

ONLINE APPENDIX TO
Bayesian Local Projections

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Abstract

This Online Appendix contains additional details and robustness exercises for the paper ‘Bayesian Local Projections’.

Keywords: Local Projections, VARs, Bayesian Techniques, Impulse Response Functions, Direct Forecasting

JEL Classification: C32; C11; C14.

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A Data Construction and Transformations

Variables Construction (link to download page behind FRED-code)

- **Real GDP: RGDP** $\equiv \log \left(\frac{GDPC1}{POP} \right)$

GDPC1 Real Gross Domestic Product, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate

POP Total Population: All Ages including Armed Forces Overseas, Thousands, Quarterly, Not Seasonally Adjusted

- **Real Consumption: RCON** $\equiv \log \left(\frac{PCND+PCESV}{GDPDEF \times POP} \right)$

PCND Personal Consumption Expenditures: Nondurable Goods, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate

PCESV Personal Consumption Expenditures: Services, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate

GDPDEF Gross Domestic Product: Implicit Price Deflator, Index 2009=100, Quarterly, Seasonally Adjusted

- **Real Investment: RINV** $\equiv \log \left(\frac{PCDG+GPDI}{GDPDEF \times POP} \right)$

PCDG Personal Consumption Expenditures: Durable Goods, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate

GPDI Gross Private Domestic Investment, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate

- **Total Hours Worked: HOUR** $\equiv \log \left(\frac{HOANBS}{POP} \right)$

HOANBS Nonfarm Business Sector: Hours of All Persons, Index 2009=100, Quarterly, Seasonally Adjusted

- **Real Compensation per Hour: WAGE** $\equiv \log (COMPRNFB)$

COMPRNFB Nonfarm Business Sector: Real Compensation Per Hour, Index 2009=100, Quarterly, Seasonally Adjusted

- **Federal Funds Rate: FFR** $\equiv \frac{FEDFUNDS}{4}$

FEDFUNDS Effective Federal Funds Rate, Percent, Quarterly, Not Seasonally Adjusted

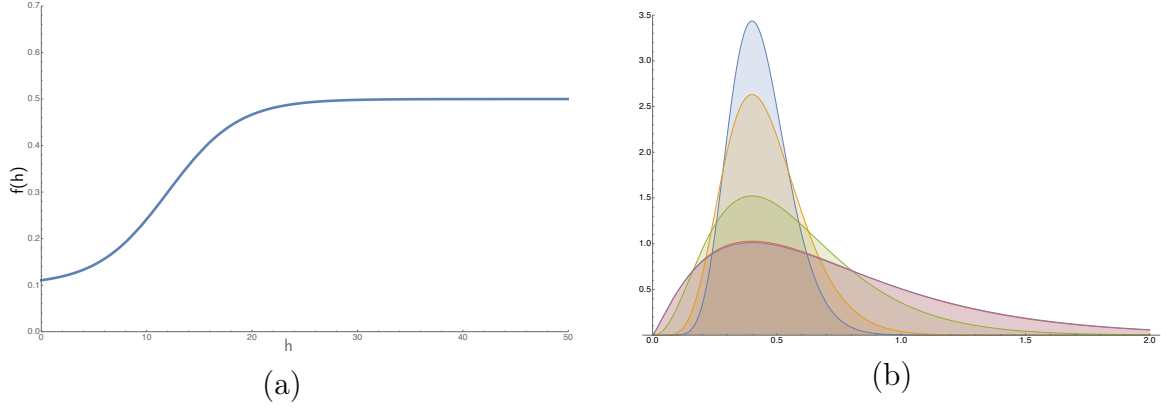
TABLE A.1: DATA AND TRANSFORMATIONS

		Transformations			
		Per Capita	DSGE	IRFs	FORECASTS
RGDP	Real Gross Domestic Product	•	log-diff	log	log
RCON	Real Consumption	•	log-diff	log	log
RINV	Real Investment	•	log-diff	log	log
HOUR	Hours Worked in Non-Farm Sector	•	log	log	log
WAGE	Real Compensation per Hour		log-diff	log	log
DEFL	GDP Deflator		log-diff	log	log
FFR	Federal Funds Rate		level/4	level/4	level/4

Note: Original data series are retrieved from FRED.

B Hyperpriors

FIGURE B.1: HYPERPRIOR FOR BLP COEFFICIENTS



Note: (a) Shifted Logistic function that regulates the variance of the hyperprior for $\lambda(h)$. (b) Hyperprior for $\lambda(h)$ at different horizons. At $h = 1$, the hyperprior has mode equal to 0.4 and standard deviation equal to 0.12 (blue line). The standard deviation increases to 0.16 at $h = 6$ (orange), to 0.30 at $h = 12$ (green), to 0.49 at $h = 24$ (red), and to 0.5 at $h = 36$ (purple).

C Additional Results Simulations

TABLE C.1: COVERAGE ACCURACY

T	Model	Interval	Horizon							Mean
			2	5	8	11	14	17	20	
80	SLP	68	0.391	0.351	0.341	0.357	0.359	0.365	0.399	0.364
		95	0.685	0.613	0.624	0.635	0.639	0.650	0.665	0.642
	LP	68	0.539	0.506	0.492	0.479	0.467	0.441	0.419	0.487
		95	0.849	0.824	0.809	0.785	0.764	0.725	0.698	0.786
	BLP	68	—	—	—	—	—	—	—	—
		95	—	—	—	—	—	—	—	—
	BVAR	68	0.627	0.545	0.582	0.616	0.641	0.683	0.711	0.625
		95	0.919	0.883	0.915	0.959	0.975	0.987	0.991	0.945
120	SLP	68	0.441	0.399	0.413	0.438	0.448	0.461	0.478	0.438
		95	0.760	0.695	0.714	0.734	0.749	0.763	0.778	0.738
	LP	68	0.557	0.549	0.543	0.554	0.525	0.504	0.503	0.536
		95	0.880	0.859	0.859	0.849	0.826	0.804	0.797	0.842
	BLP	68	0.691	0.767	0.850	0.915	0.943	0.960	0.954	0.860
		95	0.951	0.965	0.987	0.995	0.993	0.997	0.996	0.981
	BVAR	68	0.604	0.509	0.532	0.565	0.592	0.635	0.665	0.582
		95	0.914	0.859	0.888	0.927	0.955	0.973	0.979	0.926
240	SLP	68	0.455	0.399	0.463	0.512	0.507	0.537	0.562	0.488
		95	0.765	0.709	0.784	0.813	0.833	0.835	0.867	0.799
	LP	68	0.553	0.577	0.591	0.583	0.561	0.561	0.572	0.571
		95	0.867	0.885	0.894	0.882	0.893	0.875	0.873	0.881
	BLP	68	0.597	0.644	0.743	0.881	0.945	0.971	0.978	0.813
		95	0.901	0.935	0.975	0.993	0.999	0.997	0.999	0.970
	BVAR	68	0.545	0.449	0.500	0.523	0.533	0.575	0.617	0.530
		95	0.869	0.819	0.836	0.858	0.886	0.921	0.942	0.875

Note: The table reports average coverage across horizons of the 68% and 95% confidence intervals (credible sets) for SLP and LP (BLP using a VAR-based prior and BVAR) when the model is estimated with only a subset of the data generating process for $T = 80, 120$, and 240 , and $p = 5$.

