

The observer effect in quantum: the case of classification

Johan F. Hoorn^{1,2,3,4} and Johnny K. W. Ho²

¹School of Design, The Hong Kong Polytechnic University, Hung Hom, Hong Kong Special Administrative Region

²Laboratory for Artificial Intelligence in Design, Hong Kong Science Park, New Territories, Hong Kong Special Administrative Region

³Department of Computing, The Hong Kong Polytechnic University, Hung Hom, Hong Kong Special Administrative Region

⁴Department of Communication Science, Vrije Universiteit, Amsterdam, The Netherlands

Abstract

The observer effect in quantum physics states that observation inevitably influences the system being observed. Our proposed epistemic framework treats the observer as an integral part of sensory information processing within entangled quantum systems, highlighting the subjective and probabilistic aspects of observation and inference. Our study introduces a hierarchical model for fuzzy instance classification, which aligns sensory input with an observer's pre-existing beliefs and associated quantum probability-based truth values. Sensory data evolves via interaction with observer states, as described by the Lindblad master equation, and is then classified adaptively using positive operator-valued measures (POVM). Our parametrization employs measures of concurrent similarity and dissimilarity, facilitating perceptual associations and asymmetric cognition. The observer's position on a skeptic-believer spectrum modulates ambiguous matching of noisy perceptions. We show that sensory information becomes intricately entangled with observer states, yielding a wide array of probabilistic classification results. This framework lays the groundwork for a quantum-probability-based understanding of the observer effect, encouraging further exploration of quantum correlations and properties in cognitive processes.

Introduction

The issue we address is the well-known and long-debated observer effect and adjacent uncertainty principle. Even if the way the world is structured is fully deterministic, we will need to run probabilistic experiments *ad infinitum* to decide that what we observe is deterministic, especially when clean observations are confronted by technological or theoretical complexities¹. Helland² states that the knowledge of an observer is contained in a set of accessible theoretical variables, and the observer will not be sure whether this set contains what s/he needs to know. In line with this thought, the observer will never be sure if everything is known that needs to be known. We have to accept that all theorizing is probabilistic *sine qua non*. Our aim is not to circumvent this issue but to take it as our starting point and make the observer an inseparable part of scientific observation and hypothesizing³, who may be subject to various goals, beliefs, and psychological conditions⁴. As our research objective, we propose an epistemic framework that allows for a generalized, quantum-probability based interpretation⁵ by considering observation as part of the system and to provide a framework of variables to be involved in the protocol. In contrast to Sassoli de Bianchi⁶, our rationale is whether physically out there or not, all observed and hypothesized phenomena revert back to an observer's interpretation and view. What we intend is to model through quantum probability the observer as part of what is observed, object and subject being "entangled." Our proposal does not solve the insoluble of not knowing what there is to be known but does make our views part of the range of probabilities of occurrences of an object under investigation.

When hypothesizing about the dynamics of sub-atomic particles, one does a statement about their states. Spin, or spin angular momentum, is an intrinsic property of matter that characterizes the magnetic moment of a particle. The magnetic moment produces a magnetic field and allows the particle to interact with an external magnetic field. The interaction is associated to energy, enabling the use of the spin quantum number to specify the state of matter in terms of energy levels, whether or not degenerated. It is also a way to make spin an observable entity and create measurable events. Being quantum, spin may be in superposition, exhibiting all properties of quantum probability.

Like all matter, the brain is susceptible to physical laws. Mind as neurofunctions of the brain, then, may also work according to physical dynamics of firing—perhaps in a

derived way such as spintronics⁷. For example, ions in superposed collective states of (including but not limited to) spin distribution may occur in a neural area such as the amygdala, relating together attraction and aversion or trust and mistrust in some sort of balance of contrasting mental states. Here, we model mental states as quantum states and epistemic processing as state evolution through the introduction of equations of quantum dynamics and quantum measurements. The nature of quantum probability allows for inter-transformation of contrasting states in the form of parallel processing.

Epistemics is the formalization of knowledge and beliefs an agency holds⁸. To arrive at a comprehensively acquired dataset, knowledge base, and analytical interpretation with an epistemic framework that allows for non-classical effects, we will work with the following principles from quantum physics: A single object or event may be observed in multiple places and may look like different things in appearance and behaviors. When an event is observed (i.e., measured), it may behave differently from when it is not observed. All the possible states an object or event can be in may obscure the final state: superposed states introduce uncertainty about single events and require probability estimates, a percentage of likelihood. Such quantum probability is applicable to every aspect of what we regard as reality, including things larger than the sub-atomic scale (cf. ref. ^{2,6,9,10}). No matter how often a measurement is taken and no matter the mathematics involved, not one single outcome result is predictable, but the likelihood of multiple answers can be calculated. For the observer, any possible explanation that can be thought of may be prevalent at one point, but it may take a separate school of thought (interpretations, views) to account for each explanation—while respect should be paid to Occam’s razor. Any observation but also math equation is affected by actual decisions, choices, foci, and biases of the observers and their measurement devices (cf. ref. ⁶). An observer, his/her question, and measurement are always entangled with the thing observed, queried, or measured. Repeated experimentation can merely confirm a bias with higher confidence but does not necessarily render the “correct” answer. We believe that assuming physical multiverses and that “what looks different is different” disregards Occam’s razor.

Related work

Observers can be modelled in different ways. For instance, Baclawski⁹ models the observer as an independent Poisson process combined with the measurement device, including their relationship (a key aspect). The combined measurement-observer process would render (slightly) different results from the isolated measurement and this combined process could then be observed by yet another observer in the same Poissonian way.

Realpe-Gómez¹¹ models the observer as a physical process itself. Realpe-Gómez¹¹ approaches physics from a first-person perspective and formulates a (recursive) belief propagation algorithm on the basis of imaginary-time quantum dynamics as described by Schrödinger's equation and its conjugate, a two-state vector formalism. On this view, information can transition back and forth as probability distributions (cf. a wave function in imaginary time), and opposite flows of information can be mixed (*ibid.*). At laboratory level, the experimenter not only sees that A causes B; the mental representation of B makes the experimenter manipulate A. This produces a reciprocal flow of information, “a directed loop representing reciprocal causation” and demands second order differential equations to begin with in describing the observation of a physical system (*ibid.*). Realpe-Gómez¹¹ requires that an observing system like this can “extract high-level features from the raw data provided by the external environment,” which are fed to “a dynamical model of the external world,” operating on those features¹¹. Instead of the Turing machine or recursive neural network that Realpe-Gómez suggests, we introduce the quantum-probability formalization of an epistemics module developed by Hoorn¹² to tie in the first-to-third-person perspective with the physics under observation.

In Realpe-Gómez¹¹, a fundamentally classic system like an observer looks at another classic system and from a first-person perspective, that observed system may appear to be “quantum.” The observer does not induce decoherence but is him/herself the cause that phenomena appear to him/her as quantum².

Allard Guérin and Brukner¹⁰ modeled local viewpoints on the arrangement of quantum causality. Events are viewpoint-dependent because their timing is. Allard Guérin and Brukner¹⁰ studied how a quantum system evolves over time as related to a particular observer, of whom they conceive of as a “causal reference frame,” describing a given event at an observer-dependent point in time¹⁰. The event in focus is local but other events may

linger in past and future, the only demand being consistency among descriptions of those events by other observers, which is comparable to being “intersubjective.” Locally, then, when the past and when the future event happens may depend on the various observers: One may interpret causality in non-causal occurrences, owing to the time localization of events in the eyes of one specific observer. Across multiple observers, however, it would be observer-independent that A causes B. If A effectuates B, then future event B would be in a controlled superposition with current event A. Yet, such causal relationship between two events may be indefinite still.¹⁰

Helland² states that the mind of an observer in a group of observing minds that communicate with one another, under specific conditions, produce/s a quantum model of the world. Helland² describes the process of acquiring knowledge; an epistemic account of the concepts in one’s mind during decision making. While making a decision, the observer is limited by the number of variables s/he is capable of considering and observers sometimes make “imperfect” decisions. To Helland², some variable inaccessible to one’s own mind may be (in part) visible to someone else. This somewhat resembles the indirect observation in the “Wigner’s friend” scenario, where the onlooker (Wigner) may not know for sure what the experimenter (his friend) observed¹³. Wigner and friend have different concepts of reality, different representations of the physical world in their minds¹². Also, for the observer self, past observations cannot necessarily be trusted to occur in the future¹⁴. Helland² provides an epistemic interpretation—it is observer-dependent what one *can* know—to cases such as Young’s double-slit experiment, Schrödinger’s superposed random events occurring or not, or Wigner entangled with his friend’s observation of superposition. Epistemically, the answer to superposed events is “state unknown².” Seeing a wave or a particle would depend on the measurement the observer chooses. As soon as the onlooker knows the experimental result, s/he may agree with the experimenter. When all observers find intersubjective agreement, there would be an “objective” state of the world (*ibid.*).

