Esami di Nyquist 4

$$P(s) = \frac{10(1-s)^2}{s(s+1)^3}$$

$$P(j\omega) = \frac{10(1-j\omega)^2}{j\omega(j\omega+1)^3}$$

$$|P(j\omega)| = \frac{10\cdot(1+\omega^2)}{\omega\cdot(\omega^2+1)^{\frac{3}{2}}} = \frac{10}{\omega\cdot(\omega^2+1)^{\frac{3}{2}-1}} = \frac{10}{\omega\cdot(\omega^2+1)^{\frac{3-2}{2}}} = \frac{10}{\omega\cdot(\omega^2+1)^{1/2}}$$

$$\angle P(j\omega) = -2\arctan(\omega) - \frac{\pi}{2} - 3\arctan(\omega) = -\frac{\pi}{2} - 5\arctan(\omega)$$

- Dato che c'è un polo in zero:

L'ascissa dell'asintoto verticale $\sigma_a = 10\cdot[(-1-1)-(1+1+1)] = -50$

- Calcolo i limiti:

$$\lim_{\omega\to0^+}|P(j\omega)| = +\infty \qquad \lim_{\omega\to0^+}\angle P(j\omega) = -\frac{\pi}{2}$$

$$\lim_{\omega\to+\infty}|P(j\omega)| = 0 \qquad \lim_{\omega\to+\infty}\angle P(j\omega) = -\frac{\pi}{2} - \frac{5\pi}{2} = \frac{-\pi 6}{2} = -3\pi$$

- Cerco le intersezioni:

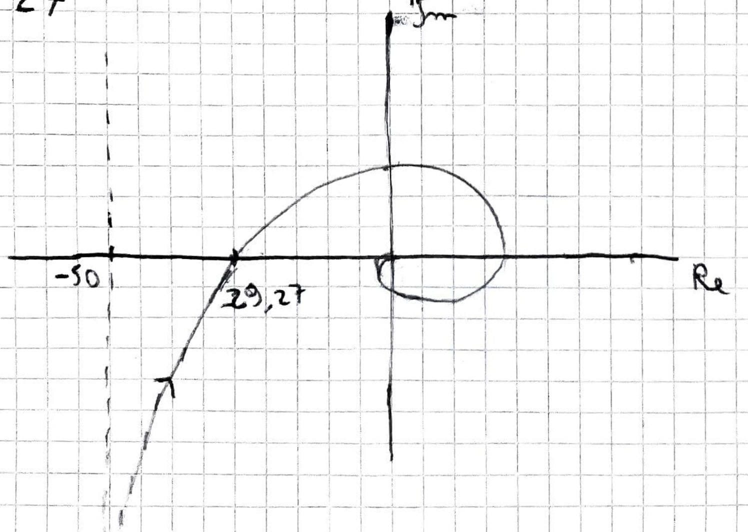
$$-\frac{\pi}{2} - 5\arctan(\omega) = -\pi \qquad 5\arctan(\omega) - \frac{\pi}{2} = 0$$

$\omega_p = 0,3249$ rad/s

$$-\frac{\pi}{2} - 5\arctan(\omega) = 0$$

$\omega_0 = 0$ rad/s

$P(j\omega_p) = -29,27$



Esami Nyquist 9

$$L(s) = \frac{s+2}{s^2(s+1)} \qquad L(j\omega) = \frac{j\omega + 2}{(j\omega)^2 (j\omega+1)}$$

$$|L(j\omega)| = \frac{\sqrt{\omega^2+4}}{\omega^2 \sqrt{\omega^2+1}} \qquad \arg L(j\omega) = -2\frac{\pi}{2} - \operatorname{arctg}\left(\frac{\omega}{1}\right) + \operatorname{arctg}\left(\frac{\omega}{2}\right)$$

