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An analysis of stock recommendations

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and

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We study the information content of stock reports when investors are uncertain about a financial analyst's incentives. Incentives may be aligned, in which case the analyst wishes to credibly convey information, or incentives may be misaligned. We find the following: Any investor uncertainty about incentives makes full revelation of information impossible. Categorical ranking systems, such as those commonly used by brokerages, arise endogenously as equilibria. Under certain conditions, analysts with aligned incentives can credibly convey unfavorable information but can never credibly convey favorable information. Finally, we compare testable implications of the model to empirical properties of stock recommendations.

1. Introduction

■ In many situations, the economic environment is sufficiently complex that decision makers are uncertain about the impact of their decisions. For instance, a legislature may be uncertain about the economic effect of proposed emission controls. Likewise, investors may be uncertain about the consequences of investing in a particular stock with their retirement savings. In these situations, decision makers often turn to experts for advice and guidance. A key difficulty facing the decision maker is that the *motives* of the expert providing advice may not be transparent. This situation commonly arises in the interaction between investors and financial research analysts.

This article examines how investor uncertainty about the motives of financial research analysts employed by securities firms affects the information content of their stock reports. Securities firms offer services that include investment banking (such as underwriting the issue of publicly traded companies, raising bank loans, and advising on mergers) and brokerage services (such as investment advice and equity research). Securities firms are required to separate their brokerage and investment banking activities because research analysts in the brokerage division may face undue pressure from the investment banking division to issue stock reports that favor the interests

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of investment banking clients over those of brokerage clients. To strengthen the "Chinese wall" separating the brokerage and investment banking divisions, Congress amended U.S. securities laws in 1988. In the wake of this legislation, the Securities and Exchange Commission (SEC) issued guidelines, and the National Association of Securities Dealers and the New York Stock Exchange issued a joint memorandum in 1991 endorsing this separation.

Nevertheless, from time to time, research analysts face pressure to "breach" the Chinese wall and issue upwardly biased stock reports that favor the interests of the firm's investment banking clients (Dugar and Nathan, 1995, 1996; Laderman, 1998; Lin and McNichols, 1998; and McNichols and O'Brien, 1997). Indeed, recently the SEC expressed renewed concern about the incentives that analysts face to bias their disclosure to investors; the SEC's office of compliance, inspections, and examinations is currently investigating the policies and procedures that securities firms have in place to ensure that analysts are appropriately shielded from the other divisions in the firm (Burns, 2000). Congress too is concerned about the potential conflict of interest and has recently been holding hearings to establish whether a conflict exists between analysts' investment banking and personal stockholding interests and their fiduciary responsibility to investors. Congress is also considering implementing more stringent disclosure requirements for analysts (Schroeder, 2001; Schack, 2001). In response to these concerns, the Securities Industry Association released "Best Practices" guidelines to enhance analyst credibility (Opdyke, 2001). In May 2002, Merrill Lynch & Co., the largest brokerage firm in the United States, agreed to pay $100 million and reform its stock research process to settle a New York state probe of allegations that it issued overly optimistic research reports on its investment banking clients (Gasparino, 2002). The high profile given to these issues highlights the importance of investors' uncertainty about an analyst's incentives when reporting on a specific company.

We model a setting where an analyst, through his expertise, obtains a private and nonverifiable signal about the firm's value. The analyst is also privately informed about the nature of the incentives he faces at that moment; for instance, whether there is the possibility of winning future investment banking business or whether he has an equity stake in the firm. The analyst, who is not obliged to truthfully report his private information, releases a stock report. Investors value the firm upon observing this report. The analyst's payoffs depend on the firm's stock price, its underlying value, and the presence of investment banking opportunities or personal stockholdings in the firm. The analyst's incentives are said to be aligned with those of investors when payoffs are maximized by a stock price that exactly reflects the analyst's information about the firm's value. Conversely, incentives are misaligned with those of investors when an analyst prefers to induce a higher stock price than is warranted by his information.

Our main findings are as follows:

(i) In Proposition 1, we show that any investor uncertainty about incentives makes it impossible for an analyst to credibly reveal good news about a firm's valuation—even when the analyst's incentives are perfectly aligned with those of investors. Thus, full revelation of information is impossible.

(ii) Proposition 2 identifies a class of equilibria consisting of categorical equity-ranking systems. This class of equilibria has the institutionally interesting property that restrictions on ranking categories, such as those commonly used by brokerages to rank stocks (e.g., buy/hold/sell), arise endogenously as equilibria. Further, Proposition 6 shows that these equilibria have the property that *all* analysts tend to issue more-favorable reports with greater frequency than less-favorable reports—even those with incentives perfectly aligned with those of investors. Nevertheless, analysts whose incentives are misaligned tend to issue favorable reports even more frequently.

(iii) In Propositions 3 and 4, we establish that it is possible for an analyst to credibly convey bad news about a firm. Thus, the institutional restriction of requiring analysts to report using a small number of equity-ranking categories may prevent the efficient transmission of information about troubled or distressed firms. Proposition 5 establishes conditions under which the restriction to using ranking categories leads to no information transmission, whereas absent these restrictions, bad news about a firm's prospects would be fully reported by analysts with aligned incentives.

(iv) The validity of our results may be tested empirically. We highlight testable implications of our model and offer a number of statistical tests using published data.

The rest of the article proceeds as follows. Section 2 reviews the related literature. Section 3 describes the model. Section 4 examines issues related to full revelation of analyst information. Section 5 studies the information that can be credibly conveyed in stock recommendations. Section 6 offers empirical implications of this analysis. Section 7 concludes.

2. Literature review

■ The nearest antecedent to our article is Benabou and Laroque (1992), who also consider the problem of the incentives of analysts to misreport their information in a cheap-talk framework. Our article differs from theirs in two respects. First, while there is uncertainty about the incentives of an analyst in both models, Benabou and Laroque exogenously impose the condition that when an analyst has aligned incentives (i.e., is an "honest" analyst), he truthfully reveals his information. In our model, in contrast, analysts with aligned incentives report in a payoff-maximizing fashion. We show that endogenizing this reporting decision matters—unlike in Benabou and Laroque, there is no equilibrium in our model where an analyst whose incentives are aligned with investors discloses his information at face value. A second key difference between the models centers on reputational concerns. Benabou and Laroque are mainly concerned with the dynamics of the disclosure strategies. As a result, their static model is simpler than ours: their model consists of a binary state space, a binary message space, and a binary action space that determines stock prices. Since our concern is with the impact of transitory investment banking opportunities on analyst incentives, our focus is on a static game, but in a richer context. In our model, states, messages, and actions are all continuous. This modelling framework allows us to explore certain comparative static properties that are institutionally relevant but cannot be addressed in Benabou and Laroque's framework.

Other work in this area has focused on situations where analysts are not directly concerned with the stock price induced by their reports, but rather are concerned with convincing investors of their expertise in forecasting. Trueman (1994) considers a reporting environment where analysts with different forecasting abilities are motivated to build reputations for forecasting accuracy. He finds that analysts with strong forecasting abilities truthfully reveal their information, whereas those with weak predictive abilities try to mimic the strong type. Ottaviani and Sorensen (1999) study information transmission in a model that has some application to analyst reporting. As in Trueman, analysts are solely concerned with investors' perception of their forecasting ability. In contrast to these articles, our model, like that of Benabou and Laroque, is applicable to situations where an analyst is mainly concerned with the impact of his report on the price of the firm's stock.

