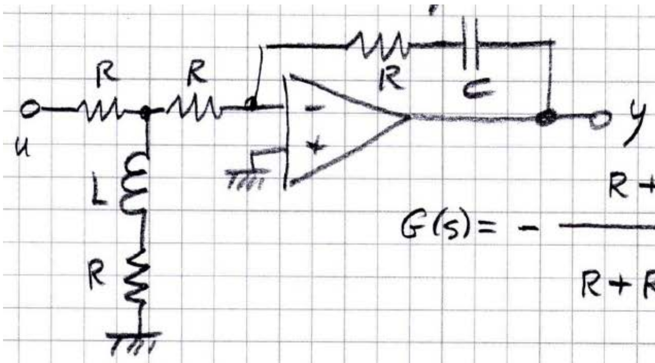
Tracce delle soluzioni

1.



1. f.d.t. $G(s) = -\dfrac{Z_f}{Z_{i,t}}$

$$G(s) = -\frac{R+\frac{1}{Cs}}{R+R+\frac{R^2}{Ls+R}} = -\frac{\frac{RCs+1}{Cs}}{\frac{2RLs+2R^2+R^2}{Ls+R}}$$

$$G(s) = -\frac{RCs+1}{Cs}\cdot\frac{Ls+R}{2RLs+3R^2} = -\frac{(RCs+1)(Ls+R)}{RC\cdot s\cdot(2Ls+3R)}$$

2. zeri: $z_1 = -\dfrac{1}{RC}$, $z_2 = -\dfrac{R}{L}$

poli: $p_1 = 0$, $p_2 = -\dfrac{3R}{2L} = -\dfrac{3}{2}\cdot\dfrac{R}{L}$

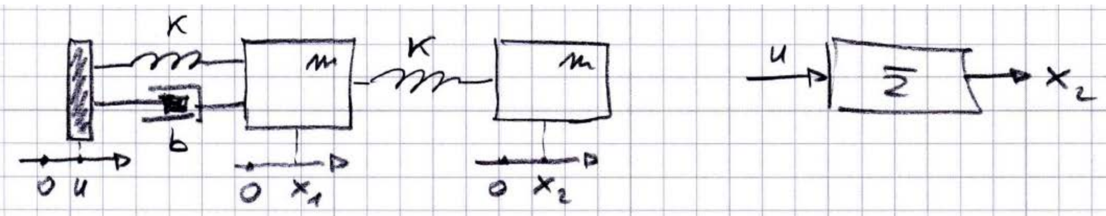
$$\text{modi} = \left\{1, \exp\left\{-\frac{3}{2}\cdot\frac{R}{L}\,t\right\}\right\}$$

3. $$G(s) = -\frac{RCLs^2+R^2Cs+Ls+R}{2RCL\cdot s^2+3R^2C\cdot s} = -\frac{RCLs^2+(R^2C+L)s+R}{2RCL\cdot s^2+3R^2C\cdot s}$$

eq. diff. $2RCL\,D^2y(t) + 3R^2C\,Dy(t) =$

$= -RCL\,D^2u(t) - (R^2C+L)\,Du(t) - R\,u(t)$

2



$$\begin{cases} m D^2 x_1 = -k(x_1 - u) - b(D x_1 - D u) + k(x_2 - x_1) \\ m D^2 x_2 = -k(x_2 - x_1) \end{cases}$$

$$\begin{cases} m D^2 x_1 = -k x_1 + k u - b D x_1 + b D u + k x_2 - k x_1 \\ m D^2 x_2 = -k x_2 + k x_1 \end{cases}$$

$$\begin{cases} m D^2 x_1 + b D x_1 + 2 k x_1 = k x_2 + b D u + k u \\ k x_1 = m D^2 x_2 + k x_2 \end{cases}$$

$$\begin{matrix} k \\ (mD^2 + bD + 2k) \end{matrix} \begin{cases} (mD^2 + bD + 2k) x_1 = k x_2 + (bD + k) u \\ k x_1 = (mD^2 + k) x_2 \end{cases}$$

$$(mD^2 + bD + 2k)(mD^2 + k) x_2 = k^2 x_2 + k(bD + k) u$$

$$(m^2 D^4 + k m D^2 + b m D^3 + b k D + 2 k m D^2 + 2 k^2 - k^2) x_2 = k(bD + k) u$$

$$\boxed{m^2 D^4 x_2 + b m D^3 x_2 + 3 k m D^2 x_2 + b k D x_2 + k^2 x_2 = k b D u + k^2 u}$$

f.d.t. $\boxed{G(s) = \dfrac{k b s + k^2}{m^2 s^4 + b m s^3 + 3 k m s^2 + b k s + k^2}}$

3.
Vedi le dispense del corso.

4.
Vedi le dispense del corso.

5.

