

# The unintended disparate effects of privacy in decision tasks



Jacob Christopher



James Kotary



My Dinh



Vincenzo Di Vito



Michael Cardei



Jinhao Liang



Saswat Das



Key Zhu



Cuong Tran

Ferdinando Fioretto University of Virginia

@FCSM-24



<https://nandofioretto.com>



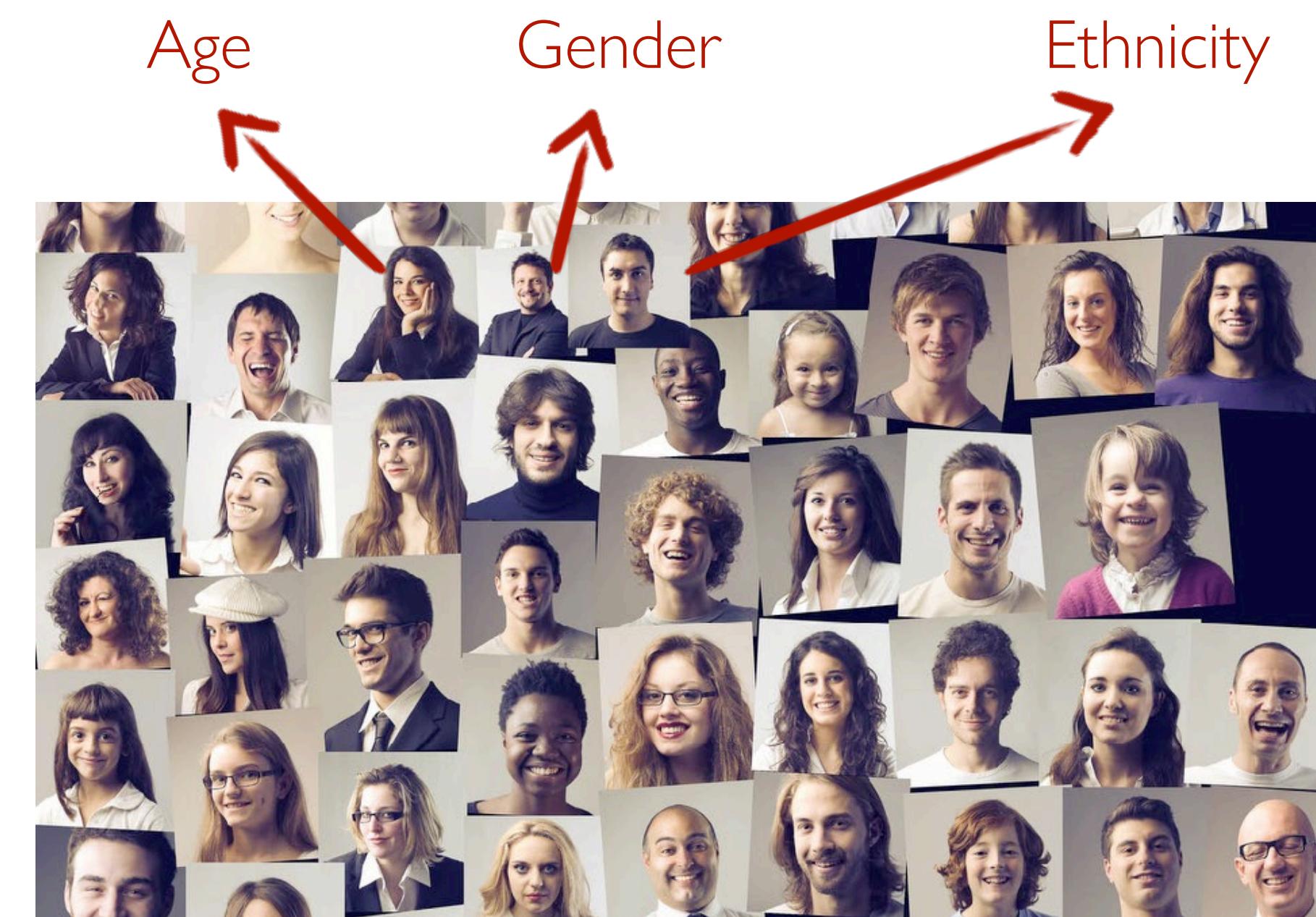
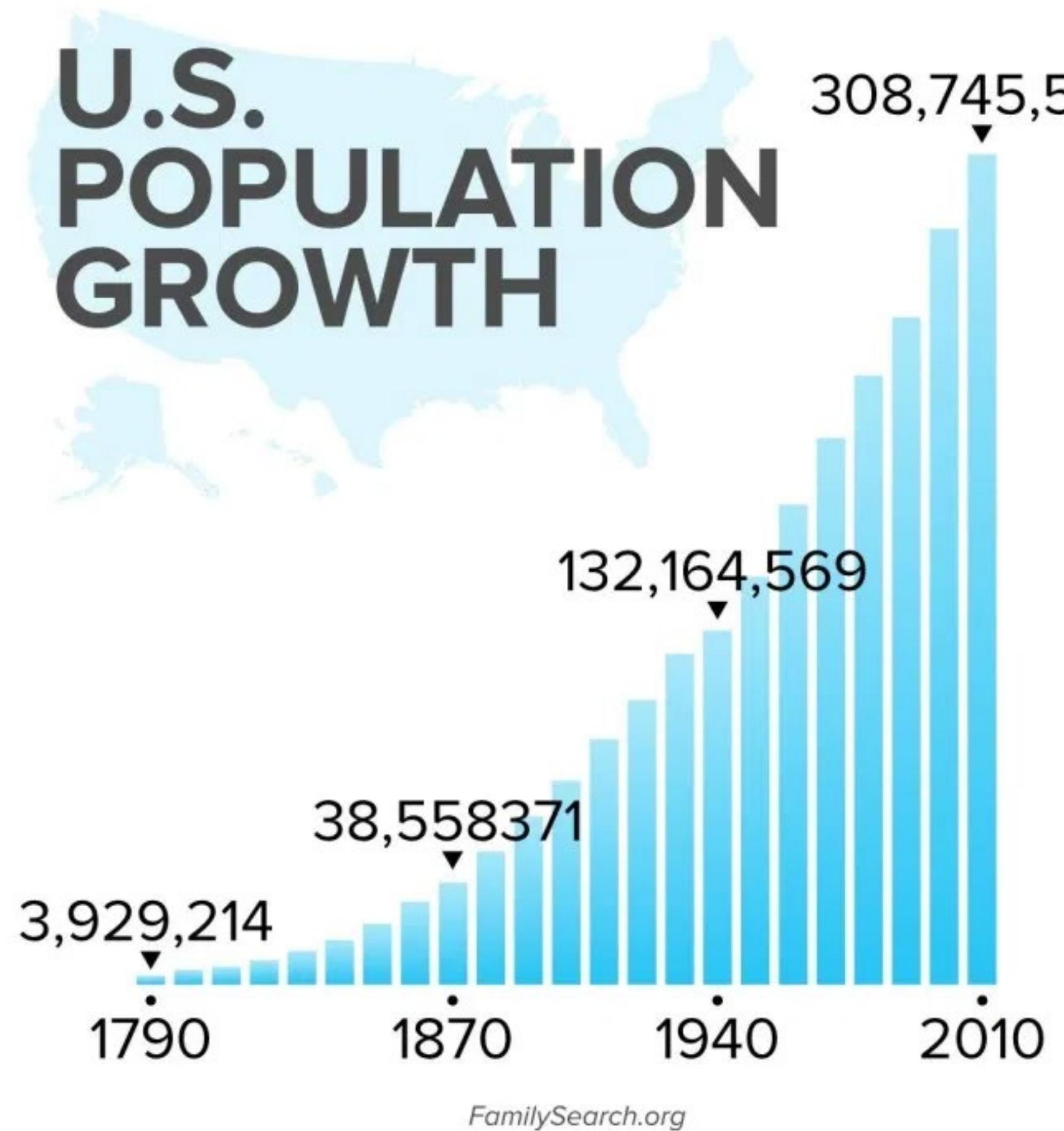
@nandofioretto



nandofioretto@gmail.com

# US Census data collection

Enumeration of the total population living the US



# US Census data collection

## Accurate count is important

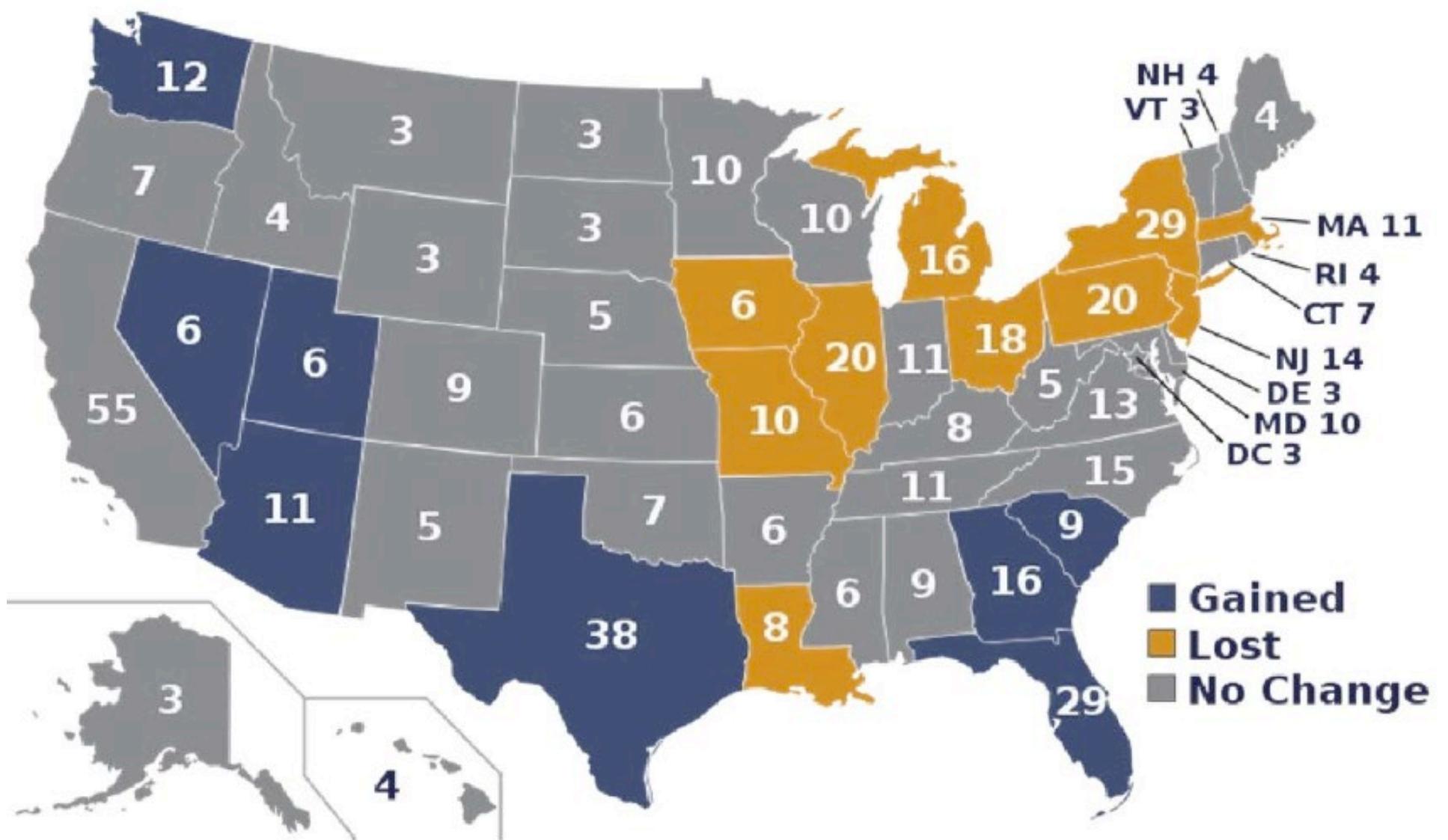
- Used to apportion multiple federal funding streams.
- \$665 billions allocated to 132 economic security programs (2022) other than health insurance or social security benefits.



Highway Planning and Construction



U.S. DEPARTMENT OF EDUCATION

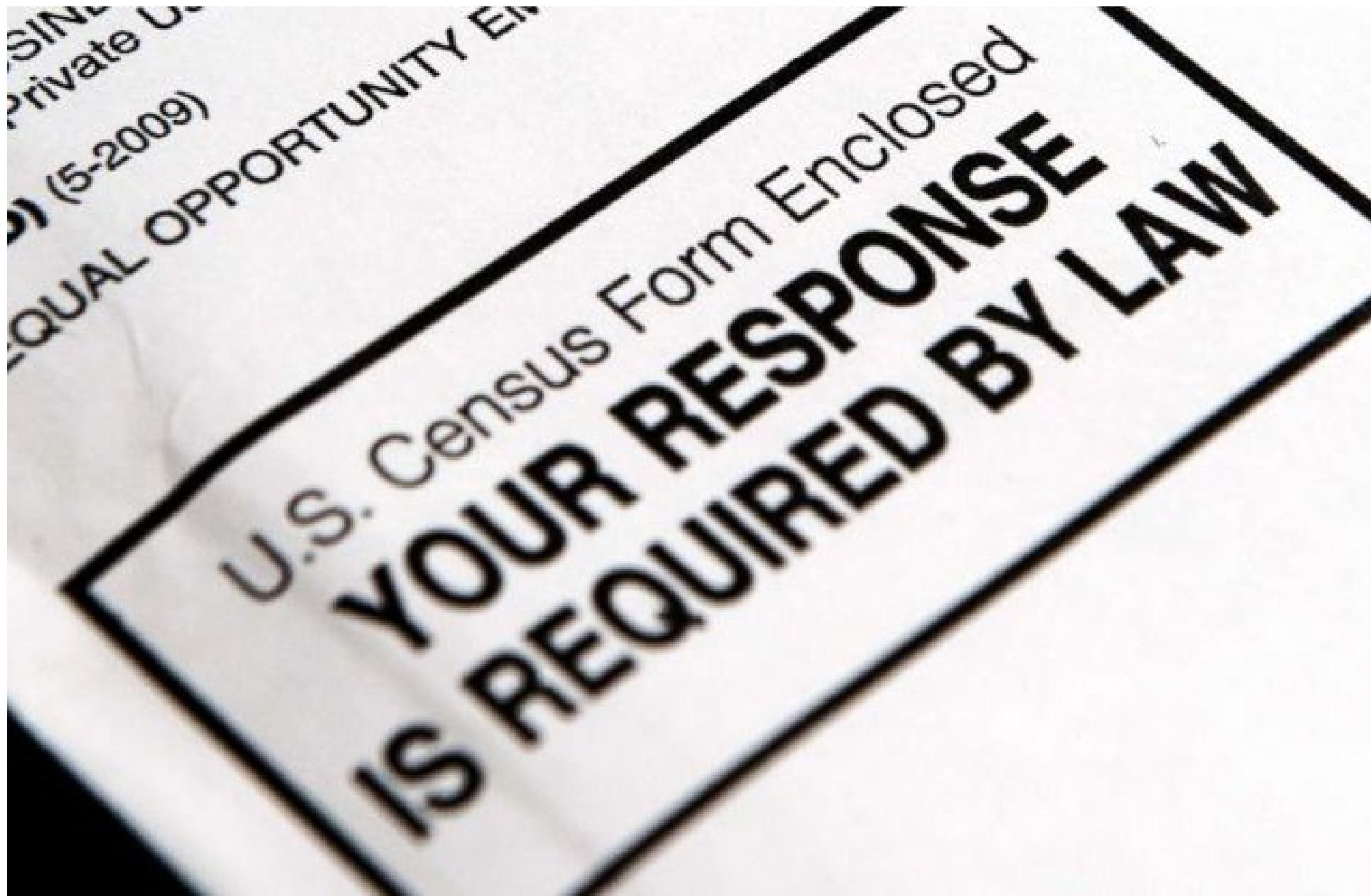


Determine the number of seats that states get in the US House of Representatives.

