Tracce delle soluzioni

1. Vedi le dispense del corso.

2. Vedi le dispense del corso per la definizione di banda e per la formula finale. Quest'ultima si deduce risolvendo un'equazione algebrica di secondo grado.

3.

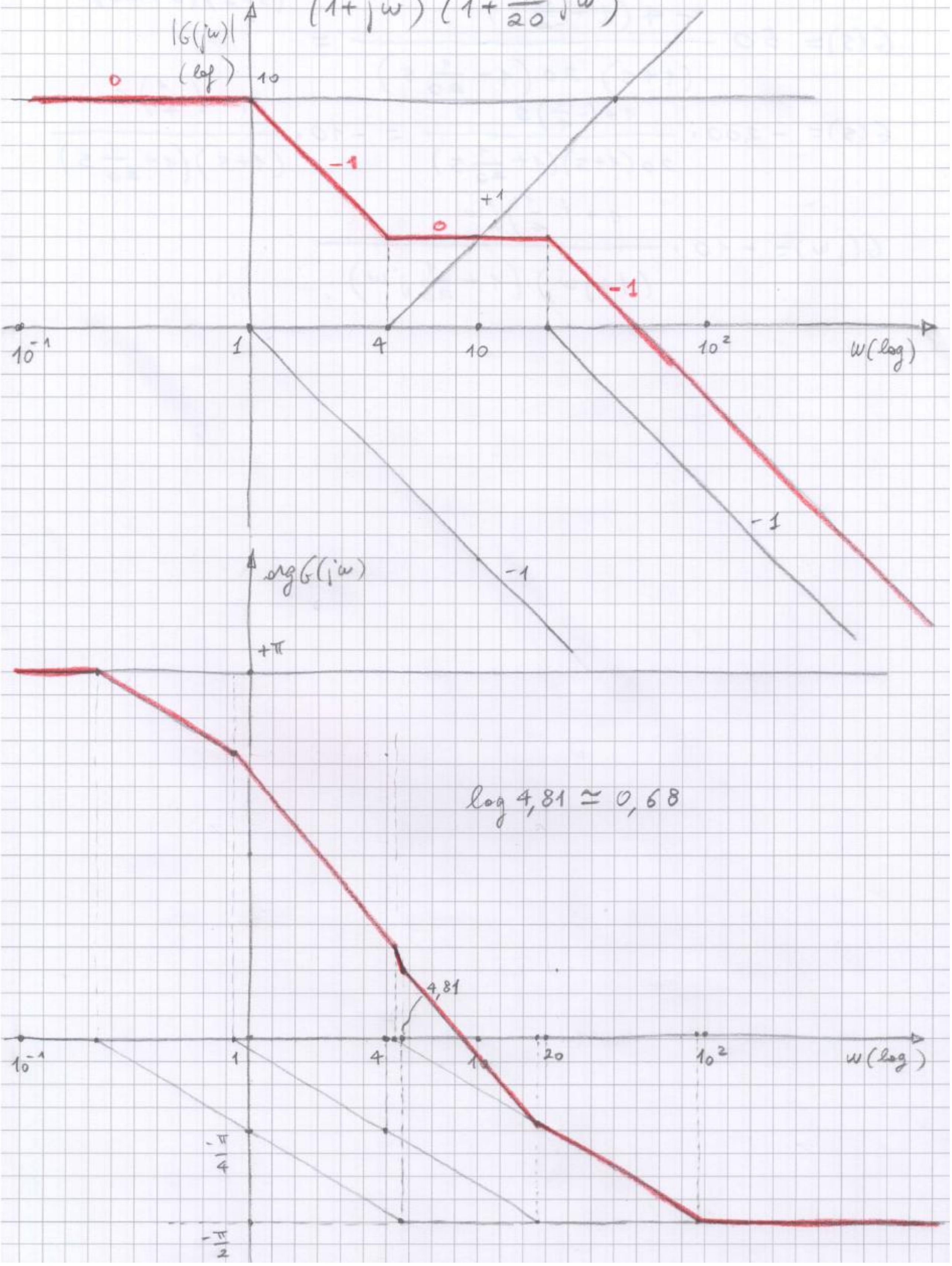
Diagrammi di Bode della f.d.t. $G(s) = 50\,\dfrac{s-4}{(s+1)(s+20)}$

$$G(s) = 50\,\frac{-4\left(1+\frac{1}{-4}s\right)}{(1+s)\cdot 20\cdot\left(1+\frac{1}{20}s\right)} =$$

$$G(s) = -200\cdot\frac{1+\left(-\frac{1}{4}\right)s}{20(1+s)\left(1+\frac{1}{20}s\right)} = -10\cdot\frac{1+\left(-\frac{1}{4}\right)s}{(1+s)\left(1+\frac{1}{20}s\right)}$$