$$G(s) = \frac{1}{s+1}, \quad u(t) = \sin t, \quad y(t) = \frac{1}{\sqrt{2}} \sin\left(t - \frac{\pi}{4}\right), \; t<0$$

eq. diff. $\quad Dy(t) + y(t) = u(t)$

1) Per $t<0$:

$$Dy(t) + y(t) = \frac{1}{\sqrt{2}} \cos\left(t - \frac{\pi}{4}\right) + \frac{1}{\sqrt{2}} \sin\left(t - \frac{\pi}{4}\right) =$$

$$= \frac{1}{\sqrt{2}} \left[\cos t \cdot \cos\frac{\pi}{4} + \sin t \cdot \sin\frac{\pi}{4} \right]$$

$$+ \frac{1}{\sqrt{2}} \left[\sin t \cdot \cos\frac{\pi}{4} - \cos t \cdot \sin\left(\frac{\pi}{4}\right) \right]$$

$$= \frac{1}{\sqrt{2}} \left[\cancel{\frac{1}{\sqrt{2}} \cos t} + \frac{1}{\sqrt{2}} \sin t \right] + \frac{1}{\sqrt{2}} \left[\frac{1}{\sqrt{2}} \sin t - \cancel{\frac{1}{\sqrt{2}} \cos t} \right]$$

$$= \frac{1}{2} \sin t + \frac{1}{2} \sin t = \sin t = u(t)$$

2) Per qualsivoglia ingresso, $y(t) \in C^{\rho-1}$ dove ρ è il grado relativo del sistema. Nel nostro caso $\rho = 1$ e quindi $y \in C^0$.

$$y(0^-) = y(0^+), \quad y(0^-) = \frac{1}{\sqrt{2}} \sin\left(-\frac{\pi}{4}\right) = -\frac{1}{2}$$

$y(t) = c\, e^{-t}, \; t \geq 0$ perché $u(t) = 0$ per $t \geq 0$.

$y(0^+) = c$, quindi $c = -\frac{1}{2}$

$$y(t) = -\frac{1}{2} e^{-t}, \; t \geq 0$$

3) $(u, y) \in \mathcal{B}$ e $u(t)$ è continua su $\mathbb{R}$ ma non così la derivata prima di $u(t)$: $u \in \overline{C^0}$ (il grado massimo dell'ingresso è 0). Dalla proprietà $u \in \overline{C^l} \iff y \in \overline{C^{\rho+l}}$, $l \in \mathbb{N}$ segue che il grado massimo di continuità dell'uscita è 1.

6.

$$G(s)=\frac{s+1}{(s+2)[(s+1)^2+4]}\ ,\quad u(t)=t\,1(t)\ ,\quad U(s)=\frac{1}{s^2}$$

$$Y(s)=\frac{s+1}{s^2(s+2)\left[(s+1)^2+4\right]}=\frac{K_{11}}{s^2}+\frac{K_{12}}{s}+\frac{K_2}{s+2}+\frac{K_3}{s+1-j2}+\frac{\overline{K}_3}{s+1+j2}$$

$$\underline{K_{11}}=\frac{s+1}{(s+2)\left[(s+1)^2+4\right]}\Bigg|_{s=0}=\frac{1}{2\cdot 5}=\frac{1}{10}$$

$$\underline{K_2}=\frac{s+1}{s^2\left[(s+1)^2+4\right]}\Bigg|_{s=-2}=\frac{-1}{4\cdot[5]}=-\frac{1}{20}$$

$$\underline{K_3}=\frac{s+1}{s^2(s+2)\,(s+1+j2)}\Bigg|_{s=-1+j2}=\frac{\cancel{-1}+j\cancel{2}^{\,1}\cancel{+1}}{(-1+j2)^2(1+j2)\,(j\cancel{4}^{\,2})}=$$

$$=\frac{1}{(1-4-4j)\,(1+j2)\cdot 2}=\frac{1}{(-3-j4)\,(1+j2)\cdot 2}$$

$$=\frac{1}{(-3-j6-j4+8)\cdot 2}=\frac{1}{2(5-j10)}=\frac{1}{50}+j\frac{2}{50}$$

$$K_{12}+K_2+K_3+\overline{K}_3=0$$

$$\underline{K_{12}}=\frac{1}{20}-\frac{1}{50}-\cancel{j\frac{2}{50}}-\frac{1}{50}+\cancel{j\frac{2}{50}}=\frac{1}{100}$$

$$y(t)=\frac{1}{10}t+\frac{1}{100}-\frac{1}{20}e^{-2t}+2e^{-t}\left[\frac{1}{50}\cos(2t)-\frac{2}{50}\sin(2t)\right]$$

$$\boxed{y(t)=\frac{1}{100}+\frac{1}{10}t-\frac{1}{20}e^{-2t}+\frac{1}{25}e^{-t}\left[\cos(2t)-2\sin(2t)\right],\quad t\geq 0}$$