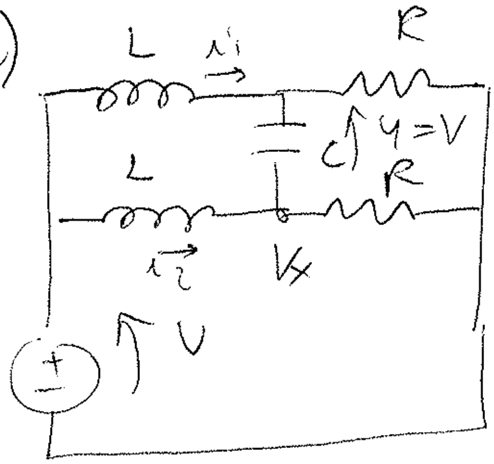
1) a)



$$i_2 + i_1 = \frac{V_x}{R} + \frac{V_x + V}{R}$$

$$R(i_2 + i_1) = 2V_x + V$$

$$V_x = \frac{-V + R(i_2 + i_1)}{2}$$

$$L \dot{i}_1 = u - V - V_x = u - \frac{V}{2} - \frac{R(i_2 + i_1)}{2}$$

$$L \dot{i}_2 = u - V_x = u + \frac{V}{2} - \frac{R(i_2 + i_1)}{2}$$

$$C \dot{V} = i_1 - \frac{(V_x + V)}{R} = i_1 - \frac{V}{2R} - \frac{i_2 - i_1}{2}$$

$$x = \begin{bmatrix} i_1 \\ i_2 \\ V \end{bmatrix} \qquad \dot{x} = \overbrace{\begin{bmatrix} -\frac{R}{2L} & -\frac{R}{2L} & -\frac{1}{2L} \\ -\frac{R}{2L} & -\frac{R}{2L} & \frac{1}{2L} \\ \frac{1}{2C} & -\frac{1}{2C} & -\frac{1}{2RC} \end{bmatrix}}^{A} x + \overbrace{\begin{bmatrix} 1/2 \\ 1/2 \\ 0 \end{bmatrix}}^{B} u$$

$$y = V = \underbrace{[0 \ 0 \ 1]}_{C} x$$

b) $X_R(1) = \mathrm{Im}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$; $X_R(2) = X_R(1) + \mathrm{Im}\begin{bmatrix} -\frac{R}{2L} \\ -\frac{R}{2L} \\ 0 \end{bmatrix} = X_R(1)$

$$X_R = \mathrm{Im}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

c) $T_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\quad T_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\hat{A} = T^{-1} A T = T^{-1} \begin{bmatrix} -\frac{R}{L} & -\frac{R}{2L} & -\frac{1}{2L} \\ -\frac{R}{L} & -\frac{R}{2L} & \frac{1}{2L} \\ 0 & -\frac{1}{2C} & -\frac{1}{2RC} \end{bmatrix} = \left[\begin{array}{c|cc} \underbrace{-\frac{R}{L}}_{A_R} & \frac{R}{2L} & -\frac{1}{2L} \\ \hline 0 & 0 & \frac{1}{C} \\ 0 & \frac{1}{2C} & -\frac{1}{2RC} \end{array}\right]$$

A_{12}, A_{NR}; $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \to 0$

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \hat{B} = T^{-1} B = \begin{bmatrix} 1/L \\ 0 \\ 0 \end{bmatrix}$$

B_R; $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \to 0$

$$\hat{C} = C T = \begin{array}{cc} C_R & C_{NR} \\ \left[\, 0 \right| & \left. 0 \;\; 1 \,\right] \end{array}$$

d) ~~[illegible]~~

$$H(s) = C_R (sI - A_R)^{-1} B_R = 0$$

2) a)

$$\ker [A-\lambda I] = \ker \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & -5 & -5 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \mathrm{Im} \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \not\ni v$$

$$\ker [A-\lambda I]^2 = \ker \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 25 & 25 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \mathrm{Im} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \ni v$$

$\Rightarrow$ v è UN AUTOV. GENERALIZZATO DI INDICE 2

b)

$$x(k) = A^k v = \lambda^k v + (A-\lambda I) v \cdot k \lambda^{k-1}$$

$$= 2^k \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} k\, 2^{k-1}$$

$$= \begin{bmatrix} 2^k \\ 0 \\ -k \cdot 2^{k-1} \\ k \cdot 2^{k-1} \end{bmatrix}$$

