Tracce soluzioni

1.
Vedi dispense del corso.

2.

a)
$$L(j\omega) = 10 \frac{j\omega + 2}{(j\omega)^2 (j\omega+1)}$$

$$\arg L(j\omega) = -\pi + \operatorname{arctg}\frac{\omega}{2} - \operatorname{arctg}\omega$$

$\omega \to 0 \quad \arg L(j\omega) \to -\pi \qquad |L(j\omega)| \to +\infty$

$\omega \to +\infty \quad \arg L(j\omega) \to -\pi \qquad |L(j\omega)| \to 0$

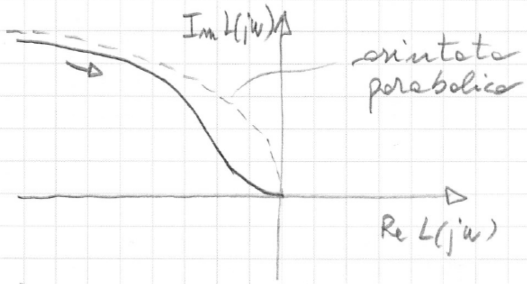
Per ω piccolo e positivo: $\arg L(j\omega) \cong -\pi + \frac{\omega}{2} - \omega = -\pi - \frac{\omega}{2} < -\pi$

$\Rightarrow$ emergenza del diagramma polare dal secondo quadrante.

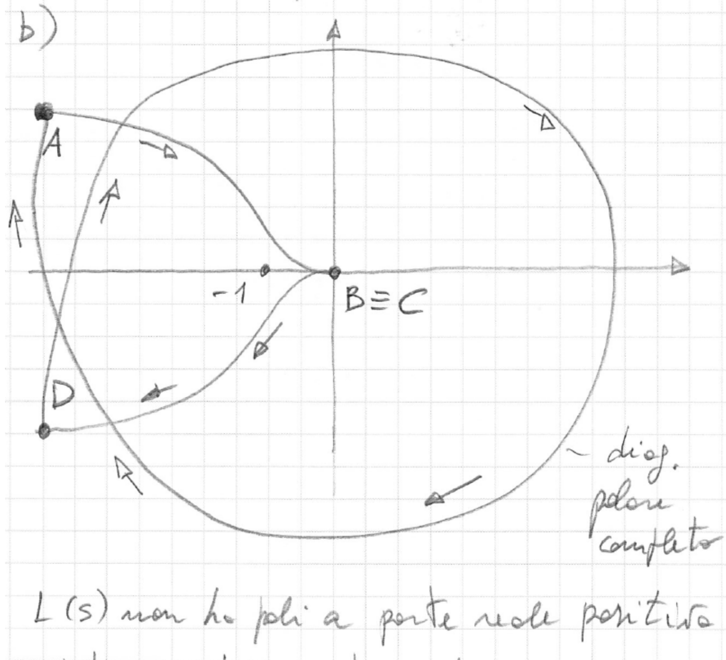
$\arg L(j\omega) = -\pi$ con $\omega > 0$: $\operatorname{arctg}\frac{\omega}{2} - \operatorname{arctg}\omega = 0$

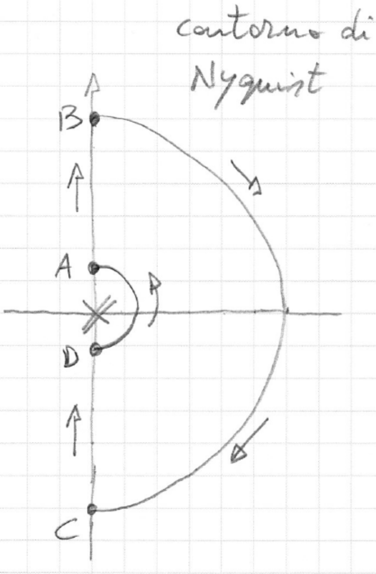
$$\frac{\frac{\omega}{2} - \omega}{1 + \frac{\omega}{2}\cdot\omega} = 0 \qquad -\frac{1}{2}\omega = 0 \qquad \text{nessuna soluzione per } \omega > 0$$

Quindi nessuna intersezione con l'asse reale negativo.



b)





L(s) non ha poli a parte reale positiva. Per il criterio di N. la stabilità asint. sussiste quando il d.p.c. non circonda né tocca il punto -1. In questo caso il d.p.c. circonda 2 volte -1. Quindi il sistema ret. è instabile.

