c_{\text{leg}}\sin\theta_l\cos\theta_l,\quad c_3=c_{\text{leg}}\sin^2\theta_t,\quad c_4=c_{\text{leg}}\sin\theta_t\cos\theta_t$$

$$k_{l,v}=k_{\text{leg}}(1-l_0/L_l)(1-y|_N/z),\quad k_{t,v}=k_{\text{leg}}(1-l_0/L_t)(1-y|_{N-1}/z)$$

$$k_{l,h}=k_{\text{leg}}(1-l_0/L_l)\left(\sum_{i=1}^{N}d_i/u-1\right),\quad k_{t,h}=k_{\text{leg}}(1-l_0/L_t)\left(1-\sum_{i=1}^{N-1}d_i/u\right)$$

$$\Phi_{i,j}(N)=\phi_i(N)\phi_j(N),\phi_i(N)=\sin\left(i\pi\left(\sum_{n=2}^{N}d_n+x_0\right)/L\right)$$

The damping and stiffness matrices are noted time-varying. It can be seen from Eq. (13) that ($k_{l,v}$, $k_{t,v}$). ($k_{l,h}$, $k_{t,h}$) are the effective vertical and horizontal stiffnesses, respectively. The effective vertical stiffnesses, ($k_{l,v}$, $k_{t,v}$) determined from a combination of the leg stiffness (k_{leg}), the displacement of the COM (u,v) and the displacement of structure (y), correspond to the vertical motion of the COM rather than any physical spring associated with the leg stiffness in the model. Therefore, the equations of motion of the human–structure system are coupled in terms of the interacting force at the contact point. With the movement of a pedestrian, the interacting force between human and structure changes with the contact points. Although the leg stiffness (k_{leg}) is constant during walking, the effective leg stiffness is nonlinear and it will be discussed later.

All the above analysis is based on the double support phase. The analysis process for the single support phase is similar to that for the double support phase. All the coefficients relating to the trailing leg will be equal to zeros in the single support phase.

2.2. Feedback mechanism

A damped compliant walking model is able to reproduce a one-step gait cycle that consists of the single and double support phases. This model requires an energy input mechanism to maintain the steady walking gait because energy has been dissipated with the system damping in the walking process. This energy consumption is distributed throughout the gait cycle [21]. One possible candidate, among many ways to provide the additional energy, is to apply a control force or torque to the system in a feedback or feed-forward manner [22]. The control force can be modeled as a varying distribution over one gait cycle as generated by human in walking or any form of convenience. The present model of this force has the requirement to maintain the total energy of the human body during walking.

Assuming the external work in horizontal direction equals to the energy loss in the process. The control force can be obtained from the following equation:

$$F_{\mathrm{ctrl}}(t)=\frac{E_0-E(t)}{\Delta u(t)} \tag{14}$$

where $F_{\mathrm{ctrl}}(t)$, E_0, $E(t)$ and $\Delta u(t)$ are the horizontal control force, initial energy input, energy of the human body and horizontal displacement increment of COM at time 't', respectively.

The total energy of the human body $E(t)$ including the potential energy and the kinetic energy is given by

$$E(t)=\frac{1}{2}m_h v_z^2+\frac{1}{2}m_h v_x^2+\frac{1}{2}k_{\mathrm{leg}}(\Delta L_l)^2+\frac{1}{2}k_{\mathrm{leg}}(\Delta L_t)^2+m_h gz \tag{15}$$

When the control force is included in Eq. (12), the force is time-varying as

$$\mathbf{F}(t)=[0,0,\ldots,-m_h g,F_{\mathrm{ctrl}}(t)]_{n+2}^{\mathrm{T}} \tag{16}$$

3. Dynamic analysis of structure

3.1. Modal analysis of the HSI system

If the pedestrian moves on a footbridge, the instantaneous mass and stiffness matrices of the HSI system should be used for the solution of the instantaneous natural frequencies of the entire system. The challenging and fascinating aspects of the study lie in the fact that a slightly damped structure system and a highly damped human body system are combined to form a new system [24–26]. In this study, the instantaneous modal properties of the entire system will be obtained by using the state-space method with the following definitions [27]:

$$\mathbf{V}=\begin{Bmatrix}\mathbf{U}\\ \dot{\mathbf{U}}\end{Bmatrix},\quad \mathbf{A}=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}\end{bmatrix},\quad \mathbf{B}=\begin{Bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\mathbf{F}(t)\end{Bmatrix} \tag{17}$$

Eq. (17) can be transformed into the state space as

$$\dot{\mathbf{V}}=\mathbf{AV}+\mathbf{B} \tag{18}$$

In such a case, the modal properties can be obtained by solving the eigenproblem stated as

$$\mathbf{A}\phi=\lambda\phi \tag{19}$$

For a damped MDOF system, the ith natural frequencies f_i and damping ratios ξ_i are respectively given as

$$f_i=\frac{1}{2\pi}|\lambda_i|,\quad \xi_i=\frac{\mathrm{Re}(\lambda_i)}{|\lambda_i|} \tag{20}$$

and they are time dependent because the damping and stiffness matrices in Eq. (17) are time dependent.

3.2. Procedure of time response analysis of the HSI system

In this paper, the dynamic responses of a footbridge subjected to pedestrian motion at every time step 't' may be obtained in the following steps:

1. Input the geometric and physical parameters of the footbridge and pedestrian.
2. Perform a modal analysis to obtain the frequencies $\omega_1,\omega_2,\ldots,\omega_n$ and the corresponding damping ratios of the footbridge alone.
3. The initial conditions of the footbridge and the pedestrian are specified, such as: (a) the initial displacement and velocity of the COM at time $t=0$; (b) the position for the first leg to touch down; (c) time increment Δt; (d) initial step number $st=1$;
4. Construct the overall mass, stiffness and damping matrices of the HSI system at time $t=0$.
5. Compute the initial acceleration of the HSI system from the initial stiffness, damping matrices and force vector

of system based on the equilibrium of force as

$$\ddot{\mathbf{U}}_0 = \mathbf{M}^{-1}\left(\mathbf{F}_0 - \mathbf{C}\dot{\mathbf{U}}_0 - \mathbf{K}\mathbf{U}_0\right)$$

