Tracce delle soluzioni

1.
Vedi dispense del corso

2.

$$\begin{cases} m D^2 x_1 = f - k x_1 - b D x_1 + k(x_2 - x_1) \\ m D^2 x_2 = -k(x_2 - x_1) \end{cases}$$

$$\begin{cases} m D^2 x_1 = f - 2k x_1 - b D x_1 + k x_2 \\ m D^2 x_2 = -k x_2 + k x_1 \end{cases}$$

$$\begin{cases} k x_2 = m D^2 x_1 + b D x_1 + 2k x_1 - f \\ m D^2 x_2 + k x_2 = k x_1 \end{cases}$$

$$\begin{array}{l} (mD^2+k) \\ k \end{array} \begin{cases} k x_2 = m D^2 x_1 + b D x_1 + 2k x_1 - f \\ (m D^2 + k) x_2 = k x_1 \end{cases}$$

$$(mD^2+k)(mD^2+bD+2k)x_1 - (mD^2+k)f = k^2 x_1$$

$$(m^2 D^4 + m b D^3 + 2km D^2 + km D^2 + kbD + \cancel{2}k^2)x_1 - \cancel{k^2 x_1} = \\ = (mD^2+k)f$$

eq. diff. $m^2 D^4 x_1 + m b D^3 x_1 + 3km D^2 x_1 + kb D x_1 + k^2 x_1 = m D^2 f + k f$

f.d.t. $G(s) = \dfrac{m s^2 + k}{m^2 s^4 + m b s^3 + 3km s^2 + kb s + k^2}$

3. Il guadagno statico è G(0)=1/k.

Gli zeri sono $z_{1,2} = \pm j\sqrt{\frac{k}{m}}$

3.

⑤ a. $\mathcal{L}[g_s(t)] = G(s)\cdot\frac{1}{s}$

$$\mathcal{L}[g_s(t)] = \frac{1}{2}\cdot\frac{1}{s} + \frac{1}{s+1} - \frac{3}{2}\cdot\frac{1}{s+2}$$

$$G(s) = \frac{1}{2} + \frac{s}{s+1} - \frac{3}{2}\cdot\frac{s}{s+2} = \frac{2s+1}{(s+1)(s+2)}$$

$$G(s) = 2\cdot\frac{s+\frac{1}{2}}{(s+1)(s+2)}$$

b. $u(t) = 1(t) + t\cdot 1(t)$, $U(s) = \frac{1}{s} + \frac{1}{s^2}$

$$Y(s) = \frac{2s+1}{(s+1)(s+2)}\cdot\left(\frac{1}{s}+\frac{1}{s^2}\right) = \frac{2s+1}{\cancel{(s+1)}(s+2)}\cdot\frac{\cancel{s+1}}{s^2}$$

$$= \frac{2s+1}{s^2(s+2)} = \frac{K_{11}}{s^2} + \frac{K_{12}}{s} + \frac{K_2}{s+2}$$

$$K_{11} = \left.\frac{2s+1}{s+2}\right|_{s=0} = \frac{1}{2} \qquad K_2 = \left.\frac{2s+1}{s^2}\right|_{s=-2} = -\frac{3}{4}$$

$$K_{12} + K_2 = 0 \Rightarrow K_{12} = \frac{3}{4}$$

$$Y(s) = \frac{1}{2}\cdot\frac{1}{s^2} + \frac{3}{4}\cdot\frac{1}{s} - \frac{3}{4}\cdot\frac{1}{s+2}$$

$$y(t) = \mathcal{L}^{-1}[Y(s)] = \left(\frac{1}{2}t + \frac{3}{4} - \frac{3}{4}e^{-2t}\right)\cdot 1(t)$$

4.

$$L(j\omega) = 8\,\frac{1-j\omega}{(j\omega+2)^3}$$

$$|L(j\omega)| = 8\,\frac{\sqrt{1+\omega^2}}{(\omega^2+4)^{3/2}}$$

$$\arg G(j\omega) = -\operatorname{arctg}\omega - 3\operatorname{arctg}\frac{\omega}{2}$$