3.
Vedi dispense del corso.

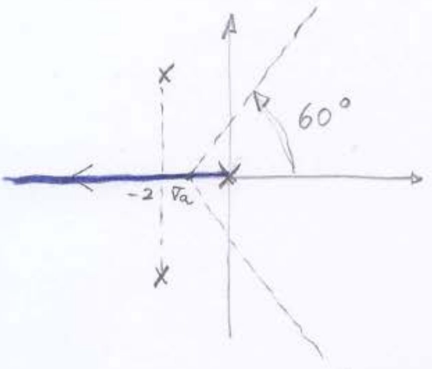
4.

a. Eq. caratteristica del sistema retroazionato

$$1+K\frac{1}{s\left[(s+2)^2+16\right]}=0\quad,\quad K>0$$

Poli ad anello aperto: $P_1=0$, $P_{2,3}=-2\pm j4$

ASINTOTI: centro in $\sigma_a=\frac{P_1+P_2+P_3}{3}=-\frac{4}{3}$



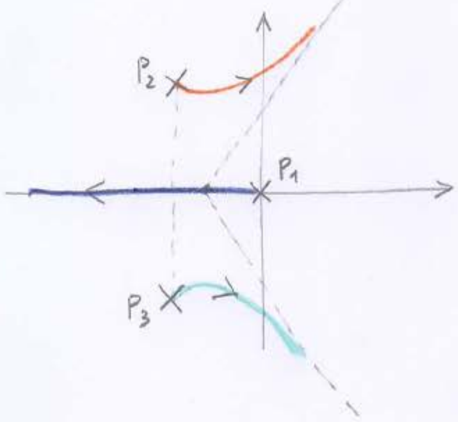
L'asse reale negativo appartiene al luogo.

RADICI DOPPIE:

$$\frac{1}{s}+\frac{1}{s+2-j4}+\frac{1}{s+2+j4}=0$$

$$3s^2+8s+20=0$$

$$s=\frac{-4\pm\sqrt{16-60}}{3}\notin\mathbb{R}$$



Quindi non esistono radici doppie reali nel luogo.

ANGOLI DI PARTENZA:

$$\theta_2=\pi-\left[\frac{\pi}{2}+\frac{\pi}{2}+\mathrm{arctg}\,\frac{2}{4}\right]=$$

$$=-\mathrm{arctg}\,\frac{1}{2}=-0{,}4636=-26{,}57^\circ$$

$$\theta_3=+26{,}57^\circ\qquad\theta_1=+180^\circ$$

b.

La configurazione dei poli retroazionati in corrispondenza del guadagno ottimo K^* è σ, $\sigma+j\omega$, $\sigma-j\omega$ $[\sigma,\omega\in\mathbb{R}]$.

Dal teorema del baricentro $3\sigma=-4\Rightarrow\sigma=-\frac{4}{3}$

Quindi $K^*\ni 1+K^*\frac{1}{\left(-\frac{4}{3}\right)\left[\left(-\frac{4}{3}+2\right)^2+16\right]}=0$

$\Rightarrow K^*\cong 21{,}9$

5.

$$C(s) = \frac{y_3 s^3 + y_2 s^2 + y_1 s + y_0}{s(s^2+9)}$$

$$L(s) = C(s)P(s) = 9 \cdot \frac{y_3 s^3 + y_2 s^2 + y_1 s + y_0}{s(s^2+9)(s+4)}$$

$$K_v = \lim_{s \to 0} s L(s) = \frac{9 \cdot y_0}{9 \cdot 4} = \frac{y_0}{4}$$

$$K_v = 4 \implies \frac{y_0}{4} = 4, \quad \boxed{y_0 = 16}$$

Il polinomio caratteristico desiderato è

$$P_d(s) = \left[(s+2)^2+1\right](s+2)(s+c) \quad \text{con } c > 2.$$

$$P_d(s) = s^4 + (6+c)s^3 + (6c+13)s^2 + (13c+10)s + 10c$$

Il polinomio caratteristico associato al controllore scelto è

$$P_c(s) = s(s^2+9)(s+4) + 9(y_3 s^3 + y_2 s^2 + y_1 s + y_0)$$

$$P_c(s) = s^4 + (4+9y_3)s^3 + (9+9y_2)s^2 + (36+9y_1)s + 9y_0$$

Si impone che $P_c(s) \equiv P_d(s)$

$$\begin{cases} 4 + 9y_3 = 6 + c \\ 9 + 9y_2 = 13 + 6c \\ 36 + 9y_1 = 10 + 13c \\ 9y_0 = 10c \quad \implies c = \frac{144}{10} = 14.4 \quad \text{ok! } c \gg 2. \end{cases}$$

$$\boxed{y_1 = 17.91, \quad y_2 = 10.04, \quad y_3 = 1.822}$$

6.