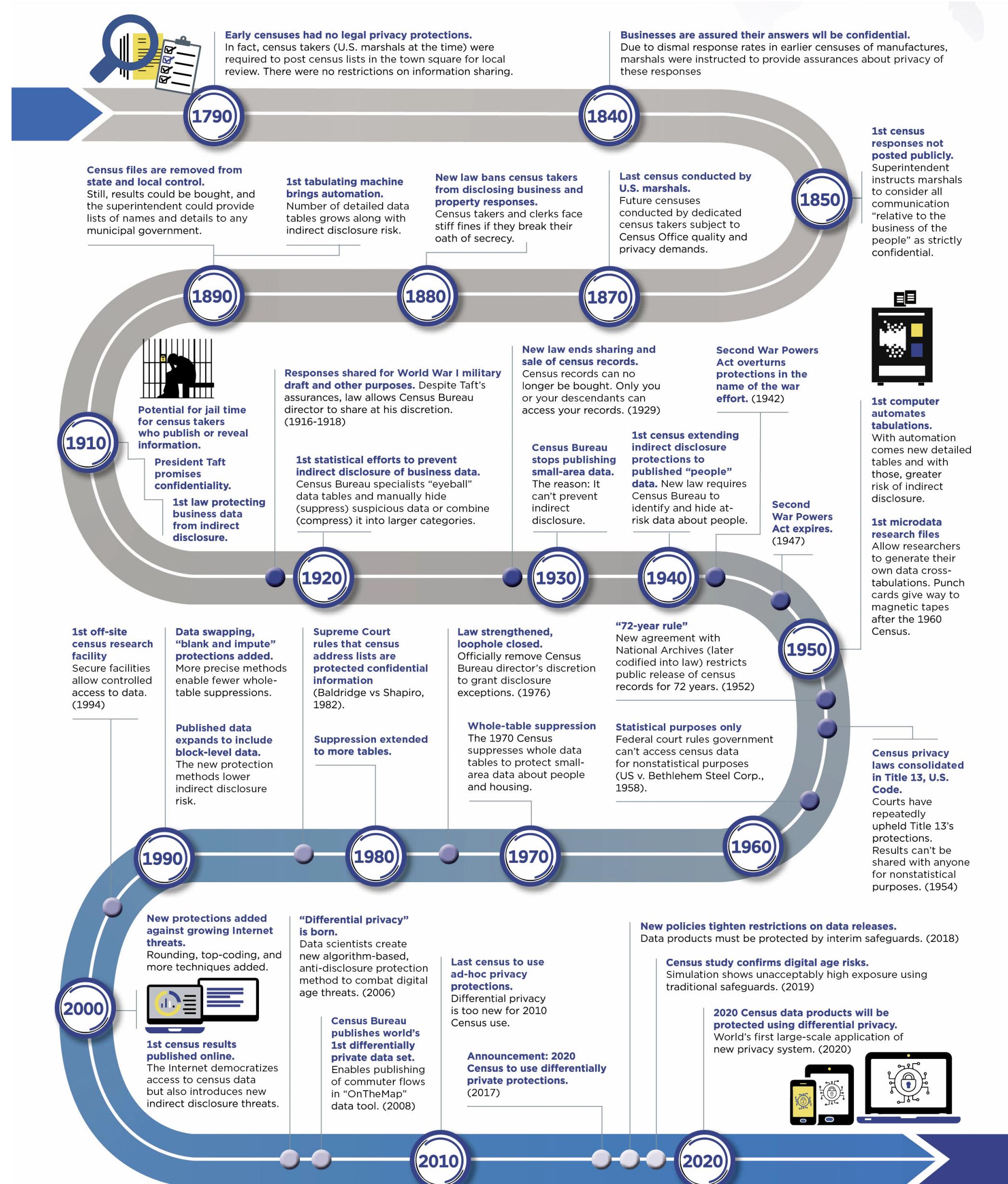
# US Census data collection

## Privacy is required by law

Because of the importance to have accuracy count congress makes the data collection mandatory.



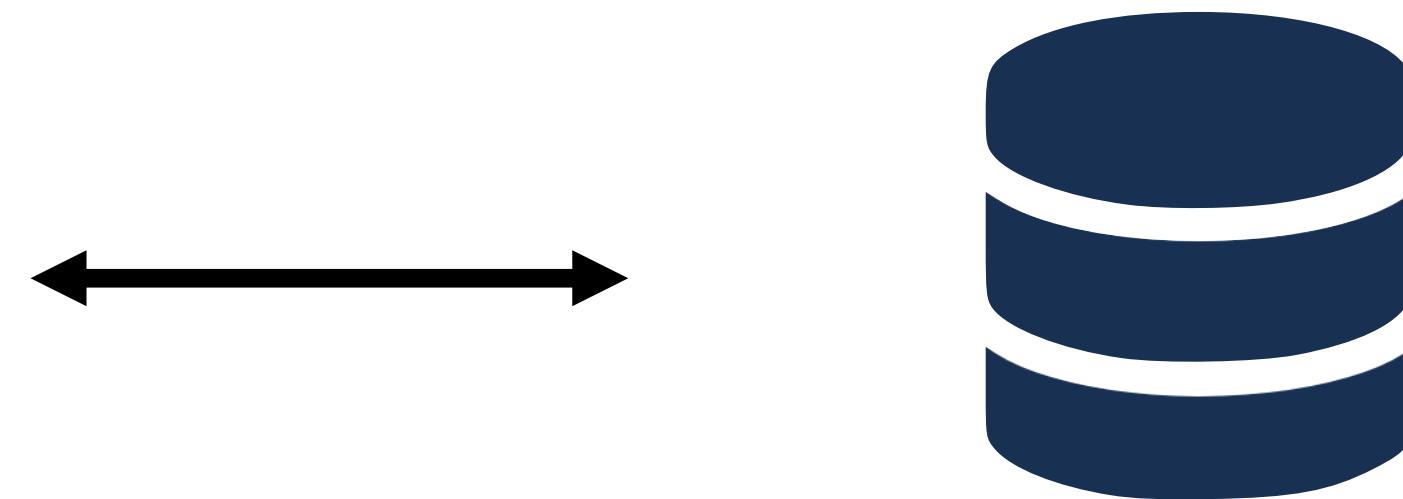
Title 13: Census is required to retain data confidentiality.



# Reconstruction Attacks



U.S. Department of Commerce  
Economics and Statistics Administration  
U.S. CENSUS BUREAU  
[census.gov](http://census.gov)



308,745,548 people in 2010 release which implements some “protection”

Commercial databases

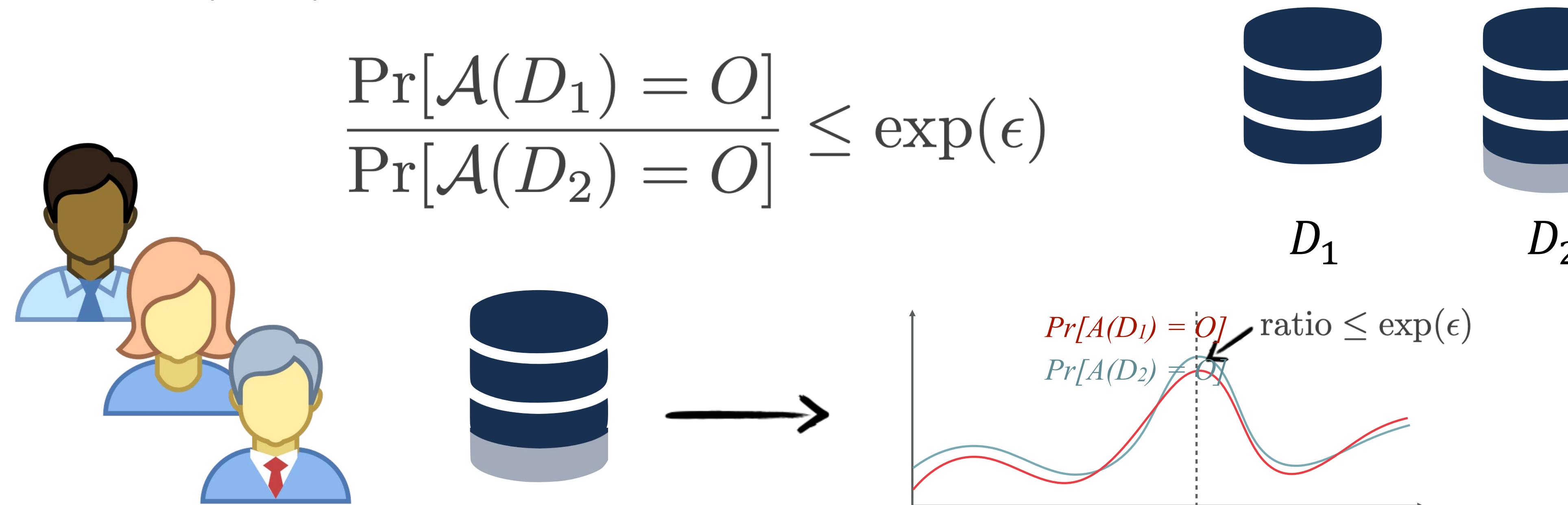
Linkage Attacks — Results from UC Census:

- Census blocks correctly reconstructed in all 6,207,027, inhabited blocks.
- Block, sex, age, race, ethnicity reconstructed:
  - Exactly: 46% of population (142M).
  - Allowing age +/- 1 year: 71% of population (219M).
- Name, block sex, age, race, ethnicity:
  - Confirmed re-identification: 38% of population.

# Differential Privacy

## Definition

A randomized algorithm  $\mathcal{A}$  is  $\epsilon$ -differentially private if, for all pairs of inputs  $D_1, D_2$ , differing in one entry, and for any output  $O$ :



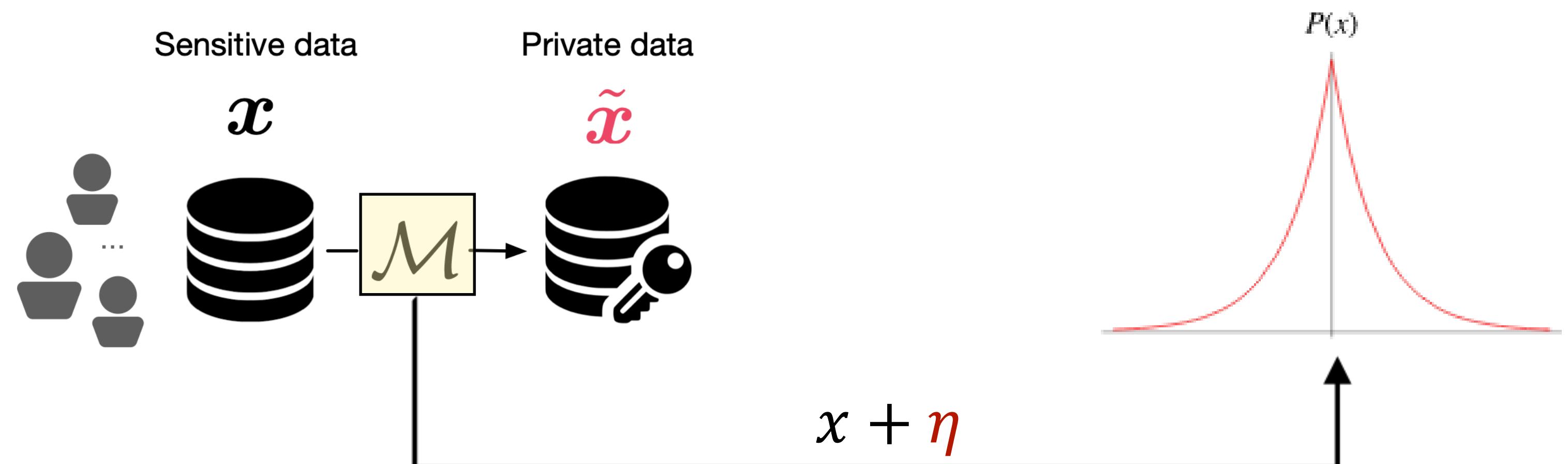
**Intuition:** An adversary should not be able to use output  $O$  to distinguish between any  $D_1$  and  $D_2$

# Differential Privacy

## Notable properties

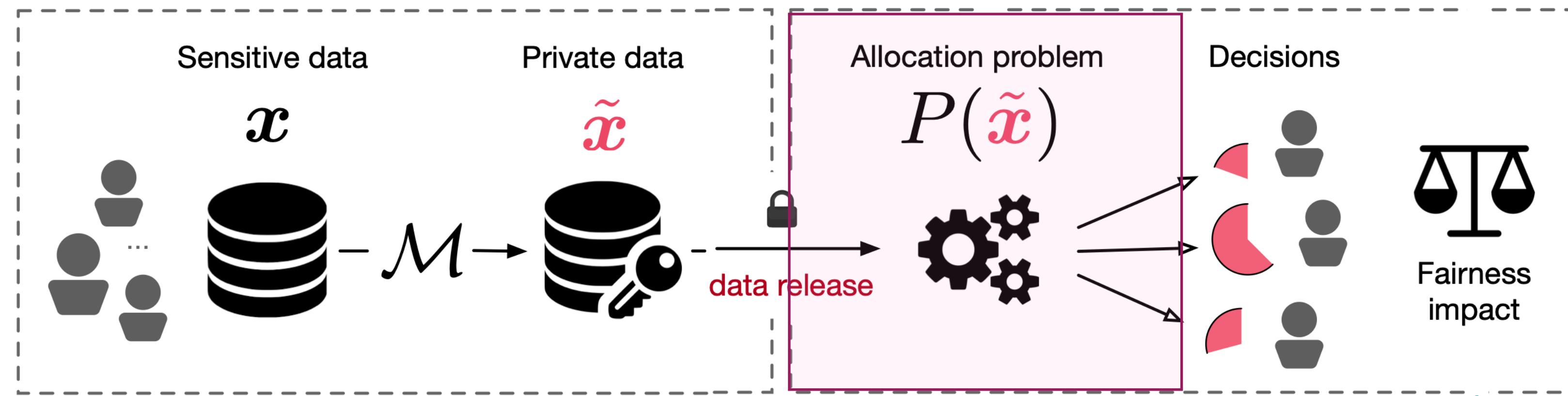
- Immune to linkage attack: Adversary knows arbitrary auxiliary information.
- Composability: If  $A_1$  enjoys  $\epsilon_1$ -differential privacy and  $A_2$  enjoys  $\epsilon_2$ -differential privacy, then, their composition  $A_1(D), A_2(D)$  enjoys  $(\epsilon_1 + \epsilon_2)$ -differential privacy.
- Post-processing immunity: If  $A$  enjoys  $\epsilon$ -differential privacy and  $g$  is an arbitrary data-independent mapping, then  $g \circ A$  is  $\epsilon$ -differential private.

