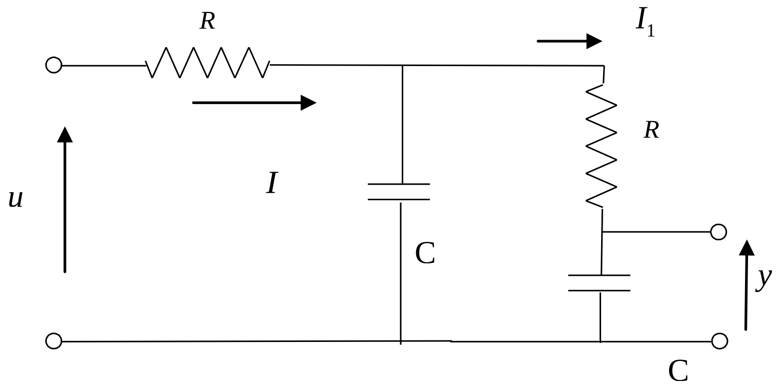
Tracce delle soluzioni

1.

1)



$$U = ZI$$

$$Z = R + \frac{RCs+1}{Cs(RCs+2)}$$

$$Y = \frac{1}{Cs}\cdot I_1 = \frac{1}{Cs}\cdot I\,\frac{\frac{1}{Cs}}{\frac{1}{Cs}+R+\frac{1}{Cs}} = \frac{1}{Cs}\cdot\frac{U}{Z}\,\frac{\frac{1}{Cs}}{\frac{1}{Cs}+R+\frac{1}{Cs}}$$

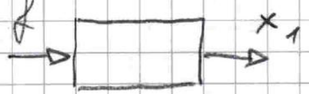
$$G(s) := \frac{Y}{U} = \frac{1}{T^2s^2+3Ts+1}$$

2) I poli del sistema sono $\frac{-3-\sqrt{5}}{2T}$, $\frac{-3+\sqrt{5}}{2T}$ e quindi i corrispondenti modi sono $\exp\left(\frac{-3-\sqrt{5}}{2T}\cdot t\right)$, $\exp\left(\frac{-3+\sqrt{5}}{2T}\cdot t\right)$.

3) L’equazione differenziale associata a G(s) è

$$T^2D^2y + 3TDy + y = u$$

2.



$$\begin{cases} mD^2x_1 = f - k(x_1 - x_2) \quad \Rightarrow \quad kx_2 = mD^2x_1 + kx_1 - f \\ mD^2x_2 = k(x_1 - x_2) - k(x_2 - x_3) \\ mD^2x_3 = k(x_2 - x_3) \end{cases}$$

$$\begin{cases} kx_3 = mD^2x_2 + 2kx_2 - kx_1 \\ mD^2x_3 + kx_3 = kx_2 \end{cases} \qquad (mD^2+k)x_3 = kx_2$$

$$(mD^2+k)(mD^2x_2 + 2kx_2 - kx_1) = k^2x_2$$

$$(mD^2+k)(mD^2+2k)x_2 - k(mD^2+k)x_1 = k^2x_2$$

$$(m^2D^4 + 2kmD^2 + kmD^2 + 2k^2 - k^2)x_2 = k(mD^2+k)x_1$$

$$(m^2D^4 + 3kmD^2 + k^2)x_2 = k(mD^2+k)x_1$$

$$(m^2D^4 + 3kmD^2 + k^2)(mD^2x_1 + kx_1 - f) = k^2(mD^2+k)x_1$$

$$(m^2D^4 + 3kmD^2 + \cancel{k^2})(mD^2+k)x_1 \cancel{- k^2(mD^2+k)x_1} = (m^2D^4 + 3kmD^2 + k^2)f$$

$$(m^2D^4 + 3kmD^2)(mD^2+k)x_1 = (m^2D^4 + 3kmD^2 + k^2)f$$

$$\text{f.d.t.}\ G(s) = \frac{m^2s^4 + 3kms^2 + k^2}{(m^2s^4 + 3kms^2)(ms^2+k)} = \frac{m^2s^4 + 3kms^2 + k^2}{m \cdot s^2(ms^2+3k)(ms^2+k)}$$

3.

Le dimostrazioni della prima e terza proprietà sono riportate sulle dispense del corso. La seconda dimostrazione richiesta si deduce facilmente a partire dalla prima proprietà.

4.

Vedi le dispense del corso.

5.

