SYSTEMATIC UNCERTAINTIES

- Goals is to ~~answer~~ address the following questions:
  - What are systematic uncertainties?
  - How do we correctly evaluate them?
  - How do we know if all systematics have been accounted for?
  - How do we treat correlations between systematic uncertainties?
  - How do we combine them?

~~In truth, no sys~~
In practice, no systematic and coherent treatment is available nor possible. We can only refer to knowledge, experience, common sense and intuition.
This is more a draft of a cookbook rather than a mathematical explanation. I will ~~give you~~ explain the main ~~coo~~ cooking ~~the~~ techniques and explain a few recipes, but then you have to explore on your own.

- Definitions of systematic uncertainties

~~a) Systematic~~
Possible definitions:

a) Systematic uncertainties are all uncertainties that are not directly due to the statistics of the data

↳ With this definition, the trigger efficiency or detector acceptance obtained from MC simulations would be systematic errors even though they have a statistical behaviour.

~~b)~~

b) Systematic uncertainties are measurement errors which are not due to statistical fluctuations of real or simulated data samples

↳ In this case the trigger efficiency or detector acceptance would be treated as a statistical uncertainty.

$\boxed{\text{Syst 1}}$

- Example: ~~Th~~ Space dependence of CUORE BI

~~Detector~~ Experiment made of ~1000 detectors.

Data taking divided in ~30 datasets of ~1 month duration.

Question: is there any space dependence of the measured bkg rate?

↳ Is a single expectation value for the bkg rate enough to model the bkg rate of all 1000 detectors?

Method: 1) Compute average bkg rate

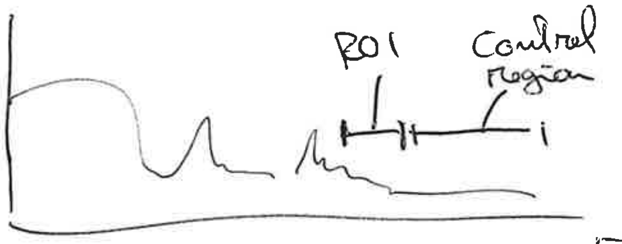
$$BR = \sum_{\text{detector } i} \sum_{\text{dataset } d} \frac{N_{di}}{m T_{di} \Delta E}$$

N_{di} = number of counts in control region

m = detector mass

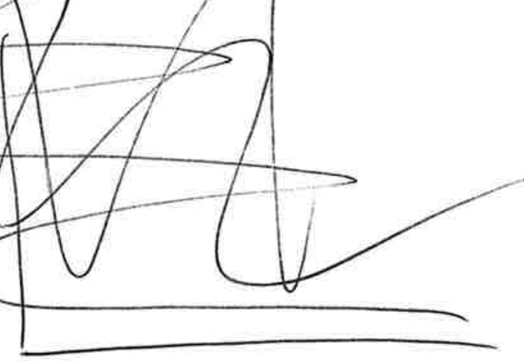
T_{di} = dataset duration

ΔE = width of control region



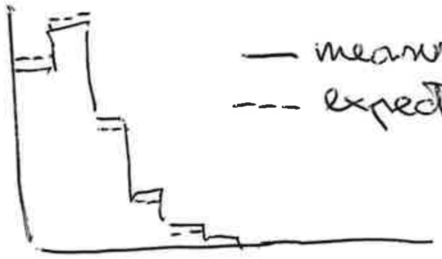
~~2) Produce distribution of bkg rate for each detector-dataset:~~

~~$BR_{di} = \frac{N_{di}}{m T_{di} \Delta E}$~~



2) Compare distribution of N_{di} ~~with~~ (made of ~30 000 entries) with ~~Poisson distribution~~ ~~combination~~ sum of corresponding Poisson distributions, ~~all with~~ produced by setting