DP algorithms rely on randomization



# Fairness in downstream decisions

## Setting



Bias:  $B_P^i(M, \boldsymbol{x}) = \mathbb{E}_{\tilde{\boldsymbol{x}} \sim M(\boldsymbol{x})}[P_i(\tilde{\boldsymbol{x}})] - P_i(\boldsymbol{x})$

Definition ( $\alpha$ -Fairness). A data-release mechanism  $M$  is said  $\alpha$ -fair w.r.t. a problem  $P$  if, for all datasets  $\boldsymbol{x} \in \mathcal{X}$  and all  $i \in [n]$

$$\xi_B^i(P, \mathcal{M}, \boldsymbol{x}) = \max_{j \in [n]} |B_P^i(\mathcal{M}, \boldsymbol{x}) - B_P^j(\mathcal{M}, \boldsymbol{x})| \leq \alpha$$

# Disproportionate impacts in decision making

## Title 1 allotment

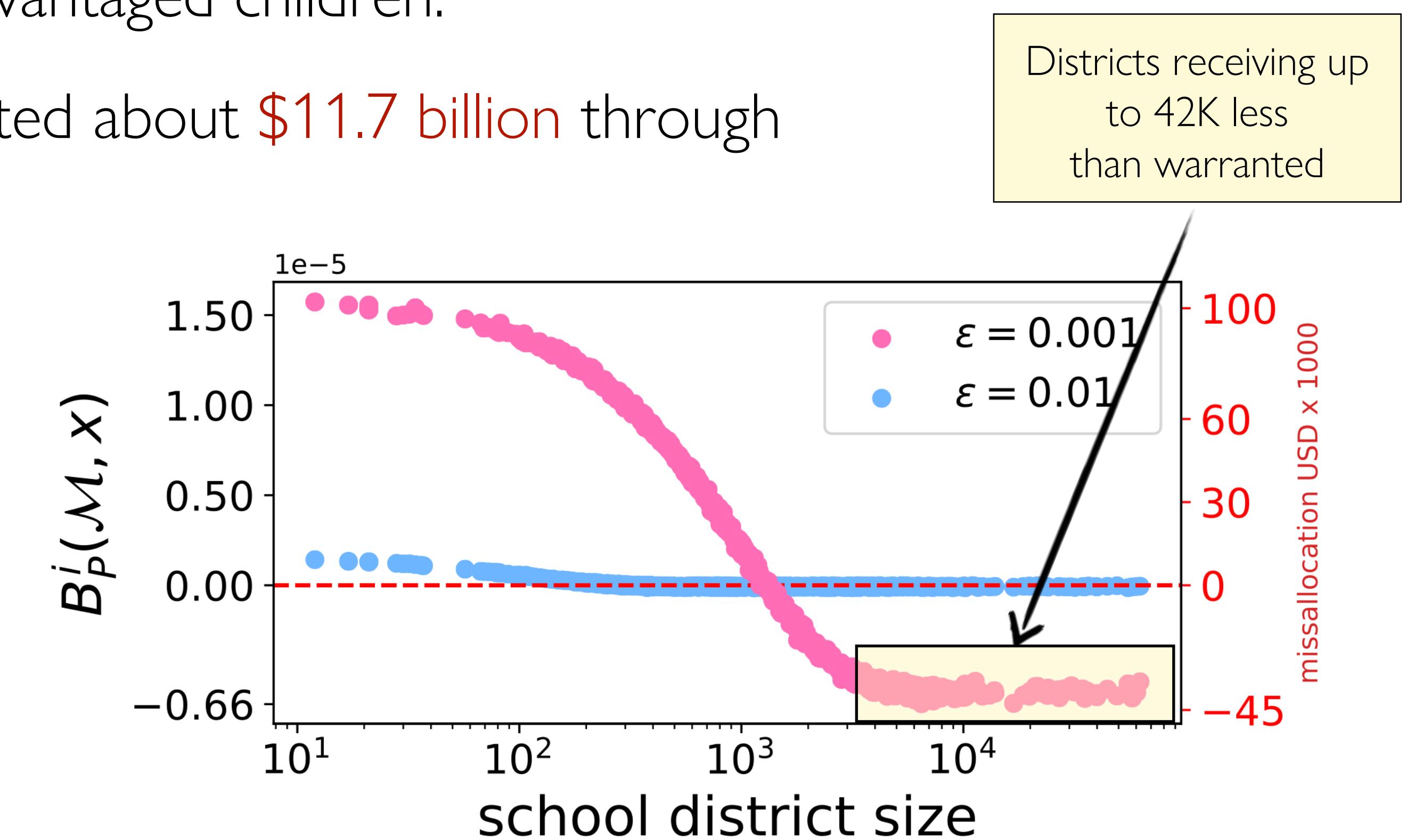
- Title 1 of the Elementary and Secondary Education Act is one of the largest U.S. program offering educational assistance to disadvantaged children.
- In the fiscal year 2021 alone, it distributed about \$11.7 billion through several types of grants.

- Allotment:

count of children 5 to 17 in district i

$$P_i^F(x) \stackrel{\text{def}}{=} \left( \frac{x_i \cdot a_i}{\sum_{i \in [n]} x_i \cdot a_i} \right)$$

student expenditures in district i



# Shape of the decision problem

## First key result

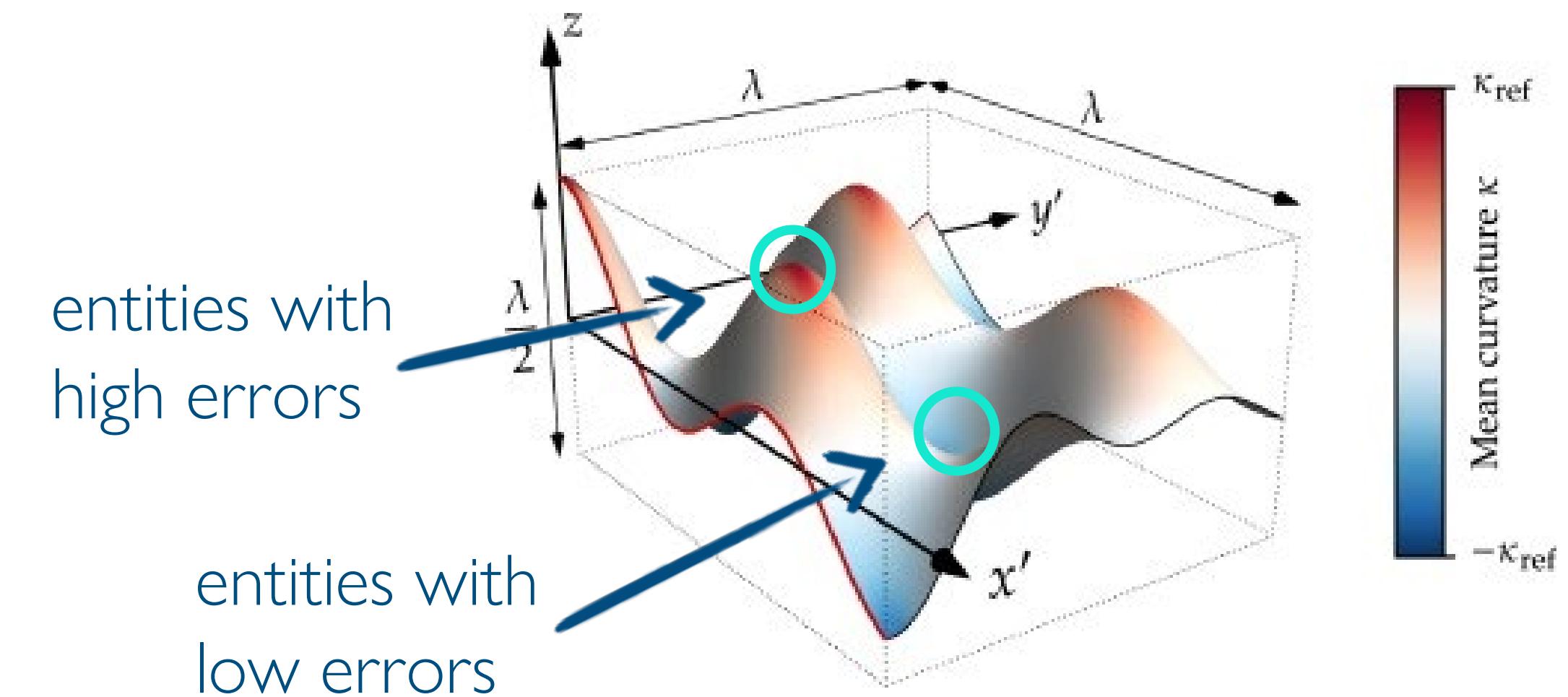
- Theorem (informal): It is the “**shape**” of the decision problem that characterizes the unfairness of the outcomes, even using an **unbiased DP mechanism**.
- The problem bias can be approximated as (when  $P_i$  is at least twice differentiable):

$$\begin{aligned} B_P^i(\mathcal{M}, \mathbf{x}) &= \mathbb{E}[P_i(\tilde{\mathbf{x}} = \mathbf{x} + \eta)] - P_i(\mathbf{x}) \\ &\approx \frac{1}{2} \boxed{\mathbf{H}P_i(\mathbf{x})} \times \boxed{\text{Var}[\eta]} \end{aligned}$$

↗      ↙

Local curvature of  
problem  $P_i$

Variance of the  
noisy input  
(depends on  $\epsilon$ )



- Fairness can be bounded whenever the problem local curvature is constant across entities, since the variance is also constant and bounded.

# Shape of the decision problem

## First key result

- Theorem (informal): It is the “**shape**” of the decision problem that characterizes the unfairness of the outcomes, even using an **unbiased DP mechanism**.
- The problem bias can be approximated as (when  $P_i$  is at least twice differentiable):

$$B_P^i(\mathcal{M}, \mathbf{x}) = \mathbb{E}[P_i(\tilde{\mathbf{x}} = \mathbf{x} + \eta)] - P_i(\mathbf{x})$$

$$\approx \frac{1}{2} \mathbf{H}P_i(\mathbf{x}) \times \text{Var}[\eta]$$

↗  
Local curvature of  
problem  $P_i$

↗  
Variance of the  
noisy input  
(depends on  $\epsilon$ )

A data release mechanism  $\mathcal{M}$  is  $\alpha$ -fair w.r.t.  $P$ , for some finite  $\alpha$ , if for all datasets  $\mathbf{x}$ , exists constants  $c_{jl}^i \in \mathbb{R}, (i \in [n], j, l \in [k])$

$$(\mathbf{H}P_i)_{j,l}(\mathbf{x}) = c_{j,l}^i \quad (i \in [n] \ j, l \in [k]).$$

- Corollary: (Perfect)-fairness cannot be achieved if  $P$  is any non-linear function, as in the case of the allocations considered.

# Disproportionate impacts in downstream decisions

## Minority language voting rights

- The *Voting Rights Act* of 1965 provides a body of protections for racial and language minorities.
- Section 203 describes the conditions under which local jurisdictions must provide minority language voting assistance during an election.
- Jurisdiction  $i$  must provide language assistance (including voter registration, ballots, and instructions) iff decision rule  $P_i^M(x)$  returns true with:

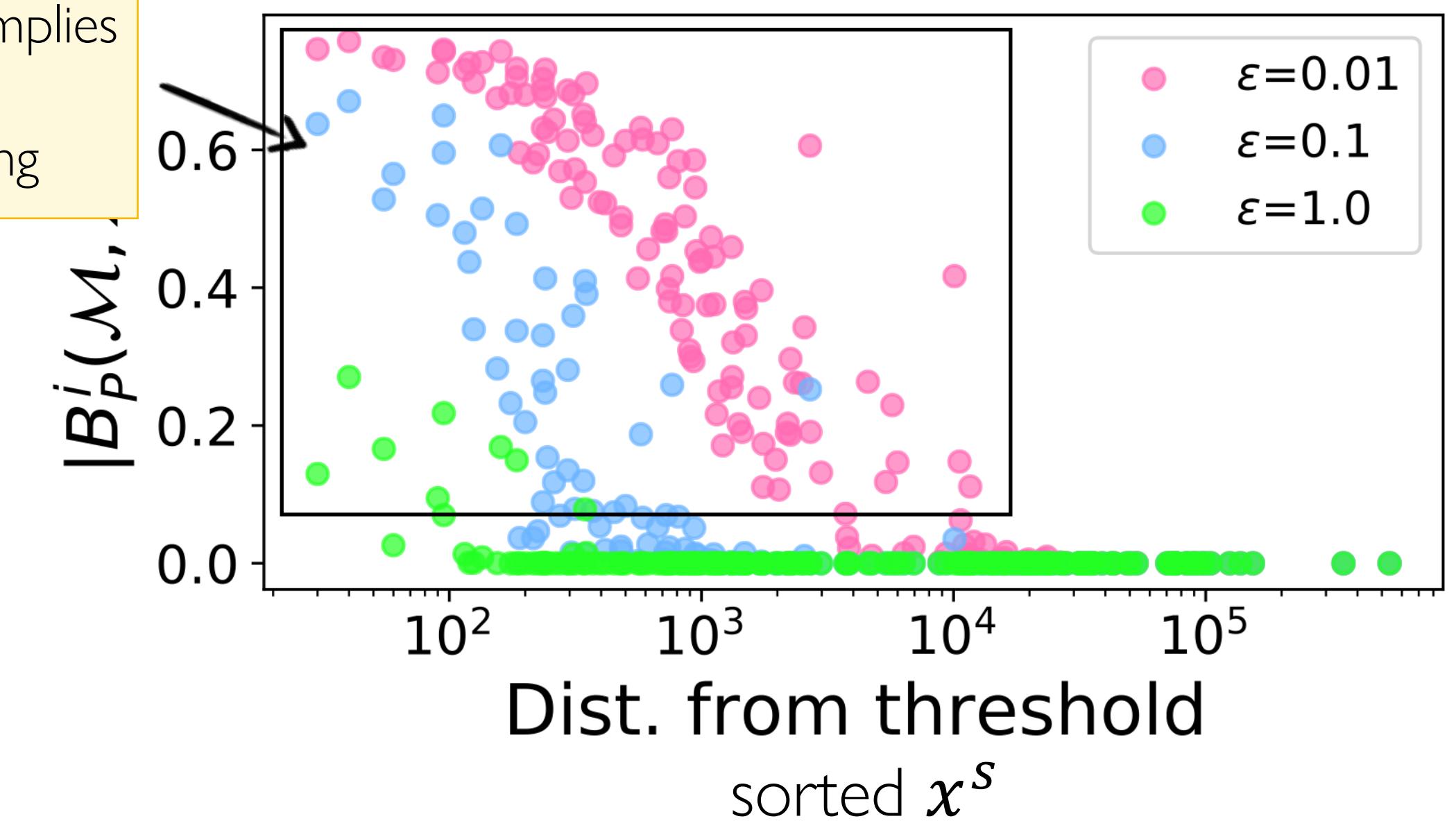
$$P_i^M(x) \stackrel{\text{def}}{=} \left( \frac{x_i^{sp}}{x_i^s} > 0.05 \vee x_i^{sp} > 10^4 \right) \wedge \frac{x_i^{spe}}{x_i^{sp}} > 0.0131.$$