$$Y_{lib} = z \cdot \frac{z}{\left(z-\frac{1}{2}\right)^2 (z^2+1)} \triangleq z \cdot Y_1(z)$$

$$Y_1(z) = \frac{z}{\left(z-\frac{1}{2}\right)^2 (z-j)(z+j)} = \frac{K_{11}}{\left(z-\frac{1}{2}\right)^2} + \frac{K_{12}}{z-\frac{1}{2}} + \frac{K_2}{z-j} + \frac{\overline{K_2}}{z+j}$$

$$K_{11} = \left.\frac{z}{z^2+1}\right|_{z=\frac{1}{2}} = \frac{2}{5}$$

$$K_2 = \left.\frac{z}{\left(z-\frac{1}{2}\right)^2 (z+j)}\right|_{z=j} = \frac{1}{2\left(-\frac{1}{2}+j\right)^2} = -\frac{6}{25} + \frac{8}{25}j$$

$$K_{12} + K_2 + \overline{K_2} = 0 \qquad K_{12} = -K_2 - \overline{K_2} = \frac{12}{25}$$

$$Y_{lib} = \frac{2}{5} \cdot \frac{z}{\left(z-\frac{1}{2}\right)^2} + \frac{12}{25} \cdot \frac{z}{z-\frac{1}{2}} + \left(-\frac{6}{25}+\frac{8}{25}j\right)\frac{z}{z-j} + \left(-\frac{6}{25}-\frac{8}{25}j\right) \cdot \frac{z}{z+j}$$

$$\left|-\frac{6}{25}+\frac{8}{25}j\right| = \frac{2}{5} \qquad \arg\left(-\frac{6}{25}+\frac{8}{25}j\right) = \pi - \operatorname{arctg}\left(\frac{4}{3}\right)$$

$$|j| = 1 \qquad \arg(j) = \frac{\pi}{2}$$

$$y_{lib}(k) = \frac{2}{5}k\left(\frac{1}{2}\right)^{k-1} + \frac{12}{25}\left(\frac{1}{2}\right)^k + 2 \cdot \frac{2}{5} \cdot \cos\left(\frac{\pi}{2}k + \pi - \operatorname{arctg}\left(\frac{4}{3}\right)\right)$$

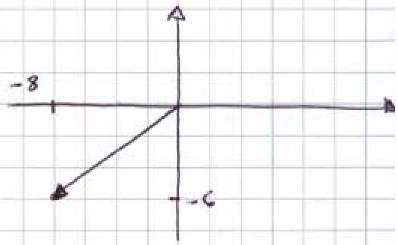
$$= \frac{4}{5}k\left(\frac{1}{2}\right)^k + \frac{12}{25}\left(\frac{1}{2}\right)^k - \frac{4}{5}\cos\left(\frac{\pi}{2}k - \operatorname{arctg}\left(\frac{4}{3}\right)\right), \quad k \geq 0$$

anche esprimibile come

$$y_{lib}(k) = \frac{4}{5}k\left(\frac{1}{2}\right)^k + \frac{12}{25}\left(\frac{1}{2}\right)^k - \frac{12}{25}\cos\left(\frac{\pi}{2}k\right) - \frac{16}{25}\operatorname{sen}\left(\frac{\pi}{2}k\right)$$

Un altro metodo risolutivo:

$$Y_{LiB}(z) = \frac{z^2}{\left(z-\frac{1}{2}\right)^2 (z^2+1)}$$



$$Y_{LiB}(z) = \frac{k_{11}}{\left(z-\frac{1}{2}\right)^2} + \frac{k_{12}}{\left(z-\frac{1}{2}\right)} + \frac{k_2}{z-j} + \frac{\overline{k_2}}{z+j}$$

$$k_{11} = \cancel{\left(z-\frac{1}{2}\right)^2} \frac{z^2}{\cancel{\left(z-\frac{1}{2}\right)^2}(z^2+1)}\Bigg|_{z=\frac{1}{2}} = \frac{1/4}{1/4+1} = \frac{1}{5}$$

$$k_2 = \cancel{(z-j)} \frac{z^2}{\left(z-\frac{1}{2}\right)^2 \cancel{(z-j)}(z+j)}\Bigg|_{z=j} = -\frac{8}{25} - \frac{6}{25} i$$

$$k_{1,2} = -k_2 - \overline{k_2} = \frac{16}{25}$$

$$Y_{LiB} = \frac{1}{5} \cdot \frac{1}{\left(z-\frac{1}{2}\right)^2} + \frac{16}{25} \cdot \frac{1}{\left(z-\frac{1}{2}\right)} + \left(-\frac{8}{25} - \frac{6}{25} j\right)\frac{1}{z-j} + \left(-\frac{8}{25} + \frac{6}{25} j\right)\frac{1}{z+j}$$

$$|p| = |j| = 1 \qquad \arg(p) = \arg(j) = \frac{\pi}{2}$$

$$|k_2| = \sqrt{\left(\frac{8}{25}\right)^2 + \left(\frac{6}{25}\right)^2} = \frac{2}{5} \qquad \arg(k_2) = -143.13^\circ$$

$$\arg(k_2) = \operatorname{arctg}\frac{3}{4} - \pi$$

$$y(k) = \frac{1}{5}(k-1)\left(\frac{1}{2}\right)^{k-2} 1(k-1) + \frac{16}{25}\left(\frac{1}{2}\right)^{k-1} 1(k-1) +$$

$$+ 2 \cdot \frac{2}{5} \cdot 1^{k-1} \cos\left(\frac{\pi}{2}(k-1) + \operatorname{arctg}\frac{3}{4} - \pi\right) 1(k-1)$$