Minimum Spanning Tree

To understand Minimum Spanning Tree, first, we need to know what a spanning tree is.

What is Spanning Tree ?

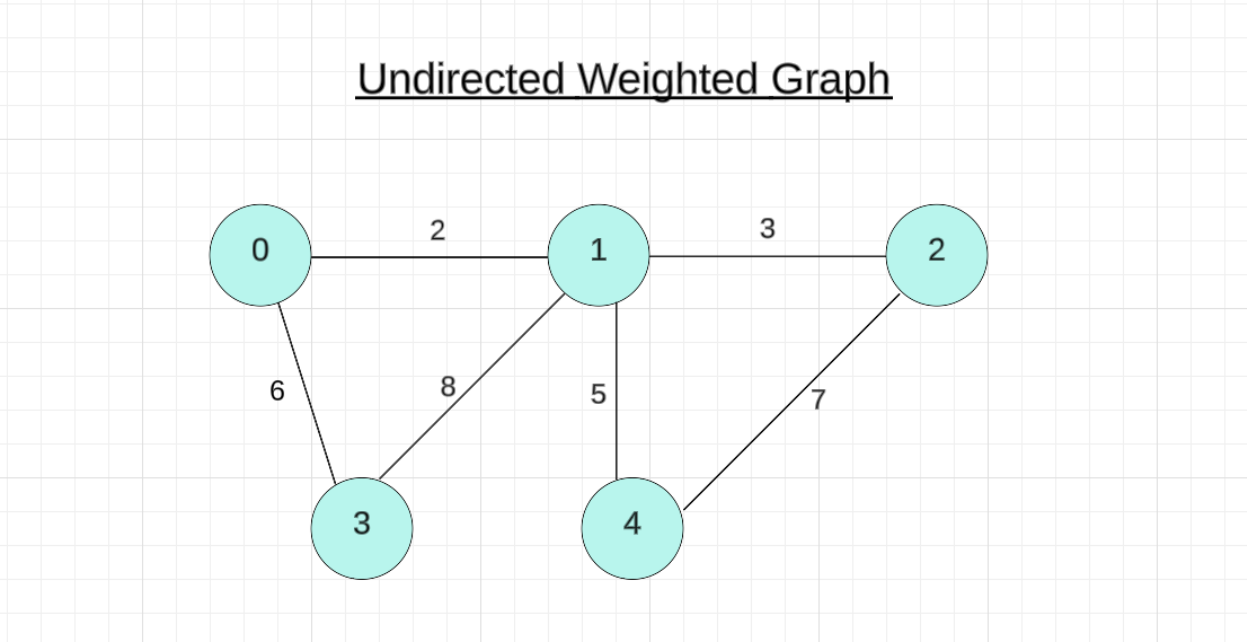
A spanning tree is a tree in which we have N nodes(i.e. All the nodes present in the original graph) and N-1 edges and all nodes are reachable from each other.

Basically, when you convert a graph into a tree and when this tree satisfies the all the condition of spanning tree, then we can say that this tree is a spanning tree. So, basically spanning tree is a subset of graph.

A spanning tree is a special kind of subgraph that is obtained by removing some of the edges from the original graph in such a way that the resulting subgraph is a tree and includes all the vertices of the original graph.