+ < 5<sup>th</sup> grade education

no. of ppl in  $i$  speaking minority language  $s$

+ limited English proficiency

Misclassification implies potentially disenfranchising



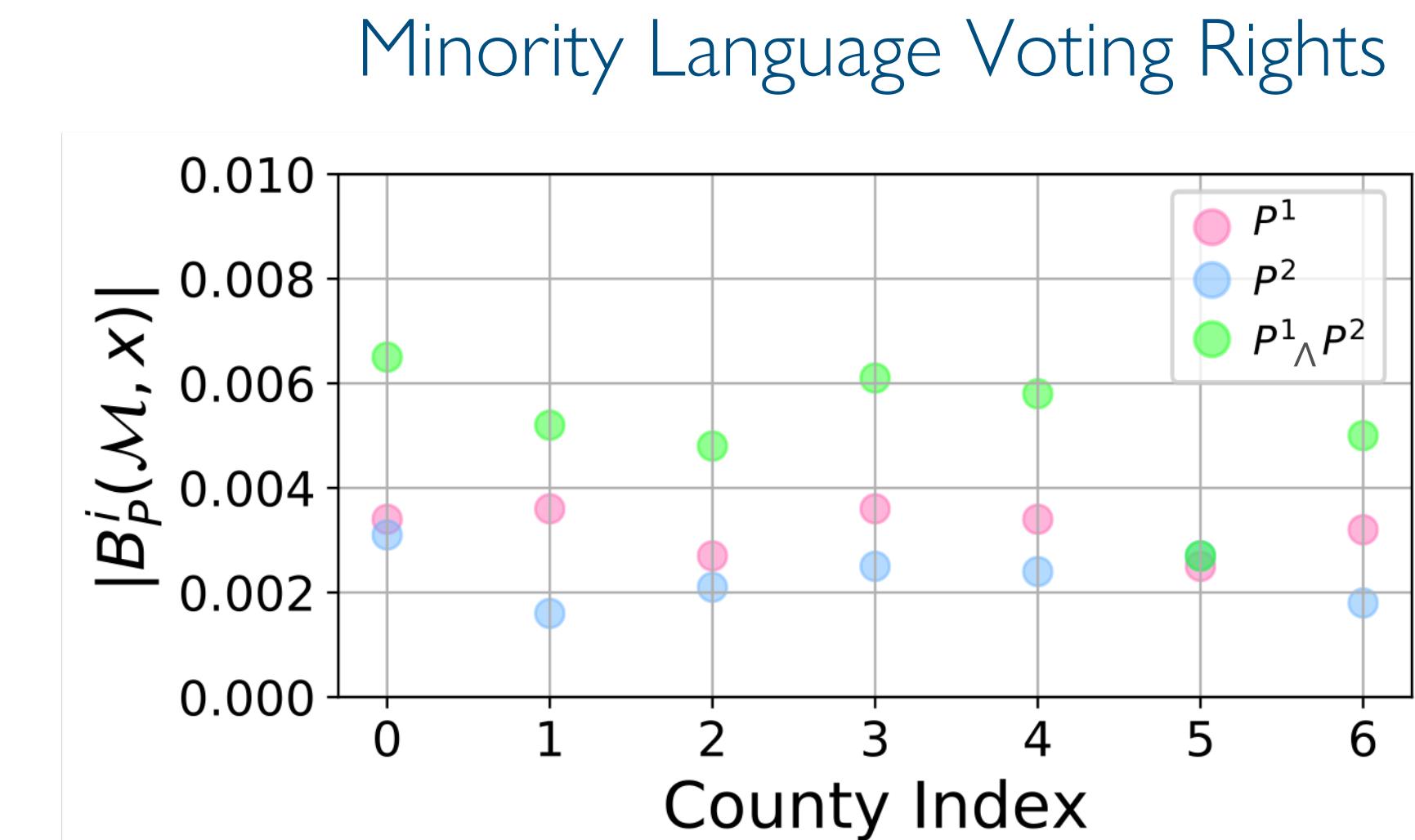
# Fairness composition

## Second key result

$$P_i^M(x) \stackrel{\text{def}}{=} \left( \frac{x_i^{sp}}{x_i^s} > 0.05 \vee x_i^{sp} > 10^4 \right) \wedge \frac{x_i^{spe}}{x_i^{sp}} > 0.0131.$$

$$P^1(x^{sp}) = \mathbb{1}\{x^{sp} \geq 10^4\}$$

$$P^2(x^{sp}, x^{spe}) = \mathbb{1}\left\{\frac{x_i^{spe}}{x_i^{sp}} > 0.0131\right\}$$



- Small bias when considered individually
- However, when they are combined using logical connector  $\wedge$ , **the resulting absolute bias increases substantially**, as illustrated by the associated green circles.

- Theorem (informal): The logical composition of two  $\alpha_1$ - and  $\alpha_2$ -fair mechanisms is  $\alpha$ -fair with  $\alpha \geq \max(\alpha_1, \alpha_2)$ .
- The unfairness induced by “composing” predicates **is no smaller than** that of their individual components.

# Shape of the decision problem

## Important conclusion

*Using DP to generate private inputs of decision problems commonly adopted to make policy determination will necessarily introduce fairness issues, despite the noise being unbiased!*

# Mitigation solution

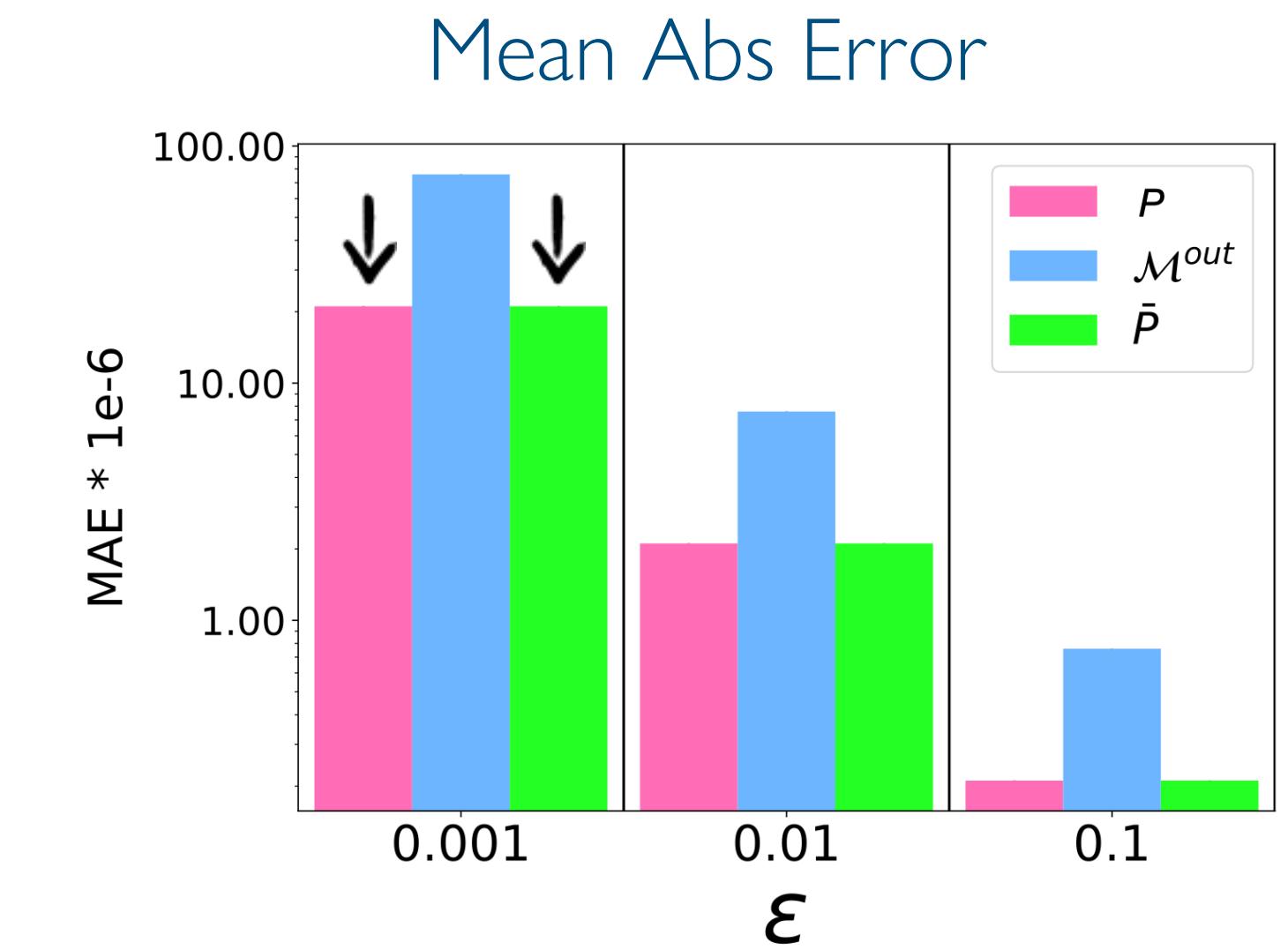
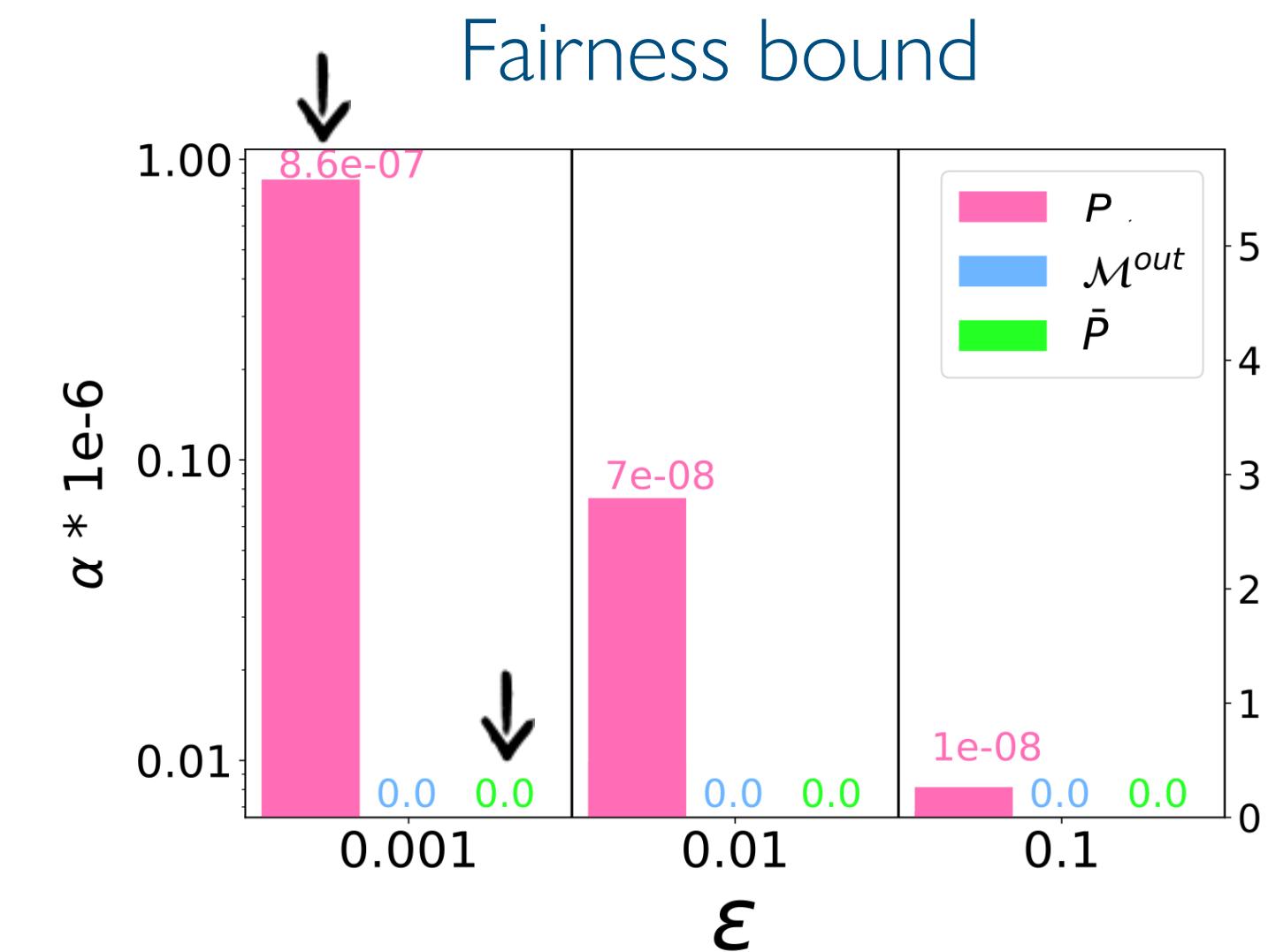
## Fair allocations