TABLE C.2: LENGTH

T	Model	Interval	Horizon							Mean
			2	5	8	11	14	17	20	
80	SLP	68	0.306	0.310	0.332	0.342	0.352	0.379	0.537	0.355
		95	0.603	0.611	0.654	0.675	0.693	0.746	1.059	0.699
	LP	68	0.576	0.660	0.665	0.662	0.650	0.635	0.598	0.635
		95	1.134	1.300	1.310	1.305	1.281	1.252	1.179	1.251
	BLP	68	—	—	—	—	—	—	—	—
		95	—	—	—	—	—	—	—	—
	BVAR	68	0.449	0.410	0.361	0.291	0.230	0.182	0.144	0.303
		95	0.899	0.857	0.799	0.701	0.616	0.545	0.489	0.713
120	SLP	68	0.292	0.303	0.327	0.340	0.350	0.379	0.518	0.349
		95	0.575	0.597	0.645	0.671	0.691	0.746	1.022	0.687
	LP	68	0.478	0.557	0.571	0.574	0.571	0.573	0.564	0.552
		95	0.943	1.097	1.125	1.131	1.125	1.128	1.111	1.089
	BLP	68	0.540	0.648	0.678	0.704	0.723	0.739	0.742	0.688
		95	1.064	1.278	1.337	1.388	1.424	1.457	1.463	1.357
	BVAR	68	0.389	0.357	0.319	0.254	0.198	0.154	0.120	0.263
		95	0.776	0.736	0.680	0.577	0.486	0.414	0.355	0.587
240	SLP	68	0.237	0.256	0.282	0.294	0.300	0.319	0.416	0.294
		95	0.467	0.504	0.556	0.580	0.591	0.629	0.820	0.578
	LP	68	0.343	0.409	0.430	0.435	0.436	0.441	0.440	0.416
		95	0.676	0.806	0.848	0.857	0.859	0.870	0.867	0.820
	BLP	68	0.363	0.431	0.457	0.469	0.474	0.482	0.479	0.455
		95	0.715	0.850	0.900	0.925	0.935	0.951	0.944	0.897
	BVAR	68	0.299	0.279	0.254	0.202	0.154	0.118	0.090	0.206
		95	0.591	0.565	0.521	0.430	0.345	0.279	0.228	0.434

Note: The table reports average length across variables of the 68% and 95% confidence intervals (credible sets) for SLP and LP (BLP using a VAR-based prior and BVAR) when the model is estimated with only a subset of the data generating process for $T = 80, 120$, and 240 , and $p = 5$.

TABLE C.3: COVERAGE ACCURACY

T	Model	Interval	Horizon							Mean
			2	5	8	11	14	17	20	
80	SLP	68	0.404	0.373	0.402	0.415	0.407	0.414	0.453	0.410
		95	0.723	0.667	0.677	0.707	0.717	0.734	0.735	0.706
	LP	68	0.561	0.559	0.521	0.526	0.502	0.491	0.465	0.527
		95	0.884	0.873	0.849	0.845	0.813	0.784	0.738	0.831
	BLP	68	—	—	—	—	—	—	—	—
		95	—	—	—	—	—	—	—	—
	BVAR	68	0.545	0.508	0.483	0.467	0.449	0.449	0.517	0.491
		95	0.858	0.855	0.857	0.875	0.885	0.880	0.895	0.874
120	SLP	68	0.437	0.406	0.451	0.448	0.457	0.475	0.499	0.450
		95	0.776	0.697	0.742	0.768	0.791	0.806	0.816	0.767
	LP	68	0.565	0.575	0.573	0.577	0.555	0.521	0.541	0.559
		95	0.890	0.884	0.881	0.866	0.853	0.845	0.831	0.866
	BLP	68	0.709	0.758	0.823	0.895	0.904	0.912	0.917	0.841
		95	0.963	0.970	0.982	0.986	0.987	0.985	0.985	0.978
	BVAR	68	0.500	0.467	0.447	0.437	0.409	0.419	0.479	0.454
		95	0.845	0.817	0.801	0.829	0.847	0.865	0.875	0.840
240	SLP	68	0.436	0.392	0.444	0.501	0.509	0.548	0.569	0.485
		95	0.763	0.712	0.792	0.815	0.837	0.839	0.877	0.804
	LP	68	0.531	0.571	0.596	0.580	0.581	0.565	0.576	0.571
		95	0.859	0.881	0.884	0.886	0.894	0.869	0.891	0.882
	BLP	68	0.595	0.631	0.757	0.865	0.928	0.948	0.958	0.803
		95	0.920	0.947	0.979	0.995	0.997	0.998	0.998	0.975
	BVAR	68	0.427	0.368	0.352	0.361	0.381	0.402	0.441	0.392
		95	0.749	0.745	0.710	0.710	0.734	0.750	0.768	0.742

Note: The table reports average coverage across horizons of the 68% and 95% confidence intervals (credible sets) for SLP and LP (BLP using a VAR-based prior and BVAR) when the model is estimated with only a subset of the data generating process for $T = 80, 120$, and 240 , and $p = 2$.

