Cloudy with a Chance of Snow: The Life of a Snowflake in a Cloud Microphysics Model

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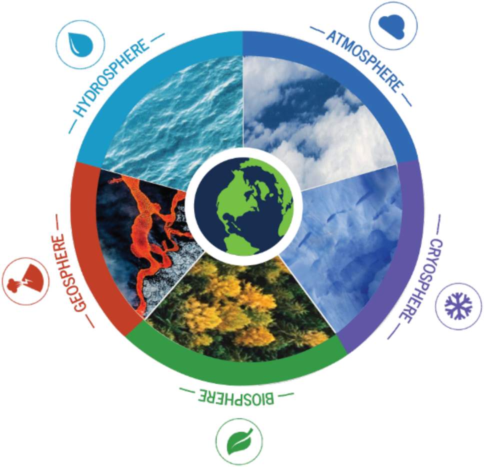
Earth System Models

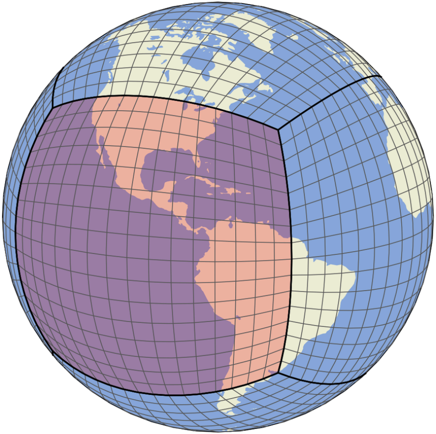
Earth system models couple many systems including

- The atmosphere
- The hydrosphere
- The cryosphere
- The geosphere
- The biosphere
- Chemistry
- Human behavior



Model Scales

Typically, in climate models, the atmosphere is discretized using grid cells that are on the order of 100 by 100 km large. This is too large to capture processes like clouds and precipitation.

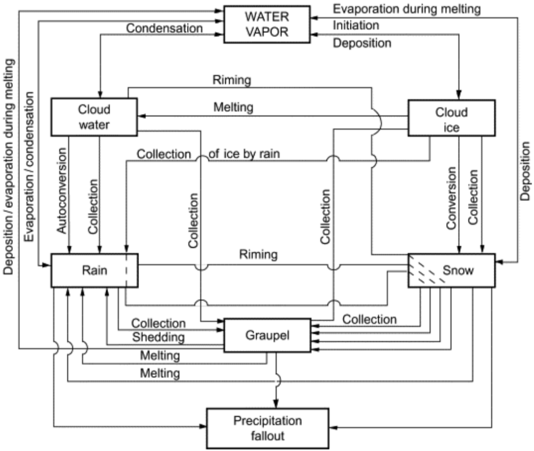


Cloud Microphysics

To capture these small scale processes, physical parametrizations are used.

Cloud microphysics models interactions between

- Water vapor
- Water droplets
- Ice crystals
- Rain
- Different forms of snow



The Governing Equations

Tracers (scalar fields) are governed by the conservation law

$$\frac{dQ}{dt} = \nabla \cdot (\mathbf{u}Q) + S(Q) \tag{1}$$

where Q is the density of the tracer and $\mathbf{u}$ is the velocity of the wind. $S(Q)$ is a source term. For this project, we focus on $S(Q)$ and so set $\mathbf{u} = 0$ for simplicity, giving

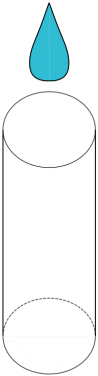
$$\frac{dQ}{dt} = S(Q) \tag{2}$$

Column Accretion

Accretion defines the rates of conversion between different categories of water in the cloud due to collisions between particles.

In first-moment physics, a falling particle will sweep out a cylinder with volume $V = ah$ where a is the normal projected surface area and h is the height.

We approximate $h \approx v_{term}\Delta t$ where v_{term} is the terminal velocity.

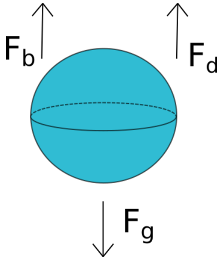


Terminal Velocity

To find the terminal velocity v_{term}, we need to determine how drag acts on the rain drop.