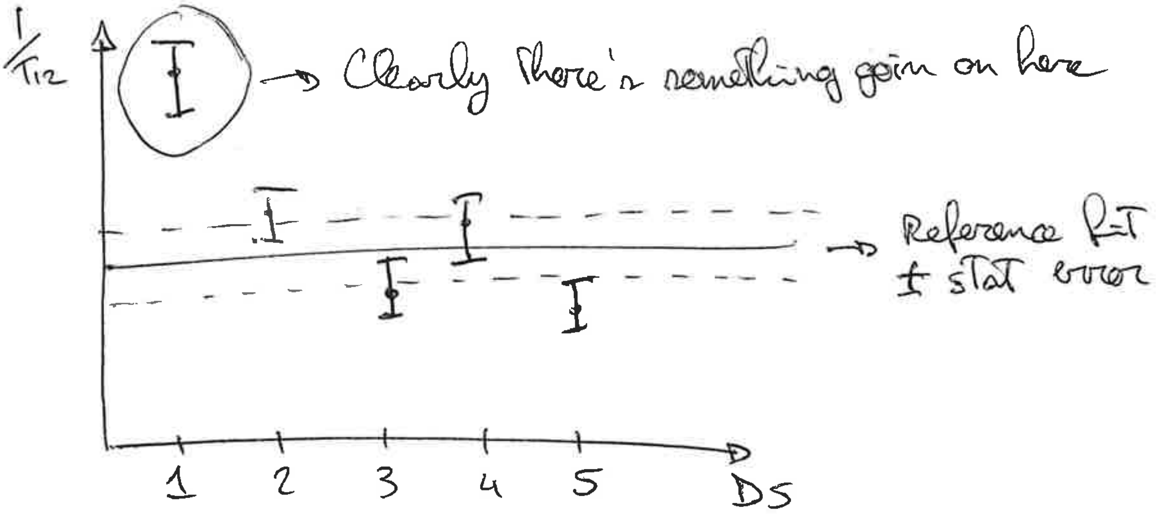
$$\lambda_{di} = BR \cdot m \cdot T_{di} \cdot \Delta E \quad \text{with same BR for all!}$$



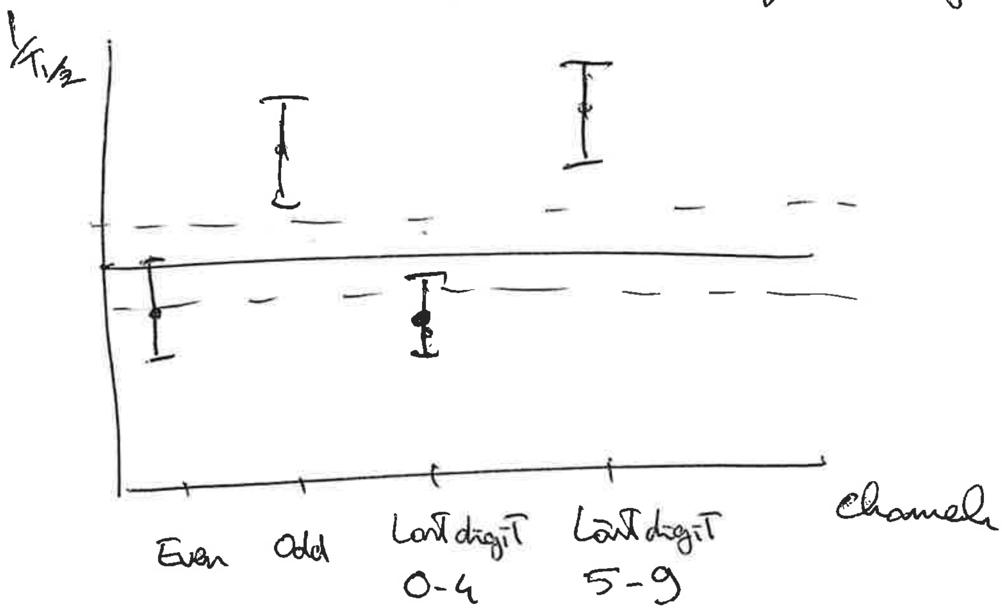
$\boxed{\text{Sgn 3}}$

- Example: Splitting data into independent subsets

→ In CUORE, we repeat the fit for each DS separately, obtaining:



→ We also split the detector geometrically, with a non-sense splitting to ~~evaluat~~ evaluate the magnitude of ~~on~~ any possible geometric systematic:



- Example: Analysis software

Most analyses involve complex software developed ~~specifically~~ for the specific situation. It is <u>crucial</u> to make sure there is no bug!

