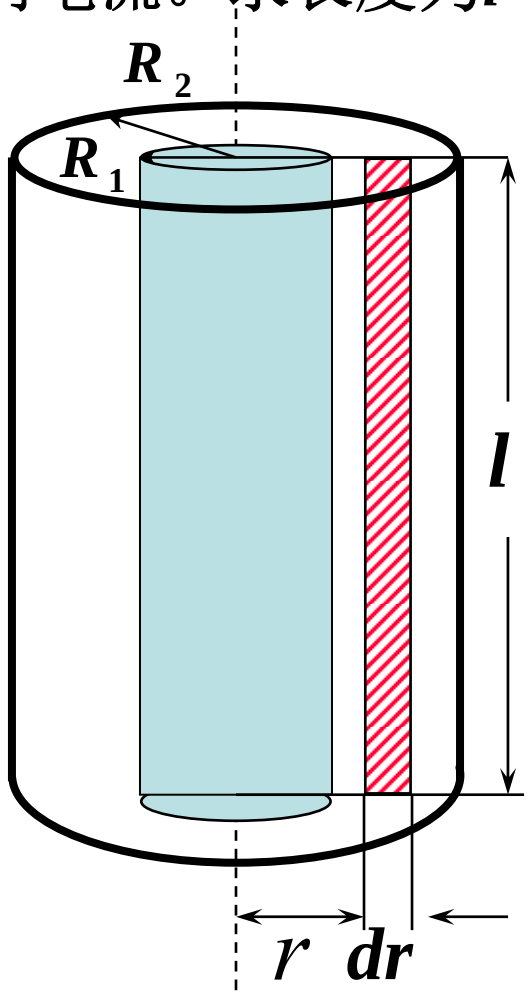
设同轴电缆内柱为半径R_1的实心导体柱，外圆筒半径为R_2而可以不计厚度，电流I均匀通过内柱，外筒流有反向电流。求长度为l的同轴电缆的磁能及自感系数。



解：电缆中的磁场分布为

$$B_1 = \frac{\mu_0 I}{2\pi R_1^2} r \quad (r < R_1)$$

$$B_2 = \frac{\mu_0 I}{2\pi} \frac{1}{r} \quad (R_1 < r < R_2)$$

$$B_3 = 0 \quad (r > R_2)$$

磁场能量为

$$W_m = \iiint_V w_m dV$$

$$= \int_0^{R_1} \frac{1}{2\mu_0} \left(\frac{\mu_0 I}{2\pi R_1^2} r\right)^2 2\pi r l dr + \int_{R_1}^{R_2} \frac{1}{2\mu_0} \left(\frac{\mu_0 I}{2\pi} \cdot \frac{1}{r}\right)^2 2\pi r l dr$$

$$W_m = \iiint_V w_m dV$$

$$= \int_0^{R_1} \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi R_1^2} r\right)^2 2\pi r l dr + \int_{R_1}^{R_2} \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi}\cdot\frac{1}{r}\right)^2 2\pi r l dr$$

$$= \frac{\mu_0 I^2 l}{4\pi R_1^4}\int_0^{R_1} r^3 dr + \frac{\mu_0 I^2 l}{4\pi}\int_{R_1}^{R_2}\frac{1}{r}dr$$

$$= \frac{\mu_0 I^2 l}{16\pi} + \frac{\mu_0 I^2 l}{4\pi}\ln\frac{R_2}{R_1}$$

由 $\frac{1}{2}LI^2 = W_m$

得自感系数为 $L = \frac{\mu_0 l}{8\pi} + \frac{\mu_0 l}{2\pi}\ln\frac{R_2}{R_1}$

单位长度同轴电缆的自感系数为 $L = \frac{\mu_0}{2\pi}\left(\frac{1}{4} + \ln\frac{R_2}{R_1}\right)$

由磁通求自感系数

要注意由粗导线构成的回路，整个回路的电流为I，回包含一匝线圈，但导线中的磁通量$d\Phi$只与一部分电流I'相交链，与I'相联系的电流只有I'/I匝，因此磁通量$d\Phi$对回路的总磁链数贡献为