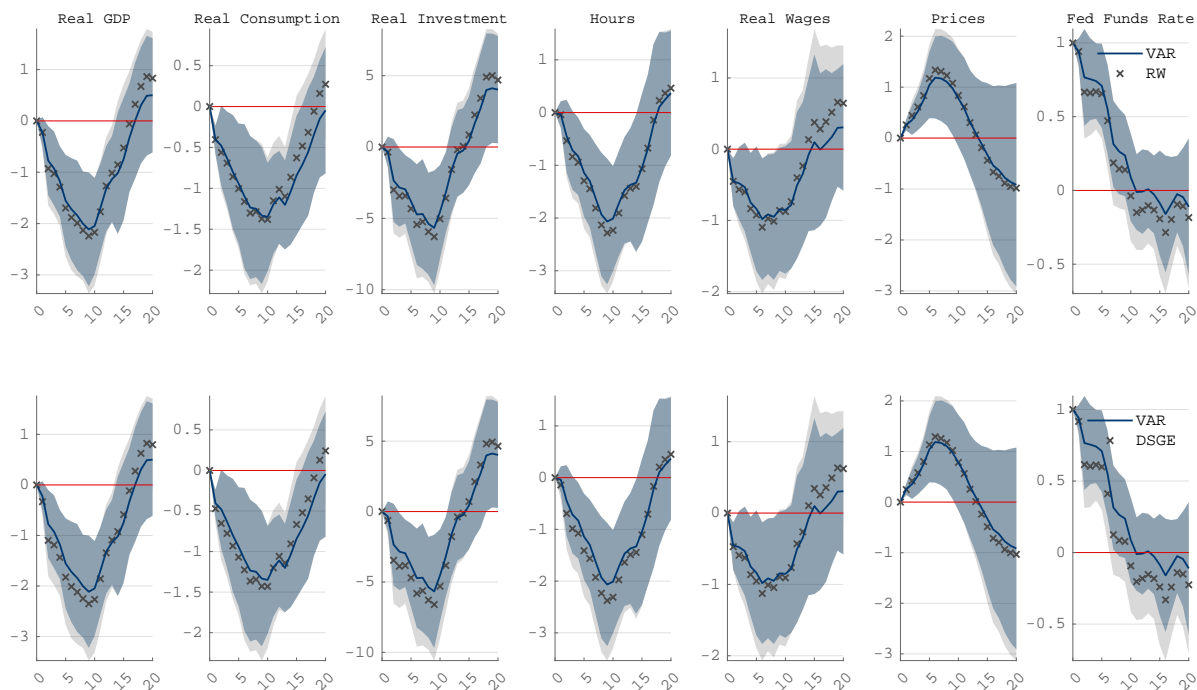
TABLE C.4: LENGTH

T	Model	Interval	Horizon							Mean
			2	5	8	11	14	17	20	
80	SLP	68	0.318	0.332	0.355	0.368	0.379	0.407	0.576	0.379
		95	0.626	0.654	0.700	0.725	0.747	0.803	1.135	0.747
	LP	68	0.569	0.665	0.672	0.669	0.654	0.654	0.618	0.643
		95	1.121	1.310	1.324	1.318	1.288	1.290	1.218	1.267
	BLP	68	—	—	—	—	—	—	—	—
		95	—	—	—	—	—	—	—	—
	BVAR	68	0.350	0.361	0.289	0.220	0.165	0.124	0.095	0.240
		95	0.709	0.761	0.664	0.562	0.477	0.410	0.358	0.582
120	SLP	68	0.300	0.318	0.347	0.358	0.367	0.394	0.536	0.364
		95	0.591	0.628	0.683	0.705	0.722	0.776	1.057	0.718
	LP	68	0.478	0.563	0.582	0.586	0.580	0.580	0.573	0.561
		95	0.942	1.110	1.148	1.155	1.142	1.143	1.129	1.105
	BLP	68	0.513	0.622	0.651	0.667	0.676	0.685	0.692	0.639
		95	1.011	1.226	1.284	1.314	1.333	1.349	1.363	1.259
	BVAR	68	0.299	0.313	0.252	0.192	0.144	0.108	0.081	0.208
		95	0.602	0.646	0.551	0.453	0.372	0.307	0.256	0.472
240	SLP	68	0.245	0.269	0.297	0.309	0.316	0.335	0.435	0.308
		95	0.482	0.530	0.585	0.608	0.623	0.659	0.857	0.606
	LP	68	0.345	0.419	0.442	0.447	0.450	0.454	0.455	0.427
		95	0.680	0.826	0.871	0.880	0.886	0.895	0.897	0.841
	BLP	68	0.357	0.436	0.463	0.470	0.480	0.488	0.487	0.450
		95	0.704	0.860	0.912	0.926	0.946	0.962	0.961	0.888
	BVAR	68	0.227	0.242	0.197	0.149	0.111	0.082	0.060	0.160
		95	0.454	0.488	0.410	0.324	0.256	0.201	0.158	0.341

Note: The table reports average length across variables of the 68% and 95% confidence intervals (credible sets) for SLP and LP (BLP using a VAR-based prior and BVAR) when the model is estimated with only a subset of the data generating process for $T = 80, 120$, and 240 , and $p = 2$.

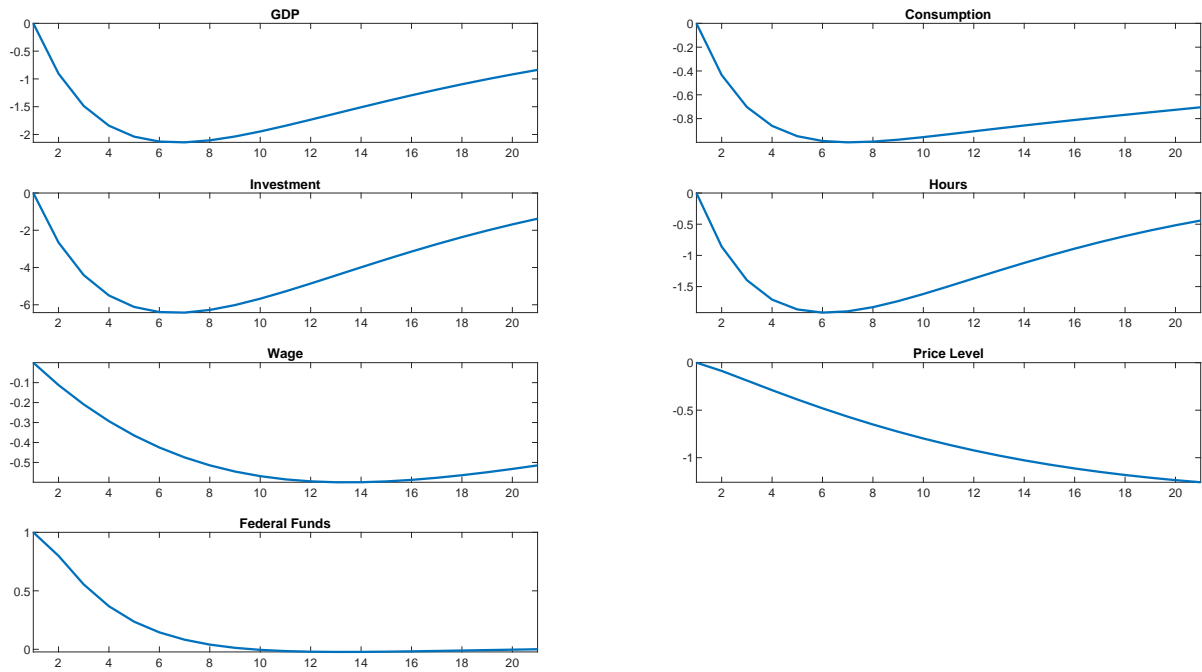
D Additional Results Empirics

FIGURE D.1: BLP RESPONSES: VAR VS RW PRIOR (TOP ROW);
VAR VS MODEL-BASED PRIOR (BOTTOM ROW)



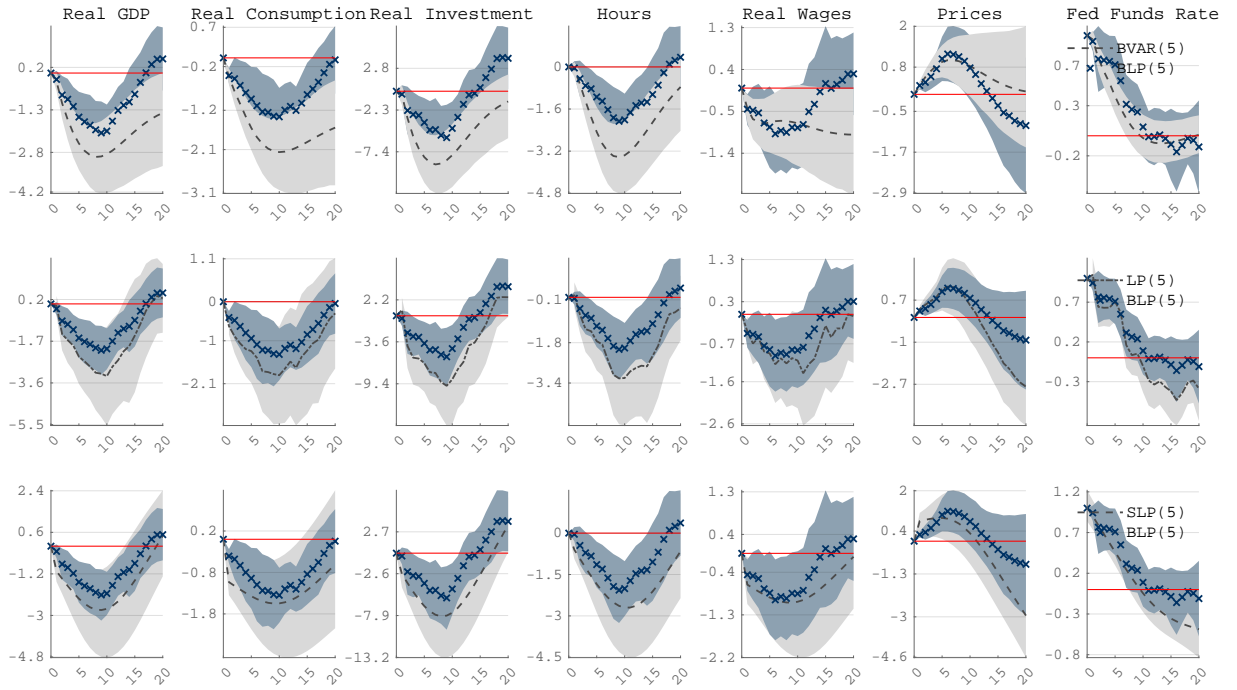
Note: BLP(5) with random walk (RW) prior (markers), and BLP(5) with VAR(5) prior (solid line) in the top row. BLP(5) with model-based (M-BASED) prior (markers), and BLP(5) with VAR(5) prior (solid line) in the bottom row. Estimation sample: 1965Q1 to 2019Q4. Pre-sample: 1954Q3 to 1964Q4. Shaded areas denote 90% posterior coverage bands.

