1.

$$T_{uy}(s) = - \frac{R + \dfrac{\frac{1}{sC} \cdot R}{\frac{1}{sC} + R}}{R + R + \dfrac{R^2}{\dfrac{\frac{1}{sC} \cdot R}{\frac{1}{sC} + R}}} = - \frac{R + \dfrac{R}{1 + RCs}}{2R + \dfrac{R^2}{\dfrac{R}{1 + RCs}}} =$$

$$= - \frac{2 + RCs}{(1 + RCs)(3 + RCs)} = \frac{-RCs - 2}{R^2C^2s^2 + 4RCs + 3}$$

eq. diff. $R^2C^2D^2y(t) + 4RC\,Dy(t) + 3y(t) = -RC\,Du(t) - 2u(t)$

zeri: $z_1 = -\dfrac{2}{RC}$ poli: $p_1 = -\dfrac{1}{RC}$, $p_2 = -\dfrac{3}{RC}$

modi $= \left\{ \exp\left\{-\dfrac{1}{RC} t\right\}, \exp\left\{-\dfrac{3}{RC} \cdot t\right\} \right\}$

2.

1. $$\begin{cases} m_1 D^2 x_1 = f - k_1 x_1 + k_2 (x_2 - x_1) + b(Dx_2 - Dx_1) \\ m_2 D^2 x_2 = -k_2(x_2 - x_1) - b(Dx_2 - Dx_1) \end{cases}$$

$$\begin{cases} m_1 D^2 x_1 = f - k_1 x_1 - k_2 x_1 - b D x_1 + k_2 x_2 + b D x_2 \\ m_2 D^2 x_2 = -k_2 x_2 - b D x_2 + k_2 x_1 + b D x_1 \end{cases}$$

$$\begin{cases} m_1 D^2 x_1 + b D x_1 + (k_1 + k_2) x_1 = f + (k_2 + bD) x_2 \\ (k_2 + b D) x_1 = m_2 D^2 x_2 + b D x_2 + k_2 x_2 \end{cases}$$

$$\left[m_1 D^2 + bD + (k_1 + k_2)\right](m_2 D^2 + bD + k_2) x_2 =$$

$$= (k_2 + bD)^2 x_2 + (k_2 + bD) f$$

$$\left[m_1 m_2 D^4 + m_1 b D^3 + m_1 k_2 D^2 + m_2 b D^3 + \cancel{b^2 D^2} + \cancel{b k_2 D} + m_2 (k_1 + k_2) D^2 + \right.$$
$$\left. + b(k_1 + \cancel{k_2}) D + k_2 (k_1 + \cancel{k_2})\right] x_2 = (\cancel{k_2^2} + \cancel{2 k_2 b D} + \cancel{b^2 D^2}) x_2$$
$$+ (k_2 + bD) f$$

$$\left[m_1 m_2 D^4 + (m_1 + m_2) b D^3 + (m_1 k_2 + m_2 k_1 + m_2 k_2) D^2 + k_1 b D + k_1 k_2\right] x_2 = (k_2 + bD) f$$

2.

$$G(s)=\frac{b s+K_2}{m_1 m_2 s^4+(m_1+m_2) b s^3+(m_1 K_2+m_2 K_1+m_2 K_2) s^2+K_1 b s+K_1 K_2}$$

3.

$m=m_1=m_2$, $K=K_1=K_2$ e $b=0$

$$G(s)=\frac{K}{m^2 s^4+3 m K s^2+K^2}$$

$$m^2 s^4+3 m K s^2+K^2=0 \qquad s^2=\frac{-3 m K \pm \sqrt{9 m^2 K^2-4 m^2 K^2}}{2 m^2}=$$

$$=\frac{-3 m K \pm m K \sqrt{5}}{2 m^2}=\frac{-3 \pm \sqrt{5}}{2} \cdot \frac{K}{m}$$

poli di Σ:

$$p_{1,2}= \pm j \sqrt{\frac{3+\sqrt{5}}{2} \cdot \frac{K}{m}} \qquad p_{3,4}= \pm j \sqrt{\frac{3-\sqrt{5}}{2} \cdot \frac{K}{m}}$$