- Note that the observed issues are **not data-driven, but problem-driven**.
- **Corollary:** If  $P$  is a linear function, then mechanism  $\mathcal{M}$  is fair w.r.t.  $P$ .
- **Linearizing the allotment problem** — General idea: Given a problem  $P_i$  derive a linear approximation  $\tilde{P}_i$  of  $P_i$

Redundant data release

$$P_i^F(x) \stackrel{\text{def}}{=} \left( \frac{x_i \cdot a_i}{\sum_{i \in [n]} x_i \cdot a_i} \right)$$

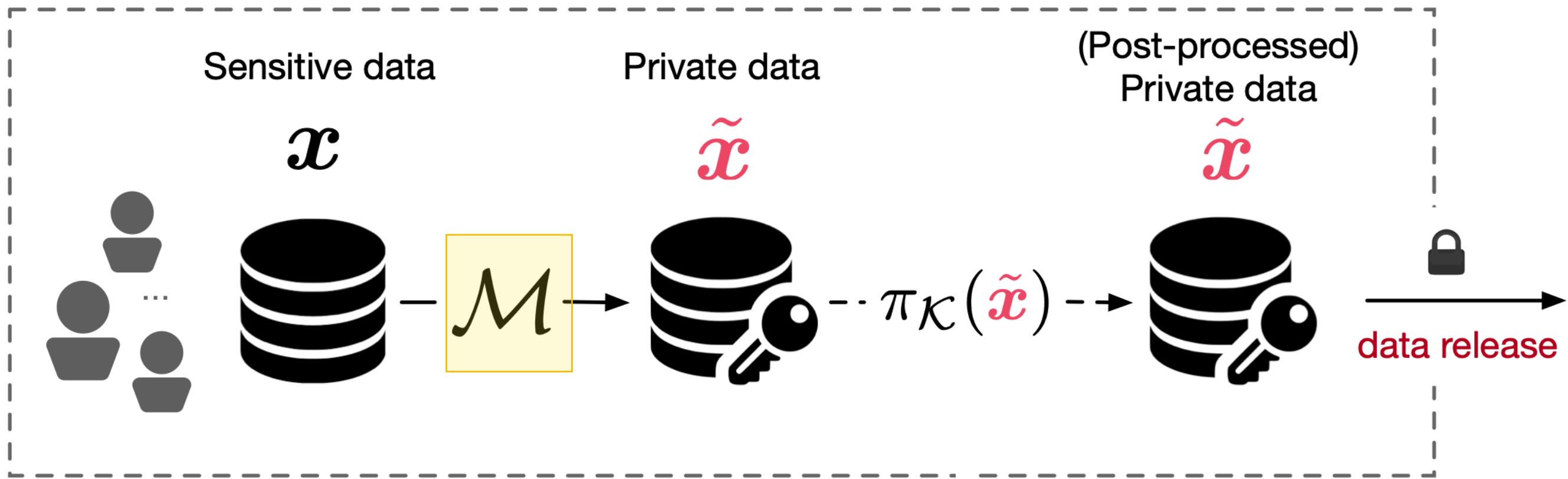
Release its (noisy) version  
as a constant



# DP Post-processing

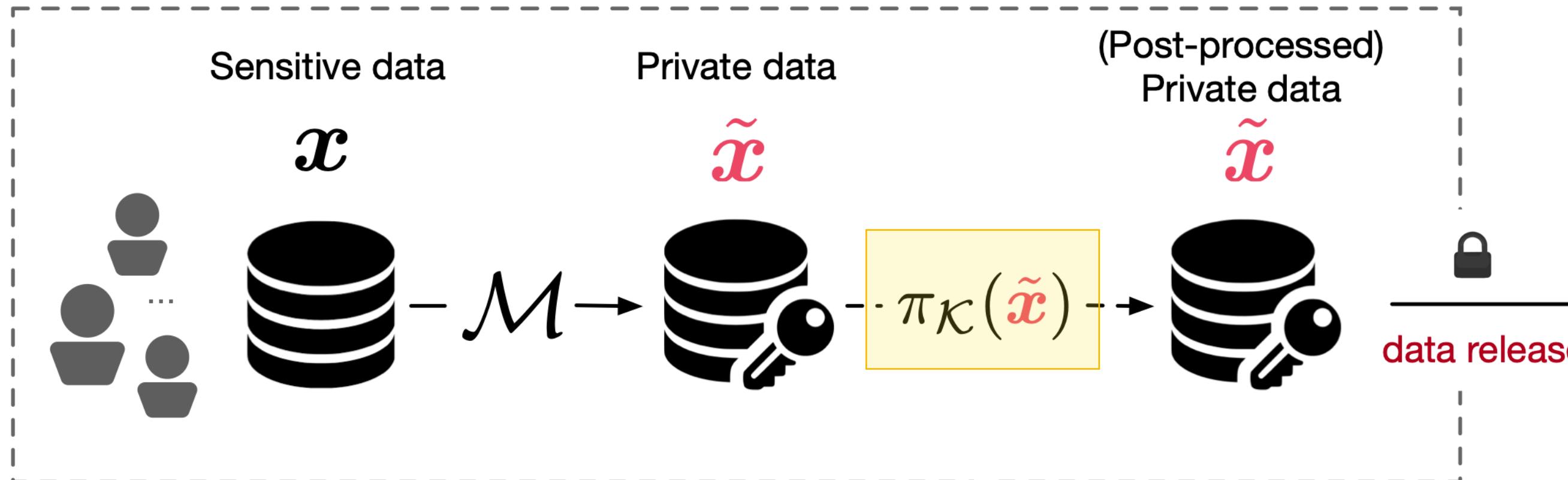
Fairness impact

# DP data release with post-processing



1. Apply noise with appropriate parameter  $\tilde{x} = x + \text{Noise}$

# DP data release with post-processing



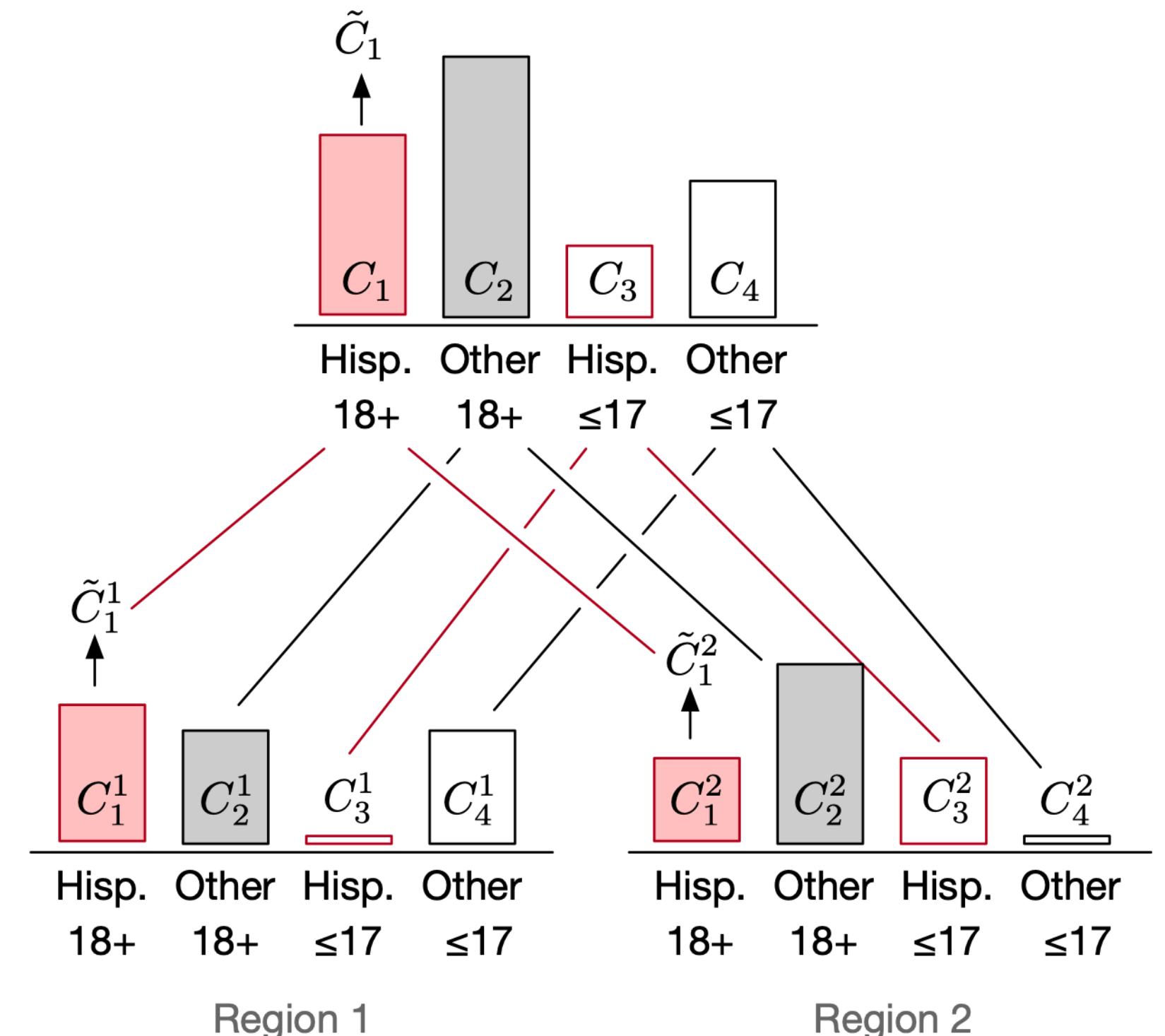
1. Apply noise with appropriate parameter  $\tilde{x} = x + \text{Noise}$

2. Post-process output  $\tilde{x}$  to enforce consistency

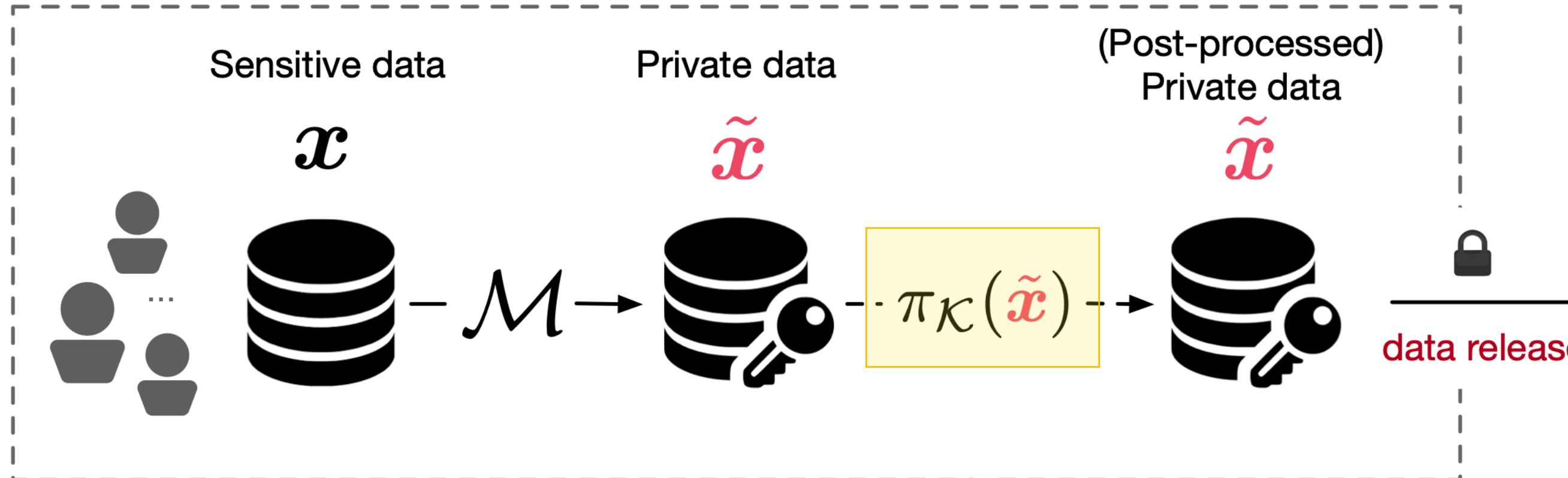
$$\pi_{\mathcal{K}}(\tilde{x}) : \operatorname{argmin}_{\mathbf{v} \in \mathcal{K}} \|\mathbf{v} - \tilde{x}\|_2$$

with feasible region defined as

$$\mathcal{K} = \left\{ \mathbf{v} \mid \sum_{i=1}^n v_i = C, \mathbf{v} \geq 0 \right\}$$



# DP data release with post-processing



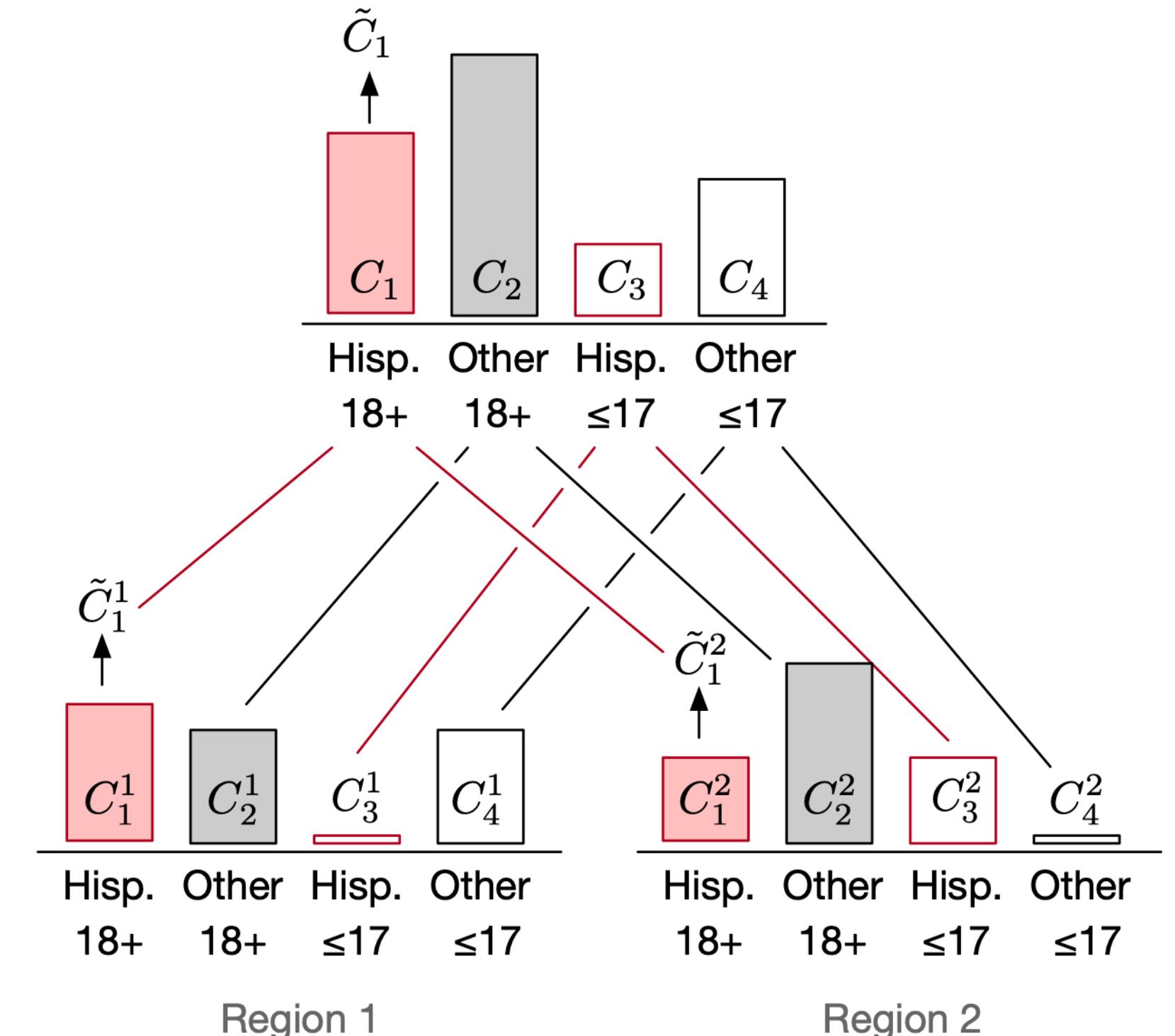
1. Apply noise with appropriate parameter  $\tilde{x} = x + \text{Noise}$