Admati and Pfleiderer (1986, 1988, 1990) also study the impact of information transmission in financial markets. Their analyses, however, purposely ignore the incentive problems between the seller of information and the buyers who use the information to make investment decisions. In particular, they assume that the seller truthfully provides information if sold directly and makes the promised investment if the information is sold indirectly. Incentive issues are focal in our study.

From a purely theoretical perspective, our article may be viewed as extending the model of Crawford and Sobel (1982) to the case where there is uncertainty about the degree of divergence in preferences between the sender and the receiver. Crawford and Sobel are interested in information transmission between a single sender and a single receiver when there is no uncertainty about the sender's incentives. They find that all equilibria are partitional. Thus analysts are unable to fully reveal their private information. In our model, when receivers (or investors) are uncertain of the sender's (or analyst's) incentives, we find a class of equilibria that is partitional and a class of equilibria where analysts with aligned incentives can credibly convey unfavorable information about a firm's value but can never credibly convey favorable information. Moreover, under certain conditions, the latter class of equilibria corresponds with greater stock price efficiency than the former class.

Finally, our article is also somewhat related to Sobel (1985) and Morris (2001). These articles also consider information transmission between a single sender and receiver when the receiver is uncertain about the sender's incentives, but they focus on the dynamics of reputation formation. As a consequence of this focus on dynamics, their modelling environments differ substantially from ours.

On the empirical front, there is considerable work on how analyst incentives affect reporting behavior: representative studies include Dugar and Nathan (1995), Francis and Soffer (1997), Lin and McNichols (1998), Michaely and Womack (1999), and Womack (1996). These articles document reporting outcomes consistent with analysts having incentives to upwardly bias their reports. There is little extant literature that relates these empirical findings to a game-theoretic model with fully optimizing agents. An important contribution of this article, therefore, is to develop a model explaining these findings.

3. Model

■ We study a financial market setting containing a firm, many investors, and an equity analyst who follows the firm. The market participants have identical uniform prior beliefs about the firm's value, θ, lying in some bounded interval, which we normalize to be [0, 1]. The analyst is employed in the brokerage division of a securities firm that offers brokerage and investment banking services. He is privately informed about the nature of the incentives, β, that he faces when preparing his stock report. Investors, in contrast, are uncertain about the analyst's incentives. We assume that β is distributed as follows:

$$\beta = \begin{cases} 0 & \text{with probability } p \\ b & \text{with probability } 1-p, \end{cases}$$

where $b, p \in (0, 1)$.

The analyst privately observes the realization of the firm's value, θ.[1] His information is neither contractible nor verifiable. The analyst costlessly issues a stock report m from some set of feasible reports M. The reporting strategy for an analyst with parameter β is a function $\mu_\beta(\theta)$ mapping his private information about firm value into a report. The report may be vague or even misleading.[2]

Investors observe the report and revise their beliefs about the firm's value, which are given by the cumulative distribution function $P(\theta \mid m)$. They then value the firm. Since we assume that the market for the firm's stock is efficient and investors are risk neutral, the firm's stock price, y, equals the firm's expected value given all publicly available information, including any information contained in the analyst's stock report m; hence, $y(m) = \int_0^1 \theta \, dP(\theta \mid m)$.

Lastly, the analyst's payoff is determined.[3] To reflect the conflict between the analyst's fiduciary responsibility to investors and obligation to investment banking clients (or potential clients), we suppose that the analyst's payoff consists of two components—a benefit associated with inflating the stock price above its true value, and a cost associated with poor performance. Specifically, the analyst's objective function is

$$U(y, \theta, \beta) = 2\beta y - (\theta - y)^2 . \tag{1}$$

[1] Of course, the analyst might have to exert effort to obtain this information. We are not concerned with incentive schemes inducing optimal effort on the part of the analyst. Such schemes are explored in Osband (1989), Hayes (1998), and Dewatripont and Tirole (1999).

[2] Since false recommendations are allowed, ours is not a "game of persuasion" (see Shin, 1994).

[3] Since we wish to focus on the information that analysts communicate via their stock reports, we do not explicitly model the possibility that analysts may trade on their own account. Admati and Pfleiderer (1986, 1990) make a similar assumption.

The analyst's incentive parameter, β, is the amount above the firm's true value that an analyst wishes to inflate the stock price. Thus, when $\beta = 0$, the analyst has purely performance-based concerns. That is, the analyst's payoffs are maximized by inducing a stock price that is equal to the firm's value, $y = \theta$. Since the analyst desires no difference between the induced stock price and the firm's value, we say that the analyst's incentives are *aligned* with those of investors. In contrast, when $\beta = b$, the analyst has mixed motives. Here, his payoffs are maximized by inducing a stock price that is somewhat above the firm's value, that is, $y = \theta + b$. Since the analyst desires a stock price that exceeds the firm's value, we say that the analyst's incentives are *misaligned* with those of investors.

All aspects of the game are common knowledge except the analyst's privately observed incentive parameter and signal of the firm's value. In analyzing the information content of stock reports, we study perfect Bayesian equilibria of this model. These require that

(i) investors' beliefs, $P(\theta \mid m)$, are formed using Bayes' rule whenever possible;
(ii) given beliefs, the analyst's reporting strategy, $\mu_\beta(\theta)$, maximizes the analyst's payoff.

We restrict attention to equilibria in pure reporting strategies. Hence, we can succinctly define the stock price occurring in equilibrium when the analyst receives signal θ and has incentives β as $Y_\beta(\theta) \equiv y(\mu_\beta(\theta))$.

Before proceeding, we discuss several features of the model. Our representation of the analyst's payoff function is consistent with the widely held view that there are generally two primary components of an analyst's compensation (Stickel, 1992; Michaely and Womack, 1999). The first component of the analyst's compensation is the analyst's ability to generate investment banking business (corresponding to the $2\beta y$ term in equation (1)). Analysts who help win investment banking business may receive a portion of the fees generated or, more commonly, a bonus that is two to four times that of analysts who do not win business (Michaely and Womack, 1999). Analysts often win this investment banking business by issuing positive recommendations that boost a firm's stock price. This component of compensation encourages an analyst to be optimistic about a firm's prospects and often creates a conflict of interest between the analyst and investors.

The second component is an analyst's performance (corresponding to the term $-(\theta - y)^2$ in equation (1)). The *Institutional Investor* All-American Research Team poll, based on a survey of money managers and institutions, is widely viewed as a measure of an analyst's standing in the industry. This poll ranks analysts on their stock-picking and earnings-forecasting ability, industry knowledge, client service, and the like. Directors of equity research at securities firms often consider analysts' rankings in this poll when setting their compensation (Michaely and Womack, 1999). Most of the factors the poll considers when evaluating an analyst's performance serve to align his interests with those of investors.

Whether or not the analyst's interests are aligned with those of investors depends upon the circumstances prevailing when the analyst issues a stock report. These circumstances change over time as changing market conditions affect the firm's and analyst's prospects. Since the presence of investment banking opportunities is highly variable, investors generally are unaware of the degree to which these opportunities are present (Michaely and Womack, 1999). Likewise, investors may be unaware of the analyst's personal stock holding in the firm. Consequently, the Investment Advisers Act of 1940 and the Standards of Professional Conduct for the Association of Investment Management and Research require analysts to disclose the presence of investment banking and other interests in their stock reports. To comply with this requirement, securities firms generally use a boilerplate clause to cover litigation contingencies and typically disclose that they *may* have investment banking relations with a company for which a report is issued; for instance, Bear Sterns included the following caveat on a stock report it issued on McKesson Corporation, dated September 4, 1996: "Bear Sterns may make markets and effect transactions, including transactions contrary to any recommendation herein, or have positions in the securities mentioned herein (or options with respect thereto) and may also have performed investment banking services for the issuers of such securities." This disclosure probably stems from imperfect observability of the analyst's incentives and the prospect of adverse litigation outcomes in the event that no disclosure is made. Since stock reports typically make the same disclosure along this dimension, it has traditionally been viewed as having questionable value.[4] Thus, investors are uncertain about an analyst's precise incentives. Representing the analyst's incentive parameter, β, as a random variable captures this key feature of the analyst reporting environment.

