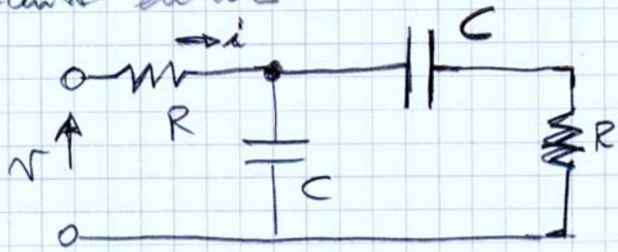
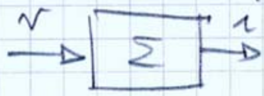
Tracce delle soluzioni

1.

1) Circuito elettrico





$T := RC$

$$V = Z_{tot}\, I \qquad I = \frac{1}{Z_{tot}}\, V$$

$$\text{f.d.t.} \equiv G(s) = \frac{1}{Z_{tot}}$$

$$Z_{tot} = R + \frac{\frac{1}{Cs}\left(\frac{1}{Cs} + R\right)}{\frac{1}{Cs} + \frac{1}{Cs} + R} = R + \frac{\frac{1}{Cs}\cdot\frac{1+RCs}{Cs}}{\frac{2}{Cs} + R} =$$

$$= R + \frac{\frac{1+RCs}{(Cs)^2}}{\frac{2+RCs}{Cs}} = R + \frac{\frac{1+RCs}{Cs}}{2+RCs} =$$

$$= R + \frac{1+RCs}{Cs(2+RCs)} = \frac{RCs(2+RCs) + 1 + RCs}{Cs(2+RCs)} =$$

$$= \frac{2RCs + (RC)^2 s^2 + RCs + 1}{Cs(RCs+2)} = \frac{(RC)^2 s^2 + 3RCs + 1}{Cs(RCs+2)}$$

$$G(s) = \frac{Cs(RCs+2)}{(RC)^2 s^2 + 3RCs + 1} = \frac{Cs(Ts+2)}{T^2 s^2 + 3Ts + 1}$$

zeri: $z_1 = 0$, $z_2 = -\frac{2}{RC}$ poli: $p_1 = \frac{-3-\sqrt{5}}{2T}$, $p_2 = \frac{-3+\sqrt{5}}{2T}$

modi: $\left\{ \exp\left\{-\frac{3+\sqrt{5}}{2T}\,t\right\}, \exp\left\{-\frac{3-\sqrt{5}}{2T}\,t\right\} \right\}$, guadagno statico $G(0) = 0$

eq. diff. $T^2 D^2 i(t) + 3T D i(t) + i(t) = CT D^2 v(t) + 2C D v(t)$

2.

(2)
$$\begin{cases} m D^2 x_1 = -k(x_1 - u) + k(x_2 - x_1) \\ m D^2 x_2 = -k(x_2 - x_1) \end{cases}$$

$$\begin{matrix} (mD^2+k) \\ k \end{matrix} \begin{cases} k x_2 = m D^2 x_1 + 2k x_1 - k u \\ (m D^2 + k) x_2 = k x_1 \end{cases}$$

$$(m D^2+k)\left[(m D^2 + 2k) x_1 - k u\right] = k^2 x_1$$

$$\boxed{m^2 D^4 x_1 + 3 k m D^2 x_1 + k^2 x_1 = k m D^2 u + k^2 u} \qquad \text{eq. diff.}$$

$$\text{f.d.t.} \quad G(s) = \frac{k m s^2 + k^2}{m^2 s^4 + 3 k m s^2 + k^2} = \frac{k(m s^2 + k)}{m^2 s^4 + 3 k m s^2 + k^2}$$

zeri: $m s^2 + k = 0$, $\quad z_{1,2} = \pm j \sqrt{\frac{k}{m}}$

poli: $m^2 s^4 + 3 k m s^2 + k^2 = 0$, $\quad s^2 = \frac{-3 \pm \sqrt{5}}{2} \cdot \frac{k}{m}$