FIGURE D.2: MODEL-BASED PRIOR:
THEORETICAL DSGE IMPULSE RESPONSES TO A MONETARY POLICY SHOCK



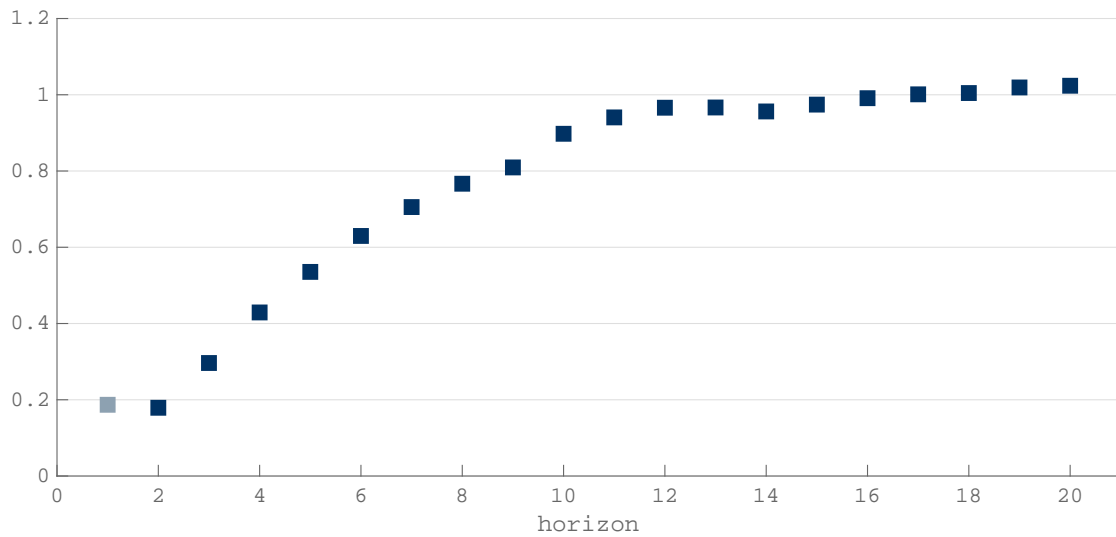
Note: The figure reports the impulse responses to a monetary policy shock at which the model-based prior is centred. They are based on the DSGE model in [Giannone et al. \(2015\)](#). Responses were normalised such that the impact response of FFR is 1%. The IRFs of GDP, Consumption, Investment, Wage and the Price Level were cumulated. Estimation sample: 1954Q3 to 2019Q4.

FIGURE D.3: EMPIRICAL IRFS: BVAR, LP AND BLP



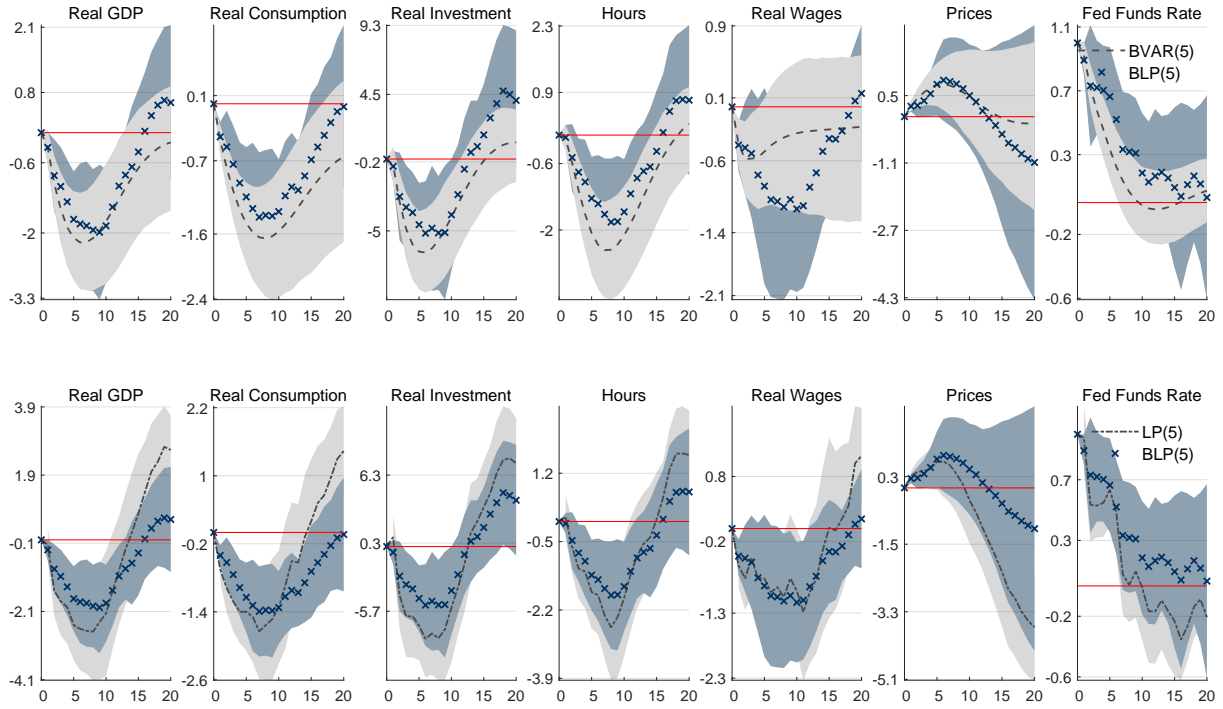
Note: Impulse response functions to a FFR innovation. Top row: BLP(5) and BVAR(5). Bottom row: BLP(5) and LP(5). Estimation sample: 1954Q3 to 2019Q4. BLP uses 1954Q3 to 1964Q4 as a pre-sample. Shaded areas denote 90% posterior coverage bands.