We assume that the analyst receives a perfect signal about the firm's value; however, our analysis would be unaffected if we were to assume instead that the analyst receives a noisy signal. Such a signal would preclude the possibility of investors using differences in the report and the realization of θ to force analysts to truthfully reveal their private information. It would, however, increase notational complexity. In addition, we have assumed that the quality of the signal the analyst receives about a firm's value is commonly known.[5] This is not likely to be the case for new and unproven analysts. Thus, our model is more appropriate for studying the information content of stock reports issued by well-established analysts.

To ensure an economically interesting tradeoff between the analyst's responsibility to investors and to investment banking clients (or potential clients), we assume that $b \in (0, 1)$. If $b \geq 1$, then the incentives to "boost" the stock price completely dominate the analyst's fiduciary responsibility to investors. To see this, notice that the highest stock price that will ever prevail is $y = 1$ (since $\theta \leq 1$), and therefore the analyst maximizes his payoff when $\beta = b \geq 1$ by inducing the highest feasible stock price regardless of the realization of θ. Thus, when $b \geq 1$, fiduciary or quality concerns are always swamped by incentives to inflate stock prices.

Throughout much of the analysis, the feasible set of messages, M, will simply be $\Re$. Note, however, that the type space consists of two components: $\theta \in [0, 1]$ and $\beta \in \{0, b\}$; thus, at first glance one might suppose that this reporting space is too restrictive for an analyst to completely disclose his type. Nonetheless, it is sufficiently rich to allow the analyst to disclose all of his private information. For example, were the analyst to report $m = \theta$ when $\beta = 0$ and report $m = \theta + 2$ when $\beta = b$, investors could perfectly infer the analyst's private information.

Finally, our model examines the influence of a report by a single analyst. In the case of issuing stock recommendations, this representation seems to roughly approximate the institutional environment. For instance, Womack (1996) finds that temporal clustering of recommendations from competing analysts is rare. In the case of the issuance of earnings forecasts, clustering is more prevalent. Thus, to the extent that interaction among analysts is important to the perceived information content of their stock reports, our model is more appropriate for stock recommendations than for earnings forecasts.

4. Price responsiveness

■ A key concern of regulators in proposing rules and procedures designed to preserve the independence of analysts is that even a *potential* conflict of interest may undermine the information content of their stock reports. In this section, we consider whether it is possible for an analyst's stock report to credibly convey information about a firm's value.

First, we study the responsiveness of stock prices to reports. Intuitively, a responsive stock price impounds, at least partially, all new information about a firm's value. In our framework, if, for some interval of firm values, all of the information available to an analyst is reflected in the stock price, then equilibrium stock prices will reflect this information and be continuous and strictly increasing over this interval of values. That is, the stock price will be *responsive* to the information of the analyst. More formally,

Definition 1. A stock price is *fully responsive* if, for some realization of β, the stock price, $Y_\beta(\theta)$, is continuous and strictly increasing almost everywhere. A stock price is *semiresponsive* if, for

[4] Indeed, recognizing that the value of this disclosure is questionable, SEC regulators and some securities firms are currently formulating proposals to strengthen written disclosure rules (Knox, 2000; Schack, 2001).

[5] In light of Michaely and Womack's (1999) finding that analyst optimism is not attributable to differences in an analyst's predictive ability but rather to incentives arising from the need to satisfy investment banking concerns, we view this perspective as conforming to the long-run outcome in the market for analyst advice.

some realization of β and some nondegenerate interval $(\underline{\theta}, \bar{\theta})$, the stock price, $Y_\beta(\theta)$, is continuous and strictly increasing for all $\theta \in (\underline{\theta}, \bar{\theta})$.

Our definition of responsiveness allows for stock prices to be responsive even when an analyst with misaligned incentives is hiding some information about the firm's value. For instance, if an analyst reports honestly when his incentives are aligned and reports the most inflated possible valuation, $\theta = 1$, when incentives are misaligned, then $Y_0(\theta)$ is strictly increasing almost everywhere and the stock price is fully responsive.

Our main result in this section is to show that when there is any possibility of misaligned incentives, however small, stock prices are never fully responsive. This result follows as a corollary of a stronger result obtained in Proposition 1. This result establishes that whenever an analyst learns sufficiently good news about the valuation of a firm he is covering, this information can *never* be fully impounded in stock price. Put differently, it is impossible for an analyst to credibly convey good news about a firm's value when there is the possibility of incentive misalignment.

In making these arguments, the following lemma is helpful; it establishes that an analyst's report containing more-positive information about a firm's value never induces a strictly lower stock price.

Lemma 1. In any equilibrium, the stock price, $Y_\beta(\theta)$, is nondecreasing in θ.

Proof. See the web Appendix at www.rje.org/main/sup-mat.html

When it is common knowledge that incentives are aligned ($\beta = 0$), there exists an equilibrium where the stock price is fully responsive.[6] In particular, suppose that an analyst truthfully discloses the firm's value; i.e., $\mu_0(\theta) = \theta$ for all θ. In this case, using Bayes' rule to form investors' posterior beliefs, we have that $y(m) = m$ for all $m \in [0, 1]$. This then implies that $Y_0(\theta) = \theta$ for all θ; hence the stock price is fully responsive. Moreover, an analyst does strictly worse than the equilibrium strategy by choosing any message $m \neq \theta$ when the firm's value is θ.

Next, consider the case where it is common knowledge that incentives are misaligned ($\beta = b$). This is covered by the model studied by Crawford and Sobel (1982).[7] It follows immediately from Lemma 1 of Crawford and Sobel that in this case, even a semiresponsive stock price is impossible. An implication of their Lemma 1 is that all equilibria consist of partitions of the state space, so no exact news can be credibly conveyed. Below we establish that partitional equilibria arise in our setting as well (Proposition 2). However, unlike the case where there is no uncertainty, in our model there always exists an equilibrium whereby *bad* news about a firm can be credibly conveyed (Propositions 3 and 4).

When incentives are uncertain, which is the focus of this article, the situation is different. To see that a fully responsive stock price is impossible, suppose to the contrary that the stock price is fully responsive. Fix a firm value $\theta' > b$. Let m' be the stock report inducing price $y = \theta'$. When $\beta = 0$, an analyst who learns that the firm's value is θ' will prefer to induce the stock price $y = \theta'$ over all other prices and report m'. Likewise, when $\beta = b$, an analyst who learns that the firm's value is $\theta'' = \theta' - b$ also will prefer to induce the price $y = \theta'$ and report m'. Thus, investor beliefs upon hearing the message m' yield an expected firm value of $E(\theta \mid m') = p\theta' + (1-p)(\theta' - b)$. But this is a contradiction, since $y(m') = E(\theta \mid m') < \theta'$.

More generally, we have shown the following.

Proposition 1. There is no equilibrium characterized by stock prices that are semiresponsive over an interval $(\underline{\theta}, \bar{\theta})$, where $\bar{\theta} > b$.

Proof. Follows from the above discussion.

A key consequence of Proposition 1 is the following.

[6] Of course, there are many other equilibria arising in this case. At the other extreme, babbling is also an equilibrium.

[7] Note, however, that the leading example of Crawford and Sobel (1982) is a special case of our model when $p = 0$.