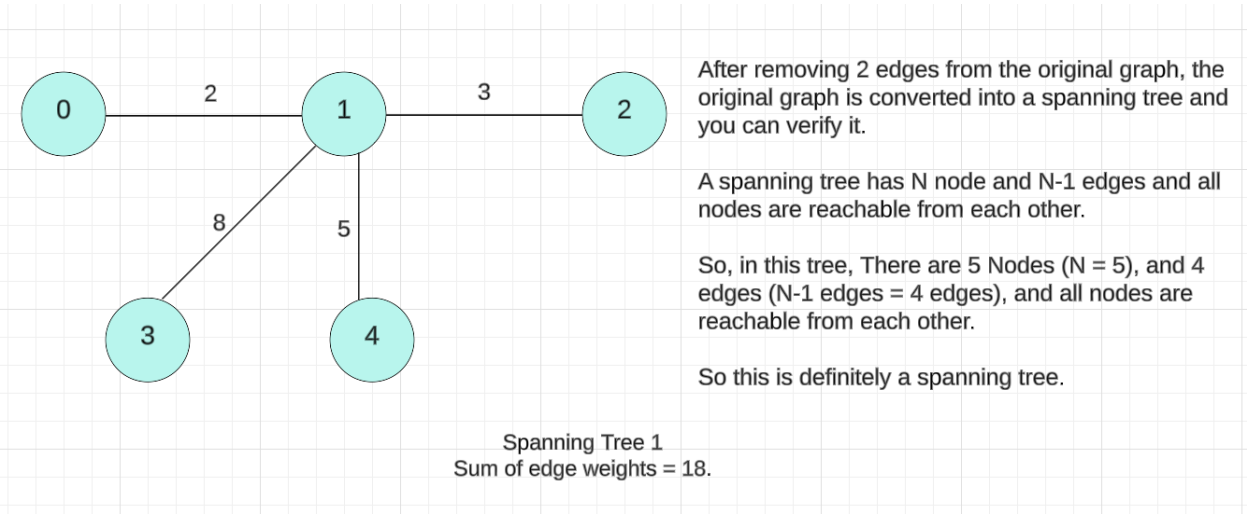
> 💡 **Note:** ***Spanning tree does not contains any cycles*** because spanning tree is obtained by removing some of the edges from the original graph in such a way that the resulting subgraph is a **tree**. Since trees are acyclic, meaning they do not contain any cycles, a spanning tree of a graph is also acyclic and does not contain any cycles. In simple word, Spanning tree is a tree and trees are acyclic.

Example: Assume we are given an undirected weighted graph with N nodes and M edges. Here in this example, we have taken N as 5 and M as 6.

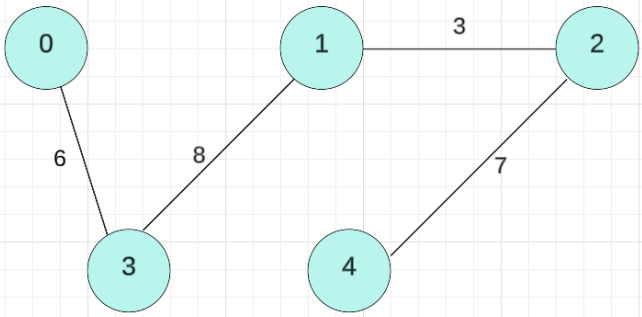


> 💡 **Note:** *Point to remember is that a graph may have more than one spanning trees.*

For the above graph, if we convert the above graph into a spanning tree then, one of the spanning tree would look like this,



We can draw more spanning trees for the above given graph(original graph). Three of them are like the following:



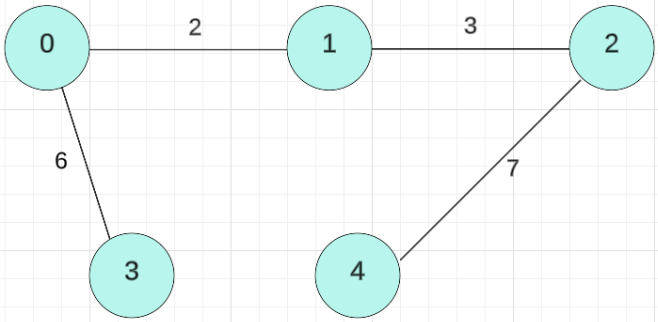
After removing 2 edges from the original graph, the original graph is converted into a spanning tree and you can verify it.

A spanning tree has N node and N-1 edges and all nodes are reachable from each other.

So, in this tree, There are 5 Nodes (N = 5), and 4 edges (N-1 edges = 4 edges), and all nodes are reachable from each other.