FIGURE D.4: EMPIRICAL OPTIMAL PRIOR TIGHTNESS



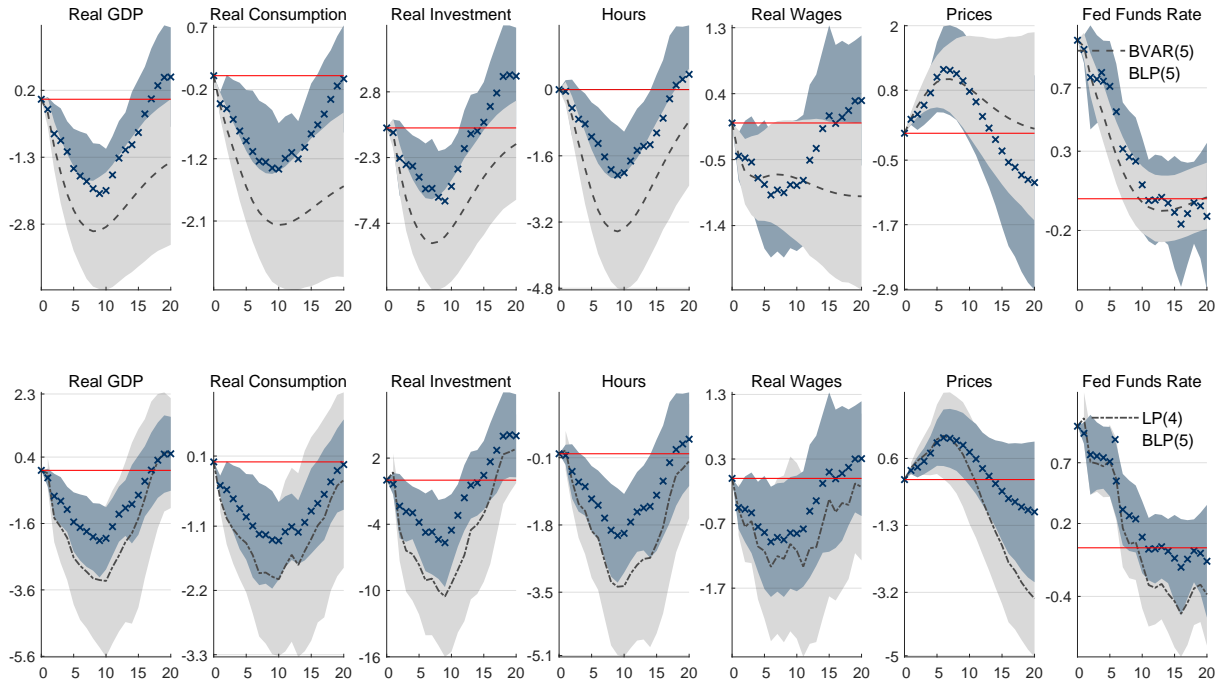
Note: The grey marker is the optimal shrinkage of the [Litterman \(1986\)](#) prior for the BVAR coefficients at $h = 1$, estimated as in [Giannone et al. \(2015\)](#). Blue markers denote the optimal tightness of the VAR prior for BLP coefficients for $h > 1$.

FIGURE D.5: EMPIRICAL IRFS ENDING SAMPLE IN 2007Q4:
BVAR, LP AND BLP



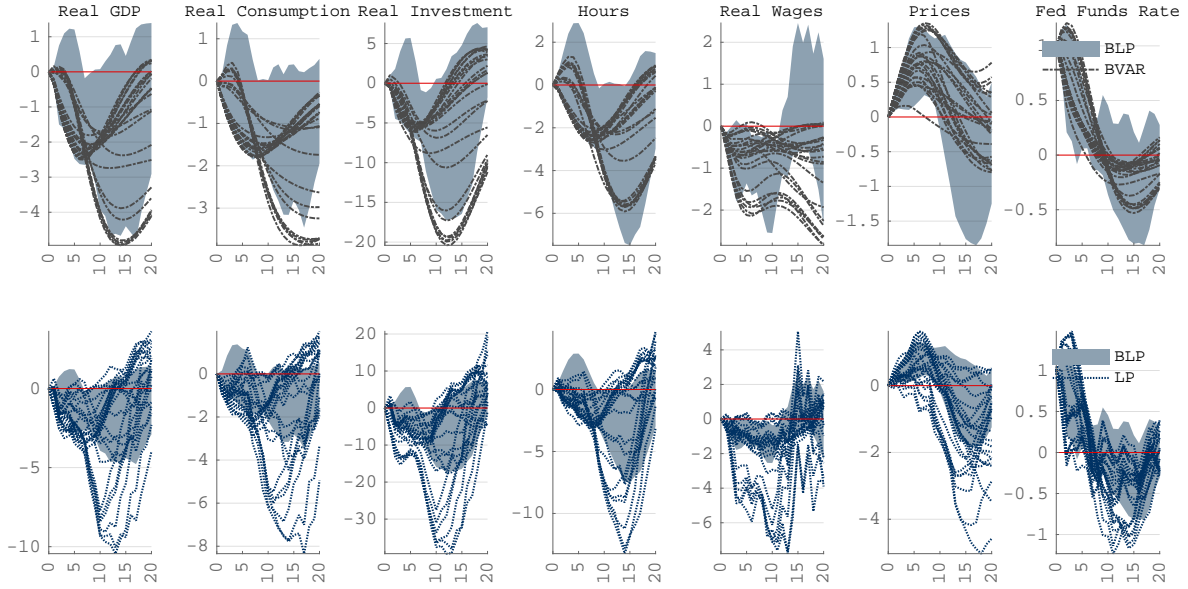
Note: Impulse response functions to a FFR innovation. Top row: BLP(5) and BVAR(5). Bottom row: BLP(5) and LP(5). Estimation sample: 1954Q3 to 2007Q4. BLP uses 1954Q3 to 1964Q4 as a pre-sample. Shaded areas denote 90% posterior coverage bands.

FIGURE D.6: EMPIRICAL IRFs:
BVAR, LP WITH LAG LENGTH SUGGESTED BY AIC AND BLP



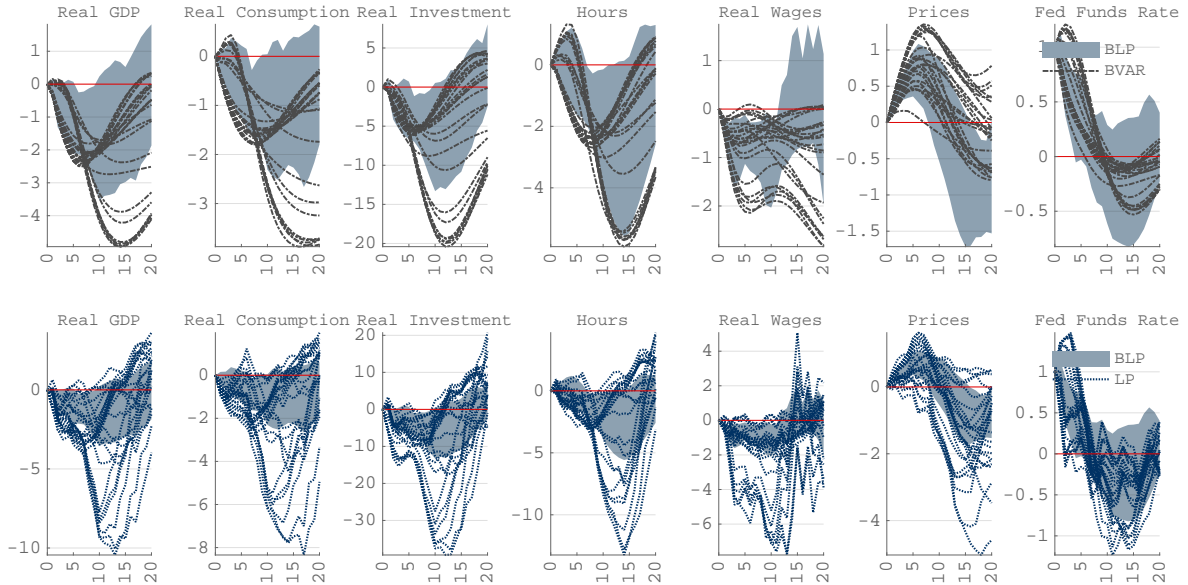
Note: Impulse response functions to a FFR innovation. Top row: BLP(5) and BVAR(5). Bottom row: BLP(5) and LP(4). Estimation sample: 1954Q3 to 2019Q4. BLP uses 1954Q3 to 1964Q4 as a pre-sample. Shaded areas denote 90% posterior coverage bands.

