Tracce delle soluzioni

1.
Vedi dispense del corso.

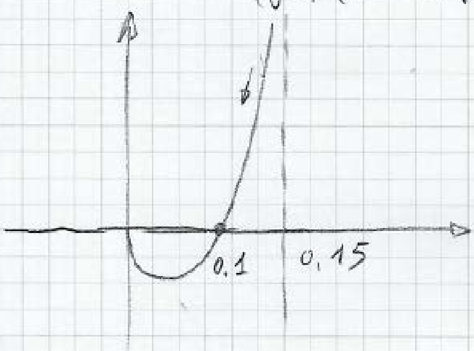
2.

$$G(s) = \frac{s-5}{s(s+10)} \qquad G(s) = \frac{-5\left(1-\frac{1}{5}s\right)}{s \cdot 10 \cdot (1+0{,}1s)}$$

$$G(s) = -\frac{1}{2} \cdot \frac{1-0{,}2 \cdot s}{s(1+0{,}1s)}$$

$$G(j\omega) = -\frac{1}{2} \cdot \frac{1-0{,}2 \cdot j\omega}{(j\omega)(1+0{,}1\,j\omega)} \qquad \tau_a = \left(-\frac{1}{2}\right)\left[-0{,}2-0{,}1\right]$$

$$= -\frac{1}{2} \cdot \left[-0{,}3\right] = 0{,}15$$



$$\omega \to 0 \quad G(j\omega) \to K \frac{1+j\tau_\Sigma \omega}{(j\omega)^h}$$

$$h = 1$$

$$G(j\omega) \to K \frac{1+j\tau_\Sigma \omega}{j\omega} = K\tau_\Sigma - j\frac{K}{\omega}$$