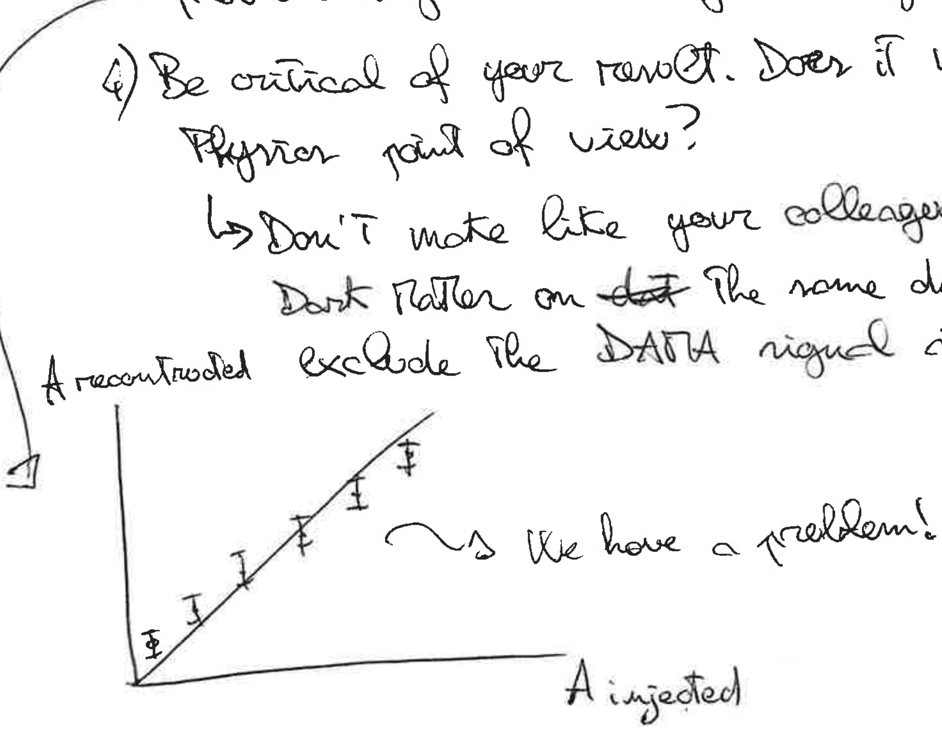
Solutions: 1) Perform a closure test using ~~artificially~~ data or Toy MC to make sure the code does what it is supposed to do!

2) Repeat the test varying the statistics of the fake data.

3) Study the fit bias as a function of the statistics, as a problem might arise only with very low or very high stat

4) Be critical of your result. Does it make sense from the physics point of view?

↳ Don't make like your colleague who discovered Dark Matter on ~~dat~~ the same data that were used to exclude the DAMA signal at 3σ!



- ~~Example~~: Tolerances

Suppose we don't know the standard deviation σ_θ of a parameter θ, but just an ~~minimum~~ allowed range $[\theta_{min}, \theta_{max}]$.
This would be the case of a hit on a scintillator strip.
~~The std~~ For a uniform distribution, the standard deviation is:

$$\sigma_\theta = \frac{\sigma_{max} - \sigma_{min}}{\sqrt{12}} \simeq 0.29\,(\sigma_{max} - \sigma_{min})$$

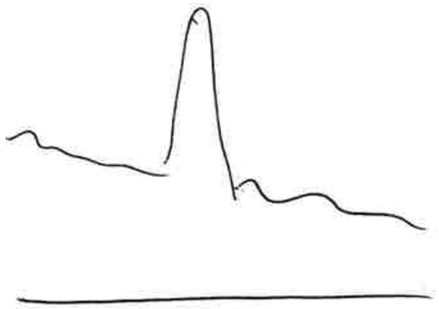
which is ~40% smaller than the naive half-range!