FIGURE D.7: STABILITY OVER SUBSAMPLES: BVAR, LP AND BLP WITH RW PRIOR



Note: BLP(5) with RW prior and BVAR(5) in the top row, and BLP(5) with RW prior and LP(5) in the bottom row. Estimation sample: 1965 to 1995 (first run); 1989 to 2019 (last run).

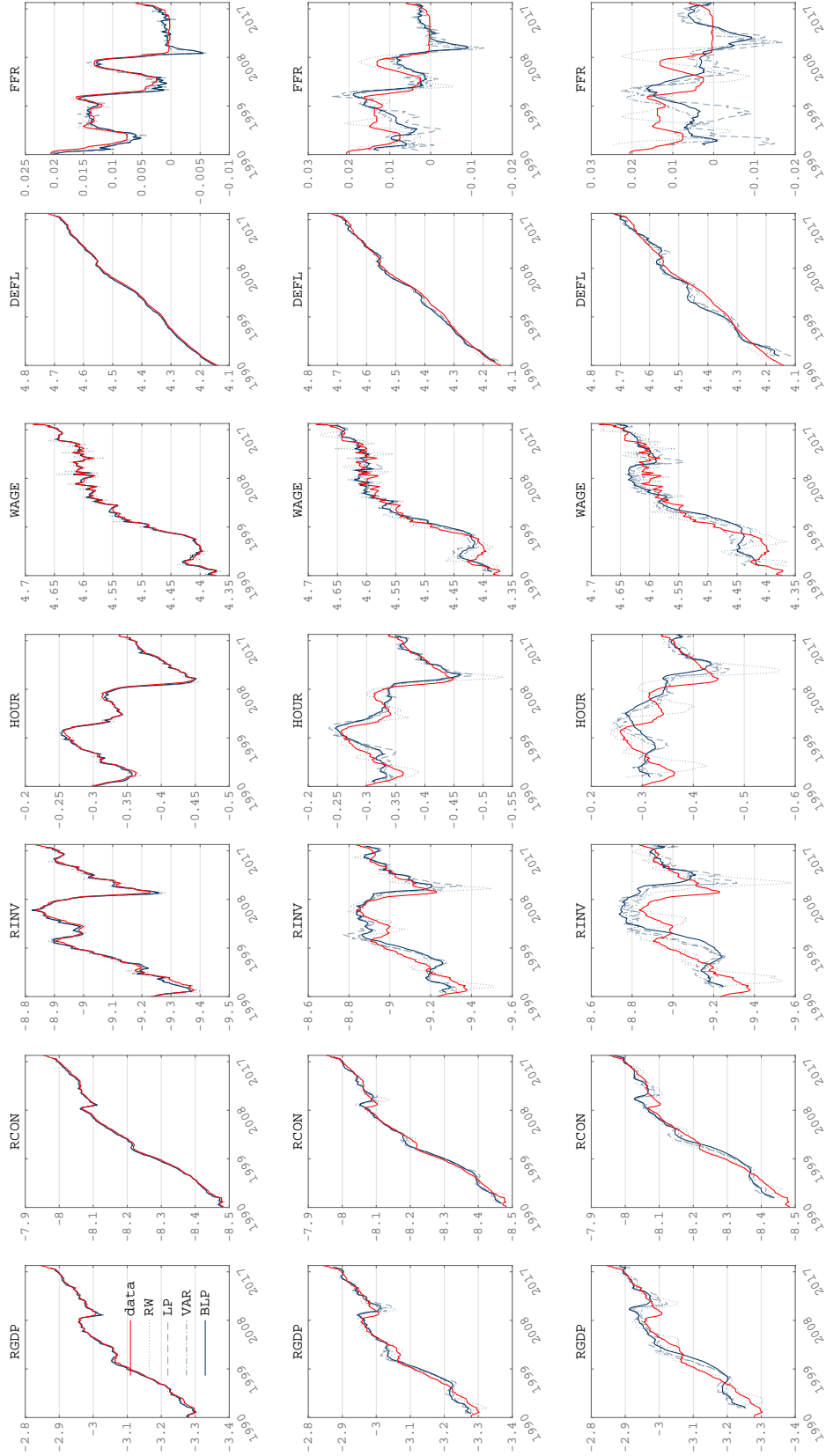
FIGURE D.8: STABILITY OVER SUBSAMPLES: VAR, LP AND BLP WITH VAR PRIOR



Note: BLP(5) with VAR(5) prior and BVAR(5) in the top row, and BLP(5) with VAR(5) prior and LP(5) in the bottom row. Estimation sample: 1965 to 1995 (first run); 1989 to 2019 (last run). Pre-Sample: 10 preceding years.