Sandberg and Delvenne¹⁵ focus on the medium through which observations are made and convey that the physical system and the instrument that observes it are connected through a cable, a laser, or an electrode, which are susceptible to thermodynamic noise (unless measured at 0 K). They envision physical system and medium as linear port-

Hamiltonian systems, assuming that energy is equally divided and that fluctuations are short-lived. From the measured data, the (earlier) state of a system can then be reconstructed¹⁵.

In the previous, authors formulate mathematics to their specific epistemics, their individual belief system, from which they construct a reality, of what they find acceptable as a means to gain knowledge and what not (e.g., whether observers can communicate, whether two features can be present at the same time, whether features are intrinsic to an object or transient, whether an experiment produces an effect that would not otherwise be present, etc.). Others may, for example, envision the observer as a causal reference frame in an observer-dependent point in time, providing quantum-coordinated transformations between two (or more) classical space-times in superposition¹⁰. Yet others take the observer as a self-printing Turing machine and design a belief-propagation algorithm from Schrödinger’s imaginary-time quantum dynamics¹¹. Our work reaches inside that causal reference frame or that self-printing Turing machine and describes what happens internally in a quantum-like observer system, its information throughput, which on the physical plane may then function as a “quantum-coordinated transformation” or as a “belief-propagation algorithm” in relation to the observed quantum systems.

In the remainder of this paper, we offer an encompassing model, in which each type of epistemics can be included as a different first-person perspective, but potentially operating under the same (observer-general) routines but with different parameterizations. Realpe-Gómez¹¹ remarks that including the observer as a physical system interacting with its observed environment, both frequentist and Bayesian probability are fused. On that note, we like to take the observer as a quantum system so to cover such range of approaches to probability: (1) Because once read out, a quantum measurement creates a materialized conditional in the mind of the observer and then may work as a classic proposition ($A \rightarrow B$); (2) Because quantum probability can be integrated with Bayesian inference (i.e., beliefs are updated after measurement); (3) Because we think the entangled system of observer and observed also should cover straightforward quantum experimentation. Hence, we hold that observation and inference are subjective, also in classic cases, and statements about the physical world express beliefs about degrees of probability even if those fall back to a

Boolean. Instruments are appendices to sense perceptions and every read-out is done by observers, including their assumptions and biases. Our epistemic approach makes no assumptions on the observed physics itself, distinct from the work of Leifer and Spekkens¹⁶, but rather models how an observer deals with (quantum) information. Because the results of measurement and observation depend largely on an investigator's choices⁶, on the decisions the experimenter makes, signal-detection processes govern decision making.

Structures of information, beliefs and observers

To address the observer effect, we rely on *Epistemics of the Virtual*¹², assuming a dynamic knowledge base with degrees of certainty about that knowledge along a truth space (true–possible–false), resting on an individual's belief system. To determine the observed entity, each concept in the belief system can be viewed as a category of features to which features of the observed sensory information are matched. These concepts are subjective to the agency and may or may not reflect actual physical entities. In Hoorn¹², the initial instance–category match (ICM) is viewed as fuzzy-template matching, aligning observed features with concepts as distributions of fuzzy templates, encoded as statistical moments such as prototypes. The distribution of a concept allows for some overlap with others, maintaining a balance to prevent confusion (cf. individual parameter settings), thus enabling fuzzy classification.

Fig. 1 outlines the classification process. Occurrences in the physical world becomes encoded sensory information when entering the mental world of the observing agency, like perception of colors originated from the wavelength of photons and brightness as the photopic or scotopic response of light irradiance. The sensory information undergoes a series of channels involving various functional components as *observers* that interact with and transform the sensory information to a classified state. When considering an observing agency that adheres to quantum principles, we propose a broader approach to representing truth values of features in the beliefs and encoded sensory information. Section A in the Methods elaborates on the detailed formulation. A feature (e.g., red) consists of a distribution of attributes (e.g., full red, pale red, and no red), each specified by a collection of quantum harmonic oscillators of truth values. The encoded sensory information encapsulates the relevance to the observing agency. Metaphorically, relevance is packets

of action potentials produced by the group of neurons that constitute the resultant neural signal. Analogous to the steep-edge feature of action potential and the positive correlation between the stimulus intensity and proportion of relevant activated neurons, relevance is modeled as the oscillator energy levels which are discrete and evenly spaced, characterized by the oscillator frequency. A higher relevance of truth value as a sign of greater stimulus is then represented by a stronger excitation of the truth-value oscillator, possessing more energy. Note that this relevance is not normalized to the conventional scale [0,1]. The total energy of the oscillators associated with the feature (all attributes and their respective truth values) indicates the feature weight. In this framework, a feature spans a Hilbert space of relevance of various attribute truth values. Rather than directly attributing a definite truth value to a feature or attribute in the classical picture, truth values are two-dimensional representations of quantum oscillators and their energy state as relevance. These oscillators can be indexed and specified with certain physical properties such as frequency. Probability amplitudes are assigned to truth-value relevance (oscillator energy levels), signifying the inherent ambiguity and indeterminacy of quantum information that permit the simultaneous existence of multiple feature attributes (e.g., full red, *and* pale red, *and* no red), truth values (true, *and* possible, *and* false), and relevance (relevant *and* irrelevant), manifested as either superpositions or mixed states, which encapsulate sampling variability of observations and fuzzy templates of beliefs, multi-perspective perceptions, and indefinite beliefs.

Apart from quantized relevance, the notion of collective, individual oscillators suggests that feature attributes and truth values may be *quantized* into *discrete* levels rather than on a continuous manifold (e.g., [0,1] for truth values) to characterize the observer agency's limited resolution (differentiability). A higher-resolution agency might distinguish the attributes of feature "red" not only between "full red" or "no red" but also intermediate levels (e.g., "pale red") by possessing extra oscillators. To describe the degree of certainty, an agency may merely use two oscillators of "entirely not true" (truth value = 0) and "entirely true" (truth value = 1) for an attribute, but higher-resolution observers may utilize additional oscillators as gradations (e.g., "perhaps true" or truth value = 0.5). These dimensions define the possible outcomes of the degree of certainty when measured. Other perceived intermediate truth values of an attribute, such as 0.25, are represented by

the collective excitation of oscillators as a composite system, where superposition (for vague, indefinite states) and a mixed state (for ambiguous states, inviting multiple definite or indefinite perspectives) are possible. In essence, the observing agency encodes sensory information and categories of concepts in the belief system as quantum states of truth-value oscillators for the range of features and attributes, presented as relevance. Each dimension of the composite Hilbert space corresponds to a particular feature–attribute–certainty–relevance combination. The entire sensory information or concept of belief spans the tensor product of all feature subspaces. The observer perceives the belief or perception distribution with the associated probabilities.

For simplicity, we assume the same observer for the entire classification process. Certain features are assigned to the observer, like accepting or rejecting a hypothesis (Accept, Reject), confirming or falsifying an observation (Confirmation, Falsification), or having a feeling about it (Trust, Mistrust). The feature pairs are not simple opposition but two unipolar scales of discrete levels subject to the observer’s resolution, sometimes showing bipolar form but commonly represent asymmetric appraisal²⁸. Such an approach handles complex and information-rich states, allowing for variation and detail²⁹. Each observer feature level is modeled as a quantum harmonic oscillator which spans a Hilbert space of arousal levels, similar to perceived information and concepts. Together, the observer is a composite system of the observer features constitute multidimensional unipolar scales, modeled as Hilbert subspaces, where each dimension corresponds to a particular observer feature–level–relevance combination. The entire observer state may be constructed from the tensor product of the subspaces.