Corollary 1. When there is a possibility that an analyst has misaligned incentives, equilibrium stock prices are never fully responsive.

One might have conjectured that when there is high probability that incentives are aligned, an analyst will disclose θ truthfully and investors will act on the report as though it were truthful.[8] Proposition 1 rules this out. An implication of this proposition is that when an analyst receives a relatively favorable signal about firm value (i.e., $\theta > b$), it is impossible for his stock report to *credibly* convey nuanced information. Instead, in any equilibrium, when an analyst receives a favorable signal about the firm's value, slight differences in the signal will almost never be reflected in stock prices.

It is worth noting that the arguments in Proposition 1 hold fairly generally. Indeed, an almost identical argument can be used to show that no semiresponsive equilibrium exists for arbitrary distributions of firm values and where analysts have preferences $U(y, \theta, \beta)$ such that $U_{11} < 0$, $U_{12} > 0$, $U_{13} > 0$, and, for each θ, there is a unique y maximizing $U(y, \theta, \beta)$.

5. Equilibrium stock recommendations

■ The previous section established the impossibility of conveying good news about a firm when there is some possibility that incentives are misaligned. In this section we study the information that can be obtained from analysts in equilibrium. We focus on two classes of equilibria.

The first class of equilibria is of interest in view of an important institutional restriction on analyst reports. Analysts are generally required to rank firms using their brokerage's particular equity ranking system (e.g., buy/hold/sell). Typically, the number of categories available to an analyst is limited to only a few.[9] In Proposition 2 we characterize a class of equilibria, which we call "categorical ranking system equilibria." For this class of equilibria, we establish conditions under which the message-space restrictions imposed institutionally have no impact on the information analysts can convey.

The existence of the second class of equilibria highlights that the institutional restriction to a few rating categories is not innocuous. In particular, we show in Propositions 3 and 4 that it is always possible for an analyst with aligned incentives to credibly convey bad news about a firm's valuation. That is, semiresponsive recommendations are possible, but only to the extent that stock prices fully impound bad news about a firm's prospects.

□ **Categorical ranking system equilibria.** Most brokerages restrict analysts to issue recommendations using one of a small number of equity-ranking categories. While the contents of an analyst's full report contains significantly more detail than simply this recommendation, it is the analyst's categorical recommendation that is widely reported in the media. Moreover, econometricians often use the categorical recommendation to summarize the contents of the analysis contained in an analyst's report when estimating the impact of analyst recommendations on stock prices. In this subsection we study the properties of this institutional feature using our model. The main result is to show that when investors remain uncertain about an analyst's incentives, even after reading the report, then the institutional practice of severely restricting the message space of the analyst to even a small number of reporting categories may *not* have an adverse effect on the informativeness of stock recommendations.

Formally, consider a class of equilibria where investors are uncertain about an analyst's incentives. That is, for any stock report, m, issued in equilibrium, investors (correctly) infer that $\Pr(\beta = 0 \mid m) \in (0, 1)$. We now establish, first, that equilibria satisfying this condition always exist, and second, that they are characterized by only a finite number of equilibrium stock prices.

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[8] Indeed, this outcome occurs in equilibrium in the models of Sobel (1985) and Benabou and Laroque (1992), where some types of analysts are exogenously constrained to truthfully reveal their private information. In contrast, in our article, as well as in that of Morris (2001), all types of agents behave optimally when choosing their report.

[9] For instance, in response to investor complaints that analysts are hiding their true sentiments in confusing multitiered scales, Morgan Stanley recently reduced the number of rating categories its analysts are allowed to use from four to three in the United States, Europe, and Japan (Chaffin, 2002).

To see that an equilibrium in this class always exists, suppose that regardless of β and θ, an analyst issues the report $m = 1/2$. Investors will not update their beliefs on the basis of this report. Further, suppose investors do not update their beliefs following *any* report issued by the analyst. In this case, an analyst can do no better than issue the report $m = 1/2$. Since stock prices reflect the expected value of the firm given the information contained in the stock report and since the investors' posterior beliefs are formed using Bayes' rule where possible, this constitutes an equilibrium for all p and β. In this equilibrium, there is only a single price response, $y = 1/2$, to the analyst's report.

To show that equilibria are characterized by only finitely many stock prices, we need two steps. The first step is to show that there are only a countable number of equilibrium stock prices. This follows from the fact that when there is *ex post* uncertainty about an analyst's incentives, we may apply the argument in Proposition 1 to establish that such equilibria are never semiresponsive.

The second step is to establish finiteness. We introduce an observation (and notation) that is used throughout the remainder of the analysis. Consider two equilibrium stock prices y_i and y_{i+1}, where $y_i < y_{i+1}$. It follows from the concavity of the analyst's objective function that there exists a firm value $\theta = a_i^\beta$ such that an analyst with incentives β is indifferent between y_i and y_{i+1} for firm value a_i^β. Thus, a_i^β is such that

$$2\beta y_i - \left(a_i^\beta - y_i\right)^2 = 2\beta y_{i+1} - \left(a_i^\beta - y_{i+1}\right)^2. \tag{2}$$

We shall refer to (2) as a "no arbitrage" condition and firm value a_i^β as the ith "cut point" for an analyst with incentives β. It is useful to note that if, for a given pair of stock prices y_i and y_{i+1}, an analyst with incentives $\beta = 0$ is indifferent for some firm value $\theta = a_i^0$, then an analyst with incentives $\beta = b$ will be indifferent for a firm value that is lower by exactly b. Hence, $a_i^b = a_i^0 - b$.

Suppose that there are a countable but infinite number of equilibrium stock prices; i.e., prices are "close" to one another. In this circumstance, an analyst with aligned incentives will release a report m_i that induces price $y_i = y(m_i)$ only when the firm's value is arbitrarily close to y_i. Further, since investors are unable to infer an analyst's incentives from the report, for an analyst with misaligned incentives, y_i is induced only when the firm's value is arbitrarily close to $y_i - b$. Thus, investors, upon hearing report m_i, infer that the firm's expected value is arbitrarily close to $y_i - (1 - p)b$. The only way to avoid this contradiction is if equilibrium stock prices are sufficiently far apart. Hence, there are only a finite number of equilibrium stock prices.

We summarize these observations in the following lemma.[10]

Lemma 2. An equilibrium in which investors cannot infer the analyst's incentives always exists. In such an equilibrium, there are only a finite number of equilibrium stock prices realized with positive probability.

Proof. See the Appendix.

Combining Lemmas 1 and 2, we may arrange the finite number, N, of equilibrium stock prices $\{y_i\}_{i=1}^N$ in ascending order in i. In such an equilibrium, an analyst will strictly prefer to induce the stock price y_i over all other prices for any θ that lies in some interval $(a_{i-1}^\beta(N), a_i^\beta(N))$. These intervals partition the space of firm values. Using these observations, Proposition 2 shows that when investors are unable to infer the analyst's incentives from the stock report, categorical ranking systems arise endogenously as equilibria.

Proposition 2. Suppose that investors cannot infer an analyst's incentives from the report. There exists a positive integer $N(p, b)$ such that every $N = 1, 2, \ldots, N(p, b)$ is associated with exactly one categorical ranking system equilibrium consisting of N ranking categories constructed as

[10] Although the statement of this lemma appears closely related to that of Lemma 1 of Crawford and Sobel (1982), the presence of uncertainty about incentives creates complications so that the method of proof in their article does not apply here. Instead, we rely on deriving a contradiction near a limit point.

follows:

(i) For all β and for all $i = 1, 2, \ldots, N-1$, $a_i^\beta(N)$ satisfies

$$2\beta y_i - \left(a_i^\beta(N) - y_i\right)^2 = 2\beta y_{i+1} - \left(a_i^\beta(N) - y_{i+1}\right)^2,$$

and $0 = a_0^\beta(N) < b < a_1^\beta(N) < \ldots < a_N^\beta(N) = 1$.