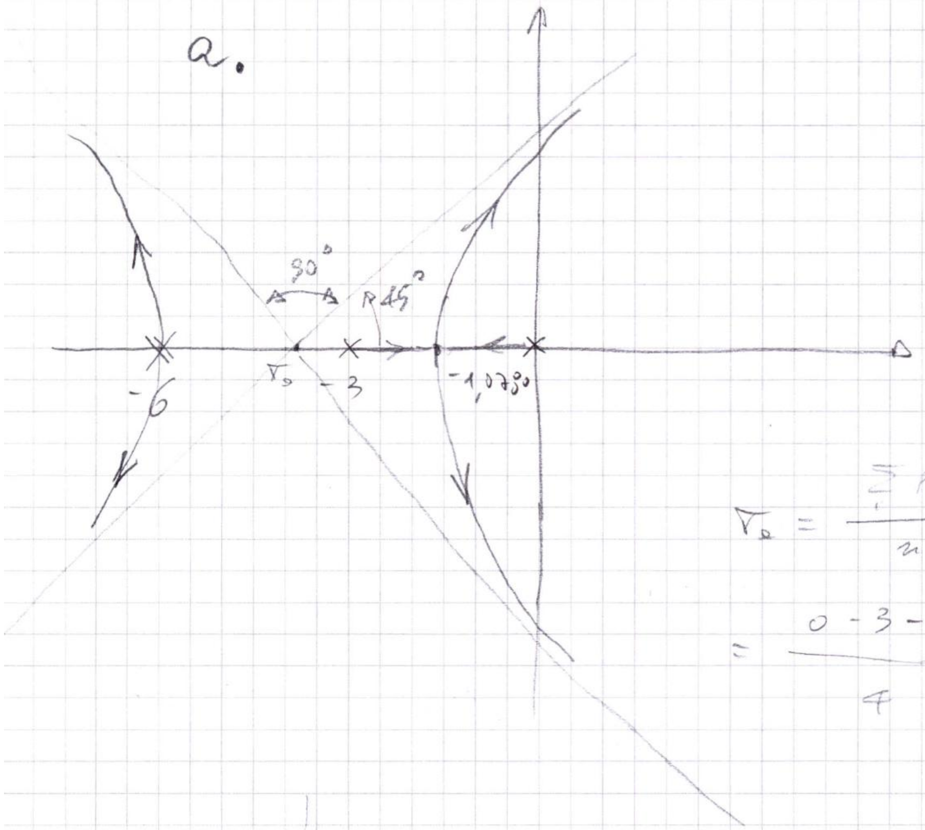
$$G(j\omega) = -10\cdot\frac{1+\left(-\frac{1}{4}\right)j\omega}{(1+j\omega)\left(1+\frac{1}{20}j\omega\right)}$$

$$G(j\omega) = -10 \cdot \frac{1 + \left(-\frac{1}{4}\right) j\omega}{(1 + j\omega)\left(1 + \frac{1}{20} j\omega\right)}$$



4.