$\mathbf{F}_g = mg$ is gravity, $\mathbf{F}_b = (\rho_a/\rho_q)mg$ is buoyancy, and $\mathbf{F}_d$ is drag, which we consider two different forms for.

m is the mass of the particle, g is gravitational acceleration, ρ_a is the density of air, and ρ_q will be the density of water or snow, as appropriate.



Drag

a) For kinetic drag, we have

$$\mathbf{F}_d = \frac{1}{2} C_d \rho_a a v_{term}^2$$

where C_d is the drag coefficient, ρ_a is the density of air, and a is the normal projected surface area. For a rain drop, this gives

$$v_{term}(r) = \sqrt{\frac{8gr}{3C_d}\left(\frac{\rho_w}{\rho_a} - 1\right)}$$

b) For Stokes drag acting on a sphere of radius r, we have

$$\mathbf{F}_d = 6\pi r \mu v_{term}$$

where μ is the viscosity of air.

For a rain drop, this gives

$$v_{term}(r) = \frac{2g(\rho_w - \rho_a)r^2}{9\mu}$$

Accretion Model

Using the column model, we will take

$$\frac{dq_c}{dt} = \int_0^\infty n_r(r)a(r)v_{term}(r)q_c E_{cp} dr \tag{3}$$

where $n_r(r)$ is the proportion of particles that have radius r, $a(r)$ is the projected normal surface area, $v_{term}(r)$ is the terminal velocity, q_c is the amount of rain or snow in the cloud, and E_{cp} is the collision efficiency. We integrate over the radius. We take

$$n_r(r) = N_r e^{-\lambda_r r} \tag{4}$$

where N_r is some scaling parameter and λ_r is a function of the other parameters.

Rain Accretion

For a sphere, $a(r) = \pi r^2$ and v_{term} has two forms as given before so for kinetic drag, we have

$$\frac{dq_r}{dt} = N_r \pi q_r E_{cp} \sqrt{\frac{8g}{3C_d}\left(\frac{\rho_w}{\rho_a} - 1\right)} \int_0^\infty e^{-\lambda_r r} r^{\frac{5}{2}} dr \tag{5}$$

and for Stokes drag, we have

$$\frac{dq_r}{dt} = \frac{2N_r g\pi(\rho_w - \rho_a) q_r E_{cp}}{9\mu} \int_0^\infty e^{-\lambda_r r} r^4 dr \tag{6}$$

These integrals can be written in terms of the Γ function

Rain Accretion Amounts

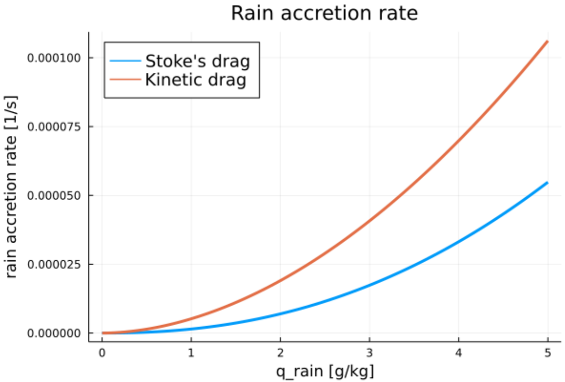


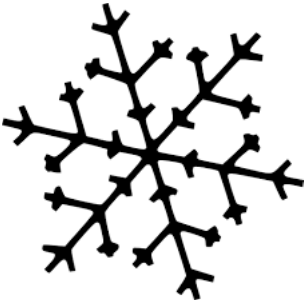
Figure: Comparison of Kinetic and Stokes drag for rain accretion rate.

Considerations for Snow

Snow is much more complicated than rain. Considerations include:

- Snow is a "cylinder", not a sphere
- Snow has a mass distribution
- Snow has holes
- Snow is hexagonal
- Snow falls at varying angles
- Snow is a "fractal"

We also note that we need a different form for Stokes drag because we no longer have a sphere



Stokes Drag for Non-Spheres

Stokes drag for a non-spherical object is given as

$$\mathbf{F}_d = 6\pi\mu v_{term} r_n K_n \tag{7}$$

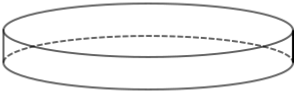
where

$$K_n = \frac{1}{3} + \frac{2r_s}{3r_n} \tag{8}$$

represents a shape factor, with $K_n = 1$ for a sphere, and r_n is the radius of the sphere with the same normal projected surface area as the non-spherical object, and r_s is the radius of the sphere with the same effective surface area as the non-spherical object.