(ii) For each β and $\theta \in [a_{i-1}^\beta(N), a_i^\beta(N))$, the stock report $m_i = y_i$ is issued. For $\theta = 1$, $m_N = y_N$ is issued.

(iii) For all $i = 1, 2, \ldots, N$,

$$\begin{aligned} y_i = {} & \pi(m_i) E\left[\theta \mid \theta \in \left[a_{i-1}^0(N), a_i^0(N)\right]\right] \\ & + (1 - \pi(m_i)) E\left[\theta \mid \theta \in \left[a_{i-1}^b(N), a_i^b(N)\right]\right], \end{aligned} \tag{3}$$

where

$$\pi(m_i) \equiv \frac{p\left(a_i^0(N) - a_{i-1}^0(N)\right)}{p\left(a_i^0(N) - a_{i-1}^0(N)\right) + (1-p)\left(a_i^b(N) - a_{i-1}^b(N)\right)}.$$

(iv) For all $m \notin \{m_i\}_{i=1}^n$, $y(m) = y_1$.

(v) $N(p, b)$ is the largest integer N satisfying

$$\begin{aligned} & \frac{1}{2}b(4N^2 - 10N + 7 - 4p(N-1)(N-2)) \\ & + \frac{1}{2}b\sqrt{8p(N-1)(2Np - 4N - 4p + 7) + (4N-5)^2} \leq 1. \end{aligned} \tag{4}$$

(vi) Further, all other equilibria are outcome equivalent to a categorical ranking system equilibrium.

Proof. See the web Appendix at www.rje.org/main/sup-mat.html.

Two aspects of this equilibrium characterization are worth noting. First, even though the analyst's message space is rich enough to allow him to fully convey his information, we find in equilibrium that an analyst, even one guided solely by performance considerations, conveys a summary message that provides only a rough guide as to the firm's value. Second, the use of a limited number of ranking categories (e.g., buy/hold/sell) often imposed upon analysts by brokerages does not necessarily lead to a significant (or indeed any) loss of information. Corollary 2 below formalizes conditions guaranteeing that an econometrician's focus on stock recommendations as a proxy for the information contained in analysts' stock reports fully characterizes the information analysts communicate in equilibrium. The emphasis on stock recommendations is typically justified in the extant empirical literature by the observation that issuing a recommendation is the final output of the analyst's activity. The other tasks that the analyst performs, such as forecasting earnings, are subordinate to the task of issuing a stock recommendation (Schipper, 1991; Womack, 1996).[11]

Corollary 2. Suppose that investors are uncertain about an analyst's incentives and that analysts are restricted to M equity-ranking categories. If the number of categories is at least $N(p, b)$, then this restriction on the message space does not harm the informativeness of stock recommendations.

[11] Nevertheless, there is work examining both recommendations and earnings forecasts when assessing the information content of stock reports; see, for instance, Francis and Sofer (1997).

□ **Semiresponsive equilibria.** We have ruled out the possibility that stock prices will fully impound an analyst's information, whatever his incentives, when the firm's underlying value is sufficiently favorable ($\theta > b$). In this subsection, we establish that when an analyst has aligned incentives, it is always possible for him to credibly convey his information when a firm's underlying value is low. In identifying equilibria with this property, it is useful to label them using the following definition.

Definition 2. A semiresponsive equilibrium is of size n when an analyst with incentives $\beta = b$ can induce exactly n stock prices in equilibrium.

Size 1 semiresponsive equilibria. First, we examine size 1 semiresponsive equilibria. In these equilibria, analysts with misaligned incentives convey no information whatsoever. Nonetheless, for low firm values, analysts with aligned incentives can credibly convey their information, and the stock price fully impounds it.

Define the most favorable news that can be credibly conveyed by an analyst with aligned incentives to be

$$\theta^* = \frac{1}{p}\left(1 - \sqrt{1-p}\right).$$

Then we observe the following.[12]

Proposition 3. There is a size 1 semiresponsive equilibrium if and only if $b \geq \theta^*$. When $b \geq \theta^*$, the following strategies characterize this equilibrium:

If $\beta = 0$, then the reporting strategy of an analyst is

$$\mu_0(\theta) = \begin{cases} \theta & \text{if } \theta < \theta^* \\ \theta^* & \text{if } \theta \geq \theta^*. \end{cases}$$

If $\beta = b$, then the reporting strategy of an analyst is

$$\mu_b(\theta) = \theta^*.$$

The pricing strategy is

$$y(m) = \begin{cases} m & \text{if } m < \theta^* \\ \theta^* & \text{otherwise.} \end{cases}$$

Proof. See the Appendix.

Two features of this equilibrium are worth noting. First, such an equilibrium is more likely to exist when the degree of incentive misalignment, b, is large than when it is small. That is, an analyst conveying bad news about a firm's prospects is more likely to be believed if the possible conflict of interest (in terms of incentives to push up stock prices) is thought to be great.[13] The intuition for why there are no size 1 semiresponsive equilibria when b is small is that analysts with aligned incentives would still like to reveal low firm values and have their reports believed. However, because the degree of incentive divergence is not too great for analysts with misaligned incentives, they will imitate the unfavorable reports of aligned analysts when firm values are extremely unfavorable. Of course, investors anticipate this possibility and discount any report accordingly. This unravels the equilibrium.

[12] In this proposition and elsewhere, we shall write the equilibrium pricing strategy without reference to the investors' posterior beliefs. Along the equilibrium path, the beliefs are implied by the strategy chosen by the analyst. Off the equilibrium path, however, beliefs are chosen freely. This implies that any stock price $y \in [0, 1]$ may be sustained for stock reports not made in equilibrium.

[13] As discussed earlier, we ensure an economically interesting tradeoff between an analyst's fiduciary obligation to investors and responsibility to investment banking clients by assuming that $b \in (0, 1)$. Nevertheless, for $b \geq 1$, a semiresponsive equilibrium of the form characterized in Proposition 3 continues to exist.

The other interesting feature of this equilibrium is that, for b sufficiently large, more information can be credibly conveyed the higher the probability that an analyst is thought to have aligned incentives, p, because θ^* is strictly increasing in p. Thus, policies that reduce the likelihood that there is a conflict of interest but do not change the level of conflict when it arises are informationally beneficial as long as the degree of misalignment is sufficiently great, that is, $b \geq \theta^*$.

While the precise characterization of θ^* in the above proposition depends on the distribution of θ and the structure of analyst preferences, semiresponsive equilibria can arise quite generally. For instance, the argument may be generalized for an arbitrary continuous distribution of θ, $F(\theta)$, on the unit interval, and analyst payoffs given by $U(y, \theta, \beta)$, where $U_{11} < 0$, $U_{12} > 0$, $U_{13} > 0$, and for each θ, there is a unique y maximizing $U(y, \theta, \beta)$. In particular, suppose that an analyst with aligned incentives maximizes his payoff when the stock price equals θ, while an analyst whose incentives are misaligned maximizes his payoff in state θ when the stock price equals $y^*(\theta, b) > \theta$. If for some $\theta^* \leq y^*(0, b)$ it is the case that

$$\frac{p(1-F(\theta^*))}{p(1-F(\theta^*))+(1-p)} E\left(\theta \mid \theta \geq \theta^*\right) + \frac{(1-p)}{p(1-F(\theta^*))+(1-p)} E(\theta) = \theta^*,$$

then a semiresponsive equilibrium of the form characterized in Proposition 3 exists. The existence of an equilibrium characterized by an interval over which prices are semiresponsive relies on the assumption that the distribution of β is discrete. Truth telling by an analyst with aligned incentives ($\beta = 0$) is disturbed by the presence of an analyst with misaligned incentives ($\beta = b$). The latter type, however, has no incentive to send a message inducing a low stock price. Hence, for low realizations of firm value, the former type can fully reveal his private information.

