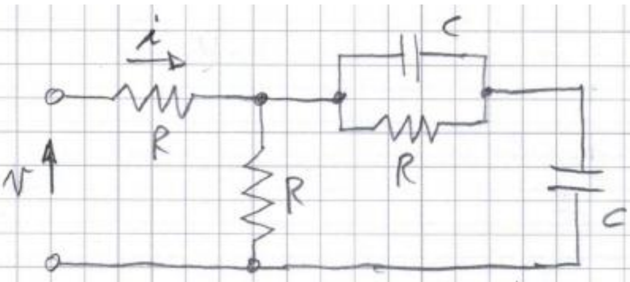
Tracce delle soluzioni

1.



$v \equiv$ tensione

$i \equiv$ corrente

$V(s) = Z_{tot} \cdot I(s)$

$I(s) = \frac{1}{Z_{tot}} \cdot V(s)$

$$Z_{tot} = R + \frac{R \cdot \left(\frac{1}{Cs} + \frac{R \cdot \frac{1}{Cs}}{R + \frac{1}{Cs}} \right)}{R + \frac{1}{Cs} + \frac{R \cdot \frac{1}{Cs}}{R + \frac{1}{Cs}}}$$

$T := RC$

Quindi $Z_{tot}(s) = R \cdot \frac{R^2C^2s^2 + 5RCs + 2}{R^2C^2s^2 + 3RCs + 1} = R \cdot \frac{T^2s^2 + 5Ts + 2}{T^2s^2 + 3Ts + 1}$

La funzione di trasferimento è $G(s) := \frac{1}{Z_{tot}(s)}$

$$G(s) = \frac{T^2s^2 + 3Ts + 1}{R(T^2s^2 + 5Ts + 2)}$$

zeri : $T^2s^2 + 3Ts + 1 = 0 \Rightarrow z_1 = -\frac{3+\sqrt{5}}{2T}, \ z_2 = \frac{-3+\sqrt{5}}{2T}$

poli : $T^2s^2 + 5Ts + 2 = 0 \Rightarrow p_1 = -\frac{5+\sqrt{17}}{2T}, \ p_2 = \frac{-5+\sqrt{17}}{2T}$

modi : $\left\{ \exp\left(-\frac{5+\sqrt{17}}{2T} \cdot t \right), \ \exp\left(\frac{-5+\sqrt{17}}{2T} \cdot t \right) \right\}$ ←

Eq. differenziale :

$$R T^2 D^2 i(t) + 5RTD i(t) + 2R \cdot i(t) = T^2 D^2 v(t) + 3TD v(t) + v(t)$$

2.

$$m D^2 x = -k(x-u) - b(Dx - Du)$$

$$m D^2 x = -kx + ku - bDx + bDu$$

$$\boxed{m D^2 x + b D x + k x = b D u + k u}$$

$$\text{f.d.t.} \quad G(s) = \frac{bs + k}{ms^2 + bs + k}$$

$ms^2 + bs + k$ è il polinomio caratteristico del sistema.

$$\Delta = b^2 - 4mk$$

Non si hanno modi armonici quando $\Delta \geq 0$ ovvero quando

$$b \geq 2\sqrt{mk}$$

3.
Vedi le dispense dell'insegnamento.

4
Vedi le dispense dell'insegnamento.

5.

$$\mathcal{L}[u(t)] = U(s) = \frac{1}{s} + \frac{1}{s^2} = \frac{s+1}{s^2}$$

$$Y(s) = G(s)\,U(s) = \frac{1}{s\,\cancel{(s+1)}\,[(s+1)^2+1]} \cdot \frac{\cancel{s+1}}{s^2} = \frac{1}{s^3\,[(s+1)^2+1]}$$

$$Y(s) = \frac{K_{11}}{s^3} + \frac{K_{12}}{s^2} + \frac{K_{13}}{s} + \frac{K_2}{s+1-j} + \frac{\overline{K_2}}{s+1+j}$$

$$K_{11} = \left.\frac{1}{(s+1)^2+1}\right|_{s=0} = \frac{1}{2} \qquad K_{12} = D\left[\frac{1}{(s+1)^2+1}\right]_{s=0} = \left.-\frac{2(s+1)}{[(s+1)^2+1]^2}\right|_{s=0} = -\frac{1}{2}$$

$$K_2 = \cancel{(s+1-j)}\left.\frac{1}{s^3\,\cancel{(s+1-j)}\,(s+1+j)}\right|_{s=-1+j} = \frac{1}{(-1+j)^3\,2j}$$

$$|K_2| = \frac{1}{2^{3/2}\cdot 2} = \frac{1}{2^{5/2}} \qquad \arg K_2 = -3\left(\frac{\pi}{2}+\frac{\pi}{4}\right) - \frac{\pi}{2} = -\frac{3}{4}\pi \ \text{mod.}\ 2\pi$$

$$K_{13} + K_2 + \overline{K_2} = 0 \qquad K_{13} = -\left(K_2 + \overline{K_2}\right) = -\left(|K_2|\,e^{j\arg K_2} + |K_2|\,e^{-j\arg K_2}\right)$$

$$K_{13} = -|K_2|\cdot\left\{\cos\arg K_2 + j\,\mathrm{sen}\,\cancel{\arg K_2} + \cos\arg K_2 - j\,\mathrm{sen}\,\cancel{\arg K_2}\right\}$$

$$K_{13} = -\frac{1}{2^{5/2}}\cdot 2\cdot\cos\left(-\frac{3}{4}\pi\right) = -\frac{1}{2^{5/2}}\cdot 2\cdot\left(-\frac{1}{\sqrt{2}}\right) = \frac{2}{2^3} = \frac{1}{4}$$

$$y(t) = \frac{1}{4}t^2 - \frac{1}{2}t + \frac{1}{4} + 2\,|K_2|\,e^{-t}\cos\left(t - \frac{3}{4}\pi\right)$$

$$y(t) = \frac{1}{4}t^2 - \frac{1}{2}t + \frac{1}{4} + 2^{-3/2}\,e^{-t}\cdot\mathrm{sen}\left(t - \frac{\pi}{4}\right)$$

La risposta forzata è anche esprimibile come

$$y(t) = \frac{1}{4}t^2 - \frac{1}{2}t + \frac{1}{4} + \frac{1}{4}e^{-t}\left(\mathrm{sen}(t) - \cos(t)\right)$$

L'ingresso è segnale discontinuo (in $t=0$). Quindi

$$y(t) \in C^{\rho-1,\infty}, \quad \rho \equiv \{\text{grado relativo}\} = 4$$

Il grado massimo di continuità di $y(t)$ è uguale a 3.

6.

Modi $\equiv \left\{ e^{-4t}, te^{-4t}, t^2 e^{-4t}, e^{-2t}\sin(2t+\varphi_1), e^{-t}\sin(2t+\varphi_2) \right\}$

poli dominanti $= -2 \pm j2$

$\omega_n = \sqrt{2^2+2^2} = 2\sqrt{2} = \ 2.828$

$\delta\omega_n = 2$; $\delta = 1/\sqrt{2} = 0.7071$

$$S = 100\exp\left(-\frac{\pi\delta}{\sqrt{1-\delta^2}}\right) = 4,3\%$$

$T_a = \dfrac{3}{2} = 1.5$ s ; $T = \dfrac{1.8}{2.828} = 0.636$ s