- **Calcolo i limiti:**

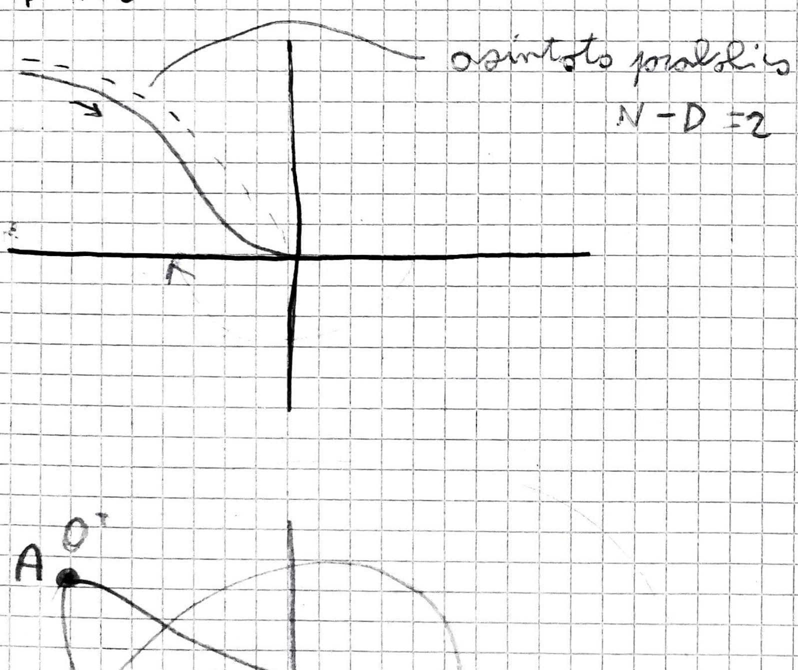
$$\lim_{\omega \to 0^+} |L(j\omega)| = +\infty \qquad \lim_{\omega \to 0^+} \operatorname{arctg} L(j\omega) = -\pi$$

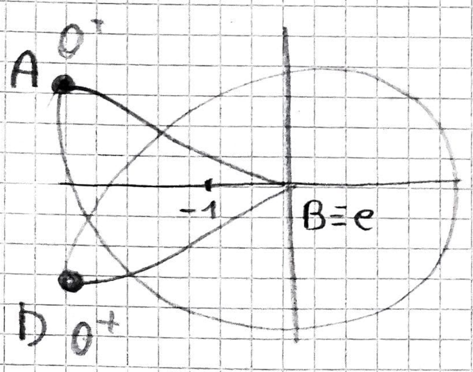
$$\lim_{\omega \to +\infty} |L(j\omega)| = 0 \qquad \lim_{\omega \to +\infty} \operatorname{arctg} L(j\omega) = -\pi - \frac{\pi}{2} + \frac{\pi}{2} = -\pi$$

- **Calcolo intersezione con l'asse reale:**

$\arg L(j\omega) = -\pi$

$\omega_p = NO$





Esami Nyquist 8

$$L(s) = 10 \cdot \frac{(s+1)(s+2)}{s^3} = 10 \cdot \frac{(j\omega+1)(j\omega+2)}{(j\omega)^3} = \frac{20(j\omega+1)(\frac{j\omega}{2}+1)}{(j\omega)^3}$$

$$|L(j\omega)| = \frac{20 \cdot \sqrt{\omega^2+1} \cdot \sqrt{\frac{\omega^2}{4}+1}}{\omega^3} \; ; \; \angle L(j\omega) = -\frac{3}{2}\pi + \operatorname{arctg}(\omega) + \operatorname{arctg}\left(\frac{\omega}{2}\right)$$

- Calcolo i limiti:

$$\lim_{\omega \to 0^+} |L(j\omega)| = +\infty \qquad \lim_{\omega \to 0^+} \angle L(j\omega) = -\frac{3}{2}\pi$$

$$\lim_{\omega \to +\infty} |L(j\omega)| = 0 \qquad \lim_{\omega \to +\infty} \angle L(j\omega) = -\frac{3}{2}\pi + \pi = \frac{-3\pi + 2\pi}{2} = -\frac{\pi}{2}$$

- Calcolo l'intersezione con l'asse reale:

Uso questa volta il criterio di Routh

$$1 + k\frac{(s+1)(s+2)}{s^3} = 0 \; ; \; s^3 + Ks^2 + 3Ks + 2K = 0$$

3	1	3K	0
2	K	2K	0
1	$\frac{3K^2 - 2K}{1}$		0

$K(3K-2) = 0$

$K = \frac{2}{3}$

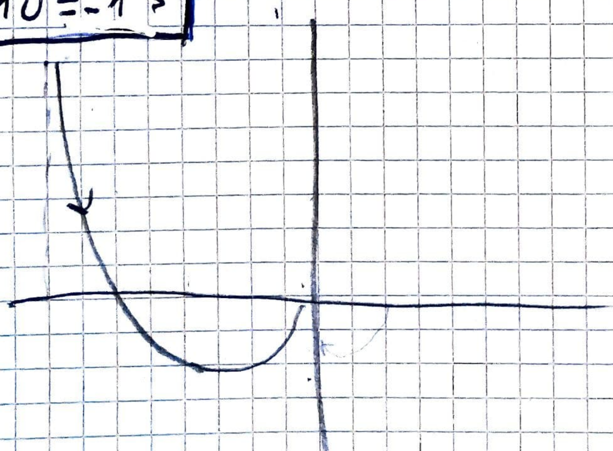
tutte permanenze di segno, radici puramente immaginarie

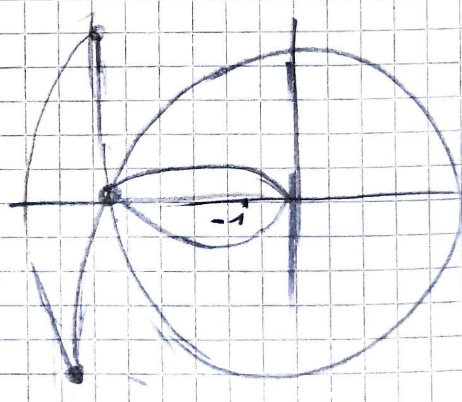
- Con la calc usando il metodo classico

$\omega_p = 1{,}414$

$L(j\omega_p) = -15$ dove interseca

$$\boxed{-k^{-1} \cdot 10 = -15}$$





-1 non è circondato nel tratto => asintoticamente stabile

Esami Nyquist 7

$$L(s) = \frac{10 s^2}{(s^3-8)(s-1)}$$

$$L(j\omega) = \frac{10 j\omega^2}{(j\omega^3-8)(j\omega-1)} = \frac{10 (j\omega)^2}{8 \cdot \left(\frac{j\omega^3}{8}-1\right)(j\omega-1)} = \frac{1{,}25 \; \omega^2}{\left(\frac{j\omega^3}{8}+1\right)(j\omega-1)}$$

$$|L(j\omega)| = \frac{1{,}25 \; \omega^2}{\sqrt{\frac{\omega^3}{64}+1} \cdot \sqrt{\omega^2+1}} \; ; \; \angle L(j\omega) = \frac{2}{2}\pi - \operatorname{arctg}\left(\frac{\omega^3}{8}\right) + \operatorname{arctg}(\omega)$$

- Calcolo i limiti:

$$\lim_{\omega \to 0^+} |L(j\omega)| = 0 \; ; \; \lim_{\omega \to 0^+} \angle L(j\omega) = \pi$$

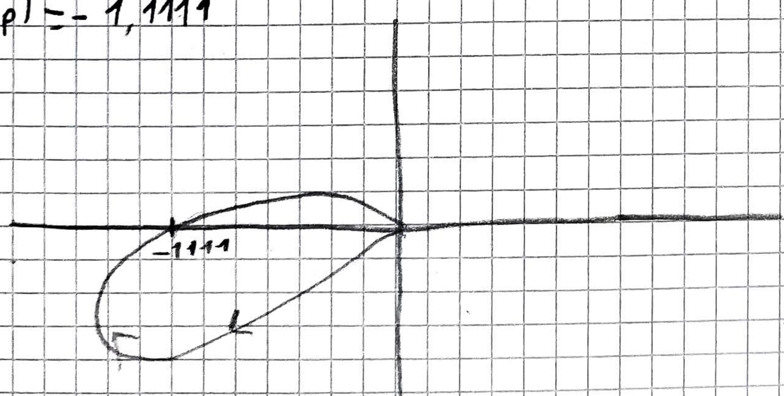
$$\lim_{\omega \to +\infty} |L(j\omega)| = \infty \; ; \; \lim_{\omega \to +\infty} \angle L(j\omega) = \pi - \frac{\pi}{2} - \frac{\pi}{2} = 0$$