- Example: Small systematics

If a systematic effect gives - even in the worst case scenario - an effect that is much smaller than the statistical uncertainty, don't waste time in calculating the corresponding systematic error!

- Example: background estimation

Suppose we take the following data, consisting of a ROI with signal + background and some sidebands with background only:



We have 2 options:

a) Side band subtraction → does not rely on possibly incorrect MC, but does not predict ~~the~~ bkg shape in ROI

b) Model bkg with MC → does not provide bkg normalization

Solution: combine the 2 methods if ~~(b)~~ (a) is not enough, meaning if the bkg shape cannot be parameterized.

$\boxed{\text{Syst 9}}$

$$\frac{1}{T_{1/2}} = G\, g_A^4\, M^2 \frac{m_{\beta\beta}^2}{m_e^2}$$

G = phase space → computed precisely

g_A = axial coupling → fixed

$m_{\beta\beta}$ = parameter of interest

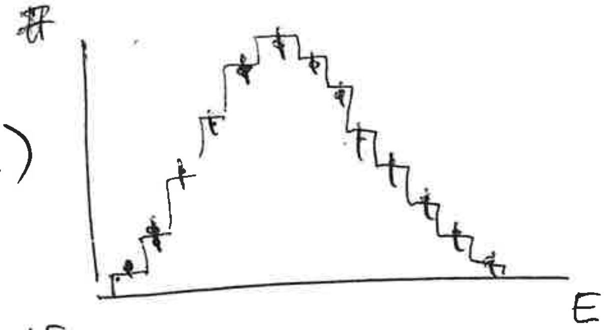
m_e = electron mass

M = Nuclear Matrix Element

↳ A single limit on $T_{1/2}$ is reflected in a "range of limits" on $m_{\beta\beta}$!

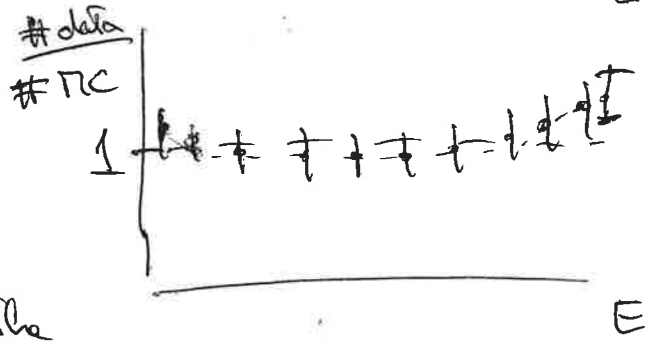
- Example: Discrepancy between data and simulation

Suppose that despite all efforts, we get stuck with a discrepancy between data and MC. How do we quantify the discrepancy?



Compute fraction of data ~~[illegible]~~ f lying in badly described region (10%)