$$\tau_\Sigma = -0{,}2 - (0{,}1) = -0{,}3$$

$$K\tau_\Sigma = -\frac{1}{2} \cdot (-0{,}3) = 0{,}15$$

$$1 + K\,G(s) = 0$$

$$1 + K\,G(j\omega) = 0$$

$$G(j\omega) = -\frac{1}{K}$$

$$1 + K\frac{s-5}{s(s+10)} = 0 \qquad s^2 + 10s + K(s-5) = 0$$

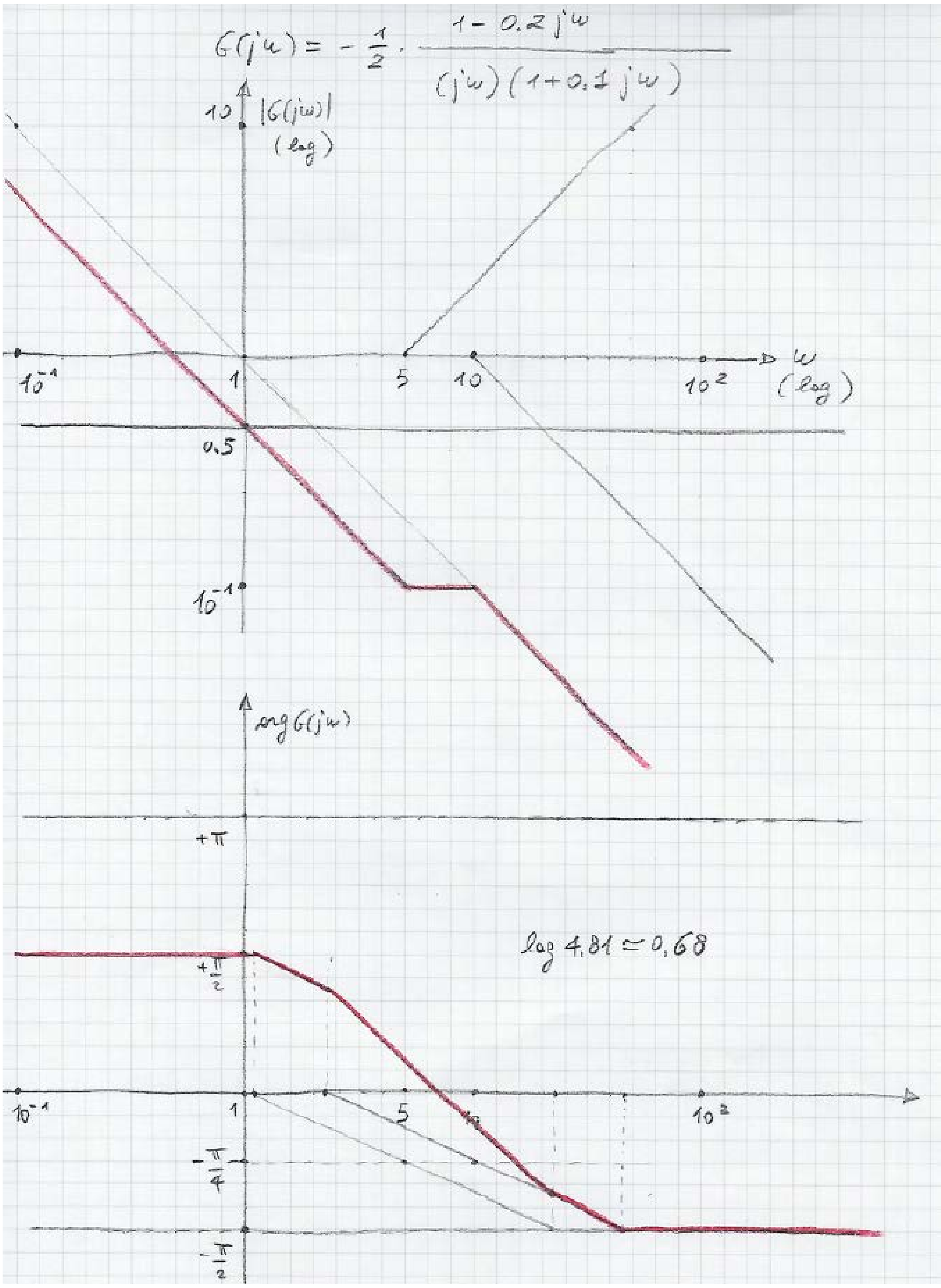
$$s^2 + (10+K)s - 5K = 0$$

$$10 + K = 0 \qquad K = -10$$

$$\Rightarrow G(j\omega) = \frac{1}{10} = 0{,}1$$

$$s^2 = 5K = -50 \qquad s = \pm j\sqrt{50}$$

$$\omega = \sqrt{50}\ \text{rad/s} = 7.07\ \text{rad/s}$$



3.
Vedi dispese dell'insegnamento.

4.

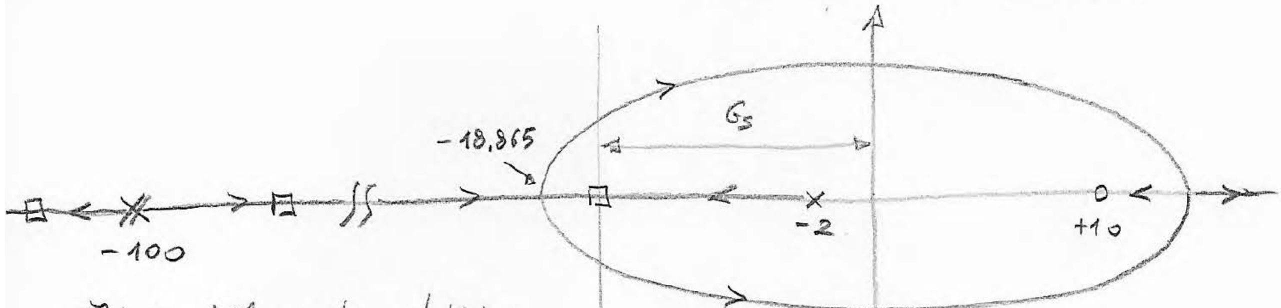
Si sceglie $b_0 = 1$, $b_1 = 1$ affinché si abbia una cancellazione dei poli dominanti del sistema controllato.

$$C(s) = K\,\frac{s^2+s+1}{(s+100)^2}\;; \qquad L(s) := C(s)\,P(s)$$

$$L(s) = K\,\frac{1-0.1\,s}{(s+2)(s+100)^2} = \underbrace{-0.1\cdot K}_{K_1 \triangleq}\,\frac{s-10}{(s+2)(s+100)^2}$$

$$1 + K_1\,\frac{s-10}{(s+2)(s+100)^2} = 0$$

Si traccia il luogo inverso delle radici ($K_1 < 0$).