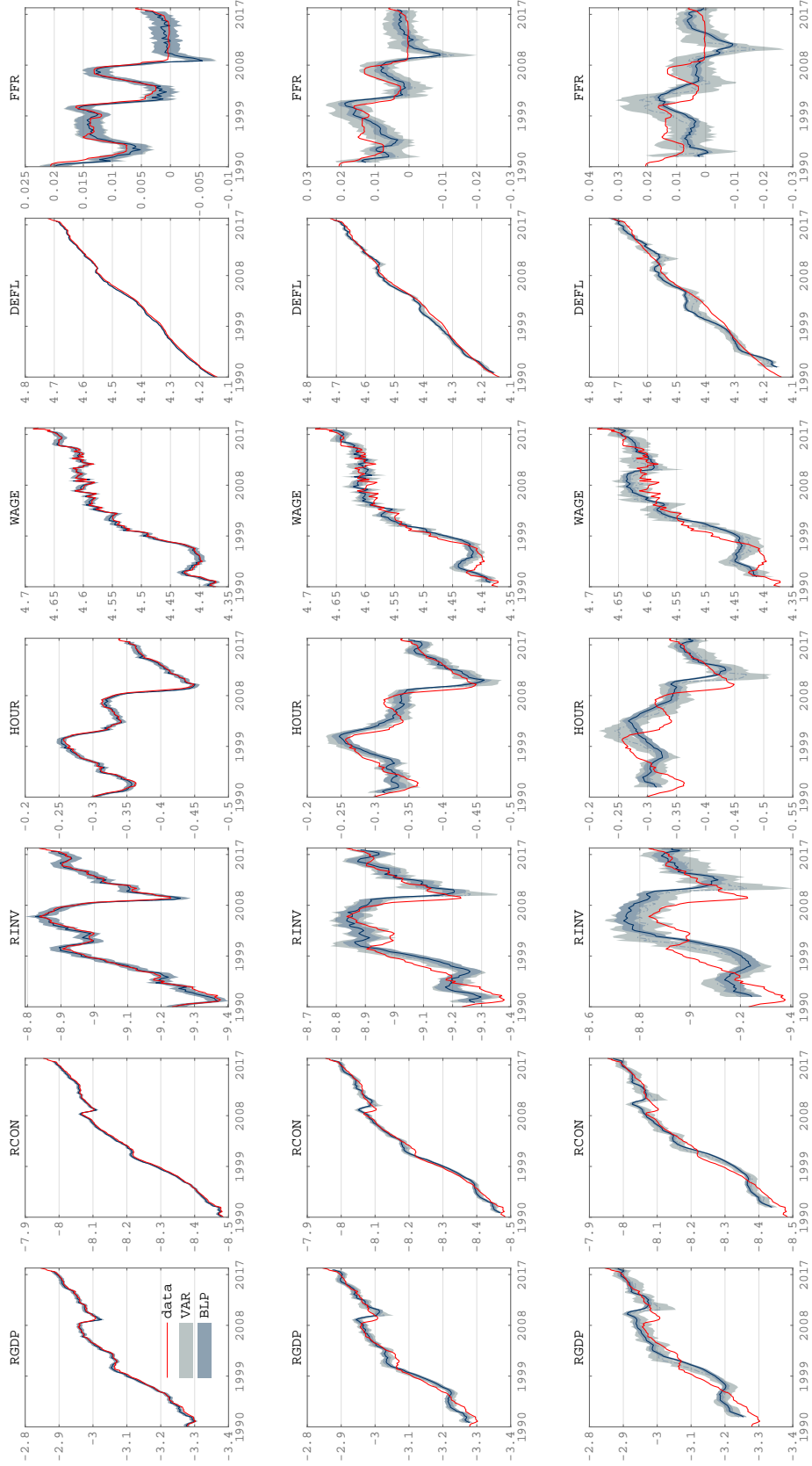
E Additional Results Forecasting

FIGURE E.1: POINT FORECASTS



Note: Point forecasts obtained with RW, LP(5), BVAR(5), BLP(5). Recursive forecasts for all methods start in 1965Q1, the forecast origins go from 1990Q1 to 2017Q4. Top panel: $h = 1$; middle panel: $h = 4$; bottom panel: $h = 8$. Forecast horizon expressed in quarters.

FIGURE E.2: DENSITY FORECASTS



Note: Density forecasts obtained with BVAR(5), BLP(5). Recursive forecasts for all methods start in 1965Q1, the forecast origins go from 1990Q1 to 2017Q4. Top panel: $h = 1$; middle panel: $h = 4$; bottom panel: $h = 8$. Forecast horizon expressed in quarters.

- Results with VAR(5) prior

TABLE E.1: RELATIVE AVERAGE RMSFE – POINT FORECAST

	LP vs RWF			VAR vs RWF			BLP vs RWF		
	$h = 1$	$h = 4$	$h = 8$	$h = 1$	$h = 4$	$h = 8$	$h = 1$	$h = 4$	$h = 8$
RGDP	1.025 (0.785)	1.249 (0.083)	1.167 (0.128)	0.956 (0.575)	1.030 (0.793)	1.026 (0.836)	0.956 (0.575)	1.145 (0.328)	1.213 (0.149)
RCON	1.006 (0.928)	0.995 (0.969)	1.163 (0.141)	0.928 (0.301)	0.822 (0.137)	0.974 (0.834)	0.928 (0.301)	0.918 (0.506)	1.181 (0.267)
RINV	1.061 (0.407)	1.121 (0.480)	0.941 (0.677)	1.031 (0.709)	0.986 (0.941)	0.888 (0.494)	1.031 (0.709)	1.146 (0.500)	1.068 (0.712)
HOURL	1.134 (0.083)	1.142 (0.394)	0.748 (0.133)	0.995 (0.944)	0.911 (0.583)	0.718 (0.058)	0.995 (0.944)	1.037 (0.826)	0.801 (0.146)
WAGE	0.685 (0.000)	0.863 (0.277)	0.850 (0.142)	0.658 (0.000)	0.777 (0.007)	0.870 (0.114)	0.658 (0.000)	0.799 (0.058)	0.821 (0.080)
DEFL	1.112 (0.155)	1.447 (0.046)	1.831 (0.038)	1.023 (0.772)	1.324 (0.011)	1.419 (0.097)	1.023 (0.772)	1.546 (0.010)	1.878 (0.031)
FFR	2.115 (0.000)	1.503 (0.010)	1.284 (0.133)	1.801 (0.000)	1.078 (0.487)	0.771 (0.092)	1.801 (0.000)	1.160 (0.185)	0.727 (0.065)

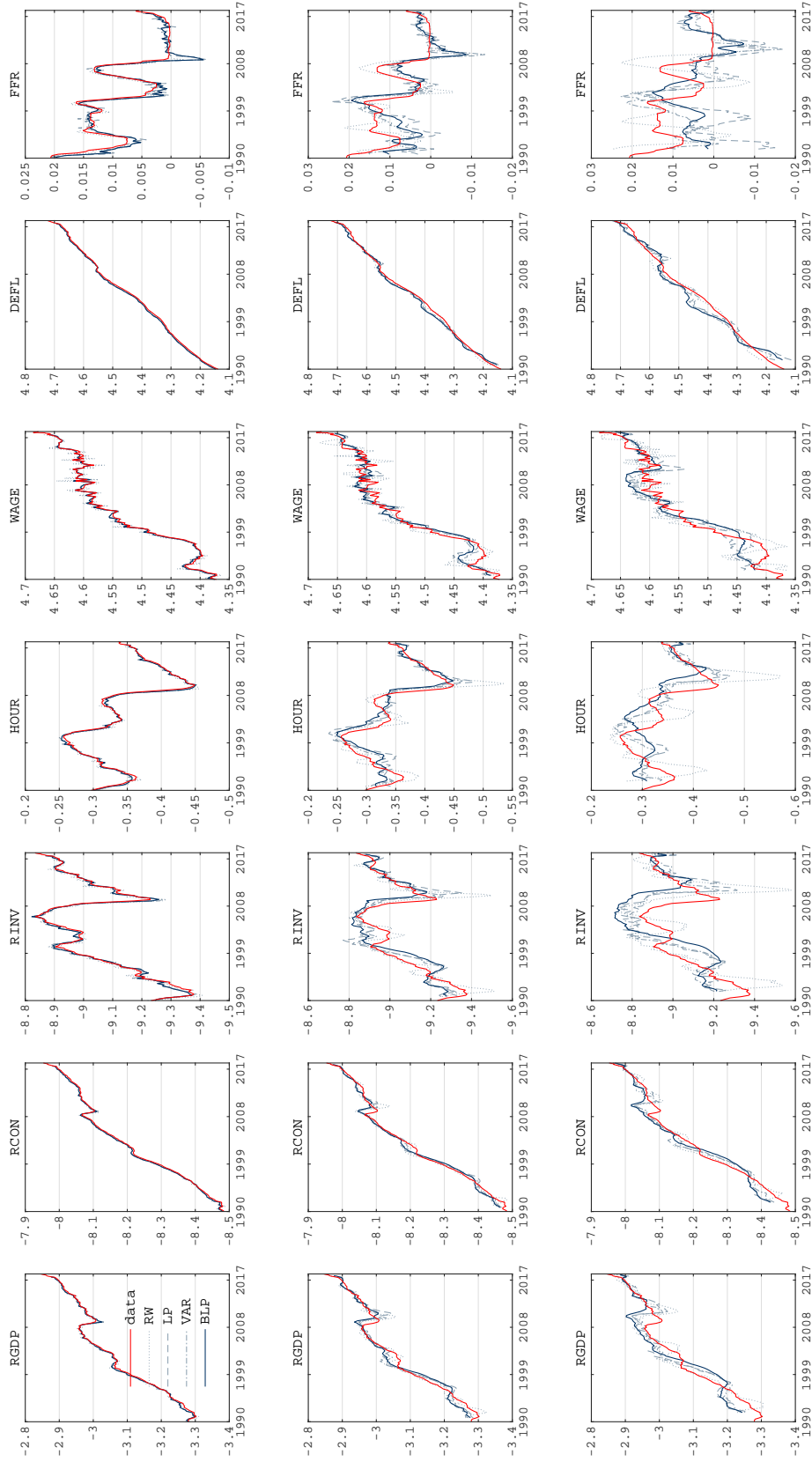
Note: RMSFE. Recursive forecasts for all methods start in 1965Q1, the forecast origins go from 1990Q1 to 2017Q4. LP, VAR and BLP are all estimated with 5 lags. The p -values of [Diebold and Mariano \(1995\)](#)'s test are reported in parentheses.