3)

a) $X_R(1) = \operatorname{Im} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$X_R(2) = X_R(1) + \operatorname{Im} A \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \operatorname{Im} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

$X_R(3) = X_R(2) + \operatorname{Im} A \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \operatorname{Im} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \operatorname{Im} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$X_R(4) = X_R(3) + \operatorname{Im} A \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \operatorname{Im} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} = \mathbb{R}^4$

$X_R(k) = \mathbb{R}^4, \ \forall k \geq 4$

b)

$x_1 \notin X_R(1), \quad x_1 \notin X_R(2), \quad x_1 \in X_3(3)$

IL NUMERO MINIMO DI PASSI È $k_1 = 3$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} = R_3 \begin{bmatrix} u(2) \\ u(1) \\ u(0) \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u(2) \\ u(1) \\ u(0) \end{bmatrix}$$

$u(2) = 0$

$u(1) = 0$

$u(0) = -1$

4) e)

$$X_R(1) = \operatorname{Im}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \quad X_R(2) = X_R(1) + \operatorname{Im}\begin{bmatrix}-1\\-2\\0\\2\end{bmatrix}$$

$$= \operatorname{Im}\begin{bmatrix}1&0\\0&1\\0&0\\0&-1\end{bmatrix}$$

$$X_R(3) = X_R(2) + \operatorname{Im}\begin{bmatrix}0\\1\\0\\-1\end{bmatrix} = X_R(2) = X_R$$

$$T_1 = \begin{bmatrix}1&0\\0&1\\0&0\\0&-1\end{bmatrix} \qquad T_2 = \begin{bmatrix}0&0\\0&0\\1&0\\0&1\end{bmatrix}$$

$$T = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&-1&0&1\end{bmatrix}, \quad T^{-1} = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&1&0&1\end{bmatrix}$$

$$\hat{A} = T^{-1}AT = T^{-1}\begin{bmatrix}-1&0&0&0\\-2&1&1&1\\0&0&-2&0\\2&-1&0&1\end{bmatrix} = \begin{array}{c} \begin{matrix}A_R & \qquad A_{12}\end{matrix} \\ \left[\begin{array}{cc|cc}-1&0&0&0\\-2&1&1&1\\ \hline 0&0&-2&0\\0&0&1&2\end{array}\right] \\ \begin{matrix}0 & \qquad A_{NR}\end{matrix}\end{array}$$

$$\hat{B} = T^{-1}B = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}\begin{matrix}\} B_R \\ \to 0\end{matrix} \qquad \hat{C} = CT = \begin{bmatrix}1 & 0 & 1 & 0\end{bmatrix}$$

b) $H(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s+1 & 0 \\ 2 & s-1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$= \frac{s-1}{(s+1)(s-1)} = \frac{1}{s+1}$$

c) $\sigma(A_n) = \{-1, 1\}$ $\quad \sigma(A_{Nn}) = \{-2, 2\}$

5) $X_R(1) = \mathrm{Im}\begin{bmatrix}0\\1\\1\end{bmatrix}$, $X_R(2) = \mathrm{Im}\begin{bmatrix}0 & a-1\\1 & 1\\1 & -3\end{bmatrix}$

$$= \mathrm{Im}\begin{bmatrix}0 & a-1\\1 & 0\\1 & -4\end{bmatrix}$$

$$X_R(3) = X_R(2) + \mathrm{Im}\begin{bmatrix}0\\0\\-a(a-1)+4\end{bmatrix} = \mathrm{Im}\overbrace{\begin{bmatrix}0 & a-1 & 0\\1 & 0 & 0\\1 & -4 & -a^2+a+4\end{bmatrix}}^{M}$$

$$\det M = -1\cdot(a-1)\left(-a^2+a+4\right)$$

$$a^2-a-4=0 \rightarrow a = \frac{1\pm\sqrt{1+16}}{2} = \frac{1\pm\sqrt{17}}{2}$$

(A,B) è RAGGIUNGIBILE per $a \notin \left\{1, \frac{1\pm\sqrt{17}}{2}\right\}$

6) $z = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\dot{z} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \overbrace{\begin{bmatrix} A_1 & 0 \\ B_2 F & A_2 \end{bmatrix}}^{\hat{A}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\sigma(\hat{A}) = \sigma(A_1) \cup \sigma(A_2)$$

$\Rightarrow$ $\hat{A}$ è di Hurwitz $\Longleftrightarrow$ A_1, A_2 sono di Hurwitz.