$$d\Psi = \frac{I'}{I} d\Phi$$

整个电流回路的磁链数为

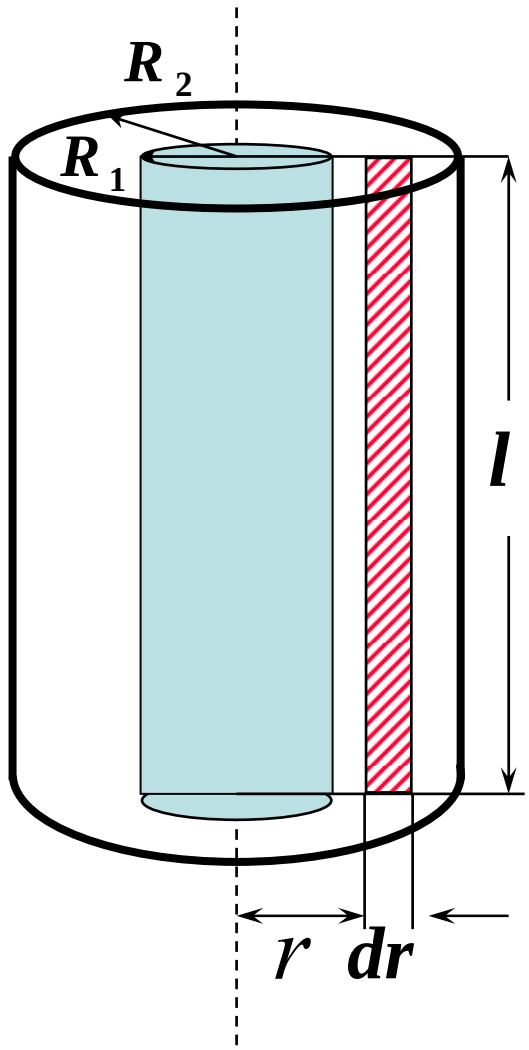
$$\Psi = \int \frac{I'}{I} d\Phi$$

本例同轴电缆，而B_1仅交链部分电流，

对磁链数的贡献为

$$\Psi_1 = \int \frac{I'}{I} B_1 ds = \int_0^{R_1} \frac{r^2}{R_1^2} \frac{\mu_0 I}{2\pi R_1^2} r \cdot l dr$$

$$= \frac{\mu_0 I l}{2\pi R_1^4} \int_{R_1}^{R_2} r^3 dr = \frac{\mu_0 I l}{8\pi}$$



本而$\boldsymbol{B}_2$交链所有电流，对磁链数的贡献为

$$\Psi_2 = \int B_2 ds = \int_{R_1}^{R_2} \frac{\mu_0 I}{2\pi}\frac{1}{r}\cdot l dr$$

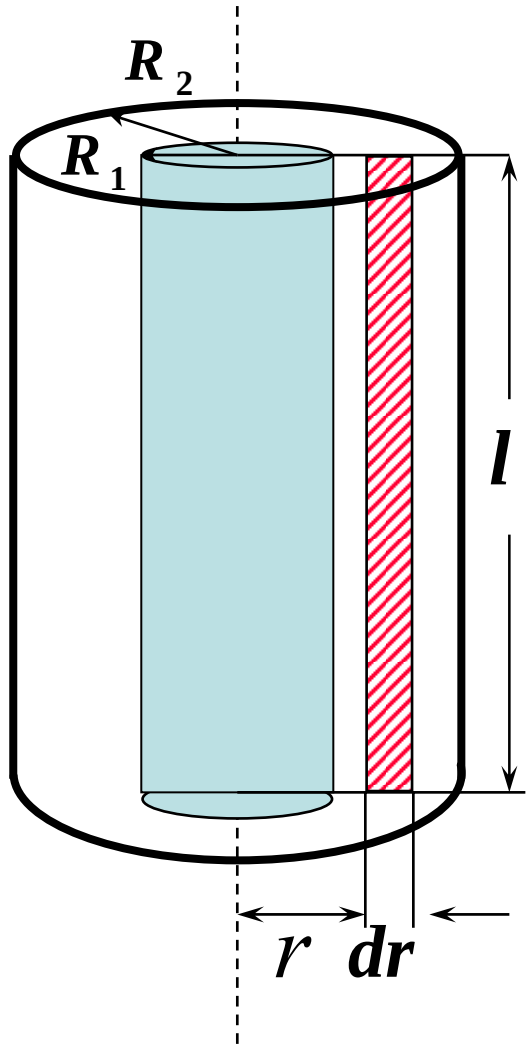
$$= \frac{\mu_0 I l}{2\pi}\ln\frac{R_2}{R_1}$$

总的磁链数为

$$\Psi_1 = \Psi_1 + \Psi_2 = \frac{\mu_0 I l}{8\pi} + \frac{\mu_0 I l}{2\pi}\ln\frac{R_2}{R_1}$$

得自感系数为

$$L = \frac{\Psi}{I} = \frac{\mu_0 l}{8\pi} + \frac{\mu_0 l}{2\pi}\ln\frac{R_2}{R_1}$$



单位长度同轴电缆的自感系数为

$$L = \frac{\mu_0}{2\pi}\left(\frac{1}{4} + \ln\frac{R_2}{R_1}\right)$$