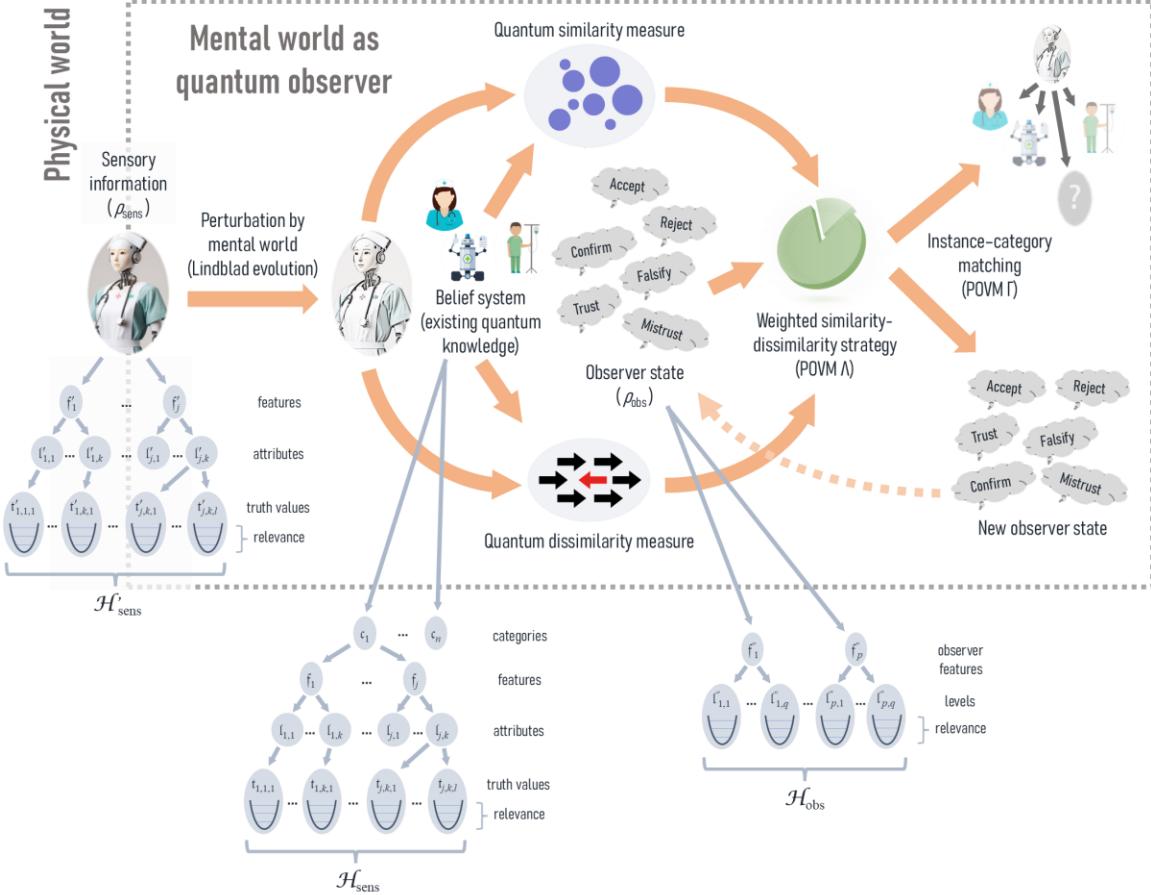


Fig. 1 Schematic diagram of the quantum observer-embedded epistemic framework. The sensory information (system) and the mental world (environment) of the machine forms a composite quantum system. The sensory information consists of a collection of quantum harmonic oscillators of feature attribute truth values that span a Hilbert space of relevance $\mathcal{H}'_{\text{sens}}$. The sensory information entangles with the mental world and undergoes a series of quantum channels (orange arrows) to get transformed to a representation of category in the belief system as the classification result. Each category has an information hierarchy similar to the sensory information, spanning a Hilbert space $\mathcal{H}_{\text{sens}}$. The mental world acts as an observer represented as another collection of oscillators where its affective state that spans the Hilbert space \mathcal{H}_{obs} influences the evolution of the sensory information, leading to particular preferential classification results. The observer state iterates over POVMs (light orange dashed arrow).

Observer effect as interactions

Prior to judgments or decisions, sensory information evolves with observer states with varying coupling strengths, strong or weak (Fig. 1). Scientific observers trying for objectivity, not to be diverted but to maintain the same state during information evolution, aim to minimize bias and personal involvement, attempting weak coupling between sensory information (the “system”) and observer (the “environment,” “reservoir,” or “bath”). Conceptually, an event of sensory information entering the observer’s “thermal bath” with individual functions may induce transient Markovian environmental fluctuations, quickly dissipated due to short correlation timescales and restoring the environment, which can be described by the Lindblad (also Lindblad–Gorini–Kossakowski–Sudarshan, or LGKS) master equation (Eq. (3))^{17,18}. The interaction Hamiltonian can detail every potential correlation (e.g., transitions) between sensory information states and observer states (see Section B of Methods). The observer’s “temperature” (Eq. (7)) determines the sensory information’s asymptotic behavior, with higher temperatures leading to more indefinite features due to wider state distributions, barring other disturbances. Apart from coupling strength (Eq. (4)), the typical timescale λ also modulates sensory information, defining the exponential distribution of the duration before ICM measurement (see Section C in Methods). Larger λ values would suggest that ICM results are less reliant on sensory information. Thus, sensory information would be continually modulated by the observer’s state. Variation of the same information state also arises in this process due to probabilistic evolution duration, presenting an observer-dependent uncertainty^{19,20}.

The interplay between sensory information and the observer persists as the observer engages in ICM and decides on the congruence of sensory features with categories (e.g., a photon is a particle and/or a wave), the approach depending on the observer. Here, signal detection rests on quantum measurement through Positive Operator-Valued Measures (POVMs) as a form of perceptual confidence (Fig. 1)^{5,21}. As shown in Fig. 2, POVM construction for ICM (Π) as depicted in Eq. (18) depends on the similarity-dissimilarity strategy (Section C(a) in Methods, also the next section), the degree of observer’s self-skepticism (Eq. (11)), candidate categories, and the observer’s state (Section C in Methods). Each POVM element is associated with the tensor product of the candidate category (or its

representation, depending on the recognition approach) and the corresponding observer state once the channel of candidate category is chosen. The category term is derived from the distribution of the truth-value relevance for the feature attributes and transformed by the observer's preference for similarity-dissimilarity measures and being a believer or a skeptic. Thus, POVM encapsulates information about candidate categories and the observer's specific measurement criteria. Including the observer state in the POVMs makes the classification outcome contingent on the observer's state at the point of measurement (see Section C(d) in Methods). Consequently, variations in interpretation upon identical sensory input are an inevitable result of the sensory-observer interplay. In a classical, Boolean ontology, the sensory information would be classified as either one or none (no match) of the candidate categories. In a non-classical ontology, the nature of POVM allows a photon may be identified as a particle in part as it is also partially a wave.

Perception transformation in similarity measures

In classification, noisy data can obscure or mislabel key features, while complex backgrounds may introduce extraneous features. A cognitively plausible measurement device may match sensory information against a category through similarity and dissimilarity measures (Fig. 1). Similarity measures increase scores for membership of features in both sensory data and candidate categories, up to the point of undetectable features, while dissimilarity measures penalize similarity with distinctive features to emphasize differentiation. Within the belief system, a feature known to be nonexistent for the category is regarded as irrelevant and assigned a low truth-value distribution (high relevance state of low truth-value oscillators). A feature perceived to have no prior knowledge of membership for the category may have a distributed truth-value distribution (comparable relevance levels over oscillators of a wide range of truth values, high dissonance) or low (mild skepticism)).

As depicted in Fig. 2, a quantum approach enables the observer to use similarity and dissimilarity measures concurrently as dual channels, with the information processing modeled as a POVM Λ with a two-level system of states $|0\rangle_{\text{sdm}}$ and $|1\rangle_{\text{sdm}}$, denoting the similarity and dissimilarity pathways, respectively (Eq. (13)–(14)), and the observer states (Section C(a) in Methods). Observer states inclined towards positive experiences like

Accept, Confirm, and Trust may bias the observer towards choosing similarity measures, and v.v. (e.g., Eq. (10)). These psychologically assisted variables pose constraints on the dimensions of the affective states, which may be associated to some neuron-level functionality²². In other words, this context-based approach may serve as an alternative route for reducing the issue of overparameterization²³. The reduced post-measurement similarity-dissimilarity strategy state ρ_{sdm} displays the probabilities for both channels, with channel weight-asymmetry resembling the Tversky index, which uses distinct parameters for feature-set operations, enabling perceptual associations and hierarchical cognition²⁴.

The classification is executed through another POVM Π , its elements reflecting the belief in candidate categories for a *given channel of similarity and dissimilarity measures* (Π_c) and *observer state* (Π_o) upon selecting different categories (Eq. (18)). One quantum interpretation of similarity and dissimilarity is the choice of the Hilbert space for POVM. Similarity is established by contrasting sensory information that intersects with the category features with *all* the features of the candidate category. For dissimilarity, *all* sensory information is used in contrast to what intersects with category features. This is a shift in the focus of comparison, contingent on the preferred similarity measure. The resulting POVM elements of Π_c comprise a mixed state of the sum of both measures weighted by the results in the POVM Λ with aligned dimensions (Eq. (13)). Detailed formulation is given in Section C(b) in Methods.

The distribution of similarity and dissimilarity measures can critically influence classification. Disjoint feature sets as a result of the two measures can widen the disparity in category identification. With noisy data and few fine-grained features recognized, along with a small effective weight for the null classification element Π_0 (Eq. (17)), categories sharing the same features may exhibit a comparable (low) degree of similarity in terms of truth-value relevance, as can categories that share distinct but comparable feature counts and truth-value distributions. This holds even when the truth-value relevance Hilbert space for a universal set of feature is assumed for POVM, as the POVM construction disregards the features-to-be-ignored by decreasing the relevance. This may reduce the resolution of assessing similarity, causing a wider spread in classification outcomes while reflecting cognitive bias in which scarce and biased information determines the inference. Conversely,

similarity measures act as a channel that retains redefined sensory information of a few common dimensions, while undetected features can distort the original perceptions during ICM: The sensory information may not maintain the excluded features but may include non-existing features, leading to more random, unspecific, and ambiguous classifications with diminished discriminative power. This also holds for purely dissimilar measures. The adaptive measurement in this context allows for a number of similarity profiles for ICM, incorporating a quantum transformation of perception in a parallelized manner.

