Bipartite Graph

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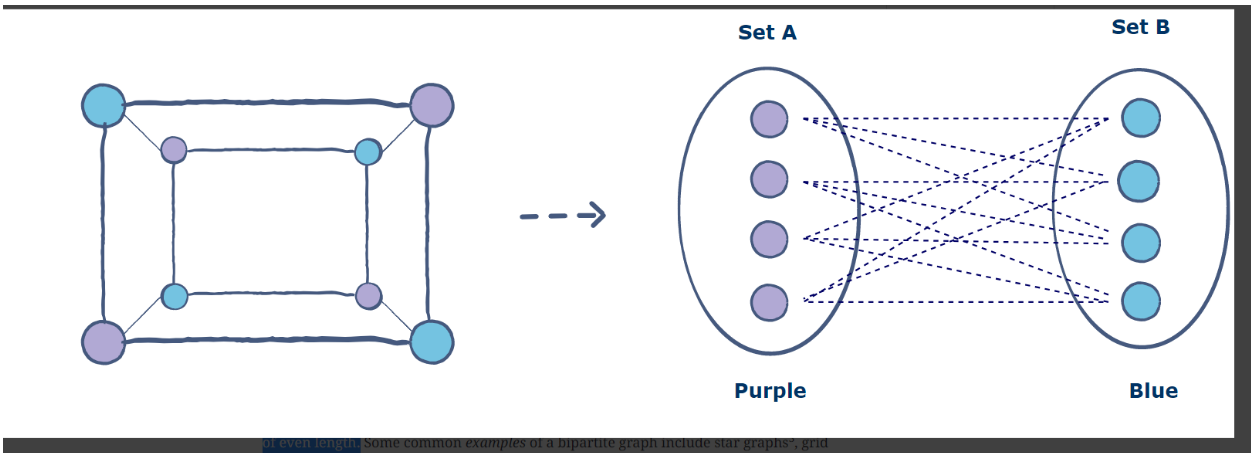
A bipartite graph is a graph whose vertices can be divided into two disjoint sets such that every edge connects a vertex of one set to a vertex of the other set. This means that there are no edges that connect vertices within the same set.



Bipartite graphs are equivalent to two-colorable graphs this means that A graph is bipartite if and only if it is two-colorable. This means that the vertices of the graph can be colored with two colors (e.g., black and white or purple and blue or yellow or green) such that no two adjacent vertices have the same color.

To see why this is true, imagine a bipartite graph where the vertices are divided into two groups, A and B. We can color all vertices in group A with one color (e.g., black) and all vertices in group B with another color (e.g., white). Since all edges connect vertices from different groups, no two adjacent vertices have the same color. Therefore, the graph is two-colorable.

Conversely, if a graph is two-colorable, we can divide its vertices into two groups based on their color. The vertices of one group will all have one color, and the vertices of the other group will all have the other color. Since no two adjacent vertices have the same color, all edges will connect vertices from different groups. Therefore, the graph is bipartite.

This equivalence between bipartite graphs and two-colorable graphs is useful for identifying bipartite graphs.

Properties of Bipartite Graph:

1. **Two-colorable:**
    a. A graph is bipartite if and only if it is two-colorable. This means that the vertices of the graph can be colored with two colors (e.g., black and white) such that no two adjacent vertices have the same color.
2. Any linear graph with no cycle is always a bipartite graph.
3. If a graph contains a cycle of even length, then it is bipartite graph.
4. If a graph contains a cycle of odd length, then it is not bipartite graph.

How to detect if a graph is Bipartite or not ?

There are several ways to detect whether a graph is bipartite or not. Here are three common methods:

1. **Graph coloring:**
    a. As I mentioned earlier, a graph is bipartite if and only if it is two-colorable. Therefore, we can try to color the graph using two colors (e.g., black and white) and check whether it is possible to do so without any adjacent vertices having the same color. If we can color the graph in this way, then it is bipartite; otherwise, it is not.
2. **Depth-first search:**
    a. Another way to detect whether a graph is bipartite or not is to perform a depth-first search (DFS) on the graph, assigning colors to the vertices as we go. We start by coloring an arbitrary vertex with one color (e.g., black) and all of its neighbors with the other color (e.g., white). Then, we recursively color all of the neighbors of the white vertices with black, and all of the neighbors of the black vertices with white. If at any point we encounter a vertex that has already been colored and its color is the same as the color we want to assign to it, then the graph is not bipartite. Otherwise, if we can color all vertices without any conflicts, then the graph is bipartite.

3. **Breadth-first search:**

    a. A similar approach to DFS is to use a breadth-first search (BFS) instead. We start by coloring an arbitrary vertex with one color (e.g., black) and all of its neighbors with the other color (e.g., white). Then, we enqueue all the white vertices and continue the BFS from there. Whenever we encounter a new vertex, we color it with the opposite color of its parent and enqueue all of its uncolored neighbors. If at any point we encounter a vertex that has already been colored and its color is the same as the color we want to assign to it, then the graph is not bipartite. Otherwise, if we can color all vertices without any conflicts, then the graph is bipartite.