$$1+\alpha\, 8\,\frac{1-s}{(s+2)^3} = 0$$ abbia radici puramente immaginarie

$$1+\alpha L(\pm j\omega) = 0 \qquad L(\pm j\omega) = -\frac{1}{\alpha}$$

e quindi l'intersezione avviene in $-\frac{1}{\alpha}$

Si pone $k \triangleq 8\alpha$

$$1+k\,\frac{1-s}{(s+2)^3} = 0 \qquad (s+2)^3 + k(1-s) = 0$$

$$s^3+6s^2+12s+8+k-ks = 0$$

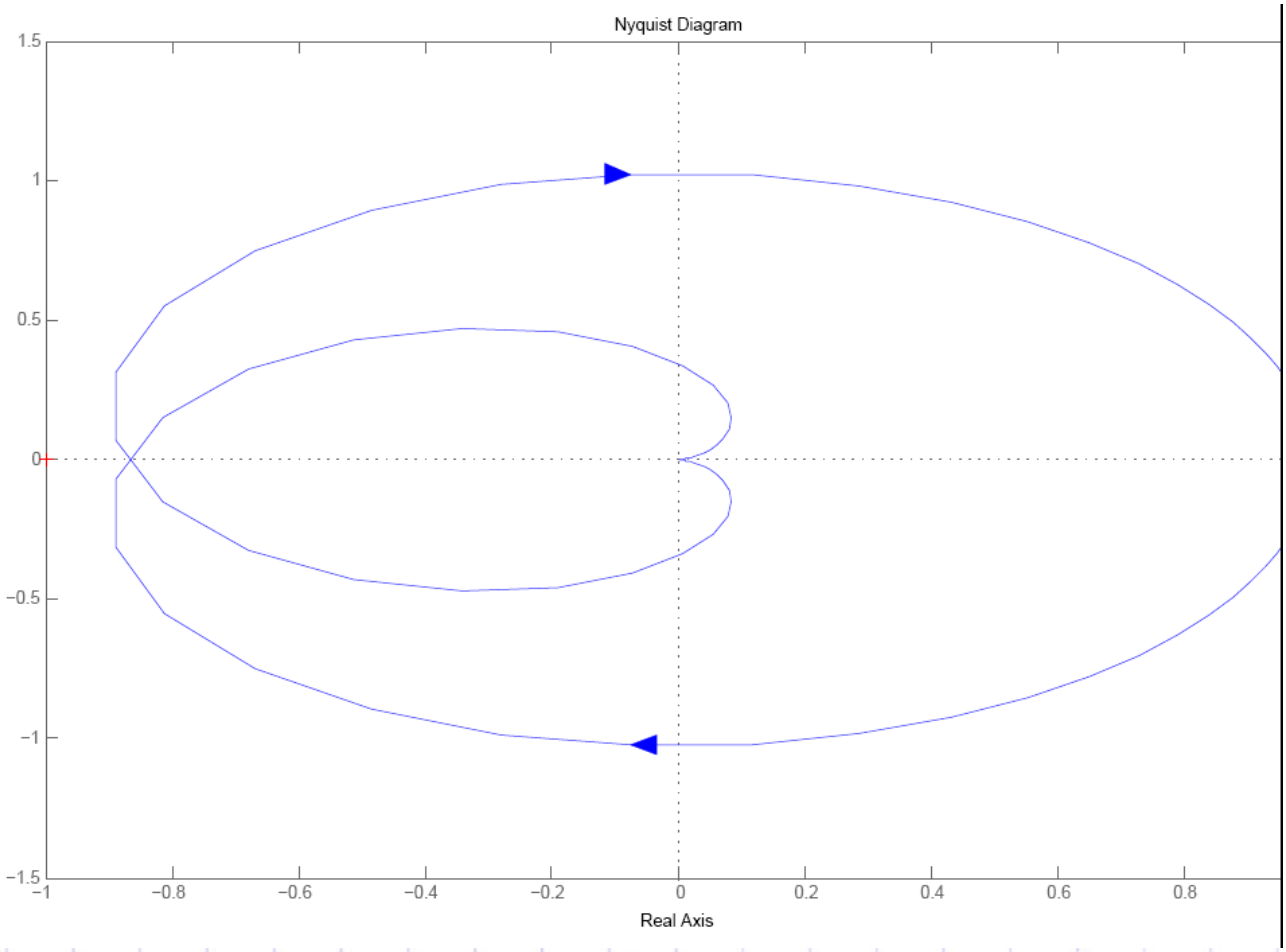
$$s^3+6s^2+(12-k)s+8+k = 0$$

3	1	$12-k$	0
2	6	$8+k$	0
1	#		

$$\# = 6(12-k)-8-k =$$
$$= 72-6k-8-k =$$
$$= -7k+64 = 0$$
$$7k = 64 \qquad k = \frac{64}{7}$$
$$8\alpha = \frac{64}{7} \Rightarrow \alpha = \frac{8}{7}$$

L'intersezione avviene in $-\frac{7}{8} \cong -0{,}875$

Il diagramma polare è quindi quello di figura:



Per il C. di Ny. il sistema retr. è asint. stabile.