Perceptions variety: believer and self-skeptic

An observer who believes in a particular theory upholds more definitive beliefs with clearer differentiation boundaries, while a (self-)skeptic maintains openness to unknown information. This is a concept-driven process related to perceptual confidence, captured by the openness degree η in the external-feature operator F_c , weighing the balance between being a believer and a self-skeptic (Eq. (11)). In dissimilarity assessments, a believer assigns a high relevance of zero truth value to features not in the category, implying their absence. Conversely, a self-skeptic, tolerating deviations from known information, treats features not explicitly included in the category as information gaps or as maximally mixed states, reflecting that “without information, anything goes” (e.g., speculations on the early development of the theory of atoms²⁵). During POVM, applying Born’s inner product of the basis, a believer operator yields a narrow category distribution due to the stringent demand of numerous matches. In contrast, a self-skeptic’s broader outcome distribution as the inner product produces finite values for all uncertain features in the candidate categories, resembling noise to the closest known match while allowing for alternative possibilities. Thus, η modulates the classification distribution.

A believer typically classifies more accurately, while a self-skeptic allows for more flexibility while tolerating noisy or incomplete information, which often features low truth values indicating uncertain or indefinite feature recognition. With a preset threshold, fewer actual features might be detected, making the sensory information resemble an amalgamation of incomplete puzzles, obscuring some relevant features. The self-skeptic’s approach of “everything is possible when unknown” suggests a strategy of ambiguous auto-completion. POVM necessarily encompasses all sensory features, setting the range

for the observer to consider potential features in the categories. A self-skeptic then assigns a flat distribution in every candidate category (or its templates) with unique features in the sensory information, representing probabilities of their existence (finite truth values) as attempts to fill in the gaps. This flat distribution temporarily alters the typical interpretation of categories for ICM. Chances may exist that some vague belief in an undiscovered feature reduces dissimilarity between sensory data and categories, facilitating identification. Owing to observer traits, this is another form of category adaptation, alongside the modified feature sets for dissimilarity measures and instance classification. Conversely, a believer accepts noisy or limited information as is, potentially leading to null identification and false negatives.

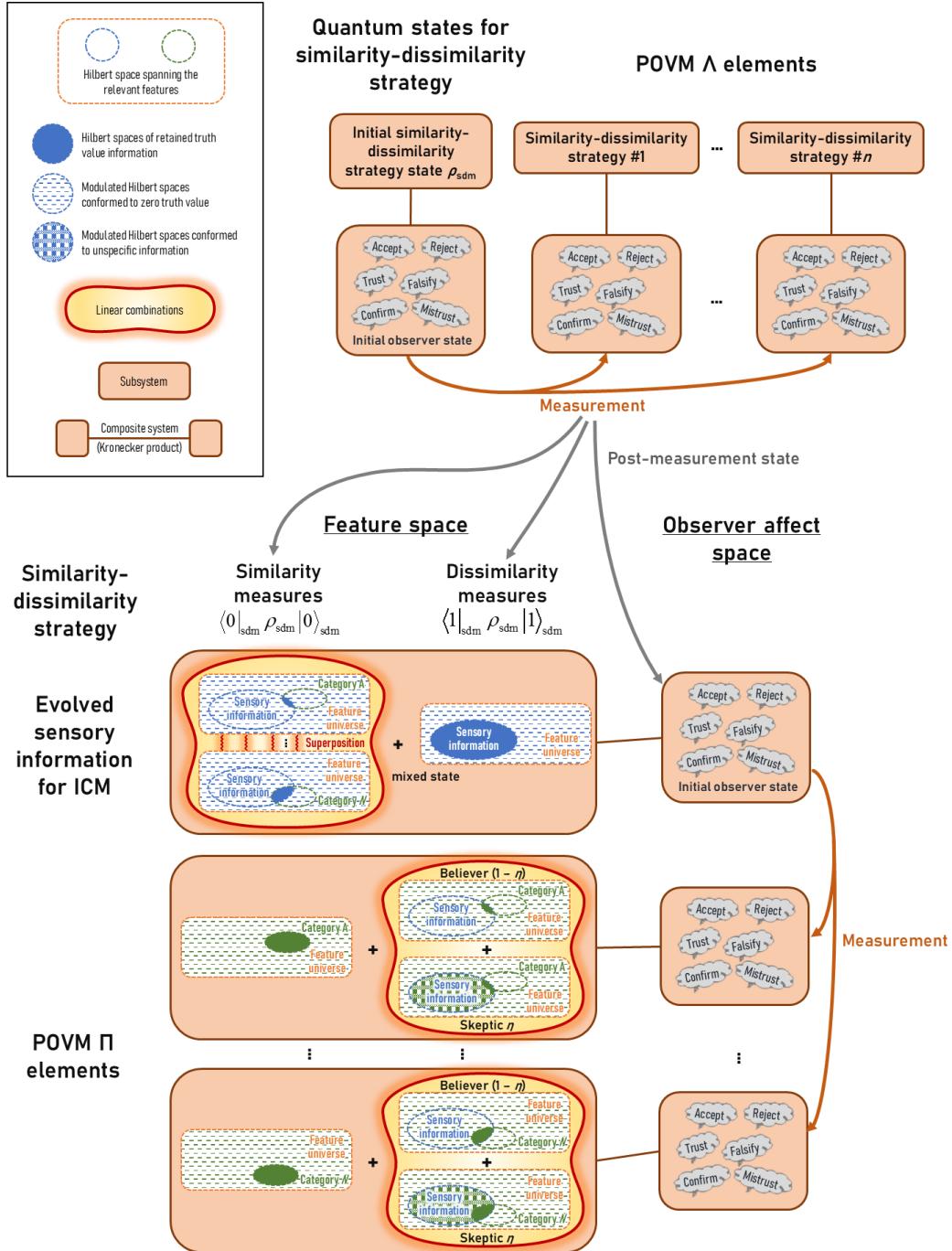


Fig. 2 The schematic two-step information processing diagram for the instance–category matching (ICM), modeled as the POVM Π (bottom), where its construct depends on the measurement result of the similarity-dissimilarity strategy (gray arrows), determined by the POVM Λ (top). Each POVM performs measurement on a composite system consisting of the concerned system (the similarity-dissimilarity strategy state or the sensory information for ICM evolved from the Lindblad master equation) and the “environment” (observer state). In the feature space, the POVM elements in Π are the candidate categories modified by the similarity and dissimilarity measures and weighted by the similarity-dissimilarity strategy. Self-skepticism is considered in the dissimilarity measures. The feature space of the POVM elements and the sensory information are aligned for all candidate categories. The observer state in Π inherits that in the measurement result of Λ (gray arrow).

Discussion

Sensory information is intricately processed by the observer, entangling with various observer states and measurement devices. Quantum probability introduces a spectrum of probabilistic classification outcomes, influenced by the observer's traits and states. Parameters throughout this process are dynamic, tailored to classification goals like rapid response or accuracy. While we did not specify the physical processes in observers, our framework highlights the multiple dimensions (including affect), causing the observer effect. Real-life variations may exceed what we discussed, with diverse instance-category matchings (ICMs) arising from the observer's mental world: k -NN, K-means, prototype matching, template matching, or a blend thereof. Different choices of interpretational thoughts, such as frequentism and Bayesian, may be possible in signal detection. In general, these alternative configurations offer more strategies that lead to different biases for classification as part of the optimization of estimation.

Note that the parameters that define the Lindblad information evolution and ICM, such as Lindblad operators, coupling strength, temperature, η , POVM elements and their relative weights, profiles of candidate categories, and so on, do not directly determine the accuracy of ICM. The ICM process is not responsible for the accuracy of classification outcomes. Sensory information typically includes unique features absent in certain candidate categories and may miss features of the intended category. The observer employs its own strategies to discern the optimal fuzzy classification amidst noise, not necessarily aiming to minimize error probability as in Helstrom measurement²⁶. Inexperienced observers with poor classification intuition can use metrics like trace distance as a threshold to externally refine its classification parameters, aiming for outcomes that meet their goals and concerns, rendering positive experiences as formalized in Silicon Coppélia²⁷ and Quantum Coppélia²⁸. Matches with these goals and concerns yield interpretations of classification accuracy, refining existing categories, learning new ones, and extends to developing and interpreting theories, protocols, experimental outcomes, and approximations in scientific research, conducted by humans or computational systems working with abstract principles. Detail assessment is integral to the classification process, influenced by the observer's concept-driven beliefs. Theory-laden approaches, tolerance for deviations, and a combined strategy of similarity and dissimilarity measures facilitate

metaphorization and generalization, promoting empirical concept-driven insights, coherence, and unification²⁹.

In *Epistemics of the Virtual*¹², following ICM, the learning process of *epistemic appraisal* takes place, involving the assessment of context, perceived realism, and metaphorical comparisons, all rooted in the observer's belief system, leading to further sensory-observer interactions. The resulting acquisition or alteration of categories and concepts is thus an outcome of the observer effect. Our framework, which outlines the observer effect in the epistemic domain through quantum probability, lays the groundwork for future research into quantum correlations between sensory information and observer states and the impact of quantum properties on intellectual inquiry.