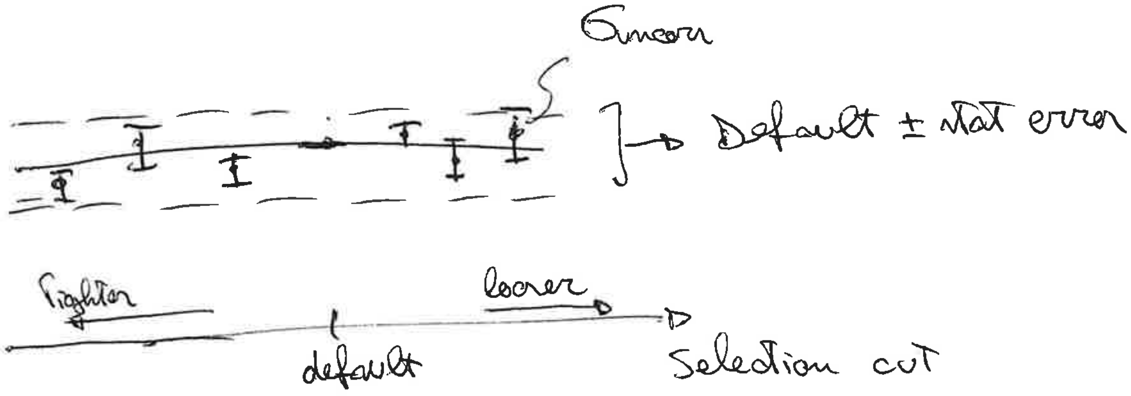
Compute relative difference d between data and MC (e.g. 20%).



⇒ The systematic uncertainty will be $\pm f \cdot d = \pm 0.2\%$.

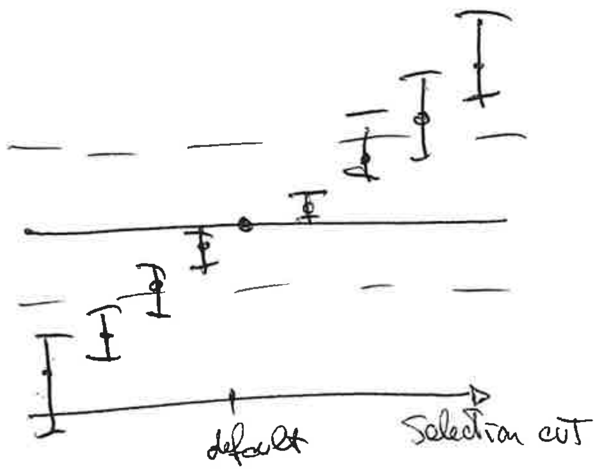
→ Notice that we are relying on the fact that the MC normalization in the "well described" region is correct!

If we plot the uncorrelated error as a function of the cut variation:



Possible outcomes of cut variation:

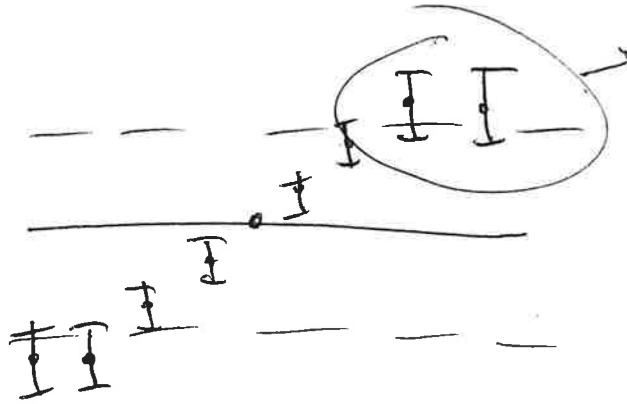
a)



→ Not possible to quote an uncertainty

→ We have a problem that we need to understand!

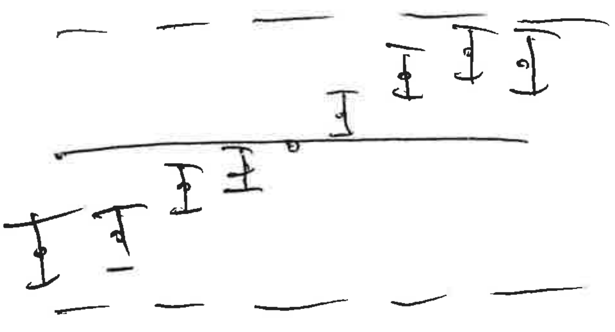
b)



→ ~~Do~~ ~~the~~ Does the trend really stabilize or is it due to a fluctuation?

If yes, quote the maximum variation, otherwise go back to a

c)



→ Quote max variation

→ If we mis-estimate the systematic, it's still smaller than the statistical uncertainty

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