TABLE E.2: LOG PREDICTIVE SCORES – DENSITY FORECAST

	Relative to LP			Relative to VAR		
	$h = 1$	$h = 4$	$h = 8$	$h = 1$	$h = 4$	$h = 8$
RGDP	0.042	0.088	-0.023	–	0.634	0.874
	(0.283)	(0.650)	(0.765)	–	(0.564)	(0.677)
RCON	0.075	0.080	-0.001	–	0.617	0.762
	(0.207)	(0.531)	(0.800)	–	(0.511)	(0.740)
RINV	-0.015	-0.021	-0.112	–	0.608	0.997
	(0.256)	(0.693)	(0.719)	–	(0.569)	(0.788)
HOUR	0.095	0.096	-0.090	–	0.569	0.959
	(0.248)	(0.610)	(0.969)	–	(0.435)	(0.987)
WAGE	0.040	0.081	0.075	–	0.748	1.050
	(0.221)	(0.484)	(1.122)	–	(0.539)	(0.894)
DEFL	0.073	-0.047	-0.020	–	0.536	0.488
	(0.520)	(0.731)	(1.083)	–	(0.563)	(1.078)
FFR	0.109	0.263	0.715	–	0.627	0.822
	(0.223)	(0.446)	(1.145)	–	(0.596)	(0.838)

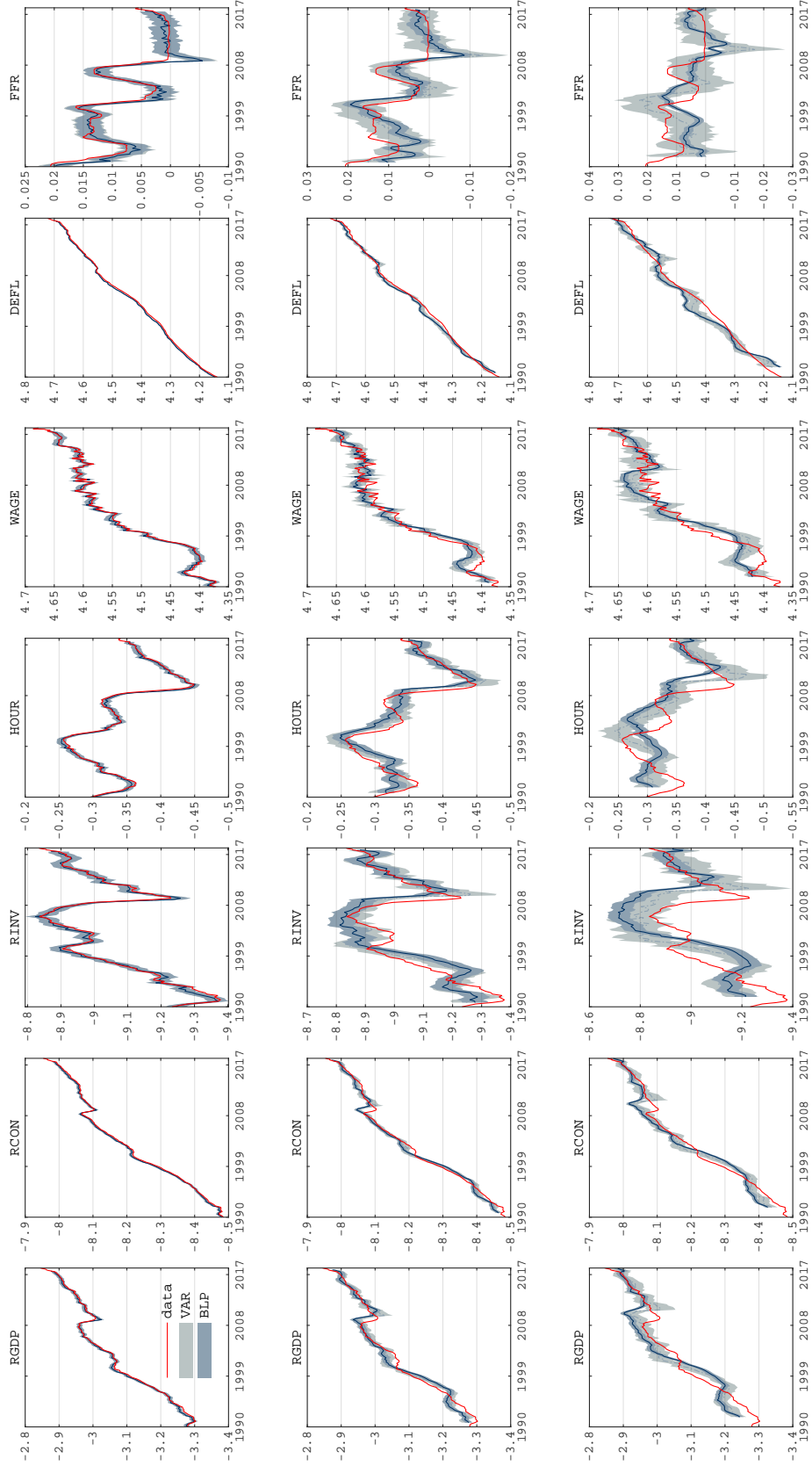
Note: Log predictive scores. Recursive forecasts for all methods start in 1965Q1, the forecast origins go from 1990Q1 to 2017Q4. LP, VAR and BLP are all estimated with 5 lags. The standard deviations are reported in parentheses.

FIGURE E.3: POINT FORECASTS



Note: Point forecasts obtained with RW, LP(5), BVAR(5), BLP(5). Recursive forecasts for all methods start in 1965Q1, the forecast origins go from 1990Q1 to 2017Q4. Top panel: $h = 1$; middle panel: $h = 4$; bottom panel: $h = 8$. Forecast horizon expressed in quarters.

FIGURE E.4: DENSITY FORECASTS



Note: Density forecasts obtained with BVAR(5), BLP(5). Recursive forecasts for all methods start in 1965Q1, the forecast origins go from 1990Q1 to 2017Q4. Top panel: $h = 1$; middle panel: $h = 4$; bottom panel: $h = 8$. Forecast horizon expressed in quarters.

References

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