Methods

A. Information states of encoded sensory information and observers

A sensory organ or device may encode a sensory feature in a Hilbert space representation. Each feature attribute receives a distribution of truth values as an independent state that takes the form of a degree of belief that the sensory apparatus indeed perceives a feature to occur at these particular attributes. The entire system is a collection of quantum oscillators of truth values. For pure states of independent oscillators, the j^{th} sensory feature of $n_{\text{sens},j} \geq 2$ attributes, where each attribute k contains $n_{\text{sens},j,k} \geq 2$ truth values (l) that each allows $n_{\text{sens},j,k,l} \geq 2$ relevance (m) states, may be represented by the density matrix

$$\rho_{\text{sens},j} = \bigotimes_{k,l} |\psi_{\text{sens},j,k,l}\rangle\langle\psi_{\text{sens},j,k,l}| \in \mathbb{C}^{(\prod_{j,k,l} n_{\text{sens},j,k,l}) \times (\prod_{j,k,l} n_{\text{sens},j,k,l})} \quad (1)$$

where $|\psi_{\text{sens},j,k,l}\rangle$ is the ket of relevance of the sensory information for the j^{th} sensory feature, k^{th} attribute and l^{th} truth value. Probability amplitudes are assigned to each state. In the quantum formulation, superpositions and mixed states are allowed. The combination of information of n_{sens} features encoded by independent sensory organs or devices makes up the complete sensory information that is represented by the tensor product

$$\rho_{\text{sens}} = \bigotimes_j \rho_{\text{sens},j} \in \mathbb{C}^{\left(n_{\text{sens}} \prod_{j,k,l} n_{\text{sens},j,k,l}\right) \times \left(n_{\text{sens}} \prod_{j,k,l} n_{\text{sens},j,k,l}\right)} \text{ acting on the Hilbert space } \mathcal{H}_{\text{sens}}.$$

The basis of the resulting density matrix contains all combinations of the specific feature, attribute, truth value and relevance, i.e., each dimension represents a series of conjunction “ $\Lambda_{j,k,l,m}$ feature j , attribute k , truth value l , relevance m .” For simplicity, we assume ρ_{sens} to be a pure state.

In the same vein do the sensory devices have their own states of n_{obs} observer features in the Hilbert space \mathcal{H}_{obs} , where the p^{th} observer feature having $n_{\text{obs},p}$ levels labeled as q , each with $n_{\text{obs},p,q}$ relevance states labeled as r , is represented by $\rho_{\text{obs},p}$, and

$$\rho_{\text{obs}} = \bigotimes_p \rho_{\text{obs},p} \in \mathbb{C}^{\left(n_{\text{obs}} \prod_{p,q} n_{\text{obs},p,q}\right) \times \left(n_{\text{obs}} \prod_{p,q} n_{\text{obs},p,q}\right)}.$$

In quantum probability, we maintain the notion that information states are fuzzy in principle and crisp occasionally.

In each process, particular subspaces may be extracted for specific processing. Subspace selection acts as part of the selection rule for the information processing. Ruling out a dimension prohibits any pathway arriving at that state, setting the probability of evolving into that state to zero. Under certain circumstances, dimensions may be too highly correlated to be separate dimensions that can hold their positive semi-definiteness. In that case, dimension reduction may be possible. The converse is also true.

Particular information and signal models can be specified based on the observer’s choice of interpretation. It includes all knowledge as pieces of information and relationships between them, which form a signal basis that can be compared against the sensory information in a way that is specified by a signal model, specifying what is regarded as a signal. The actual dimensions of the Hilbert spaces for the observer states and the sensory information are specified here. This includes the available sensory organs and devices, the precision of measures (e.g., whether it is an 8-bit or 24-bit color encoding), the cognitive differentiability in terms of the resolution of the truth values under measurement (e.g., whether it only has full belief (truth value = 1) or no belief (0), or that states of fuzzy membership are allowed). For example, the sensory feature j = “color—red” can be assigned with binary states within the range [0,1]: $k = 0$ (no red) and 1 (full red), each of them assigned with 3 possible truth values in the range of [0,1]: $l = 0, \frac{1}{2}, \text{ and } 1$,

inheriting 4 relevance values. Then, $\rho_{\text{sens},j} \in \mathbb{C}^{4^{3\times 2} \times 4^{3\times 2}}$. Note, in general, the state labels could be formats other than numeric. For observer features, for simplicity's sake, each of them will be a mere two-level system in the Hilbert space (e.g., with $p = \text{Trust}$, trust level $q = 0, 1$, each level with two possible relevance). The joint space represents the entire observer state, particularly, $\rho_{\text{obs}} = \bigotimes_{p,q} |\psi_{\text{obs},p,q}\rangle\langle\psi_{\text{obs},p,q}|$, where $p = \text{Accept, Reject, Confirm, Falsify, Trust, Mistrust, or any other observer attribute (e.g., Belief, Disbelief)}$. In this illustration, $\rho_{\text{obs}} \in \mathbb{C}^{2^{2\times 6} \times 2^{2\times 6}}$. In general, depending on design, intrinsic resolution of the observer, and the noise level, finer levels may be assigned to the truth values of the observer features. Note the non-commuting properties of tensor products here. For simplicity, we assume independent, two-state harmonic oscillators. For now, we assume a simple diagonal form of stationary sensory information.

B. State evolution in the mental world

For the sake of ease, suppose the encoded sensory information exchanges information with one observer in the observing agency but nothing else. The sensory information (the “system”) and the observer (the “environment”) form a closed system. This composite information system undergoes unitary evolution $U(t) = e^{iH_T t}$, following the Hamiltonian $H_T = H_{\text{sens}} \otimes \mathbb{1}_{\text{obs}} + \mathbb{1}_{\text{sens}} \otimes H_{\text{obs}} + H_{\text{int}}$, where $H_{\text{sens}} \in \mathcal{H}_{\text{sens}}$, $H_{\text{obs}} \in \mathcal{H}_{\text{obs}}$, and $H_{\text{int}} = \sum_j g_j S_j \otimes O_j$ are the free sensory-information Hamiltonian, free observer Hamiltonian, and interaction Hamiltonian, respectively. $\mathbb{1}_A$ is the identity operator on the Hilbert space \mathcal{H}_A . $S_j \in \mathcal{H}_{\text{sens}}$ and $O_j \in \mathcal{H}_{\text{obs}}$ are the operators of the system and observing agent of which the degrees of freedom are involved in a particular interaction. When the sensory information and the observer are modeled as the respective collection of independent quantum harmonic oscillators, the Hamiltonian of A can be written as the Kronecker sum of the N individual oscillators:

$$\begin{aligned} H_A &= \hbar \bigoplus_{n=1}^N \omega_n \left(a_n^\dagger a_n + \frac{1}{2} \mathbb{1}_n \right) \\ &= \hbar \sum_{n=1}^N \left(\bigotimes_{m < n} \mathbb{1}_m \right) \otimes \omega_n \left(a_n^\dagger a_n + \frac{1}{2} \mathbb{1}_n \right) \otimes \left(\bigotimes_{n < m \leq N} \mathbb{1}_m \right), \end{aligned} \quad (2)$$

where ω_n , a_n^\dagger and a_n^\dagger are the frequency, creation and annihilation operators of the n^{th} oscillator, respectively. The strength of each interaction is characterized by the coupling constant g_j . For the Hermitian nature of H_{int} , the terms in H_{int} forms Hermitian conjugate pairs, i.e., for all j , there exists j' such that $g_j S_{j'} \otimes O_{j'} = g_j^* S_j^\dagger \otimes O_j^\dagger$.

For convenience, Hamiltonians here follow the unit of angular frequency ($\hbar=1$). In principle, the diagonal elements describe the frequency of the phase for that state, determining the probability amplitude of the states. The off-diagonal elements characterize the transition between states, which is depicted in the oscillation of the probability amplitudes (Rabi oscillations). For an indefinite observer whose state changes over time, a non-diagonal H_{obs} may be devised. H_{int} contains off-diagonal elements that describe the interaction between the sensory information and the observer.