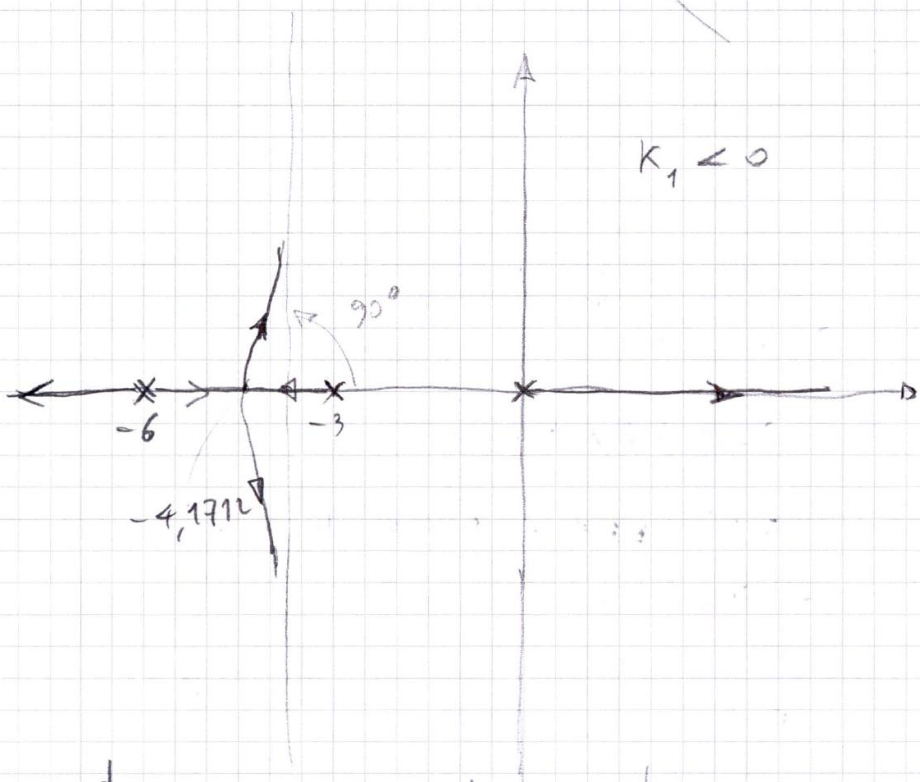
a.



$K_1 > 0$

$150 : 4 = 3,75$

$$\sigma_0 = \frac{\sum p - \sum z_i}{n - m} = \frac{0 - 3 - 6 - 6}{4} = \frac{-15}{4} = -3,75$$



$K_1 < 0$

d.

$$1 + K_1 \frac{1}{s(s+3)(s+6)^2}\Bigg|_{s=-1.0788} = 0$$

$$1 + K_1 \cdot \frac{1}{-50.19} = 0 \qquad K_1 = 50.19$$

radici doppie

$$\sum_{i=1}^{n} \frac{1}{s-p_i} - \sum_{i=1}^{m} \frac{1}{s-z_i} = 0$$

$$\frac{1}{s} + \frac{1}{s+3} + \frac{1}{s+6} + \frac{1}{s+6} = 0$$

$$\frac{1}{s} + \frac{1}{s+3} + \frac{2}{s+6} = 0$$

$$(s+3)(s+6) + s(s+6) + 2s(s+3) = 0$$

$$s^2+9s+18 + s^2+6s + 2s^2+6s = 0$$

$$4s^2+21s+18 = 0$$

$$s_{1,2} = \frac{-21 \pm \sqrt{21^2 - 16 \cdot 18}}{8} = \frac{-21 \pm \sqrt{153}}{8} = \frac{-21 \pm 12.369}{8}$$

$$= \begin{cases} -1,0788 \\ -4,1712 \end{cases}$$

b) $$1 + K_1 \frac{1}{s(s+3)(s+6)^2} = 0$$

$$(s^2+3s)(s^2+12s+36) + K_1 = 0$$

$$s^4 + 12s^3 + 36s^2 + 3s^3 + 36s^2 + 108s + K_1 = 0$$

$$s^4 + 15s^3 + 72s^2 + 108s + K_1 = 0$$

4	1	72	K_1
3	$\cancel{15}$ 5	$\cancel{108}$ 36	0
2	324	$5K_1$	0
1	$324 \cdot 36 - 25K_1$	0	0
0	$5K_1$		

$11'664 - 25K_1 > 0$

$K_1 > 0$

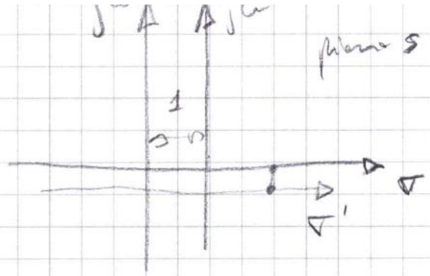
$25K_1 < 11'664$

$K_1 < \frac{11'664}{25} \simeq$

$\simeq 466,56$

$K_1 \in (0, 466,56)$

c.