modi di Σ

$$\operatorname{sen}\left(\sqrt{\frac{3+\sqrt{5}}{2} \cdot \frac{K}{m}} \cdot t+\varphi_1\right), \quad \operatorname{sen}\left(\sqrt{\frac{3-\sqrt{5}}{2} \cdot \frac{K}{m}} \cdot t+\varphi_2\right)$$

anche esprimibili come

$$\operatorname{sen}\left(1,618 \cdot \sqrt{\frac{K}{m}} \cdot t+\varphi_1\right), \quad \operatorname{sen}\left(0,618 \cdot \sqrt{\frac{K}{m}} \cdot t+\varphi_2\right)$$

3.
Vedi le dispense del corso.

4.

$$G(s) = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

$$Y(s) = G(s)\frac{1}{s} = \frac{\omega_n^2}{s\left(s+\delta\omega_n - j\omega_n\sqrt{1-\delta^2}\right)\left(s+\delta\omega_n + j\omega_n\sqrt{1-\delta^2}\right)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+\delta\omega_n - j\omega_n\sqrt{1-\delta^2}} + \frac{\overline{K}_2}{s+\delta\omega_n + j\omega_n\sqrt{1-\delta^2}}$$

$$K_1 = \cancel{s}\, G(s)\frac{1}{\cancel{s}}\Big|_{s=0} = 1$$

$$K_2 = \frac{\omega_n^2}{s\left(s+\delta\omega_n + j\omega_n\sqrt{1-\delta^2}\right)}\Bigg|_{s=-\delta\omega_n + j\omega_n\sqrt{1-\delta^2}} =$$

$$= \frac{\omega_n^2}{\left(-\delta\omega_n + j\omega_n\sqrt{1-\delta^2}\right)\left(2j\omega_n\sqrt{1-\delta^2}\right)} = \frac{1}{\left(-\delta + j\sqrt{1-\delta^2}\right)2j\sqrt{1-\delta^2}}$$

$$= \frac{1}{2\left[-(1-\delta^2) - j\delta\sqrt{1-\delta^2}\right]}$$

$$|K_2| = \frac{1}{2\sqrt{1-\delta^2}} \qquad \arg K_2 = -\arg\left[-(1-\delta^2) - j\delta\sqrt{1-\delta^2}\right] =$$

$$= -\left(-\frac{\pi}{2} - \operatorname{arctg}\frac{\sqrt{1-\delta^2}}{\delta}\right) = \frac{\pi}{2} + \operatorname{arctg}\frac{\sqrt{1-\delta^2}}{\delta}$$

$$= \frac{\pi}{2} + \arccos\delta$$

$$g_s(t) = 1 + 2|K_2|\, e^{-\delta\omega_n t}\cdot\cos\left(\omega_n\sqrt{1-\delta^2}\cdot t + \arg K_2\right) =$$

$$= 1 + \frac{1}{\sqrt{1-\delta^2}}\, e^{-\delta\omega_n t}\cdot\cos\left(\omega_n\sqrt{1-\delta^2}\, t + \frac{\pi}{2} + \arccos\delta\right)$$

$$= 1 - \frac{1}{\sqrt{1-\delta^2}}\, e^{-\delta\omega_n t}\cdot\sin\left(\omega_n\sqrt{1-\delta^2}\cdot t + \arccos\delta\right)$$

5.

1° metodo: $u(t) = 1(t) - 1(t-1)$

$$U(s) = \frac{1}{s} - \frac{1}{s}e^{-s} \qquad Y(s) = G(s)U(s) = \frac{1}{s(s+1)^2} - \frac{1}{s(s+1)^2}e^{-s}$$

$$y(t) = \mathcal{L}^{-1}\left[\frac{1}{s(s+1)^2}\right] - \mathcal{L}^{-1}\left[\frac{1}{s(s+1)^2}\cdot e^{-s}\right]$$

$$\frac{1}{s(s+1)^2} = \frac{K_1}{s} + \frac{K_{21}}{(s+1)^2} + \frac{K_{12}}{s+1}$$

$$K_1 = \left.\frac{1}{(s+1)^2}\right|_{s=0} = 1 \qquad K_{21} = \left.\frac{1}{s}\right|_{s=-1} = -1 \qquad K_1 + K_{12} = 0$$

$$K_{12} = -1$$

$$\mathcal{L}^{-1}\left[\frac{1}{s(s+1)^2}e^{-s}\right] = \mathcal{L}^{-1}\left[\frac{1}{s(s+1)^2}\right](t-1)\cdot 1(t-1)$$

$$y(t) = 1 - te^{-t} - e^{-t} - \left[1 - (t-1)e^{-(t-1)} - e^{-(t-1)}\right]\cdot 1(t-1)$$