The Lindblad master equation^{17,18} describes the dynamics of the sensory-information density-matrix during evolution under small coupling strength (weak coupling) between the sensory information and the observer with the following properties: (1) the system is approximated to be uncorrelated to its environment with an additional or modified Hamiltonian as perturbation during evolution; (2) correlation and relaxation of the environment happen at much smaller time scales compared to the system; (3) the state of the composite system may be approximated as $\rho(t) = \rho_{\text{sens}}(t) \otimes \rho_{\text{obs}}(0)$. The Lindblad master equation is given by

$$\begin{aligned}\dot{\rho}_{\text{sens}}(t) &= -i [H_{\text{sens}} + H_{\text{Ls}}, \rho_{\text{sens}}(t)] \\ &\quad + \sum_{j,\omega'} \left(L_j(\omega') \rho_{\text{sens}}(t) L_j^\dagger(\omega') - \frac{1}{2} \{ L_j L_j^\dagger(\omega'), \rho_{\text{sens}}(t) \} \right) \\ &\equiv \mathcal{L} \rho_{\text{sens}}(t)\end{aligned}\quad (3)$$

where $[\cdot, \cdot]$, $\{\cdot, \cdot\}$ are the commutator and the anti-commutator, respectively. \mathcal{L} is the Liouville superoperator. The Lamb shift Hamiltonian H_{Ls} renormalizes subtly the energy levels of the encoded sensory information, owing to the interaction with the observer but will be neglected here for simplicity. The diagonal form of the Lindblad master equation comprises a set of Lindblad (jump) operators $L_j(\omega)$, calculated by

$$L_i(\omega') = |g_i| \sqrt{\gamma_i(\omega')} V_{ij}^\dagger S_j(\omega'), \quad (4)$$

where $\gamma'(\omega)$ is a diagonal matrix, resulting from the eigen-decomposition of the positive, cross spectral density matrix $\gamma(\omega)$ that represents the non-Hamiltonian (non-Hermitian, incoherent) part of the dynamics:

$$\gamma_{kl}(\omega) \equiv \Gamma_{kl}(\omega) + \Gamma_{lk}^*(\omega) = \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} \text{Tr}_{\text{obs}} \left[e^{iH_{\text{obs}}\tau} O_k^\dagger e^{-iH_{\text{obs}}\tau} O_l \rho_{\text{obs}}(0) \right], \quad (5)$$

where

$$\Gamma_{kl}(\omega) \equiv \int_0^{\infty} d\tau e^{i\omega\tau} \text{Tr}_{\text{obs}} \left[e^{iH_{\text{obs}}\tau} O_k^\dagger e^{-iH_{\text{obs}}\tau} O_l \rho_{\text{obs}}(0) \right] \quad (6)$$

accounts for the effect of the observer. In particular, $V^\dagger \gamma(\omega) V = \text{diag}(\gamma'(\omega))$ for some unitary operator V . Tr_{obs} is the partial trace over the observer degrees of freedom (dimensions). $\rho_{\text{obs}}(0)$ denotes the initial state of the observing agency assumed in thermal equilibrium of temperature T :

$$\rho_{\text{obs}}(0) = \frac{\exp(H_{\text{obs}}/k_B T)}{\text{Tr}[\exp(H_{\text{obs}}/k_B T)]} \quad (7)$$

with a natural unit of the Boltzmann constant ($k_B = 1$). Higher temperatures give more uniform state distributions. Note that $\gamma(\omega)$ is the Fourier transform of the reservoir correlation function (cross-correlation function) $\text{Tr}_{\text{obs}} \left[e^{iH_{\text{obs}}\tau} O_k^\dagger e^{-iH_{\text{obs}}\tau} O_l \rho_{\text{obs}}(0) \right] = \langle \hat{O}_k^\dagger(\tau) O_l(0) \rangle$.

Every system operator of interaction S_k undergoes spectral decomposition:

$$S_k = \sum_{\omega'} S_k(\omega') = \sum_{\omega'} s_{k\omega'} S'(\omega') \quad (8)$$

with ω' and $S'(\omega')$ as the eigenvalues and normalized eigenoperators of the superoperator $[H_{\text{sens}}, \cdot]$ with the properties $[H_{\text{sens}}, S'(\omega')] = -\omega' S'(\omega')$ and $[H_{\text{sens}}, S'^\dagger(\omega')] = \omega' S'^\dagger(\omega')$. In practice, ω' and $S'(\omega')$ can be constructed from the eigenvalues and normalized eigenvectors of H_{sens} , denoted by ω_i and $|\phi_i\rangle$, respectively, using the relation

$$S'_k(\omega' = \omega_\alpha - \omega_\beta) = |\phi_\alpha\rangle\langle\phi_\beta| \quad (9)$$

for k enumerated by α and β .

The Markovian characteristics that reflect in the decaying reservoir correlation-function requires a continuum of oscillator frequency. For a finite number of oscillators,

each of definitive frequencies as in Eq. (2), a frequency continuum can be achieved by regularizing the Dirac delta functions in $\gamma_k(\omega)$, using a Lorentzian function with the full width at half maximum (FWHM) approximated to be the reciprocal of the correlation time-scale ($1/\tau_{\text{corr}}$).

C. Instance–category matching (ICM)

Upon making a decision, the sensory information is compared against a selection of n_c candidate categories. The duration between the reception of the sensory information and the act of ICM is modeled by an exponential distribution with a mean duration λ . The matching of instances with categories is modeled as a POVM Π . As for the POVM, a finite set of positive semi-definite operators that represent the joint state of the candidate categories $\mathfrak{C} = \{\mathfrak{c} : \mathfrak{c} \in \mathfrak{C}\}$ of size n_c and the corresponding associated observer state \mathfrak{o} are defined by $\Pi = \left\{ \Pi_m : \Pi_0 + \delta \sum_{\mathfrak{c} \in \mathfrak{C}} \sum_{\mathfrak{o} \in \Omega(\mathfrak{c})} \beta_{\mathfrak{c}, \mathfrak{o}} \Pi_{\mathfrak{c}} \otimes \Pi_{\mathfrak{o}} = \mathbb{1} \right\}$. Each category is modeled as an operator $\Pi_{\mathfrak{c}}$ for the matching process, and each of it may be associated with a number of matrices $\Pi_{\mathfrak{c}}$ of respective possible observer states $\mathfrak{o} \in \Omega(\mathfrak{c})$. The combinations of the candidate categories and observer state can exhibit a many-to-many relationship. For ease of physical interpretation, $\Pi_{\mathfrak{c}} \otimes \Pi_{\mathfrak{o}}$ is normalized by its trace. Observe that the probability of obtaining the category \mathfrak{c} upon measurement of state ρ is $\text{Tr}[(\Pi_{\mathfrak{c}} \otimes \Pi_{\mathfrak{o}}) \rho (\Pi_{\mathfrak{c}}^\dagger \otimes \Pi_{\mathfrak{o}}^\dagger)]$ as given by the Born rule. $\text{Tr}(\Pi_{\mathfrak{c}} \otimes \Pi_{\mathfrak{o}})$ sets the upper bound of the probability for obtaining \mathfrak{c} and \mathfrak{o} . This maximum value should conform with the probability rules, and thus, be normalized by its trace.

(a) Determination of the similarity-dissimilarity strategy

Algorithmically, the distribution of the similarity and dissimilarity measures as information processing pathways can be modeled as a POVM Λ . As a simple example, assume a 3-qubit 2-level state for the observer (e.g., (Dis)similarity, Trust and Mistrust), then the POVM can be written as

$$\begin{aligned}\Lambda = & \left\{ \Lambda_2 = \mathbb{1} - \Lambda_1, \right. \\ & \Lambda_1 = k \left(\varepsilon |0\rangle\langle 0| + (1-\varepsilon) |1\rangle\langle 1| \right)_{\text{sdm}} \\ & \left. \otimes \left[(1-\varepsilon) |0\rangle\langle 0| + \varepsilon |1\rangle\langle 1| \right]_{\text{trust}} \otimes \left[\varepsilon |0\rangle\langle 0| + (1-\varepsilon) |1\rangle\langle 1| \right]_{\text{mistrust}} \right\}\end{aligned}\quad (10)$$

for some $\varepsilon \in [\frac{1}{2}, 1]$ that indicate the degree of collapse of the observer state and some norm-scaling constant k . Assuming that the distribution of the post-measurement states reflects the similarity between the pre-measurement state and the POVM operator, the POVM operators can be modeled as scaled density matrices so that the outcome probability has the same form as the Hilbert–Schmidt inner product. A resultant state of $|0\rangle_{\text{sdm}}$ and $|1\rangle_{\text{sdm}}$ invokes similarity and dissimilarity measures, respectively. We denote the reduced density matrix for the similarity measure after measurement as ρ_{sdm} .

(b) Construction of ICM POVM

In similarity measures, the sensory information is tested against each of the candidate categories with the potential common features. Assuming a pure state, the sensory information undergoes a channel to form a state of (uniform) superposition. In being indefinite knowledge, superposition comprises a series of quantum states of the original sensory information with its Hilbert space truncated to the common features of each candidate category. Truncation is followed by an expansion to include the exclusive features of the candidate category with a ground state of relevance, indicating the neglection of these dimensions during the matching process. Denote the transformed sensory information in the similarity measure as $\rho_{\text{sens,sim}}$.

The Hilbert spaces of the candidate categories are retained, keeping all features. Depending on the actual recognition method, the density matrix of the candidate categories may be transformed. Further discussion is in the section of Recognition Method. In dissimilarity measures, the Hilbert spaces of the sensory information are retained, keeping all features. The Hilbert space of each candidate category is truncated to the common features of the sensory information. Upon the adaption of the recognition method, the Hilbert space is expanded by the external-feature operator F_c for the exclusive features of sensory information. The external-feature operator characterizes the believer or skeptic

nature of the observer. Assuming the belief system has the same degree of openness $\eta \in [0,1]$ for all features, F_c can represent a distribution of the two, i.e.,

$$F_c = (1-\eta) \bigotimes_{f \in c} |0\rangle_f \langle 0|_f + \eta \bigotimes_{f \in c} \frac{1}{n_f} \mathbb{1}_f \quad (11)$$

Where $|0\rangle_f$ denotes a quantum state of zero truth values (with high relevance) for feature f with a basis of n_f truth value as an indication of a believer, and $\mathbb{1}_f/n_f$ represents the maximally mixed state that attributes to a self-skeptic. Then,

$$\Pi'_c = R(\Pi''_c \otimes F_c) \quad (12)$$

where Π''_c is the operator that includes only the features of c . R is the reordering operator that can be constructed by a series of swap operators to align a consistent order of the Hilbert spaces.