Size 2 semiresponsive equilibria. Equilibria in this class may be characterized by the following strategies.

If $\beta = 0$, then the reporting strategy of an analyst is

$$\mu_0(\theta) = \begin{cases} \theta & \text{if } \theta < a_1 \\ y_1 & \text{if } \theta \in [a_1, a_2] \\ y_2 & \text{if } \theta \in (a_2, 1]. \end{cases}$$

If $\beta = b$, then the reporting strategy of an analyst is

$$\mu_b(\theta) = \begin{cases} y_1 & \text{if } \theta \in [0, a_2 - b] \\ y_2 & \text{if } \theta \in (a_2 - b, 1]. \end{cases}$$

The pricing strategy is

$$y(m) = \begin{cases} m & \text{if } m \in [0, a_1] \cup \{y_1, y_2\} \\ y_2 & \text{otherwise.} \end{cases}$$

For $m < y_1$, investors believe that the analyst has aligned incentives. If $m \in \{y_1, y_2\}$, then the posterior probability that the analyst is believed to have aligned incentives is

$$\pi(y_1) = \frac{p(a_2 - a_1)}{p(a_2 - a_1) + (1-p)(a_2 - b)}$$
$$\pi(y_2) = \frac{p(1 - a_2)}{p(1 - a_2) + (1-p)(1 - (a_2 - b))}.$$

In any equilibrium, stock prices reflect the investors' expected value of the firm; hence,

$$y_1 = \pi(y_1) E\left(\theta \mid \theta \in [a_1, a_2]\right) + (1 - \pi(y_1)) E\left(\theta \mid \theta \in [0, a_2 - b]\right)$$

$$y_2 = \pi(y_2)\, E\left(\theta \mid \theta \in [a_2, 1]\right) + (1 - \pi(y_2))\, E\left(\theta \mid \theta \in [a_2 - b, 1]\right).$$

Finally, in any equilibrium, the analyst must weakly prefer the equilibrium stock price to any other price he might induce. Thus, the following equalities must be satisfied:

$$y_1 = a_1 \tag{5}$$

$$a_2 - y_1 = y_2 - a_2. \tag{6}$$

This equilibrium also implies that $a_2 > b$.

Equation (5) together with the fact that the stock price y_1 must equal the firm's expected value implies that

$$y_1 = \frac{a_2 + pb - b - \sqrt{(1-p)(a_2 - b)(a_2 - b + 2pb)}}{p}. \tag{7}$$

Similarly, (6) together with the fact that the stock price y_1 must equal the firm's expected value implies that

$$y_2 = \frac{1 - a_2^2 + 2a_2 b - b^2 - 2pa_2 b + pb^2}{2(1 - a_2 + (1-p)b)}. \tag{8}$$

Using (7) and (8), we can rewrite (6) as $\Phi(a_2) = 0$, where

$$\Phi(a_2) = 2a_2 - y_1 - y_2.$$

We are now in a position to characterize size 2 semiresponsive equilibria.

Proposition 4. A size 2 semiresponsive equilibrium exists if and only if $b < \theta^*$. Moreover, when such an equilibrium exists, all other size 2 semiresponsive equilibria are economically equivalent.

Proof. See the Appendix.

To summarize, when there is no uncertainty about an analyst's incentives, the possibility of stock price responsiveness depends on whether incentives are aligned. When incentives are aligned, fully responsive stock prices can occur in equilibrium; when incentives are misaligned, stock prices are never responsive. In contrast, when there is uncertainty about incentives, the situation is a mixture of the above two extremes: fully responsive stock prices are impossible and, indeed, stock prices cannot be responsive to favorable information about the firm's value (i.e., $\theta > \theta^*$); however, stock prices may be responsive to relatively unfavorable information (i.e., $\theta \leq \theta^*$).

As mentioned earlier, there are a variety of reasons why analysts, their securities firms, and the firms being covered prefer not to have unfavorable information revealed. A key implication of Propositions 3 and 4 is that, in principle, the restriction to reporting a finite number of equity ranking categories may have the effect of suppressing the transmission to investors of bad news about a firm's prospects.

To see most starkly the consequence of restricting analysts to the use of categorical equity-ranking systems, consider the case where the degree of incentive misalignment is large; that is, $b > 2/(3 + \sqrt{9 - 8p})$. In this case, there is only one categorical equity-ranking system equilibrium identified in Proposition 2—no information transmission whatsoever. In contrast, when $b \in (2/(3 + \sqrt{9 - 8p}), (1/p)(1 - \sqrt{1 - p}))$, a size 2 semiresponsive equilibrium exists; thus more information is revealed in this equilibrium whether an analyst's incentives are aligned or not. When $b \geq (1/p)(1 - \sqrt{1 - p})$, only a size 1 semiresponsive equilibrium exists. Here analysts with misaligned incentives do not reveal any information, but analysts with aligned incentives credibly reveal bad news. Thus, in all cases, the restrictions on messages have the effect of reducing the amount of bad news that is revealed about a firm. The following proposition summarizes these observations.

Proposition 5. Suppose that the degree of incentive misalignment is large, $b > 2/(3 + \sqrt{9 - 8p})$; then restricting analysts to a small number of reporting categories prevents analysts with aligned incentives from conveying bad news about a firm.

Proof. See the Appendix.

6. Testable implications

■ A considerable empirical literature has arisen assessing the informativeness of stock reports based on an analyst's choice of an equity-ranking category. Our characterization of categorical equity-ranking system equilibria in Proposition 2 offers empirically testable implications, which we examine using existing data. The first implication we explore concerns the predicted frequency of various types of recommendations. The second implication examines stock price movements in response to recommendations. Since the data used in the empirical analysis of stock recommendations is in the form of categorical ranking systems, our implications will be restricted to properties of this class of equilibria.

First, Proposition 2, which is derived under the assumption that the state space is uniformly distributed, implies certain testable properties of the frequencies with which various recommendations occur. In particular, more-favorable recommendations are issued more frequently than less-favorable recommendations—regardless of analyst incentives. That is, even when analyst incentives are aligned, recommendations tend to be "optimistic" in the sense that more-favorable reports are issued more frequently than less-favorable reports. Moreover, the frequency with which nonextreme recommendations are issued is invariant to the incentives of analysts. Formally,

Proposition 6. In any categorical ranking system equilibrium where $N \geq 2$,

(i) More-favorable recommendations are issued more frequently than less-favorable recommendations, even when incentives are aligned.

(ii) The cumulative distribution of recommendations by analysts with misaligned incentives first-order stochastically dominates that of analysts with aligned incentives when recommendations are ranked according to increasing favorableness.

(iii) Each nonextreme recommendation is issued with the same frequency regardless of incentives; that is, for all $m_i \in \{m_2, \ldots, m_{N-1}\}$, $\Pr(m_i)$ is independent of β.