2. Post-process output  $\tilde{x}$  to enforce consistency

$$\pi_{\mathcal{K}}(\tilde{x}) : \underset{\mathbf{v} \in \mathcal{K}}{\operatorname{argmin}} \|\mathbf{v} - \tilde{x}\|_2$$

with feasible region defined as

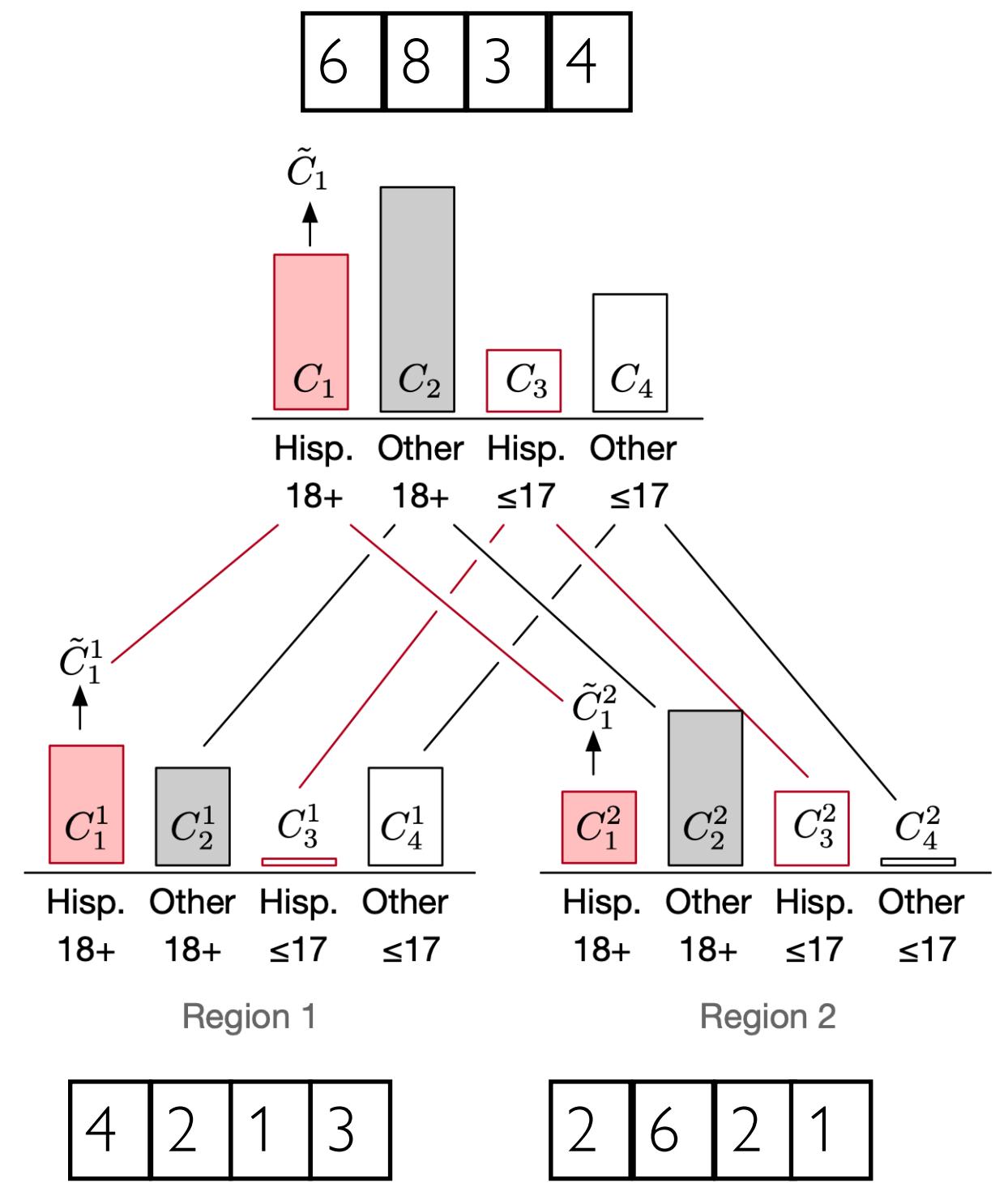
$$\mathcal{K} = \left\{ \mathbf{v} \mid \sum_{i=1}^n v_i = C, \mathbf{v} \geq 0 \right\}$$



Satisfies DP due to post-processing immunity

# DP post-processing

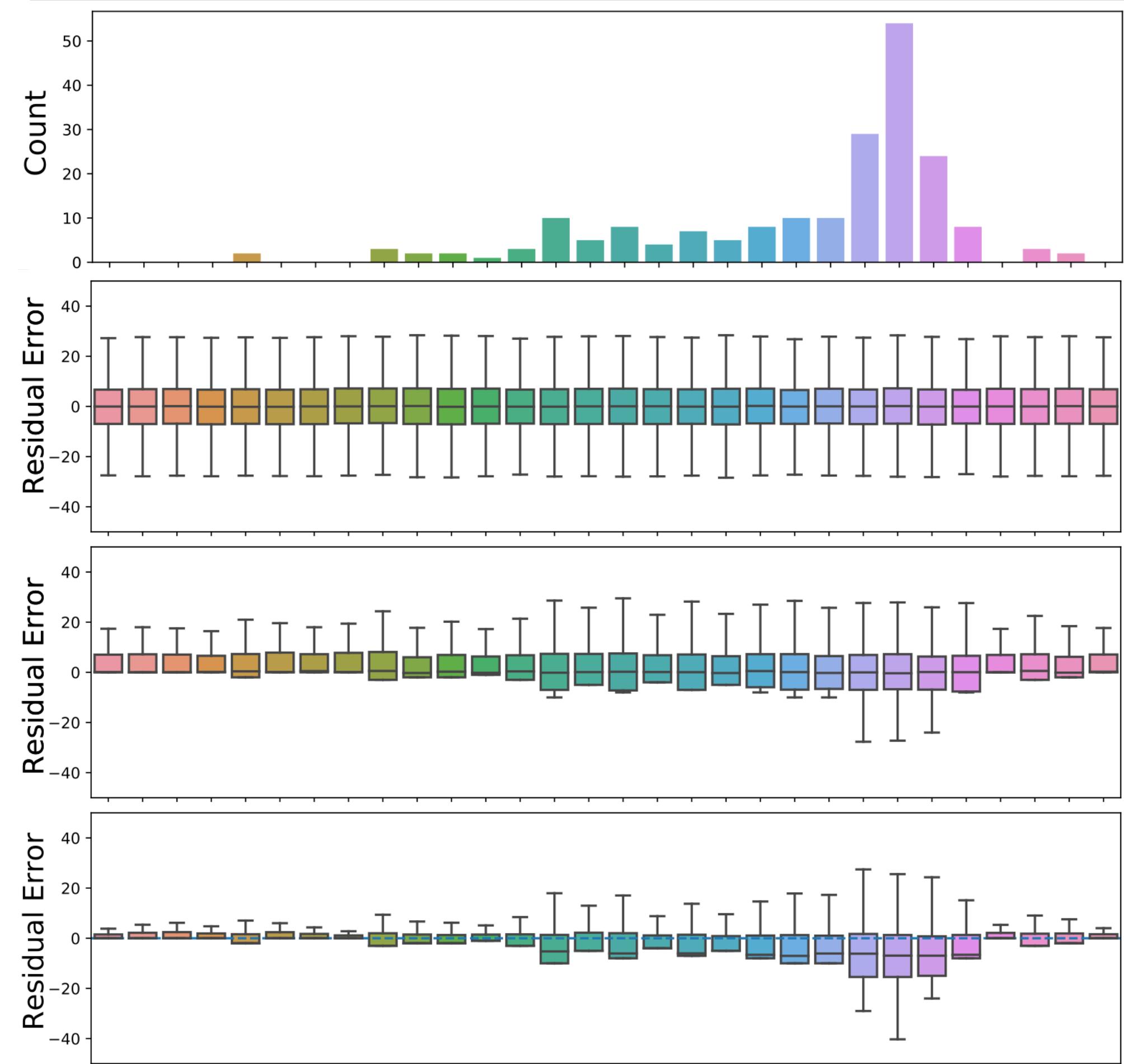
## Error and bias



$$\pi_{\geq 0} := \operatorname{argmin}_{v \geq 0} \|v - \tilde{x}\|_2$$

$$\pi_{\mathcal{K}_S} := \operatorname{argmin}_{v \in \mathcal{K}_S} \|v - \tilde{x}\|_2, \quad \mathcal{K}_S = \{v \in \mathbb{R}^n \mid \sum_i v_i = \tilde{S}, v_i \geq 0\},$$

Laplace  
mechanism



# DP post-processing

## Error and bias

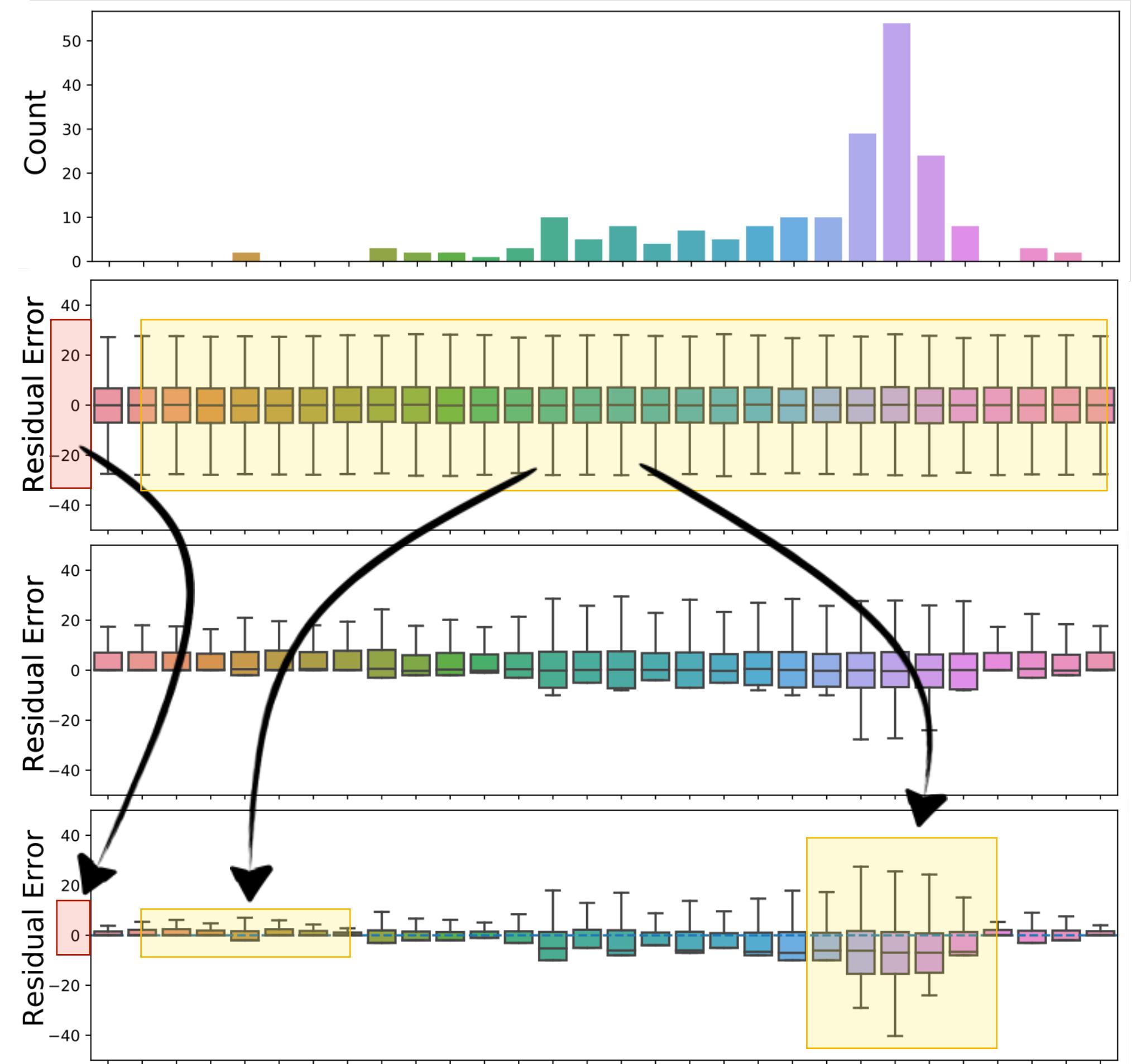
Observe that post-processing **reduces the errors.**