6. For each time step i
   6.1 Determine the time dependent position of the two feet with the followings:

$$x_l = \sum_{i=1}^{N} d_i - x_0 \quad x_t = \sum_{i=1}^{N-1} d_i - x_0$$

   6.2 Compute the compressions of two legs with Eq. (3).
   6.3 Compute the effective vertical stiffness, effective horizontal stiffness, effective horizontal damping and effective vertical damping of two legs with Eq. (13).
   6.4 Assemble the overall mass, damping, stiffness matrices and force vector $\tilde{\mathbf{F}}$ of the HSI system with Eq. (12).
   6.5 Determine the displacement and velocity increments with the Newton–Raphson method [28].
   6.6 Compute the displacement and velocity at the end of the time increment.
   6.7 Compute the length of the trailing leg with Eq. (3) and determine the step cycle if it is in the double support phase or in the single support phase. If the length of trailing leg is larger than the rest length of leg, compute the distance between the COM and the assumed contact point with the structure for the next step cycle. If the distance between the COM and the assumed contact point equal to the rest length of leg, a new step cycle starts and then set $st = st + 1$.
   6.8 Compute the control force with Eq. (14).
   6.9 Revise the overall force vector with Eq. (16).
   6.10 Determine the acceleration of HSI system at the end of time increment using the follows:

$$\ddot{\mathbf{U}}_i = \mathbf{M}^{-1}\left(\mathbf{F}_i - \mathbf{C}\dot{\mathbf{U}}_i - \mathbf{K}\mathbf{U}_i\right)$$

   6.11 Repeat steps 6.1–6.11 to obtain the dynamic responses of the HSI system for next time step until the end of time history.

4. Numerical study

To illustrate the performance of the proposed interaction model, two examples of human–structure interaction are analyzed. The results from using bipedal walking model are compared with those from using time domain force model, which is the most common existing model to analyze the dynamic behavior of footbridges.

4.1. Dynamic analyses for the footbridge under a single pedestrian

4.1.1. Example 1—walking on a stiff beam

Consider a simply supported beam [29] of length 11.0 m, width 1.25 m and thickness 0.35 m. The material properties of the beam are stiffness $EI = 1.64 \times 10^8$ N m^2, mass per unit length $m = 1.364 \times 10^3$ kg m^{-1} and all the damping ratios of the beam alone are assumed to be $\xi = 0.3\%$. The properties of the human body are: leg stiffness $k_{\mathrm{leg}} = 20$ kN m^{-1}, damping ratio of human body $\xi = 8\%$, human mass $m_h = 80$ kg, angle of attack $\theta_0 = 69^\circ$ and the initial energy input $E_0 = 812$ J. The last one is determined from trial and error to have a steady walking gait for a limited number of footstep cycles without faltering. The angle of attack θ_0 is assumed to remain constant in the process. We consider the first five natural modes of the beam in the analysis. The first two natural frequencies of the beam are $f_1 = 4.51$ Hz and $f_2 = 16.93$ Hz respectively and the modal mass are respectively $m_1 = 7040$ kg and $m_2 = 7420$ kg.

The ground reaction force (GRF) time histories generated by using bipedal walking model are plotted in Fig. 3. It clearly shows that a time-dependent damper is necessary to ensure that the ground reaction force is zero when the leg touches down at the start of each step cycle. Otherwise, the damping force generated by the pedestrian will not be zero when the leg touches down at the start of each step cycle because the axial spring velocity is not zero. As a result, the GRF is also not equal to zero.

Figs. 4 and 5 give the dynamic responses at midspan of the beam. The dash line is the dynamic responses of the beam using the time domain force histories as shown in Fig. 3. The dynamic behavior of the pedestrian can also be obtained from the equation of motion of the system in Eq. (11). Figs. 6 and 7 show the vertical displacement and acceleration of COM in walking. It can be seen that the dynamic responses of human body are steady in walking. However, if the horizontal control force was not applied, the pedestrian footstep would stumble after a limit number of step cycles. The forces generated by human across the bridge as shown in Fig. 8 are very similar to those on level rigid ground because both cases have a regular pattern of footstep forces. The control forces for each step are approximately the same as shown in Fig. 9. The above results show that the dynamic interaction of human–structure system is small when the footbridge is rigid.

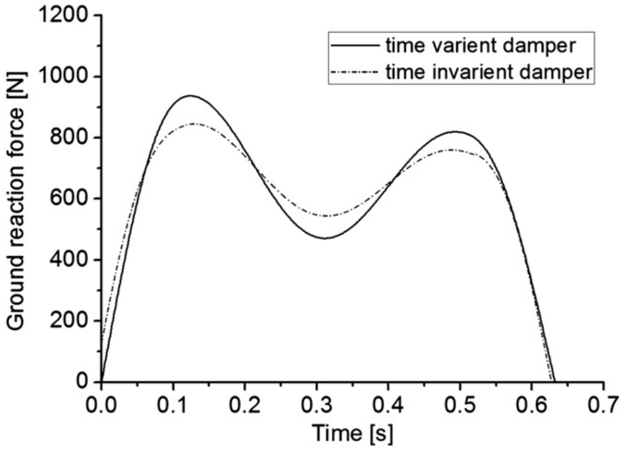

Fig. 3. Ground reaction force by bipedal walking model.

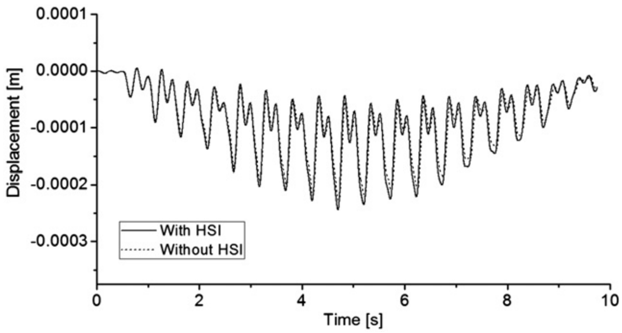

Fig. 4. Displacement at midspan of beam.