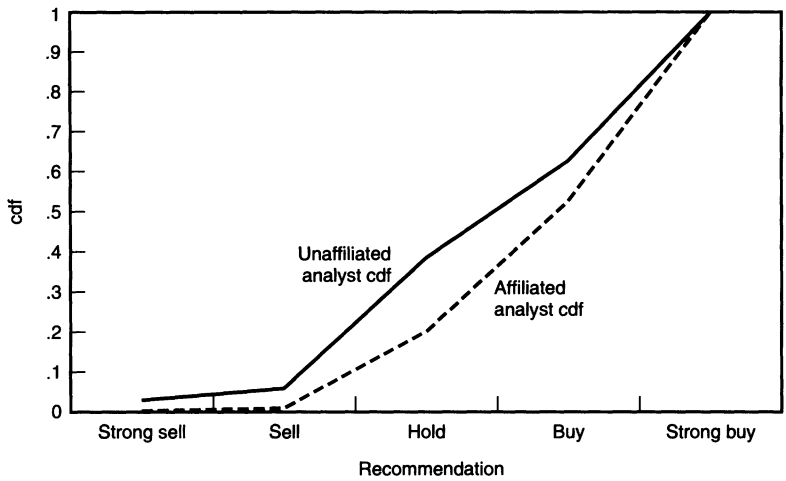
Proof. See the Appendix.

We now show that Proposition 6 is consistent with some empirical properties of stock recommendations. Lin and McNichols (1998) examined stock recommendations issued in the three-year period preceding and following companies' seasoned equity offerings. They compare, for a given company, the favorableness of stock recommendations of analysts employed by investment firms underwriting the offering (hereafter, affiliated analysts) to those of analysts who have no such affiliation (hereafter, unaffiliated analysts). As a reference, Lin and McNichols code a recommendation from most favorable (1) to least favorable (5) in a five-category ranking system.

It is useful to think of the *ex post* realization of the identity of affiliated analysts as a proxy for their incentives, β, prior to the offering. That is, for reports issued prior to the offering, one might imagine that affiliated analysts have more misaligned incentives than unaffiliated analysts, but that this fact is not apparent to investors. We should stress that, as a test of the predictions of our model, the following arguments are offered cautiously. We require a number of assumptions for our model to be applicable to this data. First and most important, we are assuming that investors are uncertain about the analyst's incentives prior to the equity offering. Second, we are assuming that the misalignment of incentives is largely caused by the desire to win investment banking business. Finally, our model literally assumes that if the analyst's employer is not awarded the company's investment banking business, then the analyst's incentives were never misaligned in

FIGURE 1

COMPARISON OF EMPIRICAL CUMULATIVE DISTRIBUTION FUNCTIONS OF AFFILIATED AND UNAFFILIATED ANALYSTS PRIOR TO EQUITY OFFERINGS



SOURCE: Data from Lin and McNichols (1998).

the first place. The last assumption, however, may be relaxed (with added complication to the equilibrium characterization but with little change in testable implications) by assuming that the incentives of analysts whose firms lose the investment banking business are *less* misaligned on average than those whose firms win. With these caveats in mind, we now turn to the data.

Figure 1 compares the empirical cumulative distribution functions (cdf) of recommendations by analysts from lead underwriting firms with recommendations issued by unaffiliated analysts. In each case, attention is confined to the latest report issued before the equity offering.

As Figure 1 shows, for analysts affiliated with underwriting firms, the predicted increasing frequency of recommendations holds. For unaffiliated analysts, more-favorable forecasts are issued more frequently than less-favorable forecasts with one exception: hold recommendations are issued more frequently than buy recommendations. Thus, the predictions of part (i) of Proposition 6 are largely supported. Further, the figure shows that the hypothesized stochastic dominance relationship is present in the data. This observation may be formalized by employing a χ^2 test of the null hypothesis of equality of the two distributions. In performing such a test, Lin and McNichols find a test statistic of 43.5, which enables us to confidently reject the null hypothesis of equality in favor of the one-sided alternative hypothesis of stochastic dominance. Thus, part (ii) of Proposition 6 is also largely consistent with the data.

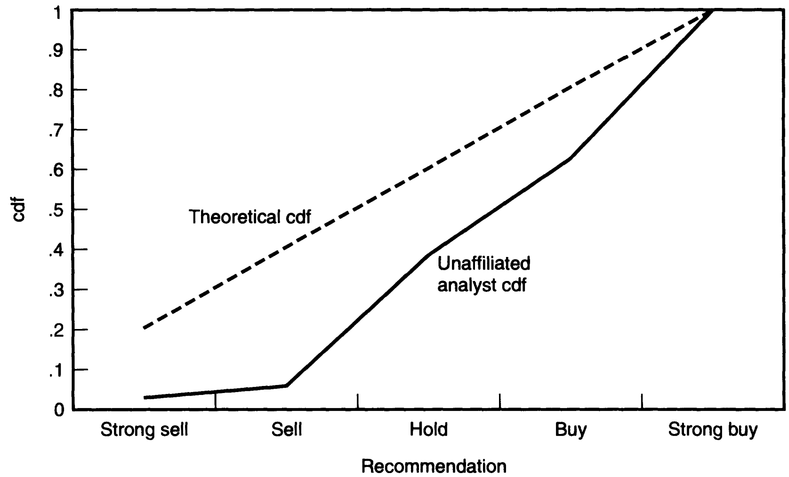
Part (iii) of Proposition 6 predicts that the distribution of observations in intermediate categories is the same for affiliated and unaffiliated analysts. To test this prediction, we perform a Wilcoxon sum of ranks test on observations in the buy, hold, and sell categories for the two types of analysts. Our null hypothesis is that these observations are drawn from the same underlying distribution. We obtain a test statistic of 5.161, which rejects the null hypothesis at conventional levels. Thus, this implication of our model is inconsistent with the data.

We can also use the data to distinguish between our model and one where there is no uncertainty about analyst incentives. Although the empirical cumulative distribution of lead underwriting analysts' recommendations is consistent with both models, the distribution of unaffiliated analysts' recommendations is not. Figure 2 compares the empirical cdf of unaffiliated analysts, who are assumed to have incentives that are aligned with those of investors, with the theoretical distribution predicted in the absence of investor uncertainty.

Notice that in the absence of investor uncertainty, the equilibrium prediction is for recommendations to be divided equally into the five categories. It is apparent that this theoretical

FIGURE 2

COMPARISON OF EMPIRICAL CUMULATIVE DISTRIBUTION FUNCTIONS OF UNAFFILIATED ANALYSTS WITH THEORETICAL PREDICTIONS IN THE ABSENCE OF UNCERTAINTY



SOURCE: Data from Lin and McNichols (1998).

prediction is not supported by the data. We can formalize this observation by performing a χ^2 test of the null hypothesis that the theoretical prediction generated the observed data. Such a test yields a χ^2 statistic of 233.65, which confidently rejects the null hypothesis. Although we have compared the empirical distribution to the equilibrium prediction when incentives are perfectly aligned, given the size of the χ^2 statistic, it seems likely that we would reject the null hypothesis for any relatively small degree of incentive misalignment.

Our model also predicts the response of stock price to recommendations. A considerable amount of empirical work on price responses to stock reports codes recommendations into three categories; thus, we examine the implications of our model in a three-category equilibrium. Given prior beliefs of investors, the stock price before the issuance of a recommendation is $E(\theta) = \frac{1}{2}$. We characterize downward or upward price movement in response to recommendations. Our model predicts the followiing.

Proposition 7. In any three-category equilibrium,

(i) sell recommendations lead to greater downward price movement than hold recommendations. Buy recommendations always lead to upward price movement;

(ii) the magnitude of a price movement in response to a sell recommendation is greater than that in response to either a hold or buy recommendation.

Proof. See the Appendix.