plane $s = \sigma + j\omega$ $\qquad z = \sigma' + j\omega$

$$\sigma' = \sigma + 1$$

$$z = \sigma + 1 + j\omega \qquad \boxed{z = s + 1}$$

$$\mathrm{Re}\, z = \sigma' \qquad \mathrm{Re}\, s = \sigma$$

$$\mathrm{Re}\, z = \mathrm{Re}\, s + 1$$

$$\mathrm{Re}\, s \le -1 \qquad \mathrm{Re}\, s \le -1 \qquad \mathrm{Re}\, s + 1 \le 0$$

$$\Updownarrow$$

$$s = z - 1 \qquad \mathrm{Re}\, z \le 0$$

$$1 + K_1 \frac{1}{s(s+3)(s+6)^2} = 0$$

$$1 + K_1 \frac{1}{(z-1)(z+2)(z+5)^2} = 0$$

$$(z^2 + z - 2)(z^2 + 10z + 25) + K_1 = 0$$

$$z^4 + 11z^3 + 33z^2 + 5z + K_1 - 50 = 0$$

4	1	33	$K_1 - 50$
3	11	5	0
2	358	$11 \cdot (K_1 - 50)$	0
1	$1790 - 121 \cdot (K_1 - 50)$	0	0
0	$11 \cdot (K_1 - 50)$		

$$1790 - 121 K_1 + 6050 > 0$$

$$7840 > 121 K_1$$

$$K_1 < \frac{7840}{121} \simeq 64.79$$

$$K_1 - 50 > 0$$

$$K_1 > 50$$

$$K_1 \in (50,\ 64.79)$$

5.

$$P(s) = \frac{1}{s^3}$$

$$C(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0}$$

$$1 + CP = 0 \qquad 1 + \frac{b_2 s^2 + b_1 s + b_0}{s^3(s^2 + a_1 s + a_0)} = 0$$

$$s^3(s^2 + a_1 s + a_0) + b_2 s^2 + b_1 s + b_0 = 0$$

$$s^5 + a_1 s^4 + a_0 s^3 + b_2 s^2 + b_1 s + b_0 = 0$$

$$P_c(s) \triangleq s^5 + a_1 s^4 + a_0 s^3 + b_2 s^2 + b_1 s + b_0$$

poli ut. desiderati : $-1, -2, -4, -5 - 6$

$$P_d(s) = (s+1)(s+2)(s+4)(s+5)(s+6)$$
$$= (s^2+3s+2)(s^2+9s+20)(s+6)$$
$$= (s^4 + 9s^3 + 20s^2 + 3s^3 + 27s^2 + 60s$$
$$+ 2s^2 + 18s + 40)(s+6) =$$
$$= (s^4 + 12s^3 + 49s^2 + 78s + 40)(s+6) =$$
$$= s^5 + 12s^4 + 49s^3 + 78s^2 + 40s +$$
$$+ 6s^4 + 72s^3 + 294s^2 + 468s + 240 =$$
$$= s^5 + 18s^4 + 121s^3 + 372s^2 + 508s + 240$$

ok!

da $P_d(s) \equiv P_c(s) \Longrightarrow \begin{cases} a_1 = 18, \quad a_0 = 121 \\ b_2 = 372, \; b_1 = 508, \; b_0 = 240 \end{cases}$

$$C(s) = \frac{372 s^2 + 508 s + 240}{s^2 + 18s + 121}$$

OK!

1. Si applichi un gradino $r(t) = 3 \cdot 1(t)$ al sistema retroazionato e si determini l'errore a regime e_r $\left[e_r := \lim_{t \to \infty} \left(r(t) - y(t) \right) \right]$ ed ~~il~~ una stima del tempo di assestamento

$e_r = 0$ perché è un sistema di tipo 3

$$T_a \simeq \frac{3}{G_a} = \frac{3}{1} \simeq 3 \text{ sec.}$$