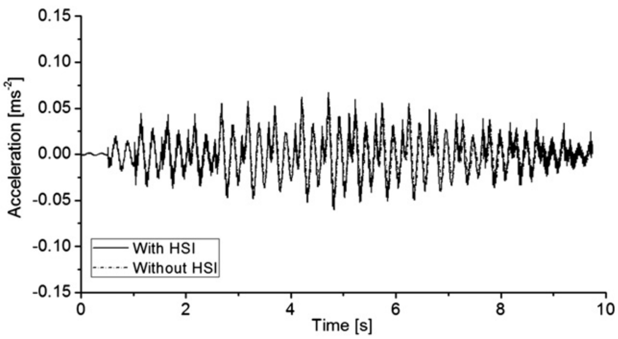

Fig. 5. Acceleration at midspan of beam.

The spectrum of acceleration at midspan of the beam is shown in Fig. 10. The footstep frequency 1.93 Hz and their multiples can be clearly identified from the spectra. This observation is consistent with the findings from other existing methods [30–32].

To study how the pedestrian can influence the dynamic properties of structures, the instantaneous values of the first four natural frequencies of the HSI system are computed from Section 3.1, and they are shown in Fig. 11. The first two frequencies are the natural frequencies of the mass of pedestrian, which are time dependent because the effective horizontal and vertical leg stiffness are varying in walking. The third and fourth frequencies are approximately equal to the first two natural frequencies of the beam. The enlarged curves show that the mass of the pedestrian has a slight effect on the natural frequency of the combined system. The results are not consistent with the report by Ellis and Ji [33] which stated that a person running and jumping on the spot cannot change the dynamic characteristics of the structure.

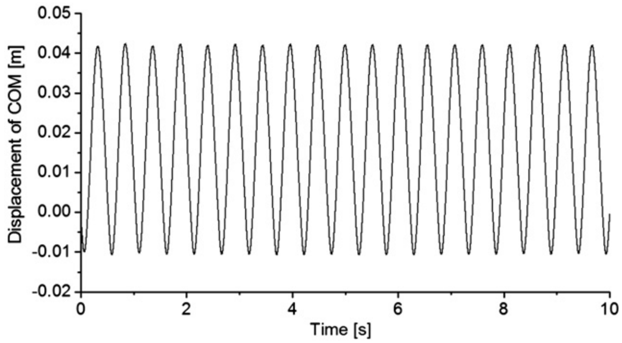

Fig. 6. Displacement of the COM in walking.

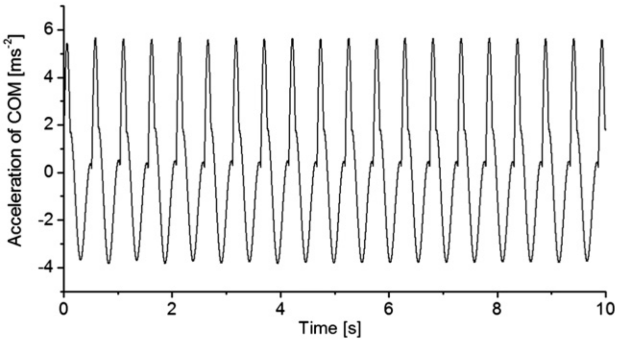

Fig. 7. Acceleration of the COM in walking.

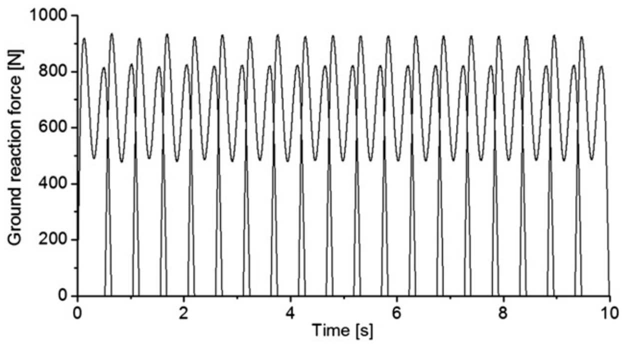

Fig. 8. Ground reaction force in walking.

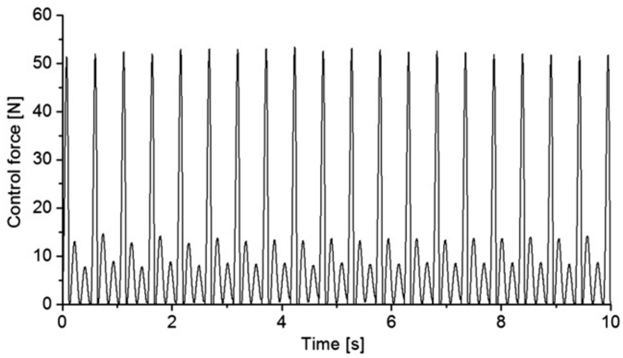

Fig. 9. Control force in walking.

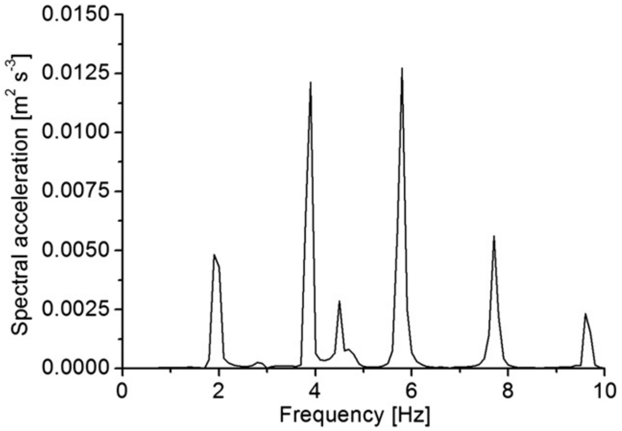

Fig. 10. Spectrum of footbridge acceleration response.