Snow as a Disc

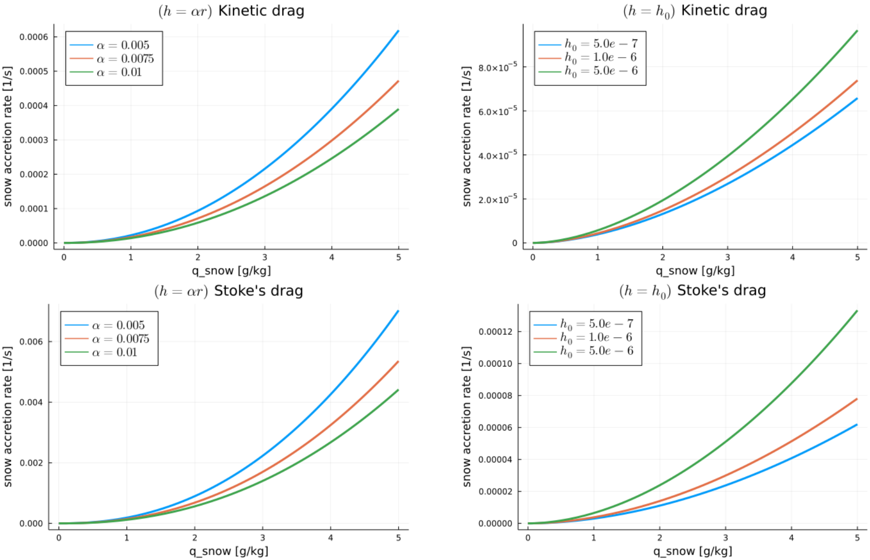
First, we model a snowflake as a cylinder with radius r and height h. We consider $h(r) = \alpha r$ for some constant of proportionality α and $h = h_0$ for some constant h_0.



Four different cases:

- Constant height, kinetic drag
- Constant height, Stokes drag
- Proportional height, kinetic drag
- Proportional height, Stokes drag

Varying the height and drag



Snow with a Given Mass Distribution

Next, we can consider the mass of a snowflake to be independent of the radius. We consider the mass to be distributed as $n_m(m) = N_m e^{-\lambda_m m}$, which gives

$$\frac{dq_s}{dt} = \int_0^\infty \int_0^\infty n_r(r) n_m(m) a(r) v_{term} q_s E_{cp} dr dm \tag{9}$$

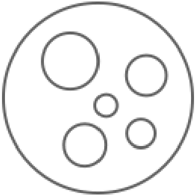
as we integrate over both the mass and density. Note that v_{term} depends on m and $a(r)$.

Snow with Holes

Snow can have holes in it. To model this, we consider some β to be the area factor. A snowflake with an area factor of β and a radius of r has a surface area of $\beta\pi r^2$. This allows us to avoid assumptions about the specific number of holes.

Three directions:

- We can assume that $\beta = \beta_0$ is constant for all snowflakes
- We can assume that β is normally distributed with $\mu = 0.64$ and $\sigma = 0.173$
- We can assume that β is a function of r with with



$$\beta(r) = \beta_{\text{min}} + (1 - \beta_{\text{min}})e^{-\lambda_\beta r} \tag{10}$$

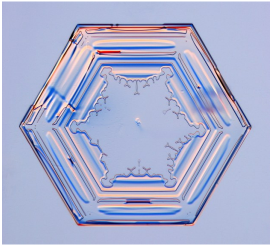
Stokes Drag with Holes

In order to deal with Stokes drag with holes, we assume that there are k equal holes of radius r_k. We then solve for r_k in terms of β and r and find r_n and r_s. Then, one can assume k as fixed, a given distribution, or as a function of r.