<u>1° metodo</u>: $u(t) = 1(t) - 1(t-1)$

$$V(s) = \frac{1}{s} - \frac{1}{s}e^{-s} \qquad Y(s) = G(s)U(s) = \frac{1}{s(s+1)^2} - \frac{1}{s(s+1)^2}e^{-s}$$

$$y(t) = \mathcal{L}^{-1}\left[\frac{1}{s(s+1)^2}\right] - \mathcal{L}^{-1}\left[\frac{1}{s(s+1)^2}\cdot e^{-s}\right]$$

$$\frac{1}{s(s+1)^2} = \frac{K_1}{s} + \frac{K_{21}}{(s+1)^2} + \frac{K_{12}}{s+1}$$

$$K_1 = \left.\frac{1}{(s+1)^2}\right|_{s=0} = 1 \qquad K_{21} = \left.\frac{1}{s}\right|_{s=-1} = -1 \qquad K_1 + K_{12} = 0$$

$$K_{12} = -1$$

$$\mathcal{L}^{-1}\left[\frac{1}{s(s+1)^2}e^{-s}\right] = \mathcal{L}^{-1}\left[\frac{1}{s(s+1)^2}\right](t-1)\cdot 1(t-1)$$

$$y(t) = 1 - te^{-t} - e^{-t} - \left[1 - (t-1)e^{-(t-1)} - e^{-(t-1)}\right]\cdot 1(t-1)$$

Per $t \in [0,1)$ $y(t) = 1 - te^{-t} - e^{-t}$

Per $t \in [1,+\infty)$ $y(t) = \cancel{1} - te^{-t} - e^{-t} \cancel{-1} + (t-\cancel{1})e^{-t}\cdot e \cancel{+ e^{-t}\cdot e}$

$$y(t) = -e^{-t} + (e-1)te^{-t}$$

<u>2° metodo</u>: Per $t \in [0,1)$ vale $u(t) = 1(t)$

e quindi $y(t) = 1 - te^{-t} - e^{-t}$

Per $t \in [1,+\infty)$ $y(t) = c_1 e^{-t} + c_2 te^{-t}$

$u(t)$ è discontinua su $(-\infty,+\infty)$, quindi $y(t) \in C^{\varrho-1}(\mathbb{R})$

ovvero $y(t) \in C^1(\mathbb{R})$

$$\begin{cases} y(1-) = y(1+) \\ Dy(1-) = Dy(1+) \end{cases}$$

Per $t \in [0,1)$ $\quad y(t) = 1 - te^{-t} - e^{-t}$

$$Dy(t) = -e^{-t} + te^{-t} + e^{-t}$$

Per $t \in [1,+\infty)$ $\quad y(t) = c_1 e^{-t} + c_2 t e^{-t}$

$$Dy(t) = -c_1 e^{-t} + c_2 \left(e^{-t} - te^{-t}\right)$$

$$y(1-) = 1 - e^{-1} - e^{-1} = 1 - 2e^{-1}$$

$$Dy(1-) = -\cancel{e^{-1}} + \cancel{e^{-1}} + e^{-1} = e^{-1}$$

$$\begin{cases} c_1 e^{-1} + c_2 e^{-1} = 1 - 2e^{-1} \\ -c_1 e^{-1} + c_2 \left(\cancel{e^{-1} - e^{-1}}\right) = e^{-1} \end{cases} \Rightarrow c_1 = -1$$

$$-e^{-1} + c_2 e^{-1} = 1 - 2e^{-1} \qquad -1 + c_2 = e - 2$$

$$\Rightarrow c_2 = e - 1$$

$$\boxed{y(t) = -e^{-t} + (e-1)\,te^{-t}}$$

6.

Soluzione

Metodo dei modi: $y(t) = c_1 e^{-t} + c_2 e^{-2t}$

$Dy(t) = -c_1 e^{-t} - 2c_2 e^{-2t}$

$$\begin{cases} c_1 + c_2 = 0 \\ -c_1 - 2c_2 = 1 \end{cases} \qquad \begin{array}{l} c_2 = -c_1 \\ -c_1 - 2(-c_1) = 1 \end{array} \qquad \begin{array}{l} c_2 = -1 \\ c_1 = 1 \end{array}$$

Quindi $\boxed{y(t) = e^{-t} - e^{-2t}}$

Metodo dell'eq. differenziale: $G(s) = \dfrac{s+3}{s^2+3s+2}$

$$D^2 y(t) + 3Dy(t) + 2y(t) = Du(t) + 3u(t)$$

Applichiamo la Trasformata di Laplace

$$s^2 Y - y(0+)s - Dy(0+) + 3(sY - y(0+)) + 2Y = 0$$

$$(s^2 + 3s + 2)Y - 1 = 0 \qquad Y = \frac{1}{s^2+3s+2}$$

$$Y(s) = \frac{1}{(s+1)(s+2)} = \frac{K_1}{s+1} + \frac{K_2}{s+2} = \frac{1}{s+1} - \frac{1}{s+2}$$

$$K_1 = \left.\frac{1}{s+2}\right|_{s=-1} = 1 \qquad K_1 + K_2 = 0 \qquad K_2 = -1$$

$$\boxed{y(t) = \mathcal{L}^{-1}[Y(s)] = e^{-t} - e^{-2t}}$$