However, **it increases unfairness!**

Laplace  
mechanism

$$\pi_{\geq 0} := \operatorname{argmin}_{v \geq 0} \|v - \tilde{x}\|_2$$

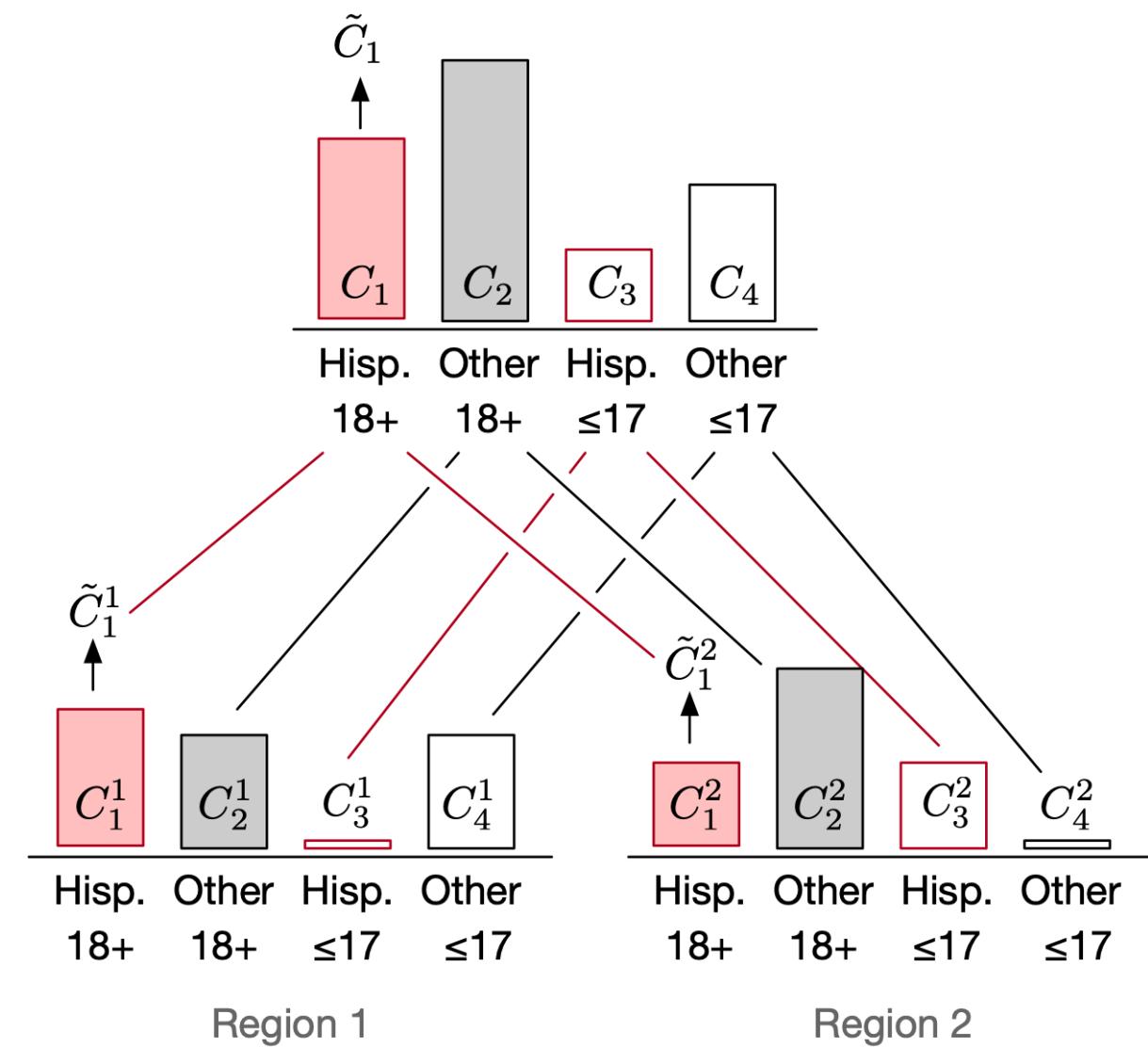
$$\pi_{\mathcal{K}_S} := \operatorname{argmin}_{v \in \mathcal{K}_S} \|v - \tilde{x}\|_2, \quad \mathcal{K}_S = \{v \in \mathbb{R}^n \mid \sum_i v_i = \tilde{S}, v_i \geq 0\},$$



# Bias of post-processing

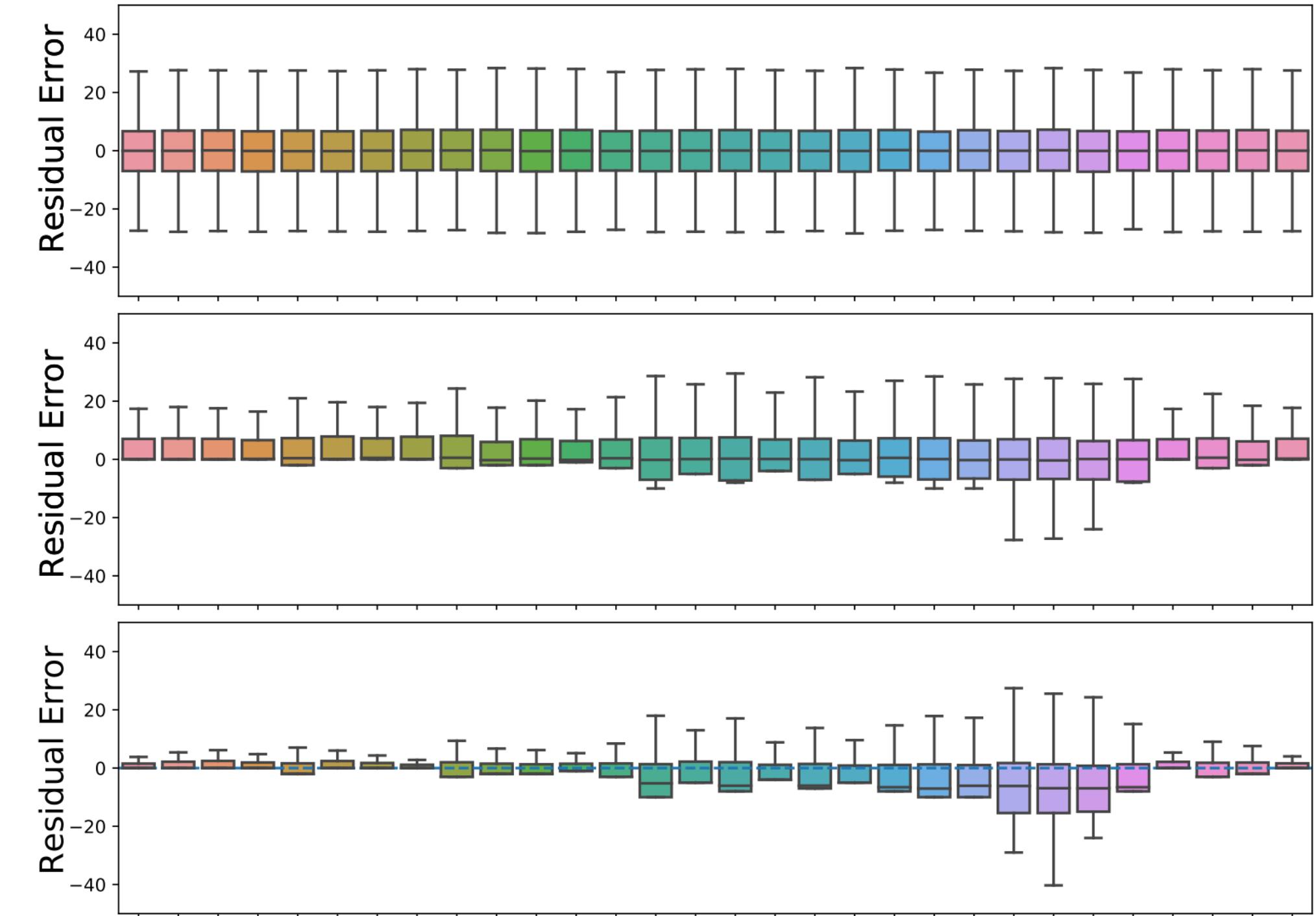
## Key result

- Thm (informal): The bias is caused by the presence of **non-negativity constraints!**



$$\pi_{\geq 0} := \operatorname{argmin}_{v \geq 0} \|v - \tilde{x}\|_2$$

$$\pi_{\mathcal{K}_S} := \operatorname{argmin}_{v \in \mathcal{K}_S} \|v - \tilde{x}\|_2, \quad \mathcal{K}_S = \{v \in \mathbb{R}^n \mid \sum_i v_i = \tilde{S}, v_i \geq 0\},$$

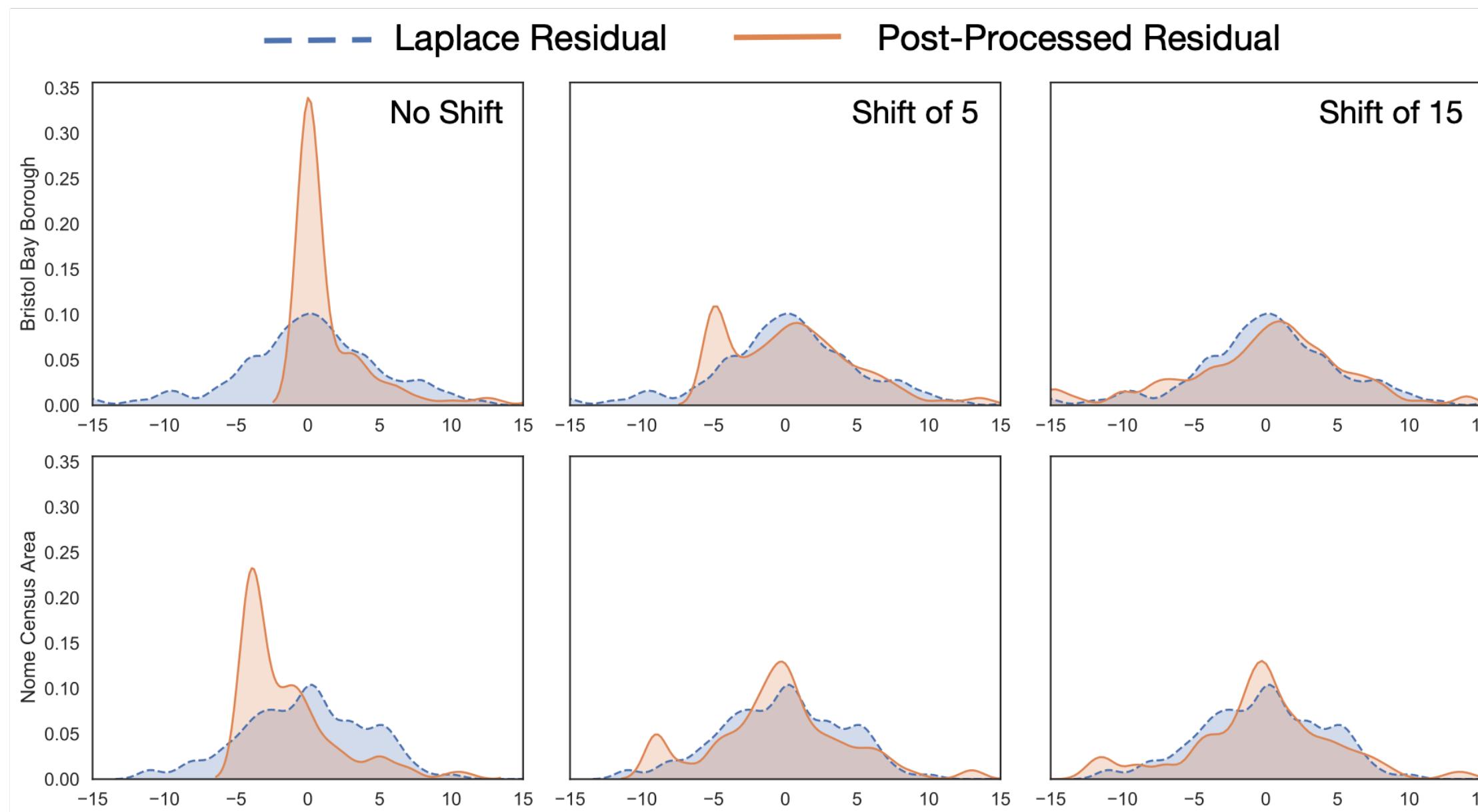


# Quantifying bias in post-processing

**Theorem:** Suppose that the noisy data  $\tilde{x}$  is the output of the Laplace mechanism with scale  $\lambda$ . The bias of the post-processed solution  $\pi_{\mathcal{K}^+}$  of program  $(L^+)$  is bounded, in  $l_\infty$  norm, by

$$\|B_{L^+}(\mathcal{M}, \mathbf{x})\|_\infty = \left\| \mathbb{E}_{\tilde{\mathbf{x}} \sim \mathcal{M}(\mathbf{x})} [\pi_{L^+}(\tilde{\mathbf{x}}) - \mathbf{x}] \right\|_\infty \leq C' \cdot \exp\left(\frac{-r_m}{\lambda}\right) \cdot \sum_{i=0}^{n-1} \frac{(r_m)^i}{i! \cdot \lambda^i},$$

where  $C'$  represents the value  $\sup_{v \in \mathcal{K}^+} \|v - \mathbf{x}\|_\infty$ , which is finite due to the boundedness of the feasible region  $\mathcal{K}^+$ .