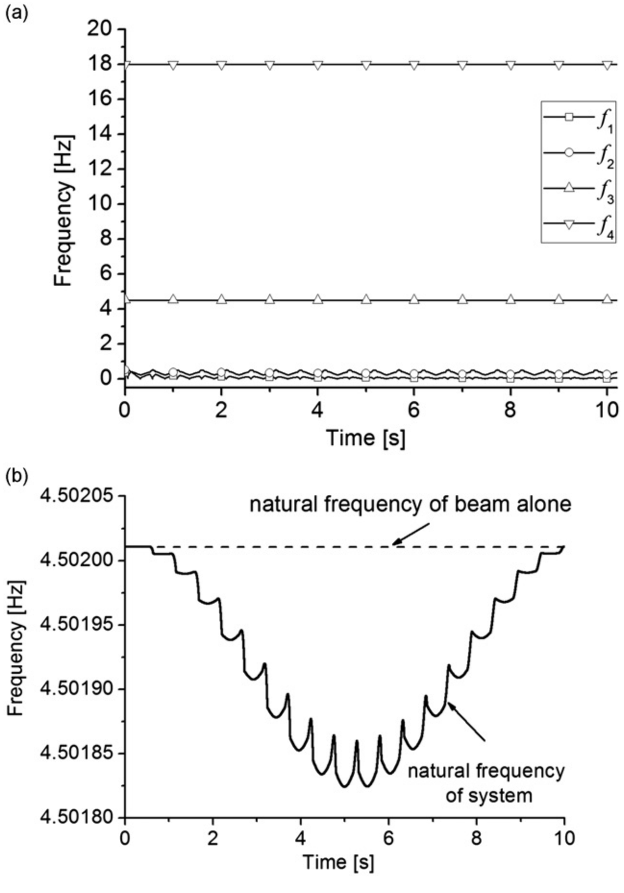

Fig. 11. Vibration frequencies of the HSI system, (a) the first four frequencies of the system and (b) enlarged detail of natural frequency, f_3, of the system.

The instantaneous values of damping of the HSI system calculated from Section 3.1 are shown in Fig. 12. Result in the figure indicates that damping increases due to the presence of walking persons. The result is consistent with recent experimental investigations [34–37].

4.1.2. Example 2—walking on a flexible beam

To further study the performance of the proposed formulation with larger interaction effect of the system, a more flexible simply supported beam subjected to a pedestrian motion is also studied. The beam has a length = 9.0 m, width = 0.8 m and thickness = 0.12 m. The material properties of the beam are mass density, $\rho = 2400\ \mathrm{kg\ m^{-3}}$ and the

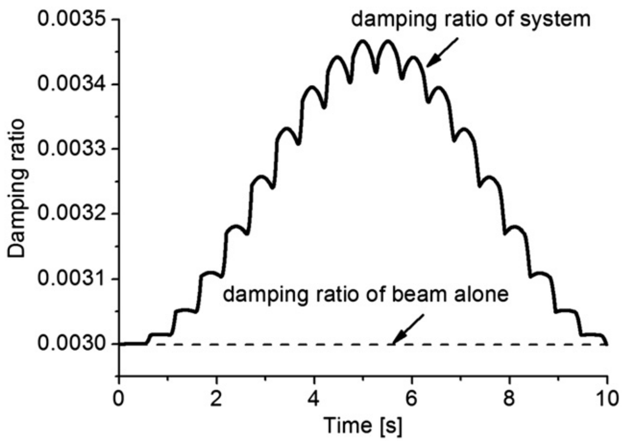

Fig. 12. Damping ratio ξ_3 corresponding to the natural frequency, f_3, of the HSI system.

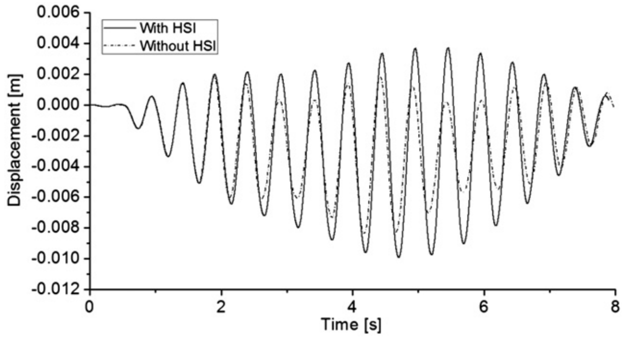

Fig. 13. Displacement at midspan of beam.

Young's modulus, $E=3\times10^{10}$ N m^{-2}. The fundamental frequency of the beam alone is 2.375 Hz. The information of the human body is exactly the same as that in the first example. The assumption on the control force for Eq. (14) remains as the static midspan deflection of the beam is up to a few millimeters only.

Figs. 13 and 14 give the time histories of the vertical displacements and accelerations at midspan of the beam. The more distinctive differences between the two sets of curves reveal that the dynamic interaction of the human–structure system is big. This is due to the fact that the compression of the legs will be larger with greater vibration of the beam. Therefore, the elastic potential energy stored in the compliant leg at the end of the single support phase of each step cycle, which serves as propulsion energy during push-off, becomes larger when the pedestrian moves towards midspan of the beam. As a result, the contact forces between the pedestrian and beam will be larger. As noted in Figs. 15 and 16, the dynamic behavior of the human body is not the same for each step cycle of walking. Moreover, the interaction footstep forces generated by the pedestrian gradually increases with the pedestrian moving towards midspan of the beam as shown in Fig. 17.

Because of the larger dynamic interaction between human and structure, much more external energy must be input to maintain the steady gait and a relatively uniform dynamic behavior of the COM in the process. It can be seen from Fig. 18 that the required control force in each step also increases gradually towards the center of beam.

These results show that the proposed formulation on the modeling of the pedestrian–bridge interaction would be suitable for the study of human–structure interaction problems. The time domain force model is not suitable for the dynamic analysis of slender structures, such as long span footbridge, as it does not consider the effects from human–structure interaction and the adjustment made by the pedestrian to maintain a steady gait.

4.2. Effect of leg stiffness, angle of attack and leg damping ratio on the dynamic response

The analysis in earlier Sections of this paper shows that the bipedal walking model has six independent parameters: body mass, angle of attack, rest leg length, leg stiffness, leg damping ratio and initial system energy E_0. To evaluate how the leg stiffness, angle of attack and leg damping ratio affect the dynamic response of footbridge, the body mass and rest leg length are fixed at $m_h=80$ kg and $l_0=1$ m respectively. The leg stiffness is varied in the range of 16–23 kN m^{-1}, angle

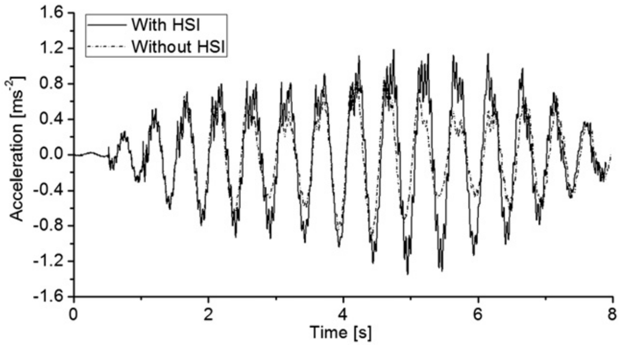

Fig. 14. Acceleration at midspan of beam.

