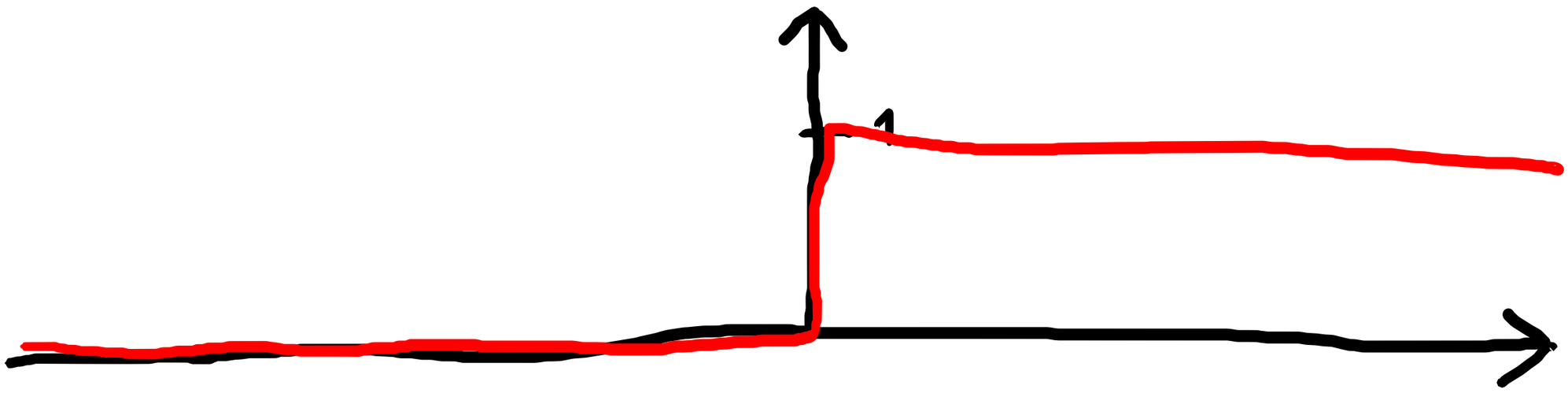
1. Granica całkowita — $0, \infty$



$f(t) = \mathbb{1}(t)\left(1 - e^{-\omega_0 t}\right)$

2. Transformata $f(t) \Rightarrow F(j\omega)$

$$F(j\omega) = \int_0^\infty f(t) e^{-j\omega t} \, dt =$$

$$= \int_0^\infty (1 - e^{-\omega_0 t}) e^{-j\omega t} \, dt = \int_0^\infty (e^{-j\omega t} - e^{-\omega_0 t} \cdot e^{-j\omega t}) \, dt =$$

$$= \int_0^\infty e^{-j\omega t} \, dt - \int_0^\infty e^{-\omega_0 t} \cdot e^{-j\omega t}$$

$$\int e^{-j\omega t}\,dt$$

$\alpha = -j\omega t$

$t = \frac{\alpha}{-j\omega} \;/\cdot d\alpha$

$dt = \frac{d\alpha}{-j\omega}$

$$\int e^{\alpha}\cdot\frac{d\alpha}{-j\omega} = \frac{1}{-j\omega}\cdot e^{\alpha} = \frac{1}{-j\omega}\cdot e^{-j\omega t}$$

$$\int e^{-\omega_0 t} \cdot e^{-j\omega t}\, dt = \int e^{(-\omega_0 t - j\omega t)}\, dt =$$

$$= \int e^{t(-\omega_0 - j\omega)}\, dt$$

$$\alpha = t(-\omega_0 - j\omega) \;/\cdot dt$$

$$d\alpha = dt\,(-\omega_0 - j\omega)$$

$$dt = \frac{d\alpha}{(-\omega_0 - j\omega)}$$

$$\int e^{\alpha} \frac{d\alpha}{(-\omega_0 - j\omega)} = \frac{1}{(-\omega_0 - j\omega)} \cdot e^{\alpha}$$

$$\frac{1}{-j\omega}\cdot e^{-j\omega t} - \frac{1}{(-\omega_0 - j\omega)}\cdot e^{t(-\omega_0 - j\omega)}\Big|_0^{V \to \lim_{V\to\infty}} =$$

$$= \frac{1}{-j\omega}\cdot e^{-j\omega V} - \frac{1}{(-\omega_0 - j\omega)}\cdot e^{V(-\omega_0 - j\omega)} - \left(\frac{1}{-j\omega}\cdot e^{0} - \frac{1}{(-\omega_0 - j\omega)}\cdot e^{0}\right) =$$

$$= 0 - \left(\frac{1}{-j\omega} + \frac{1}{(-\omega_0 - j\omega)}\right) \rightarrow \text{Rozwiązanie TRANSFORMATY}$$

$$= \frac{1}{j\omega} + \frac{1}{(-\omega_0 - j\omega)}$$

3. OBLICZENIE WIDMA

$$F(jW) = \frac{1}{jW} + \frac{1}{-W_0 - jW} = \frac{-W_0 - jW}{jW(-W_0 - jW)} + \frac{jW}{jW(-W_0 - jW)} =$$

$$= \frac{-W_0}{jW(-W_0 - jW)} = \frac{-W_0}{-jWW_0 - jW^2} = \frac{-W_0}{-jWW_0 + W^2}$$

$$|F(jW)| = \left| \frac{-W_0}{-jWW_0 + W^2} \right|$$

$$\left|\bar{F}(j\omega)\right| = \left|\frac{-\omega_0}{\omega^2 - j\omega\omega_0} \cdot \frac{\omega^2 + j\omega\omega_0}{\omega^2 + j\omega\omega_0}\right| =$$

$$= \left|\frac{-\omega_0(-j)\omega\omega_0 - \omega^2}{\omega^2\omega_0^2 + \omega^4}\right| =$$

$$= \frac{1}{\omega^2\omega_0^2 + \omega^4} \cdot \left|j\omega\omega_0^2 + \omega_0\omega^2\right| =$$

$$= \frac{1}{\omega^2\omega_0^2 + \omega^4} \cdot \sqrt{\omega^2\omega_0^4 + \omega_0^2\omega^4}$$

Wykres $\omega_0 = 10$