So this is definitely a spanning tree.

Spanning Tree 2
Sum of edge weights = 24.



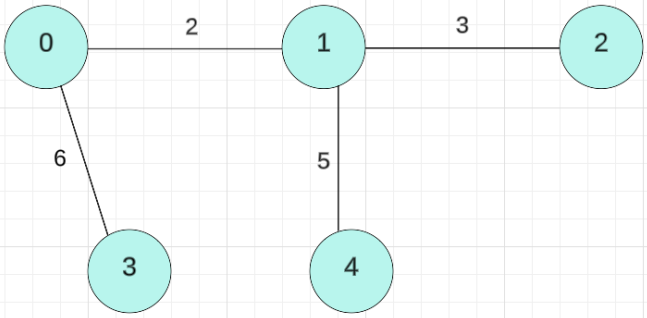
After removing 2 edges from the original graph, the original graph is converted into a spanning tree and you can verify it.

A spanning tree has N node and N-1 edges and all nodes are reachable from each other.

So, in this tree, There are 5 Nodes (N = 5), and 4 edges (N-1 edges = 4 edges), and all nodes are reachable from each other.

So this is definitely a spanning tree.

Spanning Tree 3
Sum of edge weights = 18.



After removing 2 edges from the original graph, the original graph is converted into a spanning tree and you can verify it.

A spanning tree has N node and N-1 edges and all nodes are reachable from each other.

So, in this tree, There are 5 Nodes (N = 5), and 4 edges (N-1 edges = 4 edges), and all nodes are reachable from each other.

So this is definitely a spanning tree.

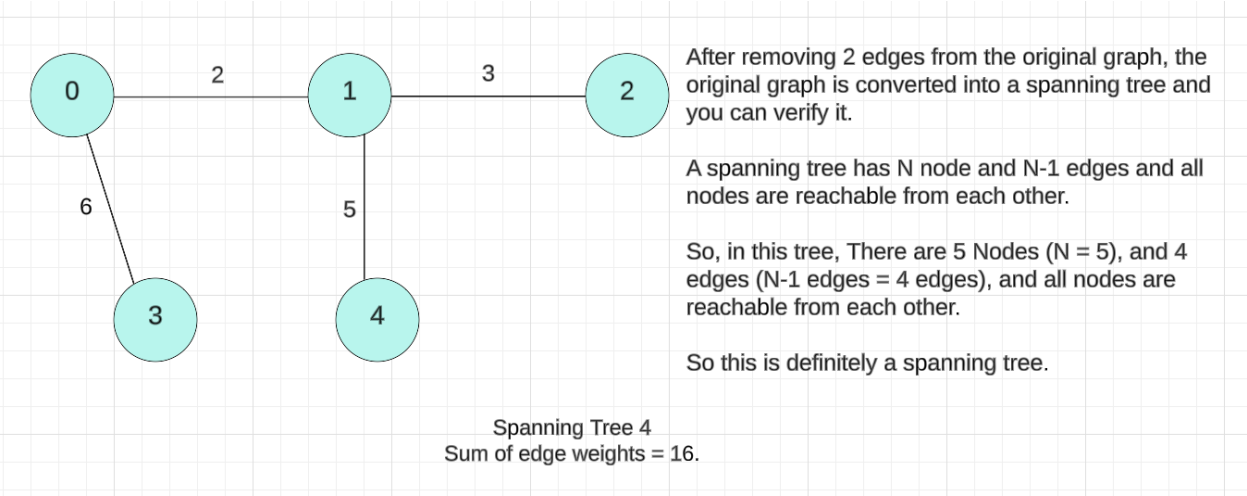
Spanning Tree 4
Sum of edge weights = 16.

All the above spanning trees contain some edge weights. For each of them, if we add the edge weights we can get the total sum of edge weights for that particular tree. Now, let's try to figure out the minimum spanning tree:

What is a Minimum Spanning Tree (AKA MST) ?

