# Tracce delle soluzioni

**1.**

> $$G := -\frac{\frac{1}{s\,C} + \frac{1}{s\,C} + \frac{\frac{1}{s\,C}\cdot\frac{1}{s\,C}}{R}}{R + R + \frac{R\,R}{\frac{\frac{1}{s\,C}\cdot R}{\frac{1}{s\,C}+R}}}$$

$$-\frac{\frac{2}{s\,C} + \frac{1}{s^2\,C^2\,R}}{2\,R + R\,s\,C\left(\frac{1}{s\,C}+R\right)}$$

$$-\frac{2\,s\,C\,R+1}{s^2\,C^2\,R^2\,(3+s\,C\,R)}$$

$$G(s) = \frac{-2RCs - 1}{(RC)^3 s^3 + 3(RC)^2 s^2}$$

eq. differenziale

$$(RC)^3 D^3 y(t) + 3(RC)^2 D^2 y(t) = -2(RC)Du(t) - u(t)$$

zeri : $z_1 = -\frac{1}{2RC}$

poli : $p_1 = 0 \quad p_2 = 0 \quad p_3 = -\frac{3}{RC}$

modi $= \left\{1,\ t,\ \exp\left\{-\frac{3}{RC}t\right\}\right\}$

**2.**

$$\begin{cases} m D^2 x_1 = f - k x_1 - b D x_1 + k(x_2 - x_1) \\ m D^2 x_2 = -k(x_2 - x_1) \end{cases}$$

$$\begin{cases} (m D^2 + b D + 2k) x_1 = k x_2 + f \\ k x_1 = (m D^2 + k) x_2 \end{cases}$$

$$(m D^2 + b D + 2k)(m D^2 + k) x_2 = k^2 x_2 + k f$$

$$\boxed{m^2 D^4 x_2 + b m D^3 x_2 + 3 k m D^2 x_2 + k b D x_2 + k^2 x_2 = k f}$$

$$\text{f.d.t.} \quad G(s) = \frac{k}{m^2 s^4 + b m s^3 + 3 k m s^2 + k b s + k^2}$$

**3.**
Vedi le dispense del corso.

**4.**
Vedi dispense dell'insegnamento.

**5.**

$$Y(s) = G(s)\,U(s) = G(s)\,\frac{1}{s} = \frac{4}{s\left[(s+1)^2+1\right]^2}$$

$$Y(s) = \frac{4}{s\,(s+1-j)^2\,(s+1+j)^2}$$

$$= \frac{K_1}{s} + \frac{K_{21}}{(s+1-j)^2} + \frac{K_{22}}{s+1-j} + \frac{K_{31}}{(s+1+j)^2} + \frac{K_{32}}{s+1+j}$$

$$= \frac{K_1}{s} + \left\{ \frac{K_{21}}{(s+1-j)^2} + \frac{\overline{K_{21}}}{(s+1+j)^2} \right\} + \left\{ \frac{K_{22}}{s+1-j} + \frac{\overline{K_{22}}}{s+1+j} \right\}$$

$$K_1 = 1 \qquad K_{21} = \left.\frac{4}{s\,(s+1+j)^2}\right|_{s=-1+j} = \frac{4}{(-1+j)(2j)^2} = \frac{1}{1-j}$$

$$|K_{21}| = \frac{1}{\sqrt{2}} \qquad \arg K_{21} = -\arg(1-j) = -\left(-\frac{\pi}{4}\right) = \frac{\pi}{4}$$

$$K_{31} = \overline{K_{21}}$$

$$K_{22} = D\left[\frac{4}{s\,(s+1+j)^2}\right]_{s=-1+j} = -4\cdot\left.\frac{(s+1+j)^2 + s\cdot 2\cdot(s+1+j)}{s^2\,(s+1+j)^4}\right|_{s=-1+j} =$$

$$= -4\,\frac{(2j)^2 + (-1+j)\,2\,(2j)}{(-1+j)^2\,(2j)^4} = -\,\frac{-1+(-1+j)\,j}{(-1+j)^2}$$

$$= -\,\frac{-2-j}{(-1+j)^2} = \frac{2+j}{(-1+j)^2} \qquad K_{32} = \overline{K_{22}}$$

$$|K_{22}| = \frac{\sqrt{5}}{2} \qquad \arg K_{22} = \arg(2+j) - 2\arg(-1+j) = \operatorname{arctg}\left(\tfrac{1}{2}\right) - 2\left(\tfrac{\pi}{2}+\tfrac{\pi}{4}\right)$$

$$= \operatorname{arctg}\left(\tfrac{1}{2}\right) - \pi - \tfrac{\pi}{2}$$

$$y(t) = 1 + 2\,|K_{21}|\cdot t\cdot e^{-t}\cos(t + \arg K_{21}) + 2\,|K_{22}|\,e^{-t}\cos(t + \arg K_{22})$$

$$= 1 + 2\,\frac{1}{\sqrt{2}}\,t\,e^{-t}\cos\left(t+\frac{\pi}{4}\right) + 2\,\frac{\sqrt{5}}{2}\,e^{-t}\cos\left(t + \operatorname{arctg}\left(\tfrac{1}{2}\right) - \pi - \frac{\pi}{2}\right)$$

$$\boxed{= 1 + \sqrt{2}\cdot t\,e^{-t}\cos\left(t+\frac{\pi}{4}\right) - \sqrt{5}\,e^{-t}\operatorname{sen}\left(t + \operatorname{arctg}\left(\tfrac{1}{2}\right)\right)}$$

anche esprimibile come $y(t) = 1 + t\,e^{-t}(\cos t - \operatorname{sen} t) - e^{-t}(\cos t + 2\operatorname{sen} t)$

Il gradino $u(t) = 1(t)$ è funzione discontinua. Quindi $y(t) \in C^{\rho-1,\infty}$ dove $\rho = 4$ è il grado relativo. Il grado massimo di continuità di $y(t)$ è 3.

**6.**

$$G(s) = \frac{1-s}{(s+1)^2}$$ Ingresso $u(t)=0$ per $t \geq 0$

$y(0+) = 2 \qquad Dy(0+) = 1$

Metodo dei modi

$$y(t) = c_1 e^{-t} + c_2 t e^{-t}$$

$$Dy(t) = -c_1 e^{-t} + c_2 e^{-t} + c_2 t \cdot (-1) e^{-t}$$

$$Dy(t) = -c_1 e^{-t} + c_2 e^{-t} - c_2 t e^{-t}$$

$$\begin{cases} c_1 = 2 \\ -c_1 + c_2 = 1 \end{cases} \Rightarrow c_2 = 1 + c_1 = 3$$

$$y(t) = 2e^{-t} + 3te^{-t}$$

metodo dell'eq. differenziale

$$G(s) = \frac{1-s}{s^2+2s+1}$$

$$D^2y + 2Dy + y = -Du + u$$

$$D^2y + 2Dy + y = 0 \quad , t \geq 0$$

Si applica la tras. di Laplace

$$s^2Y - y(0+)s - Dy(0+) + 2\left(sY - y(0+)\right) + Y = 0$$

$$s^2Y - 2s - 1 + 2sY - 4 + Y = 0$$

$$(s^2+2s+1)Y = 2s+5$$

$$Y(s) = \frac{2s+5}{s^2+2s+1} = \frac{2s+5}{(s+1)^2} = \frac{K_{11}}{(s+1)^2} + \frac{K_{12}}{s+1}$$

$$K_{11} = 2(-1)+5 = 3 \qquad K_{12} = 2$$

$$y(t) = 3te^{-t} + 2e^{-t}$$