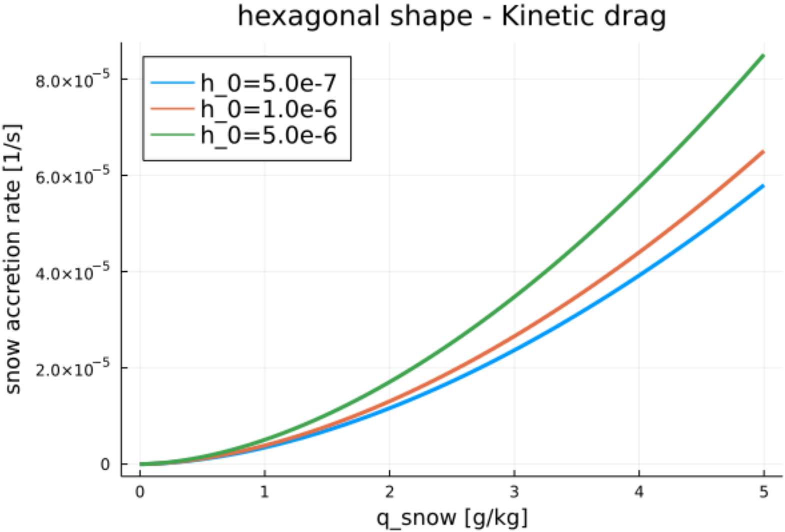
Hexagonal Snow

We model snowflakes taking the form of hexagonal plates. The kinetic model of drag is shape independent. Thus, when compared to the cylinder, we see little change in the integral form of $\frac{dq_s}{dt}$. They differ by a simple factor of $\frac{3\sqrt{3}}{2\pi}$.

We can also find the Stokes drag for the hexagonal prism. This changes $\frac{dq_s}{dt}$ considerably.



Snow accretion: Hexagonal model



Fractal Snow

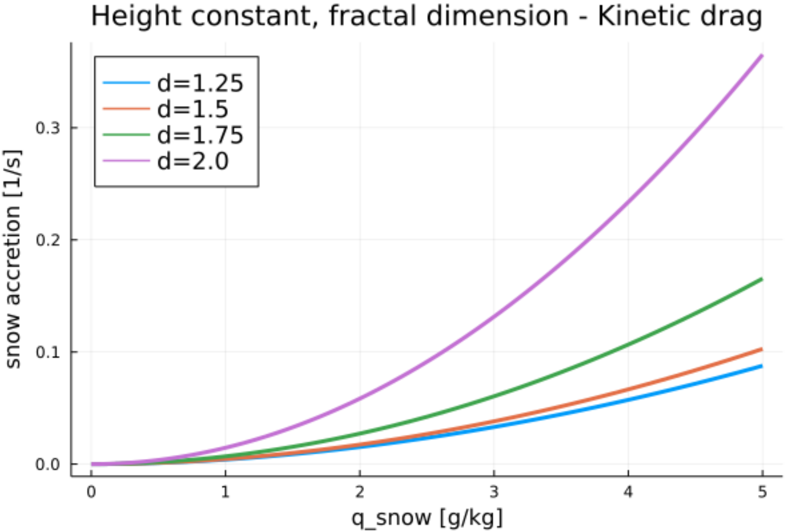
Snow is a fractal in some sense. To capture this, we assume that, instead of scaling quadratically, instead

$$a(r) = Cr^d \tag{11}$$

where C is a constant and $d \in [1, 2]$ is the "dimension" of the snow flake. If $d = 1$, the snowflake is a line, and if $d = 2$, the snowflake is fully two dimensional.

We then assume that all the snowflakes have the same dimension d_0, or that d is uniformly distributed between 1 and 2. Additionally, to handle fractal Stokes drag, we assume that $h \ll r$.

Snow accretion: Fractal snow



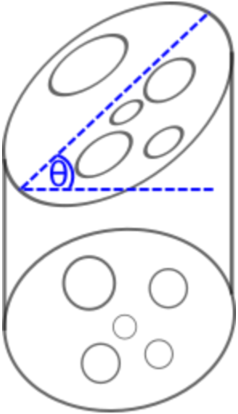
Angled Snow

Snow does not fall straight down. If we consider θ to be the angle that a snowflake is falling at, then the projected normal surface area will instead be

$$\tilde{a}(r) = a(r)\cos(\theta) \tag{12}$$

for $\theta \in [0, \frac{\pi}{2}]$.

We assume that $\theta = \theta_0$ is the same for all snowflakes, or that θ is uniformly distributed between 0 and $\frac{\pi}{2}$.



Conclusion & Outlook

In this project, we modeled snow in various ways, taking into account factors such as

- Different drag forms
- Holes in the snow
- Fractal shapes
- Angle it falls at

There are many further things that we could consider:

- Definite integrals
- Time Dependence
- More detailed holes
- Heretogeneous Snow

Acknowledgements

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Thank you for listening!

Questions?