In above example, we have drawn all the possible spanning trees and Among all possible spanning trees of a graph, **the minimum spanning tree (MST) is the one that has the minimum possible sum of all the edge weights.** The edge weights represent the costs or distances associated with each edge in the graph. Therefore, the MST is the spanning tree that connects all the vertices of the graph using the minimum possible total cost or distance.

So, for above graph, the minimum spanning tree is given below,



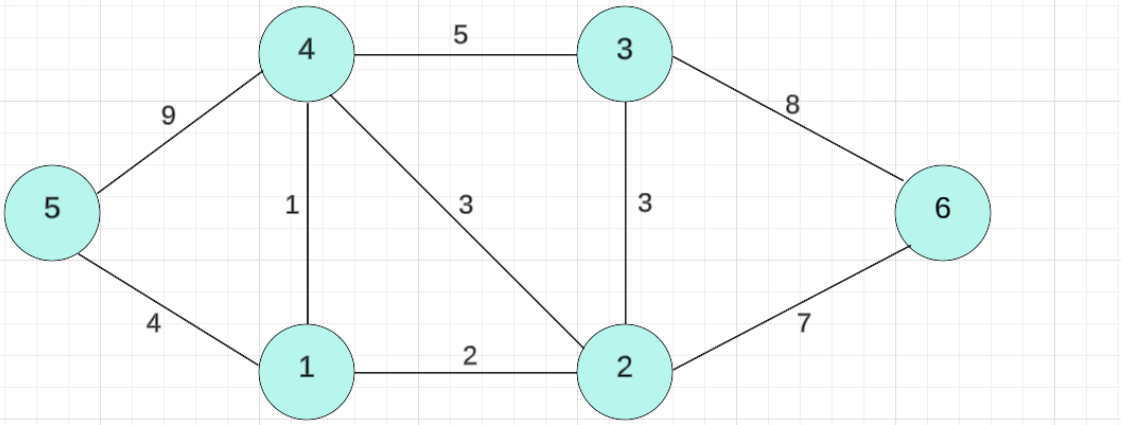
> 💡 **Note:** *There may exist multiple minimum spanning trees for a graph like a graph may have multiple spanning trees.*

Practice Problem:

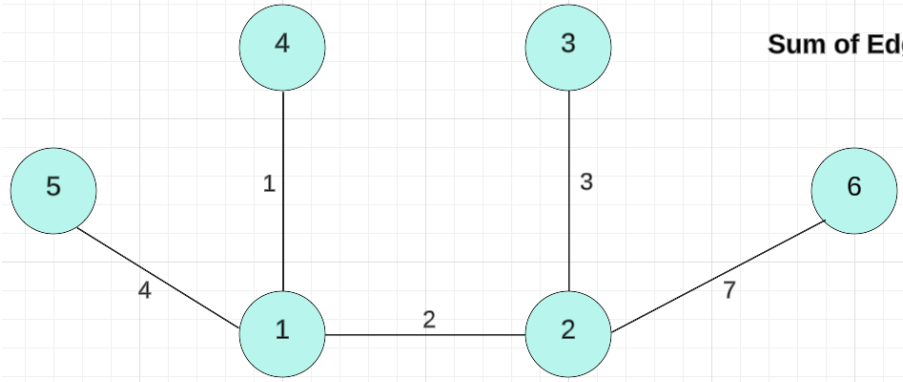
Practice Problem

After learning the concept of the Spanning Tree and Minimum Spanning Tree, you can try this practice problem to check your understanding.

Try to figure out the Minimum Spanning Tree (MST) for the below given undirected weighted graph.



Answer: Minimum Spanning Tree (MST) for above graph is,



Sum of Edge Weights = 17.

How to find Minimum Spanning Tree (MST) for a given graph ?

There are several algorithms that can be used to find the Minimum Spanning Tree (MST) of a given graph. Two of the most popular algorithms are:

1. Prim’s Algorithm.
2. Kruskal’s Algorithm.