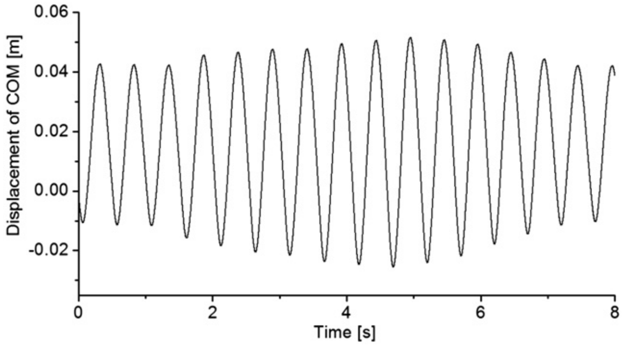

Fig. 15. Displacement of the COM in walking.

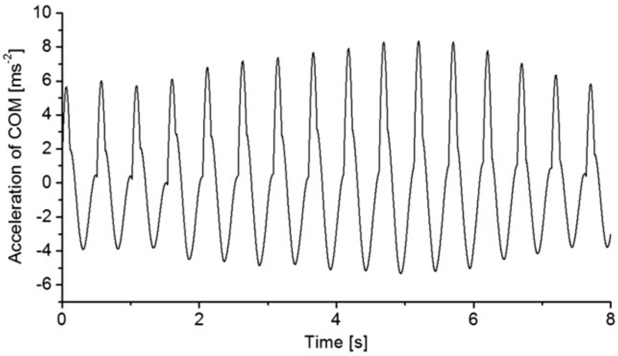

Fig. 16. Acceleration of the COM in walking.

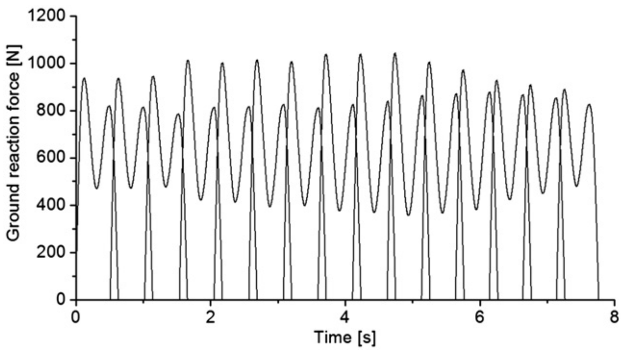

Fig. 17. Ground reaction force in walking.

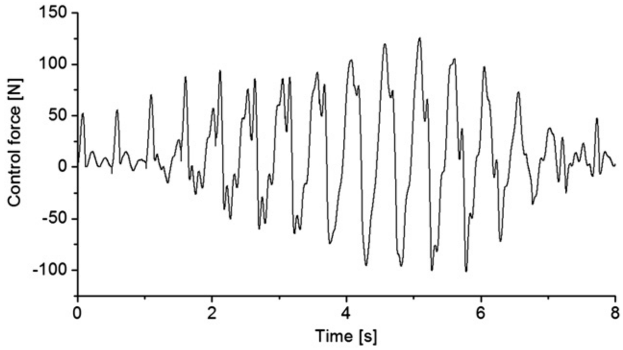

Fig. 18. Control force in walking.

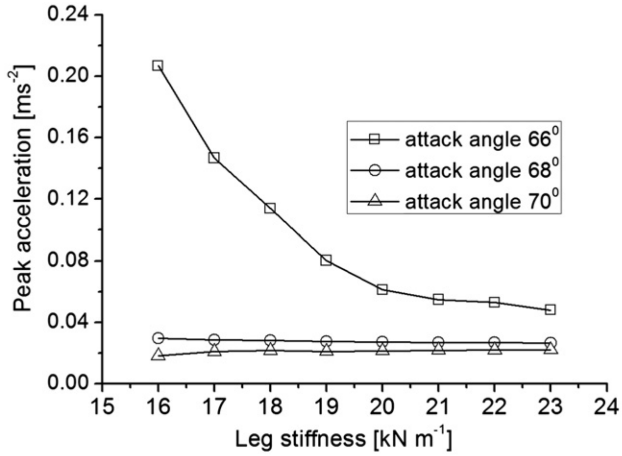

Fig. 19. Effect of leg stiffness on the dynamic response of structure (damping ratio 0.08).

Table 1
Step frequency and average speed on rigid ground (angle of attack $\theta_0=66^\circ$, leg damping ratio $\xi=0.08$).

Leg stiffness (kN m^{-1})	16	17	18	19	20	21	22	23
Step frequency (Hz)	1.66	1.68	1.69	1.70	1.75	1.77	1.78	1.82
Average horizontal speed (m s^{-1})	1.09	1.12	1.14	1.17	1.21	1.25	1.27	1.32

of attack is varied in the range of 66–70° and the damping ratio is varied in the range of 0.05–0.08 in this study. Each parameter is examined individually with all other parameters remain constant.

Considering a simply supported footbridge subjected to pedestrian motion, the following data is assumed: flexural stiffness of footbridge $EI=9.78\times10^9$ N m^2, mass per unit length $m=2.84\times10^3$ kg m^{-1}, length of footbridge $L=42$ m, all the damping ratios of structure $\xi=0.01$. The information is derived from the example presented by Li et al. [38]. The fundamental frequency of the footbridge modeled as a beam is 1.65 Hz.