Different categories may contain different sets of features, and the sensory information may contain a set of features unidentical to the categories, implying distinct Hilbert spaces for the above operations. However, the POVM operators must share identical Hilbert spaces. Therefore, every Π_c should involve non-repeating features from all candidate categories, i.e., $\Pi_c \in \bigotimes_{f,c} \mathcal{H}_f$, $f \in c$, $\forall c \in \mathbb{C}$, $f \neq f'$. In general, an expansion of Hilbert space is performed on Π_c with a state of high relevance of zero truth value to form a set of POVM operators of consistent Hilbert space. Denote the POVM elements representing similarity and dissimilarity measures as $\Pi'_{c,sim}$ and $\Pi'_{c,diss}$. The resulting Π_c can be written as

$$\Pi_c = \langle 0|_{sdm} \rho_{sdm} |0\rangle_{sdm} \Pi'_{c,sim} + \langle 1|_{sdm} \rho_{sdm} |1\rangle_{sdm} \Pi'_{c,diss}. \quad (13)$$

The sensory information undergoes parallel channels corresponding to the similarity and dissimilarity measures. Assuming the original sensory information for dissimilarity measures with an aligned Hilbert space with zero truth value for any extra dimensions, denoted as $\rho_{sens,diss}$, the resultant density matrix resembles the form in Eq. (13):

$$\rho'_{sens} = \langle 0|_{sdm} \rho_{sdm} |0\rangle_{sdm} \rho_{sens,sim} + \langle 1|_{sdm} \rho_{sdm} |1\rangle_{sdm} \rho_{sens,diss}. \quad (14)$$

Relative values of $\beta_{c,o}$ represent the tendency of the particular identification, also known as bias. Given trace-normalized $\Pi_c \otimes \Pi_o$, $\beta_{c,o}$ effectively scales $\Pi_c \otimes \Pi_o$ such that the maximum probability of obtaining c upon measurement is $\text{Tr}(\beta_{c,o} \Pi_c \otimes \Pi_o) = \beta_{c,o} \text{Tr}(\Pi_c \otimes \Pi_o) = \beta_{c,o}$. A higher value of $\beta_{c,o}$ biases the attribution towards a particular category c . Therefore, $\beta_{c,o}$ may be regarded as part of the parametrization of the outcome distribution. More specifically, $\beta_{c,o} = \beta_c \beta_{o|c}$, which is the tendency of obtaining the post-measurement state with label (c, o) , which is the product of the tendency of the state with c and the tendency of the state with o given c ; the same as conditional probability up to a proportionality constant. These parameters come from the belief system, be that classic Boolean, Bayesian, quantum Bayesian, etc.

Π_0 represents the result of “no match” (noise). The closure condition requires that the sum of the POVM matrices forms the identity matrix. Then, a natural selection of Π_0 would be the remaining component from $\beta_{c,o} \Pi_c \otimes \Pi_o$ for all $c \in \mathbb{C}$ and $o \in \mathfrak{O}(c)$ within the identity matrix. However, this remaining component does not always yield a positive semi-definite Π_0 . To construct a physical Π , we retain the closure condition and the relative values of $\beta_{c,o}$, regularized as follows:

1. The set of trace-normalized $\Pi_c \otimes \Pi_o$ is calculated.
2. A probability distribution for Π_0 and Π_c based on $\{\beta_c\}$, $\{\beta_{o|c}\}$, and β_0 (the bias parameter for every category and null identification) is formed from the belief system such that

$$\beta_0 + \sum_{c \in \mathbb{C}} \sum_{o \in \mathfrak{O}(c)} \beta_{c,o} = 1. \quad (15)$$

3. A minimizer is defined for a positive real δ with the closest ratio of null identification to positive matches to the nominal probability distribution with the objective function

$$o(\delta) = \left[\text{Tr} \left(\mathbb{1} - \delta \sum_{c \in \mathbb{C}} \sum_{o \in \mathfrak{O}(c)} \beta_{c,o} \Pi_c \otimes \Pi_o \right) - \delta \beta_0 \right]^2, \quad (16)$$

subject to the constraint of positive semi-definiteness of

$$\Pi_0 = \mathbb{1} - \delta \sum_{c \in \mathfrak{C}} \sum_{o \in \Omega(c)} \beta_{c,o} \Pi_c \otimes \Pi_o, \quad (17)$$

specifically, non-negative eigenvalues or any equivalent conditions. Note that the value of δ under unrestricted conditions is the number of dimensions of the Hilbert space.

4. Then,

$$\Pi = \left\{ \Pi_m : \Pi_m = \begin{cases} \mathbb{1} - \delta \sum_{c \in \mathfrak{C}} \sum_{o \in \Omega(c)} \beta_{c,o} \Pi_c \otimes \Pi_o, & m = 0 \\ \delta \beta_{c,o} \Pi_c \otimes \Pi_o, & m = (c, o), c \in \mathfrak{C}, o \in \Omega(c) \end{cases} \right\}. \quad (18)$$

Whether the minimizer is under the restricted conditions depends on Π_c , Π_0 , and $\beta_{c,o}$. For example, the chosen set may account for unphysical information superposition as a whole, leading to unphysical absence-of-signal identification. Then, given the same set of candidate categories, the overall weight (signal threshold) of the categories as represented by all $\beta_{c,o}$ should be adjusted for a physical interpretation of not identifying a proper category. This may imply some sort of dimension redundancy for Π_0 , meaning that not identifying a category results into a deterministic state (extremely biased outcome probability distribution). This means that the chosen set of candidate categories and their corresponding bias play a role in determining the outcome distribution, or more specifically, the threshold of category identification. That is, an alternative of regularization is to reconsider the set for candidate categories and thus the POVM. The conditions mimic a multiple-alternative forced choice (mAFC) scenario³⁰, where one option is chosen out of $n_{\mathfrak{C}} + 1$ alternatives for a single piece of encoded sensory information.

(c) Recognition method

Each candidate category c possesses a characteristic distribution: A random sample of size N_c is generated as the matching templates $|\tau'_{c,k}\rangle = W_{c,k} U_{c,k} |\bar{\tau}_c\rangle$, where $|\bar{\tau}_c\rangle$ represent the expectation (prototype) of the category with n_c dimensions, assumed to be pure. The

operator is then modeled as $\Pi''_{\mathfrak{c}} = k_{\mathfrak{c}} \sum_{k=1}^{N_{\mathfrak{c}}} |\tau'_{\mathfrak{c},k}\rangle\langle\tau'_{\mathfrak{c},k}|$, where $k_{\mathfrak{c}}$ is the normalization constant to ensure $\text{Tr}(\Pi''_{\mathfrak{c}}) = 1$. $U_{\mathfrak{c},k}$ is a random unitary matrix that transforms the prototype to templates. In particular, based on the Lie algebra and assumed two-level states, $U_{\mathfrak{c},k}$ can be represented by $n_{\mathfrak{c}}^2 - 1$ parameters $\xi_{\mathfrak{c},k,j}$, which corresponds to the generators $T_{\mathfrak{c},j}$ of $\mathfrak{su}(n_{\mathfrak{c}})$ which are traceless skew-Hermitian matrices:

$$U_{\mathfrak{c},k} = \exp\left(i \sum_{j=1}^{n_{\mathfrak{c}}^2-1} \xi_{\mathfrak{c},k,j} T_{\mathfrak{c},j}\right) \quad (19)$$

where

$$T_{\mathfrak{c},j} = \bigotimes_{r=1}^{\log_2 n_{\mathfrak{c}}} \sigma_{r(j)} \neq \mathbb{1} \quad (20)$$

and $\sigma_{r(j)}$ are the Pauli matrices (including the identity matrix) that define the j^{th} generator. Without loss of generality, $\xi_{\mathfrak{c},k,j}$ can be generated by the multivariate normal distribution with zero mean. The random unitary weight operator $W_{\mathfrak{c},k}$ modulates the perceptual importance of a feature to \mathfrak{c} by transforming the probability amplitudes in $U_{\mathfrak{c},k} |\bar{\tau}_{\mathfrak{c}}\rangle$. A state associated with an activated state of a lower-weighted sensory feature should receive lower probability since that sensory feature is less likely to be recognized. $W_{\mathfrak{c},k}$ is defined similarly to $U_{\mathfrak{c},k}$ but in general with non-zero mean parameters. Unitary transformations preserve the purity of the templates.

If the belief system contains a range of certain knowledge, this layer reveals their distributions. $\Pi''_{\mathfrak{c}}$ resembles a statistical mixture of a sampling distribution for the candidate category \mathfrak{c} (cf. a particular single instance). This formalism allows a fuzzy representation of a concept even if the basis of the Hilbert space poses constraints on doing so. The ambiguity of mixed states allows the concept to be expressed in multiple possibilities. The above formalism is also applicable to ambiguous observer states $\Pi_{\mathfrak{o}}$.