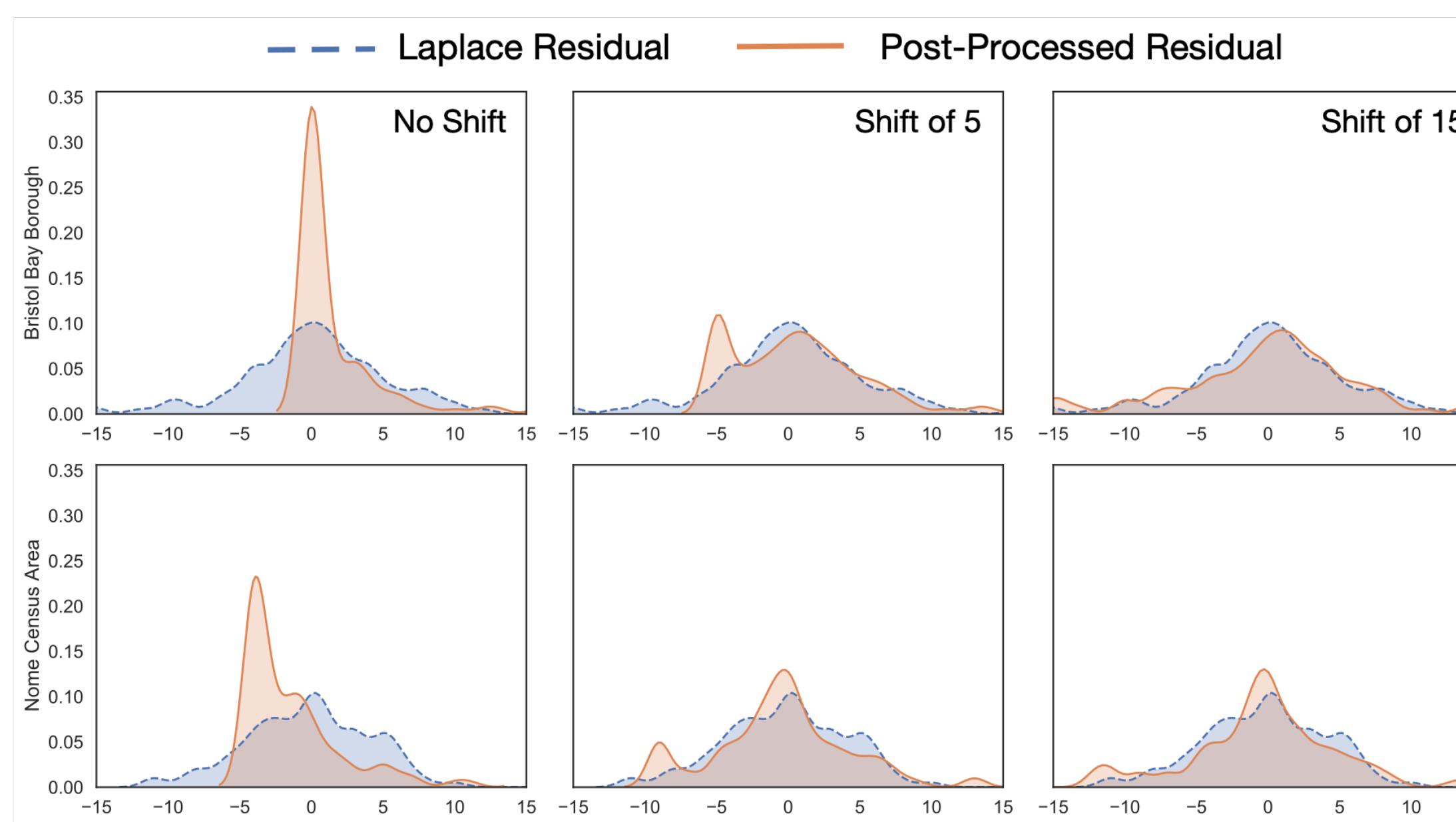


There is an  $\ell_1$ -ball of radius  $r_m = \min_i x_i$  and centered in  $\mathbf{x}$  which is a feasible subspace where there is no bias.

Shifting increases the value of  $r_m$  and the bias progressively disappear.

# Practical considerations

- Post-processing reduces the variance of the noise differently in different “regions”. Regions with many subregions (e.g., counties, census blocks, etc.) will have more variance than regions with few subregions.
- It creates situations where counties will be treated fundamentally differently in decision processes.



Aggregating the counts for

**Arizona** (pop: 2.37ML in 15 counties)

**Texas** (pop: 8.89ML in 254 counties)

Variance

186.67

200.01

~6.5% difference  
which may affect allocations!

# DP post-processing

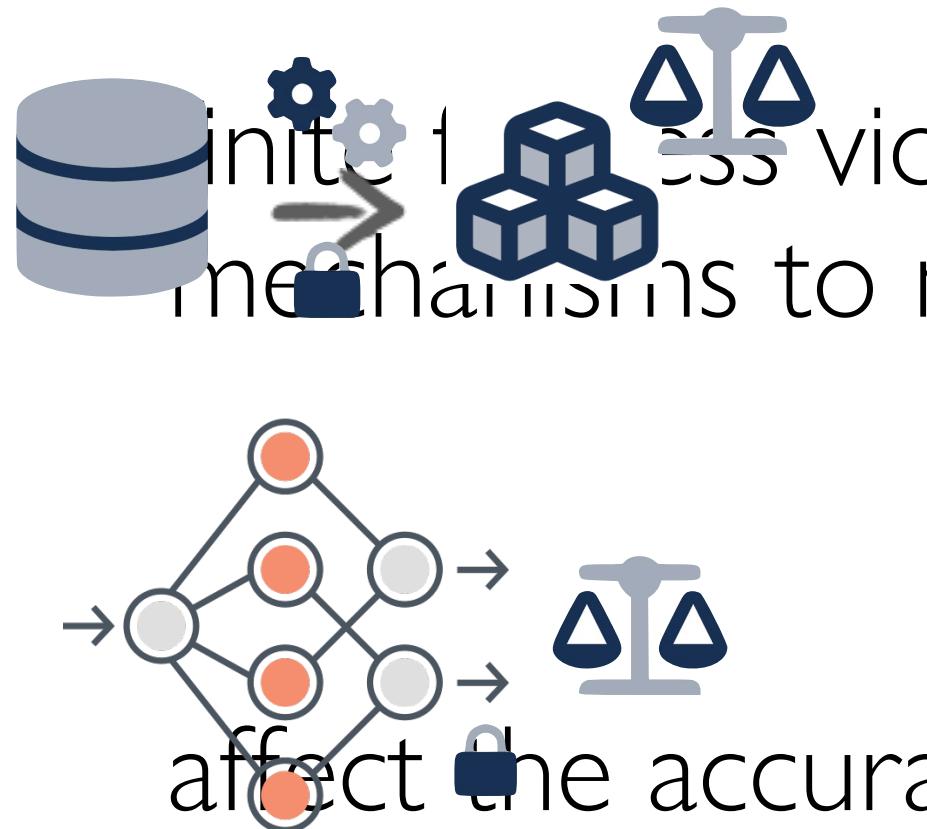
## Important conclusion

*Although post-processing reduces errors,  
its application to policy determinations  
should take into account fairness issues.*

# Conclusions

## Unintended effects of DP on decisions and learning tasks

- Motivated by the use of rich datasets combined with black-box algorithms
- Proved that several problems with significant societal impacts (allocation of funding, language assistance) **exhibit inherent unfairness** when applied to a DP release of the census data.



### Differential Privacy and Fairness in Decisions and Learning Tasks: A Survey

Ferdinando Fioretto, Cuong Tran, Pascal Van Hentenryck, Keyu Zhu

[Watch video](#)



- Exciting research direction that requires close cooperation between multiple areas and can transform the way we approach ML and decision making to render these algorithms more aligned with societal values.

# The unintended disparate effects of privacy in decision tasks

# Thank you!

**Ferdinando Fioretto** University of Virginia



<https://nandofioretto.com>



nandofioretto@gmail.com



@nandofioretto

# References (1/2)



"Differential Privacy of Hierarchical Census Data: An Optimization Approach".

Ferdinando Fioretto, Pascal Van Hentenryck, Keyu Zhu. In Artificial Intelligence (AIJ), 2021.

"Differentially Private Empirical Risk Minimization under the Fairness Lens".

Cuong Tran, My H. Dinh, Ferdinando Fioretto. In Conference on Neural Information Processing Systems (NeurIPS), 2021.

"Decision Making with Differential Privacy under the Fairness Lens".

Cuong Tran, Ferdinando Fioretto, Pascal Van Hentenryck, Zhiyan Yao. In International Joint Conference on Artificial Intelligence (IJCAI), 2021.

"Bias and Variance of Post-processing in Differential Privacy".

Keyu Zhu, Pascal Van Hentenryck, Ferdinando Fioretto. In AAAI Conference on Artificial Intelligence (AAAI), 2021.

"A Fairness Analysis on Private Aggregation of Teacher Ensembles".

Cuong Tran, My H. Dinh, Kyle Beiter, Ferdinando Fioretto. CoRR abs/2109.08630 [cs.LG], 2021.

"Post-processing of Differentially Private Data: A Fairness Perspective".

Keyu Zhu, Ferdinando Fioretto, Pascal Van Hentenryck. In International Joint Conference on Artificial Intelligence (IJCAI), 2022.

"Differential Privacy and Fairness in Decisions and Learning Tasks: A Survey".

Ferdinando Fioretto, Cuong Tran, Pascal Van Hentenryck, Keyu Zhu. In International Joint Conference on Artificial Intelligence (IJCAI), 2022.

See <https://web.ecs.syr.edu/~ffiorett/publications.html> for papers links.

# References (2/2)



"A Fairness Analysis on Private Aggregation of Teacher Ensembles".

Cuong Tran, My H. Dinh, Kyle Beiter, Ferdinando Fioretto. In the Workshop of Privacy-Preserving Artificial Intelligence (PPAI)–at AAAI, 2022.

"Fairness Increases Adversarial Vulnerability".

Cuong Tran, Keyu Zhu, Ferdinando Fioretto, Pascal Van Hentenryck CoRR abs/2211.11835 [cs.LG], 2022.

"Pruning has a disparate impact on model accuracy".

Cuong Tran, Ferdinando Fioretto, Jung-Eun Kim, Rakshit Naidu. In Conference on Neural Information Processing Systems (NeurIPS), 2022.

"SF-PATE: Scalable, Fair, and Private Aggregation of Teacher Ensembles".

Cuong Tran, Keyu Zhu, Ferdinando Fioretto, Pascal Van Hentenryck. CoRR abs/2204.05157 [cs.LG], 2022.

"Personalized Privacy Auditing and Optimization at Test Time".

Cuong Tran, Ferdinando Fioretto. CoRR abs/2302.00077 [cs.LG], 2023.

"Privacy and Bias Analysis of Disclosure Avoidance Systems".

Keyu Zhu, Ferdinando Fioretto, Pascal Van Hentenryck, Saswat Das, Christine Task CoRR abs/2301.12204

See <https://web.ecs.syr.edu/~ffiorett/publications.html> for papers links.

# DP Post-processing

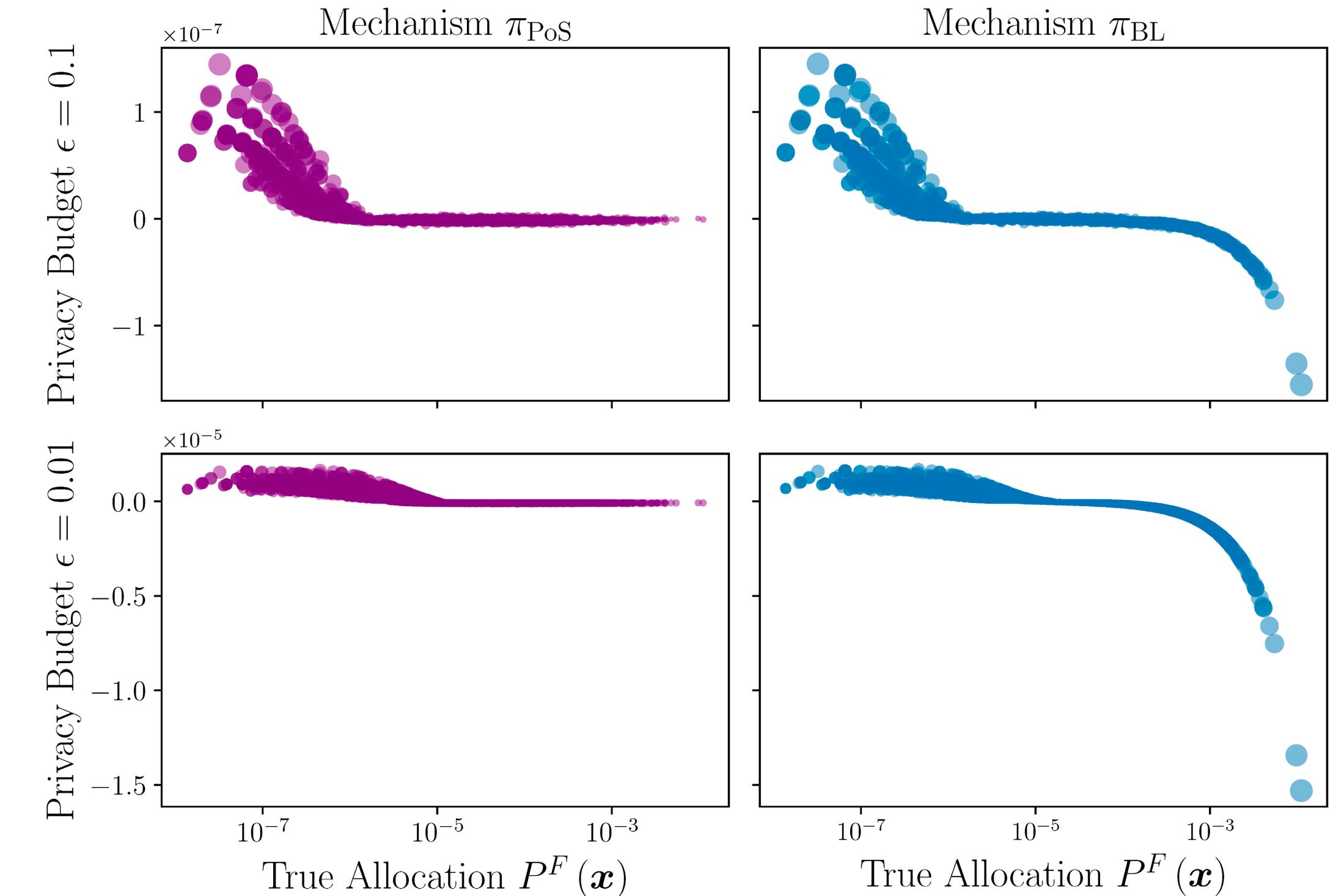
## Mitigating solution

**Definition 4** (Projection onto Simplex Mechanism (PoS)).  
*The projection onto simplex mechanism outputs the allocation as follows.*

$$\pi_{\text{PoS}}(\tilde{\mathbf{x}}) := \arg \min_{\mathbf{v} \in \Delta^n} \|\mathbf{v} - P^F(\tilde{\mathbf{x}})\|_2 \quad (P_{\text{PoS}})$$

**Theorem (informal).** For any DP dataset  $\tilde{\mathbf{x}}$  the PoS mechanism generates the unique optimal solution to program  $\pi_\alpha^*(\tilde{\mathbf{x}}) := \arg \min_{\mathbf{v} \in \Delta_n} \|\mathbf{v} - P^F(\tilde{\mathbf{x}})\|_{\leqslant} \sim$   $(P_\alpha)$

which closely approximate the optimal post-processing mechanism  $\pi^* := \min_{\pi \in \Pi_{\Delta_n}} \|\mathbb{E}_{\tilde{\mathbf{x}}} [\pi(\tilde{\mathbf{x}}) - P^F(\mathbf{x})]\|_{\leqslant}$



Privacy Budgets	$\epsilon = 0.1$		$\epsilon = 0.01$		$\epsilon = 0.001$	
Mechanisms	$\pi_{\text{BL}}$	$\pi_{\text{PoS}}$	$\pi_{\text{BL}}$	$\pi_{\text{PoS}}$	$\pi_{\text{BL}}$	$\pi_{\text{PoS}}$
$\alpha$ -fairness	3.00E-07	<b>1.50E-07</b>	1.70E-05	<b>1.75E-06</b>	8.06E-04	<b>2.23E-05</b>
Cost of Privacy	1.62E-05	<b>1.41E-05</b>	1.33E-03	<b>1.04E-03</b>	5.90E-02	<b>3.49E-02</b>