Fig. 19 shows that the leg stiffness has small effect on the dynamic response of structure except for the case when the attack angle equals to 66°. When the attack angle equals to 66° with a leg stiffness of 16 kN m^{-1}, the peak acceleration response reaches its maximum. On checking the results in Table 1, the step frequency corresponding to an angle of attack of $\theta_0=66^\circ$ with leg stiffness 16 kN m^{-1} coincides with the fundamental frequency of structure $f_s=1.65$ Hz. This case is a resonant situation and is a critical situation for the footbridge. The results indicate that the leg stiffness has significant effect on the dynamic response of structure when the step frequency is close to the natural frequency of the structure.

The dynamic interaction of human–structure system in the resonant case can be clearly observed from the control force time history in Fig. 20. The interacting force between human and structure increases with the pedestrian moving towards the center of footbridge as noted earlier in Figs. 15 and 16. The reason is that the displacement and acceleration of footbridge is much larger in the resonant situation as shown in Fig. 21. As a result, more external energy must be input to maintain the steady walking and uniform dynamic behavior of the COM in each step cycle.

Fig. 22 shows the attack angle effect on the dynamic response of footbridge. It shows that when the attack angle increases, the peak acceleration of footbridge changes slightly except at 66°, the lower end of the leg stiffness range. When

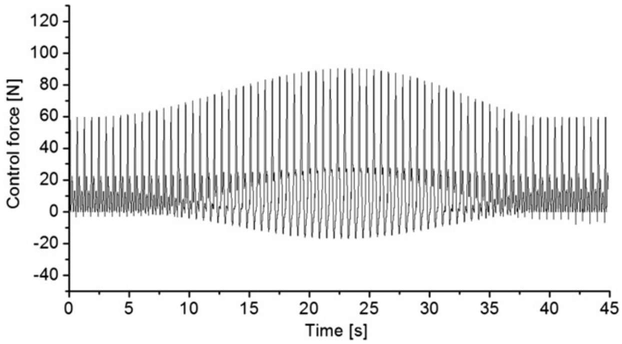

Fig. 20. Control force in walking (angle of attack 66°, stiffness 16 kN m^{-1}, damping ratio 0.08).

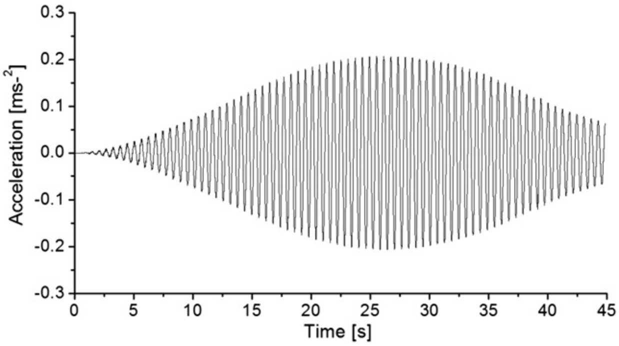

Fig. 21. Vertical acceleration at midspan of beam (angle of attack 66°, stiffness 16 kN m^{-1}, damping ratio 0.08).

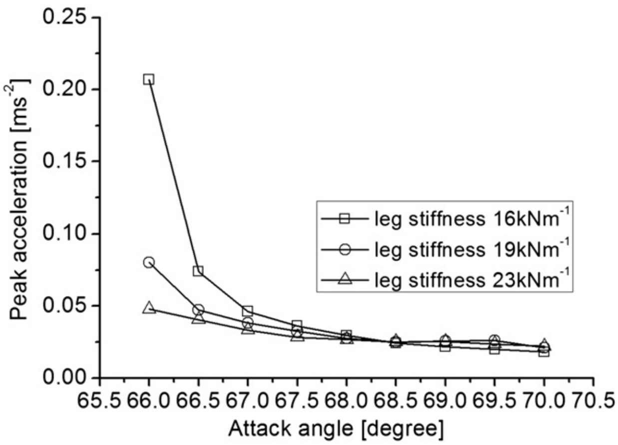

Fig. 22. Effect of attack angle on the dynamic response of structure (damping ratio 0.08).

the attack angle is larger than 68°, the curves from different leg stiffness are close together indicating that the attack angle has little effect on the dynamic response of footbridge.

Fig. 23 shows the effect of leg damping ratio on the dynamic response of footbridge. The angle of attack is fixed at 68° in this study. When the damping ratio of the bipedal walking model increases, the peak accelerations of footbridge increase almost linearly and this is not according to engineering sense. However, inspection of the variation of other parameters shows that the step frequency decreases with an increase in the leg damping when other parameters keep constant as shown in Fig. 24. The step frequency with larger damping is closer to the natural frequency of footbridge creating larger vibration amplitude. Also the GRF increases with the leg damping as shown in Fig. 25. Therefore, the response of the footbridge increases with the damping ratio of the model. For example, there is around 10 percent increase in the peak response at leg stiffness of 16 kN m^{-1} if the pedestrian damping ratio is changed from 5 to 8 percent. This indicates that the damping ratio of the bipedal model slightly affects the dynamic response of footbridge.

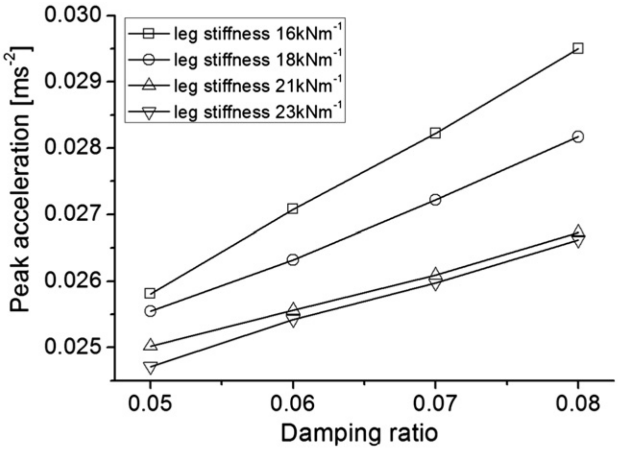

Fig. 23. Effect of damping on the dynamic response of structure (attack angle 68°).

