# ES 1

martedì 26 novembre 2019 21:42

a)


$$L\,\dot{i}(t) = u(t) + v_1(t) + v_2(t) - R\,i(t)$$
$$C\,\dot{v}_1(t) = -i(t)$$
$$C\,\dot{v}_2(t) = -i(t)$$

$$x(t) = \begin{bmatrix} i \\ v_1 \\ v_2 \end{bmatrix}$$

$$\dot{x}(t) = \overbrace{\left[\begin{array}{c|c|c} -R/L & 1/L & 1/L \\ \hline -1/C & 0 & 0 \\ \hline -1/C & 0 & 0 \end{array}\right]}^{A} x(t) + \overbrace{\begin{bmatrix} 1/L \\ 0 \\ 0 \end{bmatrix}}^{B} u(t)$$

$$y(t) = R\,i(t) = \underbrace{\begin{bmatrix} R & 0 & 0 \end{bmatrix}}_{C} x(t) + \underbrace{0}_{D}\,u(t)$$

b) $X_R(1) = \operatorname{Im} B = \operatorname{Im} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$X_R(2) = \operatorname{Im}\left[B, AB\right] = \operatorname{Im}\begin{bmatrix} 1/L & -R/L^2 \\ 0 & -1/CL \\ 0 & -1/CL \end{bmatrix} = \operatorname{Im}\begin{array}{c} \phantom{0}\quad M \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \end{array}$$

$$X_R(3) = X_R(2) + \operatorname{Im} AM = X_R(2) + \operatorname{Im}\begin{bmatrix} 2/L \\ 0 \\ 0 \end{bmatrix} = X_R(2)$$

$$X_R = X_R(2) = \operatorname{Im}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

c) $T = \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{array}\right]$, $T^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

$T_1$ $T_2$

$$\hat{A} = T^{-1}AT = T^{-1}\begin{bmatrix} -R/L & 2/L & 1/L \\ -1/C & 0 & 0 \\ -1/C & 0 & 0 \end{bmatrix} = \left[\begin{array}{cc|c} -R/L & 2/L & 1/L \\ -1/C & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right]$$

$A_R$

zeri strutt.

$A_{NR}$

$$\hat{B} = T^{-1}B = \left[\begin{array}{c} 1/L \\ 0 \\ \hline 0 \end{array}\right] \begin{array}{l} \}\ B_R \\ \\ \}\ \text{zeri str.} \end{array}$$

$$\hat{C} = CT = \left[\begin{array}{cc|c} R & 0 & 0 \end{array}\right]$$

$C_R$ $C_{NR}$

d) $H(s) = C_R (sI - A_R)^{-1} B_R + D$

$$= [R \ 0] \begin{bmatrix} s + R/L & -2/L \\ 1/C & s \end{bmatrix}^{-1} \begin{bmatrix} 1/L \\ 0 \end{bmatrix}$$

$$= \frac{s \ R/L}{s(s + R/L) + 2/LC}$$

# ES 2

martedì 26 novembre 2019 21:59

a)
$$\chi_A(\lambda) = \det \begin{bmatrix} \lambda-1 & 0 & 0 \\ 0 & \lambda-1 & 1 \\ 1 & 0 & \lambda-1 \end{bmatrix}$$

$$= (\lambda-1)^3 \qquad \sigma(A) = \{1\}$$

$$\ker(A-I) = \ker \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} = \operatorname{Im} \begin{array}{c} v_1 \\ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \end{array}$$

$$\ker(A-I)^2 = \ker \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \operatorname{Im} \begin{array}{c} \quad v_2 \\ \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \end{array}$$

$$\ker(A-I)^3 = \ker 0 = \mathbb{R}^3 \qquad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\mu_A(\lambda) = (\lambda-1)^3$$

b)
$$A^k v_2 = 1^k v_2 + k\, 1^{k-1} (A-I) v_2$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ -k \\ 0 \end{bmatrix}$$

$$A^k v_3 = 1^k v_3 + k\, 1^{k-1} (A-I) v_3 + \frac{k(k-1)}{2} 1^{k-2} (A-I)^2 v_3$$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -k \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{k(k-1)}{2} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{k(k-1)}{2} \\ -k \end{bmatrix}$$

$$A^k = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -k & \frac{k(k-1)}{2} \\ 0 & 1 & -k \end{bmatrix} \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^{-1}}_{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{k(k-1)}{2} & 1 & -k \\ -k & 0 & 1 \end{bmatrix}$$

c) NON è né ASINT. STABILE, né semplicemente stabile

ES 3

martedì 26 novembre 2019 22:12

$$X_R(1) = \text{Im}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{bmatrix}$$

$$X_R(2) = \text{Im}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \text{Im}\begin{array}{c} \begin{array}{ccc} & & M \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$X_R(3) = X_R(2) + \text{Im}\, AM = X_R(2) + \text{Im}\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} = X_R(2) = X_R$$

b) $x_1 \notin X_R(1)$

$x_1 \in X_R(2) \Rightarrow$ servono 2 passi

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = [B, AB]\begin{bmatrix} u(1) \\ u(0) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} u(1) \\ u(0) \end{bmatrix}$$

un controllo è dato da

$$u(1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad u(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

ES 4) a)

martedì 26 novembre 2019 22:18

$$R = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \quad q = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad P^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_c = PAP^{-1} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & -1 & -1 \end{bmatrix} \quad P^{-1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & -1 & 0 \end{bmatrix}$$

$$b_c = Pb = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

b) $d(\lambda) = (\lambda+1)^4 = 1 + 4\lambda + 6\lambda^2 + 4\lambda + 1$

$$f_c = \begin{bmatrix} 0 & -4 & -5 & -4 \end{bmatrix}$$

$$f = f_c P = \begin{bmatrix} -4 & -4 & -5 & -9 \end{bmatrix}$$

# ES 3)

martedì 26 novembre 2019 22:26

$$\begin{bmatrix} \dot{x}(t) \\ \dot{z}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} A & B \\ 0 & Q \end{bmatrix}}_{M} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}$$

$$\sigma(M) = \sigma(A) \cup \sigma(Q)$$

GLI AUTOV. DI $M$ SONO TUTTI a parte reale $< 0$ $\iff$ GLI AUTOVALORI DI $A$ e $Q$ sono a parte reale $< 0$

<u>Controesempio</u>

$A = 0 \in \mathbb{R}$, $B = 1 \in \mathbb{R}$,
$Q = 0 \in \mathbb{R}$

$$M = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\chi_M(\lambda) = \mu_M(\lambda) = \lambda^2 \Longrightarrow$$

il sistema $\dot{x}(t) = M x(t)$ non è semplicemente stabile.