- Calcolo l'intersezione con l'asse reale negativo:

$$\pi - \operatorname{arctg}\left(\frac{\omega^3}{8}\right) + \operatorname{arctg}(\omega) = -\pi$$

$\omega_p = 2\sqrt{2}$ rad/s

$L(j\omega_p) = -1{,}1111$



$$1 + L(s) = 0$$

$$(s^3-8)(s-1) + 10 s^2 = 0$$

$$s^4 - s^3 - 8s + 8 + 10 s^2 = 0$$

$$s^4 - s^3 + 10 s^2 - 8s + 8 = 0$$

Esami di Nyquist 1

$$L(s) = 2\,\frac{1+5s}{(1+s)^2\,(1+0{,}5\,s)^2}$$

$$L(j\omega) = 2\,\frac{1+5j\omega}{(1+j\omega)^2\cdot(1+0{,}5j\omega)^2}$$

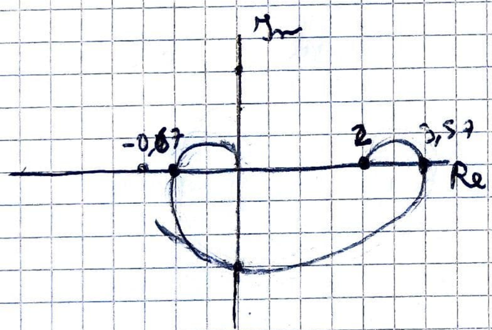
$$|L(j\omega)| = \frac{2\cdot\sqrt{1^2+(5\omega)^2}}{(1+\omega^2)\cdot(1+\frac{1}{4}\omega^2)} \qquad \angle L(j\omega) = \text{arctg}\left(\frac{5\omega}{1}\right) - 2\,\text{arctg}\left(\frac{\omega}{1}\right) - 2\,\text{arctg}\left(\frac{\omega}{2}\right)$$

- Calcolo i limiti:

$$\lim_{\omega\to 0^+} |L(j\omega)| = \frac{2\cdot\sqrt{1}}{1} = 2$$

$$\lim_{\omega\to 0^+} \angle L(j\omega) = 0$$

$$\lim_{\omega\to +\infty} |L(j\omega)| = 0 \qquad \lim_{\omega\to +\infty} \angle L(j\omega) = -\frac{3}{2}\pi$$



Cerco le intersezioni con l'asse reale (pos e neg)

$$\text{arctg}(5\omega) - 2\,\text{arctg}(\omega) - 2\,\text{arctg}\left(\frac{\omega}{2}\right) = -\pi \quad \text{(negativo)}$$

$$\text{arctg}(5\omega) - 2\,\text{arctg}(\omega) - 2\,\text{arctg}\left(\frac{\omega}{2}\right) = 0 \quad \text{(positivo)}$$

Negativo:

$$\text{arctg}(5\omega) + \pi = 2\,\text{arctg}(\omega) + 2\,\text{arctg}\left(\frac{\omega}{2}\right)$$

$$\tan\left(\text{arctg}(5\omega) + \pi\right) = \tan\left(2\,\text{arctg}(\omega) + 2\,\text{arctg}\left(\frac{\omega}{2}\right)\right)$$

- Calcolatrice:

$\omega_1 = 3{,}91508$ rad/s (negativo)

$\omega_2 = 0{,}37038$ rad/s (positivo)

$L(j\omega_1) = \underline{-0{,}6899330} - 1{,}617j$

$L(j\omega_2) = \underline{3{,}5787775} + 5{,}2781j$

<u>Studiare la stabilità del sistema utilizzando il criterio di Nyquist</u>

$L(s)$ non ha poli a parte reale negativa. Dal grafico il punto -1 non circonda il punto -1 $\Rightarrow$ Sistema è asintoticamente stabile

<u>Margine d'ampiezza</u>:

$$M_A = \frac{1}{|L(j\omega_1)|} \simeq 1{,}95$$

Esami di Nyquist 2

$$L(s) = \frac{k}{(s+1)(s+2)(s+3)} \qquad L(j\omega) = \frac{10}{(j\omega+1)(j\omega+2)(j\omega+3)}$$