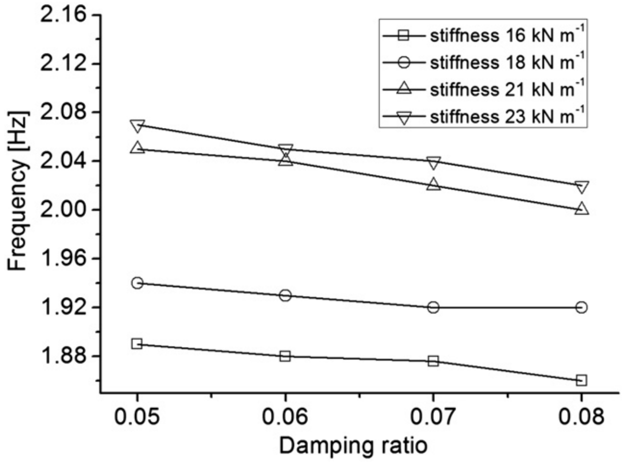

Fig. 24. Step frequency with different leg damping (attack angle 68°).

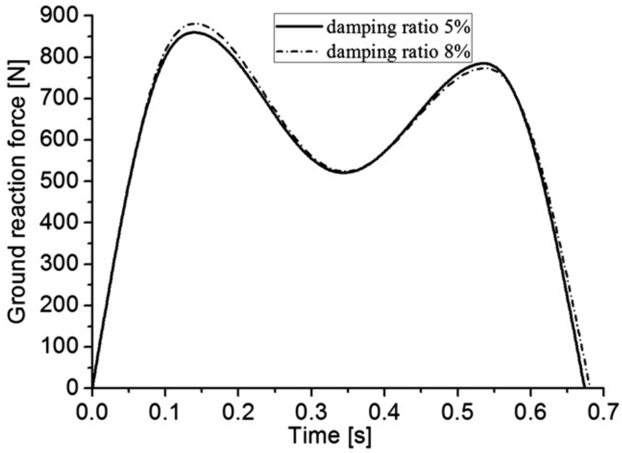

Fig. 25. Ground reaction force with different leg damping, (leg stiffness 16 kN m^{-1}, attack angle 68°).

5. Discussions

The bipedal walking model with damped compliant leg can reproduce a one-step gait cycle that consists of the single and double support phases. It can be adopted to describe the human–structure interaction in this study. The current bipedal walking model is improved with the assumption of time dependent damping and a feedback control force applied by pedestrian to maintain the steady gait. The leg axial velocity is not zero when the leg touches down at the start of the gait cycle. This implies that the leading leg damping must be zero at the start of the cycle. A control force in a feedback manner is applied by the pedestrian to compensate for energy dissipated with the system damping in walking and to regulate the walking behavior of the pedestrian. As mentioned previously, a control force is only one possible candidate, among many ways, to provide the additional energy. Many tests have shown that the leg stiffness can be adjusted to enable similar or uniform gait cycle in the process [39–42]. It should be noted that real humans might not adjust the leg properties in this way. However, how the leg stiffness is adjusted is still not clear and not reported. The dynamic behavior of COM is noted to be uniform in each step cycle with the horizontal control force applied by pedestrian. Nevertheless, the model in its current form can show how humans might interact with the bridge. The modeling procedure has more general application when the gait conditions vary dynamically, the leg parameters are also varied actively. Both previously published mechanisms and the present actuator mechanism represent the potential complexities of human walking, and they have been successful in the sense to illustrate that stable walking under lateral and vertical perturbation is possible using a simple control law. Such simple models may break down in regimes of high frequency or high amplitude perturbation whereby an extension of the present model could possibly be viable for further study.

The response of the combined HSI system is noted to be larger than the corresponding response without HSI (human loading is treated as external force only). This is derived from the case when a single pedestrian walks on the beam, and it seems to contradict with results published for a real footbridge [34,36]. It should be noted that the vibration responses induced by a crowd are influenced by the correlation between individual pedestrians. Therefore, the response induced by multiple persons in the HSI system would be smaller than the corresponding response without HSI (human loading is treated as external force only).

This paper shows that single pedestrian has little influence on the dynamic properties of the structure. Hence, the mass of single pedestrian would not contribute significantly to the overall properties of the structure when analyzing the dynamic behavior of footbridges. The effect of a crowd on the dynamic behavior of structure may be larger than that from a single pedestrian with a larger modal mass ratio of the pedestrians to the structure. However, it should be noted that the crowd cannot be simply treated as a lump mass or a distribution of the mass of pedestrians on the structure. Therefore, the effect of a crowd on the dynamic behavior of a structure has to be studied in future.

6. Conclusions

The proposed interaction model provides a means to understand how the structural vibration can influence the walking behavior of a pedestrian and how a pedestrian can respond to the vibration level of structure to maintain a steady gait, which in turn affects the dynamic responses of the structure. For slender structures subjected to a pedestrian, the leg stiffness has significant effect on the dynamic response of structure. The peak accelerations of footbridge decreases when the attach angle increases, while the damping ratio slightly affects it. The dynamic interaction will increase at a larger vibration level of structure. More external energy must be input to maintain the steady gait and uniform dynamic behavior of the COM in each step cycle. The proposed model could be used for the dynamic analysis of slender structures, such as a slender footbridge, under possibly, crowd-induced excitation.

Acknowledgments

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Appendix A. Equations of motion of the human–structure interaction system

The kinetic energy T, the potential energy V and the variation of virtual work done by non-conservative force are

$$T=\frac{1}{2}m_h\dot{z}^2+\frac{1}{2}m_h\dot{u}^2+\frac{1}{2}\int_0^L m\left(\sum_{i=1}^{\infty}\dot{Y}_i(t)\phi_i(x)\right)^2\mathrm{d}x \tag{A-1a}$$

$$V=\frac{1}{2}k_{\text{leg}}\left(\sqrt{\left(\sum_{i=1}^{N}d_i-u\right)^2+\left(z-y|_N\right)^2}-l_0\right)^2+\frac{1}{2}k_{\text{leg}}\left(\sqrt{\left(u-\sum_{i=1}^{N-1}d_i\right)^2+\left(z-y|_{N-1}\right)^2}-l_0\right)^2+m_hgz$$
$$+\frac{1}{2}\int_0^L EI\sum_{i=1}^{\infty}Y_i(t)^2\left(\frac{\mathrm{d}^2(\phi_i(x))}{\mathrm{d}x^2}\right)^2\mathrm{d}x \tag{A-1b}$$

$$\begin{aligned}\delta W&=-\alpha(t)c_{\text{leg}}v_l\delta(\Delta L_l)-(1-\alpha(t))c_{\text{leg}}v_t\delta(\Delta L_t)-\int_0^L c_s\dot{y}''\delta y''\mathrm{d}x\\&=Q_1\delta q_1+Q_2\delta q_2+\cdots+Q_n\delta q_n+Q_{n+1}\delta z+Q_{n+2}\delta u\end{aligned} \tag{A-1c}$$