Il margine di ampiezza è

$$M_A = \frac{1}{\left|-\frac{7}{8}\right|} = \frac{8}{7} \simeq 1,14$$

5.
Vedi dispense dell'insegnamento

6.

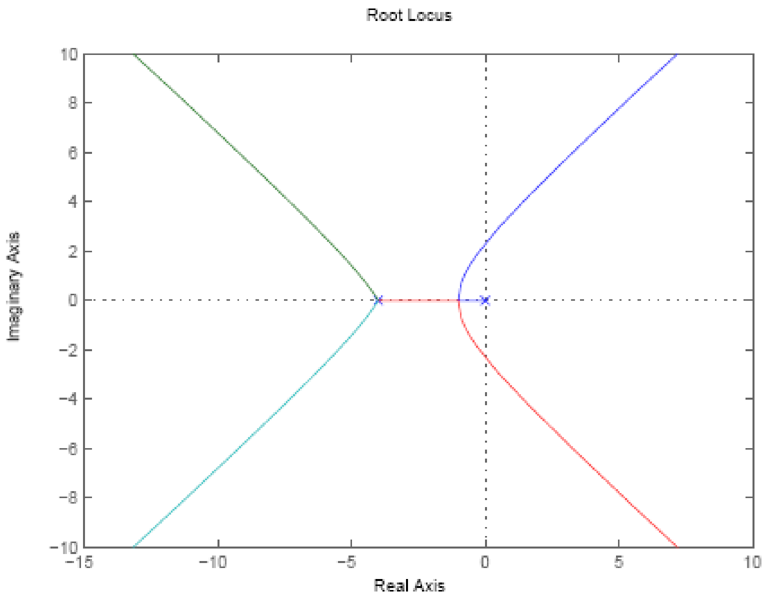
a) Intersezione degli asintoti:

$$\nabla_a = \frac{-4 \cdot 3 + 0}{4} = -3$$

Angoli deli asintoti: $+45°,\ -45°,\ +135°, -135°$
Angolo di partenza dal polo in 0: $+180°$
Angolo di partenza dal polo triplo in -4: $0°,\ +120°,\ -120°$
Radici doppie:

$$\frac{3}{s+4} + \frac{1}{s} = 0 \ \Rightarrow\ 4\,s + 4 = 0 \ \Rightarrow\ s = -1$$

Luogo delle radici:



b) Equazione caratteristica:

$$1 + K\,\frac{1}{s\,(s+4)^3} = 0 \ \Rightarrow\ s^4 + 12\,s^3 + 48\,s^2 + 64\,s + K = 0$$

Criterio di Routh:

$$\begin{array}{c|cccc}
\mathbf{4} & 1 & 48 & K & 0 \\
\mathbf{3} & 3 & 16 & 0 & 0 \\
\mathbf{2} & 128 & 3\,K & 0 & \\
\mathbf{1} & 128 \cdot 16 - 9\,K & 0 & & \\
\mathbf{0} & 3\,K & 0 & &
\end{array}$$

Condizione per la stabilità asintotica:

$$\begin{cases} 2048 - 9\,K > 0 \\ 3\,K > 0 \end{cases} \quad \Rightarrow\ K \in (0, 227.\bar{5})$$

Calcolo delle intersezoni:

$$128\, s^2 + 3 \cdot 227.\bar{5} = 0 \;\Rightarrow\; s = \pm j \sqrt{\frac{16}{3}} \simeq \pm j\, 2.309$$

c) Grado di stabilità massimo nella radice doppia $s = -1$:

$$1 + K^* \left.\frac{1}{s\,(s+4)^3}\right|_{s=-1} = 0 \;\Rightarrow\; K^* = 27$$

7.

$$L(s) = C(s)\,P(s) = k\,\frac{1+\tau s}{1+\alpha\tau s}\cdot\frac{8}{(s+2)^4}$$

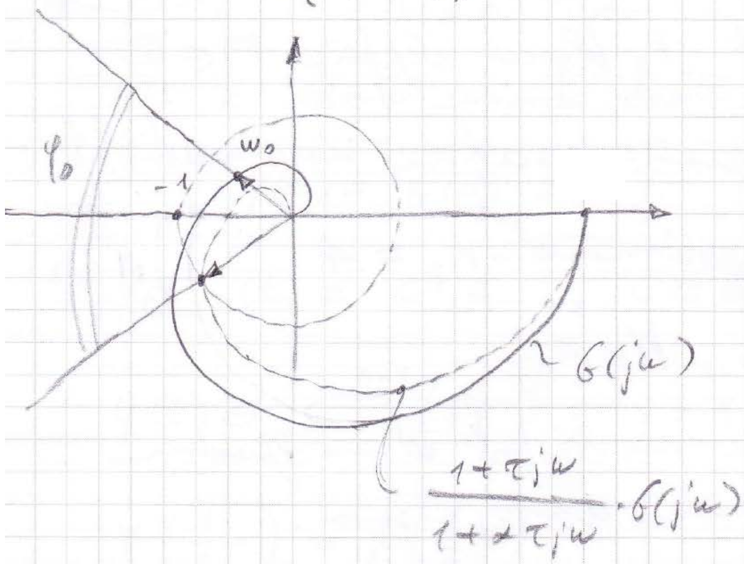
$$L(0) = k\cdot\frac{\cancel{8}\,1}{\cancel{16}\,2} = \frac{k}{2} \qquad K_p = L(0)$$