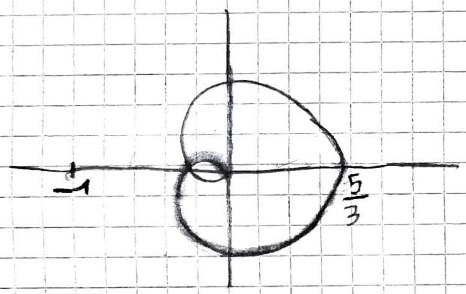
$$|L(j\omega)| = \frac{10}{\sqrt{\omega^2+1}\cdot\sqrt{\omega^2+4}\cdot\sqrt{\omega^2+9}}$$

$$\underline{/L(j\omega)} = -\operatorname{arctg}(\omega) - \operatorname{arctg}\left(\frac{\omega}{2}\right) - \operatorname{arctg}\left(\frac{\omega}{3}\right)$$

- Calcolo dei limiti:

$$\lim_{\omega\to 0^+} |L(j\omega)| = \frac{10}{6} = \frac{5}{3}\ ; \quad \lim_{\omega\to 0^+} \underline{/L(j\omega)} = 0$$

$$\lim_{\omega\to +\infty} |L(j\omega)| = 0\ ; \quad \lim_{\omega\to +\infty} \underline{/L(j\omega)} = -\frac{3}{2}\pi$$



Cerco le intersezioni

$$-\operatorname{arctg}(\omega) - \operatorname{arctg}\left(\frac{\omega}{2}\right) - \operatorname{arctg}\left(\frac{\omega}{3}\right) = -\pi \quad \text{neg}$$

$$-\operatorname{arctg}(\omega) - \operatorname{arctg}\left(\frac{\omega}{2}\right) - \operatorname{arctg}\left(\frac{\omega}{3}\right) = 0 \quad \text{pos}$$

$\omega_p = +3{,}3166$ (neg)

$\omega = 0$ (pos)

$$|L(j\omega_p)| = \frac{1}{6}$$

Margine di ampiezza:

$$M_A = \frac{1}{\frac{1}{6}} = 6$$

Margine di fase:

$$|L(j\omega_e)| = 1 \iff \frac{100}{\sqrt{(1+\omega_e^2)}\cdot\sqrt{4+\omega_e^2}\cdot\sqrt{9+\omega_e^2}} = 1 \quad \text{e. trova la } \omega_e$$

$\omega_e = 1$ rad/s

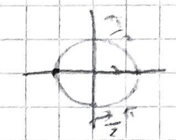
$$\arg L(j\omega_e) = -\frac{\pi}{2}$$

$$M_F = 180^\circ + \arg L(j\omega_e) = 90^\circ$$

Esami Nyquist 6

$$L(j\omega) = \frac{\cancel{1/6}}{\cancel{1/6}} \cdot \frac{(1-j\omega)}{\left(\frac{j\omega}{2}+1\right)^4}$$

$$|L(j\omega)| = \frac{\sqrt{1+\omega^2}}{\left(\frac{\omega^2}{4}+1\right)^2} \qquad \underline{/L(j\omega)} = -4\cdot \arctan\left(\frac{\omega}{2}\right) - \arctan(\omega)$$



- **Calcolo i limiti:**

$$\lim_{\omega\to 0^+} |L(j\omega)| = 1 \qquad \lim_{\omega\to 0^+} \underline{/L(j\omega)} = 0$$

$$\lim_{\omega\to +\infty} |L(j\omega)| = 0 \qquad \lim_{\omega\to +\infty} \underline{/L(j\omega)} = -2\pi - \frac{\pi}{2} = -\frac{5}{2}\pi$$

- **Calcolo l'intersezione con l'asse reale**

$$4\arctan\left(\frac{\omega}{2}\right) + \arctan(\omega) = \pi$$

$$\tan\left(4\arctan\left(\frac{\omega}{2}\right)\right) = \tan\left(\pi - \arctan(\omega)\right)$$

$$\tan\left(4\arctan\left(\frac{\omega}{2}\right)\right) = -\omega$$

...

$\omega_p = 1{,}25$

$L(j\omega_p) = 0{,}82$