The variation of displacement of two legs can be obtained as

$$\delta(\Delta L_l) = -\cos\theta_l \delta u + \sin\theta_l \left(\delta z - \sum_{i=1}^{N} \phi_i(N)\delta Y_i \right) \tag{A-2a}$$

$$\delta(\Delta L_t) = \cos\theta_t \delta u + \sin\theta_t \left(\delta z - \sum_{i=1}^{N} \phi_i(N-1)\delta Y_i \right) \tag{A-2b}$$

The variation of curvature of the beam can be obtained as

$$\delta y'' = \sum_{i=1}^{\infty} \frac{\mathrm{d}^2(\phi_i(x))}{\mathrm{d}x^2} \delta Y_i(t) \tag{A-3}$$

Substituting Eq. (A-2) and (A-3) into Eq. (A-1) gives

$$\begin{aligned}\delta W = \sum_{i=1}^{\infty} \left(\alpha(t)c_{\text{leg}}v_l\phi_i(N)\sin\theta_l + (1-\alpha(t))c_{\text{leg}}v_t\phi_i(N-1)\sin\theta_t - \int_0^L c_s \left(\sum_{i=1}^{\infty} \dot{Y}_i(t) \frac{\mathrm{d}^2(\phi_i(x))}{\mathrm{d}x^2} \right) \frac{\mathrm{d}^2(\phi_i(x))}{\mathrm{d}x^2} dx \right) \delta Y_i(t) \\ +(-\alpha(t)c_{\text{leg}}v_l\sin\theta_l - (1-\alpha(t))c_{\text{leg}}v_t\sin\theta_t)\delta z + (\alpha(t)c_{\text{leg}}v_l\cos\theta_l - (1-\alpha(t))c_{\text{leg}}v_t\cos\theta_t)\delta u\end{aligned} \tag{A-4}$$

According to Eq. (A-1)

$$Q_i = \alpha(t)c_{\text{leg}}v_l\phi_i(N)\sin\theta_l + (1-\alpha(t))c_{\text{leg}}v_t\phi_i(N-1)\sin\theta_t - \int_0^L c_s \left(\sum_{i=1}^{\infty} \dot{Y}_i(t) \frac{\mathrm{d}^2(\phi_i(x))}{\mathrm{d}x^2} \right) \frac{\mathrm{d}^2(\phi_i(x))}{\mathrm{d}x^2} \mathrm{d}x \tag{A-5a}$$

$$Q_{N+1} = -\alpha(t)c_{\text{leg}}v_l\sin\theta_l - (1-\alpha(t))c_{\text{leg}}v_t\sin\theta_t \tag{A-5b}$$

$$Q_{N+2} = \alpha(t)c_{\text{leg}}v_l\cos\theta_l - (1-\alpha(t))c_{\text{leg}}v_t\cos\theta_t \tag{A-5c}$$

The Lagrangian equations of motion of human–structure interaction system are given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_i}\right) + \frac{\partial V}{\partial q_i} = Q_i \quad (i = 1,2,\ldots,n) \tag{A-6a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{z}}\right) + \frac{\partial V}{\partial z} = Q_{n+1} \tag{A-6b}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{u}}\right) + \frac{\partial V}{\partial u} = Q_{n+2} \tag{A-6c}$$

Substituting Eq. (A-1) and (A-5) into Eq. (A-6) results in the system equations-of-motion

$$\begin{aligned} M_n\ddot{Y}_n + 2\xi_n\omega_n M_n\dot{Y}_n + \omega_n^2 M_n Y_n + \alpha(t)c_1\phi_n(N)\left(\sum_{i=1}^{\infty}\dot{Y}_i(t)\phi_i(N)\right) \\ + \Big(1-\alpha(t)\Big)c_2\phi_n(N-1)\left(\sum_{i=1}^{\infty}\dot{Y}_i(t)\phi_i(N-1)\right) - \Big(1-\alpha(t)\Big)c_2\phi_n(N-1)\dot{z} \\ -\alpha(t)c_1\phi_n(N)\dot{z} + \alpha(t)c_3\phi_n(N)\dot{u} - (1-\alpha(t))c_4\phi_n(N-1)\dot{u} - k_{t,v}\phi_n(N-1)z - k_{l,v}\phi_n(N)z = 0 \end{aligned} \tag{A-7a}$$

$$\begin{aligned} m_h\ddot{z} + \Big(\alpha(t)c_1 + (1-\alpha(t))c_2\Big)\dot{z} + \Big((1-\alpha(t))c_4 - \alpha(t)c_3\Big)\dot{u} - \alpha(t)c_1\left(\sum_{i=1}^{\infty}\dot{Y}_i(t)\phi_i(N)\right) \\ - \Big(1-\alpha(t)\Big)c_2\left(\sum_{i=1}^{\infty}\dot{Y}_i(t)\phi_i(N-1)\right) + k_{l,v}z + k_{t,v}z + m_h g = 0 \end{aligned} \tag{A-7b}$$

$$\begin{aligned} m_h\ddot{u} + ((1-\alpha(t))c_4 - \alpha(t)c_3)\dot{z} + \big(c_{leg} - \alpha(t)c_1 - (1-\alpha(t))c_2\big)\dot{u} + \alpha(t)c_3\left(\sum_{i=1}^{\infty}\dot{Y}_i(t)\phi_i(N)\right) \\ - \Big(1-\alpha(t)\Big)c_4\left(\sum_{i=1}^{\infty}\dot{Y}_i(t)\phi_i(N-1)\right) + k_{t,h}u - k_{l,h}u = 0 \end{aligned} \tag{A-7c}$$

The equations in a matrix form can be expressed as follows:

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F} \tag{A-8}$$

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