Per $t \in [0,1)$ $\quad y(t) = 1 - te^{-t} - e^{-t}$

Per $t \in [1,+\infty)$ $\quad y(t) = \cancel{1} - te^{-t} - e^{-t} \cancel{-1} + (t\cancel{-1})e^{-t}\cdot e \cancel{+ e^{-t}\cdot e}$

$$\boxed{y(t) = -e^{-t} + (e-1)te^{-t}}$$

2° metodo: Per $t \in [0,1)$ vale $u(t) = 1(t)$

e quindi $\quad y(t) = 1 - te^{-t} - e^{-t}$

Per $t \in [1,+\infty)$ $\quad y(t) = c_1 e^{-t} + c_2 t e^{-t}$

$u(t)$ è discontinua su $(-\infty,+\infty)$, quindi $y(t) \in C^{\rho-1}(\mathbb{R})$

ovvero $y(t) \in C^1(\mathbb{R})$

$$\begin{cases} y(1-) = y(1+) \\ Dy(1-) = Dy(1+) \end{cases}$$

Per $t \in [0,1)$ $\quad y(t) = 1 - te^{-t} - e^{-t}$

$$Dy(t) = -e^{-t} + te^{-t} + e^{-t}$$

Per $t \in [1,+\infty)$ $\quad y(t) = c_1 e^{-t} + c_2 t e^{-t}$

$$Dy(t) = -c_1 e^{-t} + c_2 \left(e^{-t} - te^{-t}\right)$$

$$y(1-) = 1 - e^{-1} - e^{-1} = 1 - 2e^{-1}$$

$$Dy(1-) = -\cancel{e^{-1}} + \cancel{e^{-1}} + e^{-1} = e^{-1}$$

$$\begin{cases} c_1 e^{-1} + c_2 e^{-1} = 1 - 2e^{-1} \\ -c_1 e^{-1} + c_2 \left(\cancel{e^{-1} - e^{-1}}\right) = e^{-1} \end{cases} \Rightarrow c_1 = -1$$

$$-e^{-1} + c_2 e^{-1} = 1 - 2e^{-1} \qquad -1 + c_2 = e - 2$$

$$\Rightarrow c_2 = e - 1$$

$$\boxed{y(t) = -e^{-t} + (e-1)te^{-t}}$$

6.

1. $$T_{ry}(s) = K\frac{L(s)}{1+L(s)} = 1.5\frac{\dfrac{10}{(s+1)(s+5)}}{1+\dfrac{10}{(s+1)(s+5)}} = 1.5\frac{10}{(s+1)(s+5)+10} = \frac{15}{s^2+6s+15}$$

eq. diff.: $D^2y(t) + 6Dy(t) + 15y(t) = 15r(t)$

2. Dal confronto $\dfrac{15}{s^2+6s+15} = \dfrac{\omega_n^2}{s^2+2\delta\omega_n s+\omega_n^2}$

$$\omega_n = \sqrt{15} = 3.873 \quad \Rightarrow \quad T_s \simeq \frac{1.8}{\omega_n} = 0.46 \ \text{sec.}$$

$$2\delta\omega_n = 6 \ \Rightarrow \ \delta\omega_n = 3 \ \Rightarrow \ T_a = \frac{3}{\delta\omega_n} = 1 \ \text{sec.}$$

$$\delta = \frac{3}{\omega_n} = 0,7746 \quad \Rightarrow \quad S = 100\exp\left(-\frac{\delta\pi}{\sqrt{1-\delta^2}}\right) \cong 2.1\%$$