$$3{,}5 = \frac{k}{2} \Rightarrow k = 7$$

$$L(s) = \frac{1+\tau s}{1+\alpha\tau s}\cdot\frac{56}{(s+2)^4} \stackrel{\Delta}{=} \frac{1+\tau s}{1+\alpha\tau s}\cdot G(s)$$

$$G(s) = \frac{56}{(s+2)^4} \qquad G(j\omega) = \frac{56}{(j\omega+2)^4}$$

$$|G(j\omega)| = \frac{56}{(\omega^2+4)^2} \qquad \arg G(j\omega) = -4\,\mathrm{arctg}\,\frac{\omega}{2}$$



$$\arg G(j\omega) = -\pi$$

$$-4\,\mathrm{arctg}\,\frac{\omega}{2} = -\pi$$

$$\omega_p = 2 \text{ rad/sec}$$

$$|G(j\omega_p)| = 0{,}8750$$

$$G(j\omega_p) = -0.8750$$

Scelgo $\omega_0 = 2{,}5$ rad/sec

$$|G(j\omega_0)| = 0{,}5330 \qquad \varphi_0 = -\arg G(j\omega_0) - \pi + M_F =$$

$$= 4\,\mathrm{arctg}\,\frac{2.5}{2} - \pi + \frac{30}{180}\cdot\pi = 0{,}9662 \text{ rad/sec}$$

$$\cos\varphi_0 > |G(j\omega_0)| \qquad 0{,}5684 > 0{,}5330 \quad \text{ok!}$$

$$M \triangleq \frac{1}{|G(j\omega_0)|} = 1,8761 \qquad \varphi \triangleq \varphi_0$$

$$\tau = \frac{M - \cos\varphi}{\omega_0 \sin\varphi} = \frac{1,8761 - 0,5684}{2,5 \cdot 0,8227} = 0,636 \text{ sec}$$

OK

$$\alpha = \frac{M\cos\varphi - 1}{M(M - \cos\varphi)} = \frac{1,8761 \cdot 0,5684 - 1}{1,8761 \cdot (1,8761 - 0,5684)} = 0,0271$$

Determinazione di F:

$$F \cdot \frac{L(0)}{1 + L(0)} = 1 \qquad F \cdot \frac{3,5}{1 + 3,5} = 1$$

$$F = \frac{1 + 3,5}{3,5} = \frac{4,5}{3,5} = 1,2857$$

8.

Funzione di trasferimento zeta

$$H(z) = \frac{z+1}{z^2+1}$$

$$U(z) = \frac{z}{z-1}, \quad Y(z) = H(z)\,U(z)$$

$$Y(z) = \frac{z+1}{z^2+1} \cdot \frac{z}{z-1} = z \cdot \frac{z+1}{(z-1)(z^2+1)} =: z \cdot Y_1(z)$$

$$Y_1(z) = \frac{z+1}{(z-1)(z^2+1)} = \frac{z+1}{(z-1)(z-j)(z+j)}$$

$$= \frac{K_1}{z-1} + \frac{K_2}{z-j} + \frac{\overline{K_2}}{z+j}$$

$$K_1 = \left.\frac{z+1}{z^2+1}\right|_{z=1} = 1 \qquad K_2 = \left.\frac{z+1}{(z-1)(z+j)}\right|_{z=j} = -\frac{1}{2}$$

$$\overline{K_2} = -\frac{1}{2}$$

$$Y(z) = \frac{z}{z-1} - \frac{1}{2} \cdot \frac{z}{z-j} - \frac{1}{2} \cdot \frac{z}{z+j}$$

Per $k \geq 0$

$$y(k) = 1 + 2 \cdot |j|^k \left\{ \left(-\frac{1}{2}\right) \cdot \cos[\arg(j) \cdot k] - 0 \cdot \sin[\arg(j) \cdot k] \right\}$$

$$= 1 - \cos\left(\frac{\pi}{2} \cdot k\right)$$