(d) Measurement process

For ICM, the templates in each category are matched against the sensory information. The belief system (e.g., frequentism) produces a set of decision rules for signal detection (e.g., if $p < .05$, reject H_0), governing to which category the sensory information belongs or maybe to none at all (H_0 not rejected). Each operator Π_m corresponds to a measurement outcome, which is typically a distribution representing the candidate category, or Π_0 if no match. The set of probability of the measurement outcome subject to the density matrix of the composite system $\rho = \rho'_{\text{sens}} \otimes \rho_{\text{obs}}(0)$ is determined by the Born rule:

$$\mathcal{P}_{\text{ICM}} = \left\{ p_m : p_m = \mathbb{P}(m | \rho) = \text{Tr}(\delta\beta_m \Pi_m \rho), \Pi_m \in \Pi \right\}. \quad (21)$$

Clearly, frequentism is but one approach to build up an ontology. Naïve Bayes, PAC-Bayes, fuzzy or other non-classical propositional logic provide alternative perspectives on what is valid information or not. The likelihood ratios represent the strength of internal response and the decision criteria are governed by the relative probabilities between the choices. Regardless, the final choice the observer makes is probabilistic; based on the probability distribution $\mathcal{P}'_{\text{ICM}}$.

The final choice is accompanied by acting the chosen matrix on the state density matrix with Kraus operators representing the chosen measurement:

$$K_m^\dagger K_m = \begin{cases} \Pi_0, & m = 0 \\ \delta\beta_{c,o} \Pi_c \otimes \Pi_o, & \text{otherwise} \end{cases}. \quad (22)$$

For real matrices, K_m can be calculated as the matrix' square root. If a measurement is done, the new state after ICM would be:

$$\rho_{\text{ICM}} = \frac{K_m \rho K_m^\dagger}{\text{Tr}(K_m \rho K_m^\dagger)}. \quad (23)$$

Partial trace can be used to extract the particular observer or sensory state:

$$\rho_{A,\text{ICM}} = \text{Tr}_{A'} \rho_{\text{ICM}}, \quad (24)$$

where A and A' are complement with each other, for instance, if $A = \text{sens}$, $A' = \text{obs}$.

Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

References

- 1 Crutchfield, J. P. in *Inside Versus Outside*. (eds Harald Atmanspacher & Gerhard J. Dalenoort) 235-272 (Springer Berlin Heidelberg).
- 2 Helland, I. S. A Simple Quantum Model Linked to Decisions. *Foundations of Physics* **53**, 12 (2023). <https://doi.org/10.1007/s10701-022-00658-7>
- 3 Polychronakos, A. P. Quantum mechanical rules for observed observers and the consistency of quantum theory. *Nature Communications* **15**, 3023 (2024). <https://doi.org/10.1038/s41467-024-47170-2>
- 4 Stewart, A. J. & Plotkin, J. B. The natural selection of good science. *Nature Human Behaviour* **5**, 1510-1518 (2021). <https://doi.org/10.1038/s41562-021-01111-x>
- 5 Trueblood, J. S., Yearsley, J. M. & Pothos, E. M. A Quantum Probability Framework for Human Probabilistic Inference. *Journal of experimental psychology. General* **146**, 1307-1341 (2017). <https://doi.org/10.1037/xge0000326>
- 6 Sassoli de Bianchi, M. The Observer Effect. *Foundations of Science* **18**, 213-243 (2013). <https://doi.org/10.1007/s10699-012-9298-3>
- 7 Barnes, S. E., Opris, I., Noga, B. R., Huang, S. & Zuo, F. in *Modern Approaches to Augmentation of Brain Function* (eds Ioan Opris, Mikhail A. Lebedev, & Manuel F. Casanova) 433-446 (Springer International Publishing, 2021).
- 8 Contreras-Tejada, P., Scarpa, G., Kubicki, A. M., Brandenburger, A. & La Mura, P. Observers of quantum systems cannot agree to disagree. *Nature Communications* **12**, 7021 (2021). <https://doi.org/10.1038/s41467-021-27134-6>
- 9 Baclawski, K. in *2018 IEEE Conference on Cognitive and Computational Aspects of Situation Management (CogSIMA)*. 83-89.
- 10 Allard Guérin, P. & Brukner, Č. Observer-dependent locality of quantum events. *New Journal of Physics* **20**, 103031 (2018). <https://doi.org/10.1088/1367-2630/aae742>
- 11 Realpe-Gómez, J. Modeling observers as physical systems representing the world from within: Quantum theory as a physical and self-referential theory of inference. *arXiv preprint arXiv:1705.04307* (2018).
- 12 Hoorn, J. F. Epistemics of the virtual. *Epistemics of the Virtual*, 1-241 (2012).
- 13 Wigner, E. P. in *The Scientist Speculates* (ed I. J. Good) (Heinemann, 1961).
- 14 Allard Guérin, P., Baumann, V., Del Santo, F. & Brukner, Č. A no-go theorem for the persistent reality of Wigner's friend's perception. *Communications Physics* **4**, 93 (2021). <https://doi.org/10.1038/s42005-021-00589-1>
- 15 Sandberg, H. & Delvenne, J.-C. The Observer Effect in Estimation with Physical Communication Constraints. *IFAC Proceedings Volumes* **44**, 12483-12489 (2011). <https://doi.org/https://doi.org/10.3182/20110828-6-IT-1002.02312>
- 16 Leifer, M. S. & Spekkens, R. W. Towards a formulation of quantum theory as a causally neutral theory of Bayesian inference. *Physical Review A* **88**, 052130 (2013). <https://doi.org/10.1103/PhysRevA.88.052130>
- 17 Breuer, H.-P. & Petruccione, F. in *The Theory of Open Quantum Systems* (eds Heinz-Peter Breuer & Francesco Petruccione) 109–218 (Oxford University Press, 2007).

- 18 Manzano, D. A short introduction to the Lindblad master equation. *AIP Advances* **10**, 025106 (2020). <https://doi.org/10.1063/1.5115323>
- 19 Geurts, L. S., Cooke, J. R. H., van Bergen, R. S. & Jehee, J. F. M. Subjective confidence reflects representation of Bayesian probability in cortex. *Nature Human Behaviour* **6**, 294-305 (2022). <https://doi.org/10.1038/s41562-021-01247-w>
- 20 Balsdon, T., Mamassian, P. & Wyart, V. Separable neural signatures of confidence during perceptual decisions. *eLife* **10**, e68491 (2021). <https://doi.org/10.7554/eLife.68491>
- 21 Pouget, A., Drugowitsch, J. & Kepecs, A. Confidence and certainty: distinct probabilistic quantities for different goals. *Nature Neuroscience* **19**, 366-374 (2016). <https://doi.org/10.1038/nn.4240>
- 22 Webb, T. W., Miyoshi, K., So, T. Y., Rajananda, S. & Lau, H. Natural statistics support a rational account of confidence biases. *Nature Communications* **14**, 3992 (2023). <https://doi.org/10.1038/s41467-023-39737-2>
- 23 Larocca, M., Ju, N., García-Martín, D., Coles, P. J. & Cerezo, M. Theory of overparametrization in quantum neural networks. *Nature Computational Science* **3**, 542-551 (2023). <https://doi.org/10.1038/s43588-023-00467-6>
- 24 Tversky, A. Features of similarity. *Psychological review* **84**, 327 (1977).
- 25 Pullman, B. *The atom in the history of human thought*. (Oxford University Press, USA, 2001).
- 26 Helstrom, C. W. Quantum detection and estimation theory. *Journal of Statistical Physics* **1**, 231-252 (1969). <https://doi.org/10.1007/BF01007479>
- 27 Hoorn, J. F., Baier, T., Maanen, J. v. & Wester, J. Silicon Coppélia and the Formalization of the Affective Process. *IEEE Transactions on Affective Computing* **14**, 255-278 (2023). <https://doi.org/10.1109/TAFFC.2020.3048587>
- 28 Ho, J. K. W. & Hoorn, J. F. Quantum affective processes for multidimensional decision-making. *Scientific Reports* **12**, 20468 (2022). <https://doi.org/10.1038/s41598-022-22855-0>
- 29 Kuhn, T. S. in *Metaphor and Thought* (ed Andrew Ortony) 533-542 (Cambridge University Press, 1993).
- 30 Hautus, M. J., Macmillan, N. A. & Creelman, C. D. *Detection theory: A user's guide*. (Routledge, 2021).

Acknowledgements

This research is funded by the Laboratory for Artificial Intelligence in Design (Project Code: RP2-3) under the InnoHK Research Clusters, Hong Kong Special Administrative Region Government.

Author contributions

J.F.H. designed the study. J.K.W.H. and J.F.H. conceptualized the study. J.F.H. and J.K.W.H. provided the methodology. J.K.W.H. and J.F.H. wrote the manuscript. All authors reviewed the manuscript.

Correspondence and requests for materials should be addressed to Johan F. Hoorn.

Competing interests

The authors declare no competing interests.