Calcolo della radice doppia:

$$\frac{1}{s+2} + \frac{2}{s+100} - \frac{1}{s-10} = 0 \iff s^2 - 14s - 620 = 0$$

$$s_{1,2} = -18.865\,,\; 32.865$$

Sul luogo inverso la radice doppia è in -18.865.

$$T_a = \frac{3}{G_s} \Rightarrow G_s = \frac{3}{T_a} = \frac{3}{0{,}2} = 15\ s^{-1}$$

È quindi possibile ottenere $S = 0$ e $T_a = 0{,}2$ s imponendo come polo singolo dominante -15:

$$1 + K\,\frac{1-0.1\,s}{(s+2)(s+100)^2}\bigg|_{s=-15} = 0 \Rightarrow \boxed{K = 37570}$$

$$L(0) = 1.8785$$

$$F\cdot\frac{L(0)}{1+L(0)} = 1 \Rightarrow \boxed{F = \frac{1+L(0)}{L(0)} = 1.532}$$

5.

La specifica a) equivale a $\frac{1}{1+K_p}=\frac{1}{50} \quad \Leftrightarrow \quad K_p=49$. Dato che $K_p=K\frac{5}{2}$ si ottiene $K=\frac{98}{5}$

Definiamo

$$L(s):=KP(s)=1960\frac{s+1}{(s+2)^2(s+10)}$$

$$L'(s):=C(s)P(s)=1960\frac{1+\alpha\tau s}{1+\tau s}\frac{s+1}{(s+2)^2(s+10)}$$

Si propone di progettare α e τ mediante le formule di inversione.

$$L(j\omega)=1960\frac{j\omega+1}{(j\omega+2)^2(j\omega+10)}$$

$$\arg L(j\omega)=\text{arctg }\omega-2\text{ arctg }\frac{\omega}{2}-\text{ arctg }\frac{\omega}{10}$$

$$|L(j\omega)|=1960\frac{\sqrt{1+\omega^2}}{(4+\omega^2)\sqrt{100+\omega^2}}$$

Il diagramma polare di $L(j\omega)$ è riportato in figura.

Si determina (per tentativi) ω_0 (sarà la pulsazione critica di $L'(j\omega)$):

$\omega_0=10$ rad/s

$\arg L(j\omega_0)=-2,0611$ rad $\quad\Rightarrow\quad \varphi_0=0,2951$ rad

$|L(j\omega_0)|=13,393$

verifica validità di ω_0 : $\left(|L(j\omega_0)|,\varphi_0\right)\in C$?

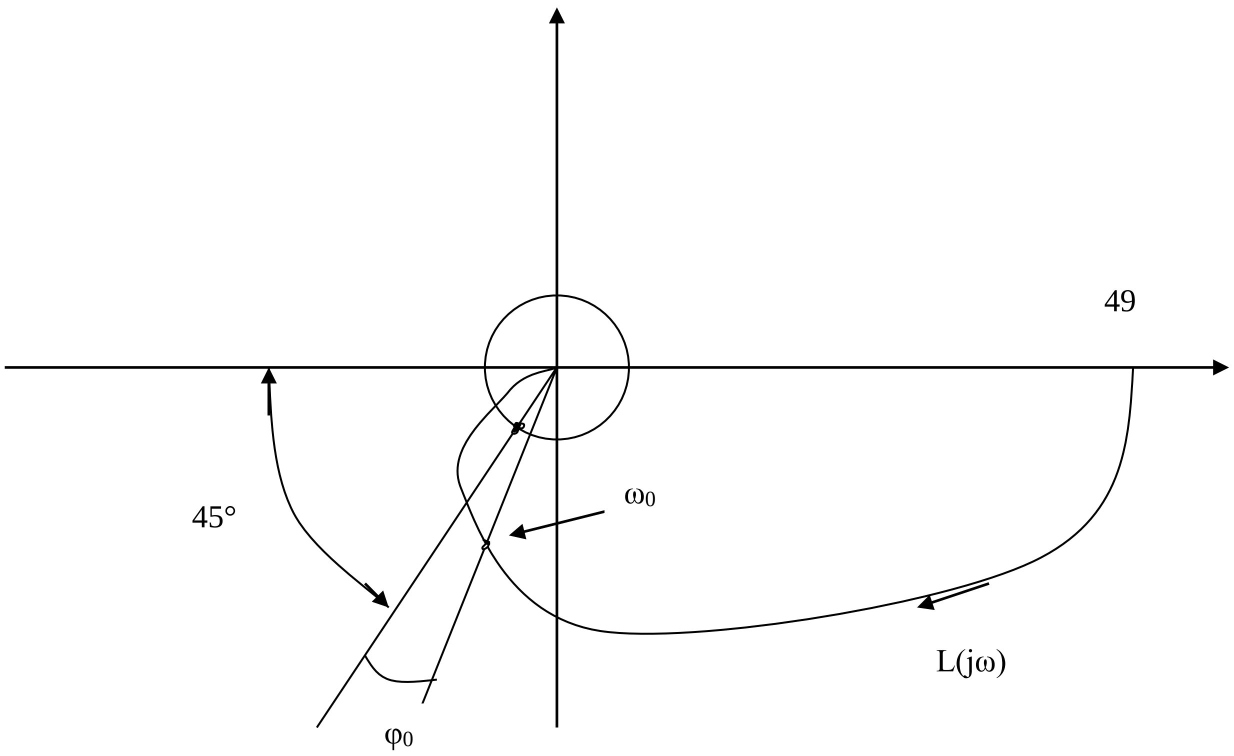
sì, perchè $\cos\varphi_0>1/|L(j\omega_0)|$: 0,9568 > 0,0747.