$$p_{1,2} = \pm j \sqrt{\frac{3+\sqrt{5}}{2}} \cdot \sqrt{\frac{k}{m}} \qquad p_{3,4} = \pm j \sqrt{\frac{3-\sqrt{5}}{2}} \cdot \sqrt{\frac{k}{m}}$$

modi: $\text{sen}\left(\sqrt{\frac{3+\sqrt{5}}{2}} \cdot \sqrt{\frac{k}{m}} \cdot t + \varphi_1\right)$, $\text{sen}\left(\sqrt{\frac{3-\sqrt{5}}{2}} \cdot \sqrt{\frac{k}{m}} \cdot t + \varphi_2\right)$

guadagno statico $G(0) = 1$.

3.
Vedi dispense del corso

Parte B

4.
Vedi dispense del corso

5.

$$Y(s) = G(s)\frac{1}{s} = \frac{1}{s(s+2)(s+1-j)(s+1+j)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+1-j} + \frac{\overline{K_3}}{s+1+j}$$

$$K_1 = \left.\frac{1}{(s+2)\left[(s+1)^2+1\right]}\right|_{s=0} = \frac{1}{4}$$

$$K_2 = \left.\frac{1}{s\left[(s+1)^2+1\right]}\right|_{s=-2} = \frac{1}{(-2)[2]} = -\frac{1}{4}$$

$$K_3 = \left.\frac{1}{s(s+2)(s+1+j)}\right|_{s=-1+j} = \frac{1}{(-1+j)(-1+j+2)(-\cancel{1}+j+\cancel{1}+j)} =$$

$$= \frac{1}{(-1+j)(1+j)\,2j} = \frac{1}{[j^2-1]\,2j} = \frac{1}{(-2)\,2j} = \frac{1}{-4j}$$

$$K = \frac{j}{-4j^2} = \frac{1}{4}j \qquad |K_3| = \frac{1}{4} \quad \arg K_3 = +\frac{\pi}{2}$$

$$g_s(t) = \frac{1}{4} - \frac{1}{4}e^{-2t} + 2|K_3|\,e^{-t}\cdot\cos\left(t+\frac{\pi}{2}\right) =$$ OK!! ✓

$$= \frac{1}{4} - \frac{1}{4}e^{-2t} + \frac{1}{2}e^{-t}\left[-\sin t\right] = \frac{1}{4} - \frac{1}{4}e^{-2t} - \frac{1}{2}e^{-t}\sin t$$

$$g_s(t) \in \overline{C}^{\rho-1} \qquad \rho = 3 \qquad g_s(t) \in \overline{C}^{2}$$

$$g(t) = D\,g_s(t) = \left(-\frac{1}{4}\right)(-2)e^{-2t} - \frac{1}{2}(-1)e^{-t}\sin t - \frac{1}{2}e^{-t}(\cos t) =$$

$$= \frac{1}{2}e^{-2t} + \frac{1}{2}e^{-t}\sin t - \frac{1}{2}e^{-t}\cos t$$ OK!

6.

1. $$G_{ry}(s)=\frac{L(s)}{1+L(s)}=\frac{\dfrac{16}{s(s+5)}}{1+\dfrac{16}{s(s+5)}}=\frac{16}{s(s+5)+16}=\frac{16}{s^2+5s+16}$$

eq. diff.: $D^2y(t)+5Dy(t)+16y(t)=16r(t)$

2. Dal confronto $\dfrac{16}{s^2+5s+16}=\dfrac{\omega_n^2}{s^2+2\delta\omega_n s+\omega_n^2}$

$\omega_n=\sqrt{16}=4 \quad \Rightarrow \quad T_s \simeq \dfrac{1.8}{\omega_n}=0,45 \ \text{ sec.}$

$2\delta\omega_n=5 \ \Rightarrow \ \delta\omega_n=2.5 \quad \Rightarrow \ T_a=\dfrac{3}{\delta\omega_n}=1,2 \ \text{ sec.}$

$\delta=\dfrac{2.5}{4}=0,625 \quad \Rightarrow \quad S=100\exp\left(-\dfrac{\delta\pi}{\sqrt{1-\delta^2}}\right)\cong 8{,}1\%$