Si definisce $M:=|L(j\omega)|$ e $\varphi:=\varphi_0$ e si impone, mediante le formule di inversione, che

$$\frac{1}{M}e^{-j\varphi}=\frac{1+\alpha\tau j\omega_0}{1+\tau j\omega_0}$$

Quindi:

$$\begin{cases}\alpha=\dfrac{M\cos\varphi-1}{M(M-\cos\varphi)}=0.0709\\[2ex] \tau=\dfrac{M-\cos\varphi}{\omega_0\text{ sen }\varphi}=4.276\text{ s}\end{cases}$$


49
45°
ω0
L(jω)
φ0

6.

$$Y_{lib} = \frac{1}{\left(z-\frac{1}{2}\right)^2 (z^2+1)} = \frac{1}{\left(z-\frac{1}{2}\right)^2 (z-j)(z+j)} =$$

$$= \frac{K_{11}}{\left(z-\frac{1}{2}\right)^2} + \frac{K_{12}}{z-\frac{1}{2}} + \frac{K_2}{z-j} + \frac{\overline{K}_2}{z+j}$$

$$K_{11} = \left.\frac{1}{(z^2+1)}\right|_{z=\frac{1}{2}} = \frac{1}{\frac{1}{4}+1} = \frac{4}{5}$$

$$K_2 = \left.\frac{1}{\left(z-\frac{1}{2}\right)^2 (z+j)}\right|_{z=j} = \frac{1}{\left(j-\frac{1}{2}\right)^2 \cdot 2j} = \frac{8}{25}\left(1+\frac{3}{4}j\right) = \frac{8}{25} + \frac{6}{25}j$$

$$K_{12} + K_2 + \overline{K}_2 = 0 \qquad K_{12} = -K_2 - \overline{K}_2 = -\frac{8}{25} - \cancel{\frac{6}{25}j} - \frac{8}{25} + \cancel{\frac{6}{25}j}$$

$$K_{12} = -\frac{16}{25} \qquad |K_2| = \frac{2}{5} \quad \arg K_2 = \operatorname{arctg}\frac{3}{4}$$

$$Y_{lib.}(k) = \frac{4}{5}\cdot(k-1)\left(\frac{1}{2}\right)^{k-2}\cdot 1(k-1) - \frac{16}{25}\left(\frac{1}{2}\right)^{k-1}\cdot 1(k-1)$$

$$+ 2\cdot\frac{2}{5}\cdot\cos\left[\frac{\pi}{2}(k-1) + \operatorname{arctg}\frac{3}{4}\right]\cdot 1(k-1)$$

anche esprimibile come

$$Y_{lib}(k) = \frac{16}{5}k\left(\frac{1}{2}\right)^k\cdot 1(k-1) - \frac{112}{25}\left(\frac{1}{2}\right)^k\cdot 1(k-1) + \frac{16}{25}\sin\left(\frac{\pi}{2}k\right)\cdot 1(k-1) + \frac{12}{25}\cos\left(\frac{\pi}{2}k\right)\cdot 1(k-1)$$

oppure

$$Y_{lib.}(k) = \frac{16}{5}k\left(\frac{1}{2}\right)^k - \frac{112}{25}\left(\frac{1}{2}\right)^k + \frac{16}{25}\sin\left(\frac{\pi}{2}k\right) + \frac{12}{25}\cos\left(\frac{\pi}{2}k\right) + 4\,\delta(k),\ k\geq 0$$