These predicted price responses are consistent with the findings of Womack (1996), Francis and Soffer (1997), and Barber et al. (2001). The magnitudes of the predicted price responses are also consistent with the asymmetric market reaction to added-to-buy and added-to-sell recommendations documented empirically by Elton, Gruber, and Grossman (1986) and Womack (1996), among others.

A test of our hypothesis would be to examine the price response to recommendations *preceding* an equity offering when investors are likely to be uncertain about an analyst's incentives. We would expect no difference in the immediate price response to recommendations issued by analysts who are subsequently revealed to have differing incentives. Over the long run, however, we predict that a stock recommendation issued by an analyst with aligned incentives will perform better than the same recommendation issued by an analyst with misaligned incentives.

These predictions are inconsistent with a model in which there is no uncertainty about analyst incentives. This test remains for future research.

7. Discussion

■ The broad array of conflicting incentives affecting financial analysts employed by securities firms leads to an environment where investors are skeptical of the motives of analysts issuing stock reports. As a consequence of this skepticism, strategic "filtering" of information contained in the reports to correct for bias often occurs. We establish that the presence of uncertainty about an analyst's incentives and the strategic responses to it by both investors and analysts lead to a situation where an analyst's information about firm value is not fully impounded in stock price even if most analysts have aligned incentives.

Two classes of equilibria emerge in this situation. The first class, which we call categorical ranking system equilibria, correspond to the equity-ranking categories (e.g., buy/hold/sell) used by brokerages to rank stocks. These equilibria have the property that *all* analysts tend to issue more-favorable reports with greater frequency than less-favorable reports—even analysts whose incentives perfectly align with those of investors. Nevertheless, analysts whose incentives are misaligned tend to issue favorable reports even more frequently. These and several other implications of our model accord well with empirical findings in this area.

The second class, which we call semiresponsive equilibria, have the property that analysts with aligned incentives are able to effectively communicate unfavorable information about a firm's value, but not favorable information. This is because reports of favorable information are imitated by analysts with misaligned incentives, whereas unfavorable reports are not. Thus, it is only for unfavorable reports that all relevant information is impounded in stock price. This information can be considerable: we identify conditions where brokerages imposing restrictions on the recommendations that an analyst may issue leads to informational loss.

As a first step to understanding the effects of investor uncertainty about the analyst's incentives, we analyzed a simple setting. However, several extensions of the model seem warranted. Specifically, investors sometimes report cross-checking analyst reports with other data to determine more precisely the degree of bias. Such cross-checking is absent from our model; however, one reasonable way that it might be added would be to allow for the existence of two or more analysts simultaneously issuing reports. Likewise, there is interaction between management of the company about whom recommendations are being made and the analyst. Management incentives and information obviously differ from those of analysts (and investors), so it would seem fruitful to extend the model to allow investors to integrate information from these two sources. Exploring these avenues remains for future research.

In conclusion, we see the analyst reporting environment as a natural setting in which to explore strategic information transmission when there is uncertainty about a sender's incentives to report information. Nevertheless, the model we examine is sufficiently general that our findings about the nature and amount of information transmission in this context are applicable to a number of other institutional settings.

Appendix

■ Proofs of Lemma 2 and Propositions 3–7 follow.

Proof of Lemma 2. Before beginning with the proof, the following facts are useful. Using equation (2), one may readily verify that

Fact 1. $a_i^0 - a_i^b = b$.

Next notice that when θ and β are such that $y_i = \theta + \beta$, the analyst does strictly worse by inducing a stock price other than y_i. That is, for firm value θ, the most preferred stock price for an analyst with incentives β is y_i. Hence,

Fact 2. In any equilibrium, $\mu_\beta(y_i - \beta) = m_i$.

We now proceed with the proof. Suppose to the contrary that there exists an equilibrium with an infinite number of equilibrium stock prices. Define the set of all equilibrium prices to be W. Since all prices lie in the interval [0, 1], it follows from the Bolzano-Weierstrass Principle (Shilov, 1996) that the set W has a limit point y^*. Hence, there exist an infinite number of distinct equilibrium prices in a $\gamma-$ neighborhood of y^* for arbitrarily small $\gamma > 0$.

Let $y_1 < y_2 < y_3$ be three such prices. Let a_1^β and a_2^β be the "cut points" associated with these prices for an analyst with incentives β. Since $\partial^2(2\beta y - (\theta - y)^2)/\partial y^2 < 0$, it follows that for all $\theta < a_1^\beta$, an analyst with incentives β strictly prefers y_1 to y_2 or y_3. Likewise for all $\theta \in (a_1^\beta, a_2^\beta)$, an analyst with incentives β prefers to induce y_2 to y_1 or y_3, and so on. Now, as $\gamma \to 0$, $y_1 \to y_3$. Hence $a_1^0 \to a_2^0$ and $a_1^b \to a_2^b$. Hence $y_2 \to pa_1^0 + (1-p)a_1^b$. From Fact 1, $a_1^b = a_1^0 - b$, hence $y_2 \to a_1^0 - (1-p)b$. From Fact 2, an analyst with aligned incentives will induce y_2 when $\theta = a_1^0 - (1-p)b$. Notice, however, that $a_1^0 - (1-p)b < a_1^0$ for $p, b > 0$. This contradicts the fact that y_1 is preferred by an analyst with aligned incentives to y_2 for all $\theta < a_1^0$. *Q.E.D.*

Proof of Proposition 3. We first show that if $b \geq \theta^*$, the above strategies indeed comprise an equilibrium. To see this, notice that when $\beta = 0$, an analyst can do no better than to induce a price $y = \theta$ when $\theta < \theta^*$ and prefers θ^* to any lower price when $\theta \geq \theta^*$. It follows therefore that investors are able to perfectly infer the firm's value from any stock report $m \in [0, \theta^*)$ and the stock price is responsive to this information. In contrast, when $\beta = b$, the analyst prefers to induce price θ^* to any price below θ^* for all realizations of θ.

Next, we show that there is a size 1 semiresponsive equilibrium only if $b \geq \theta^*$. Suppose the contrary is true. Then, to be a size 1 semiresponsive equilibrium, it must be true that there exists a value θ' such that revelation occurs when $\theta < \theta'$ and $\beta = 0$, and pooling occurs otherwise. At θ', it must be the case that an analyst with aligned incentives is indifferent between revealing, and obtaining price $y(\theta') = \theta'$, and playing the pooling price. But since, by revealing, the analyst obtains his most preferred stock price in state θ', it then follows that the pooling price must also equal θ'. Hence, in any size 1 semiresponsive equilibrium, it must be the case that

$$\frac{p(1-\theta')}{p(1-\theta') + (1-p)}\left(\frac{1+\theta'}{2}\right) + \frac{(1-p)}{p(1-\theta') + (1-p)}\left(\frac{1}{2}\right) = \theta'.$$

The unique solution to this equation is $\theta' = \theta^*$. Notice, however, that since $b < \theta^*$, it follows that for a sufficiently small $\varepsilon > 0$, when $\theta \in [0, \varepsilon]$, an analyst with misaligned incentives can profitably deviate by inducing price $\theta + b$ using the appropriate revealing report. This yields the analyst's most preferred price and hence is an improvement over θ^*, which is the price called for in the putative equilibrium. This is a contradiction. *Q.E.D.*

Proof of Proposition 4. To prove the result, we use the following lemma.

Lemma A1. The function $\Phi(a_2)$ has the following properties:

(i) $\Phi(b) < 0$ if and only if $b < \theta^*$.
(ii) $\Phi(1) > 0$ for all b and p.
(iii) $\Phi(a_2)$ is strictly increasing in a_2.

Proof of Lemma A1. We prove each part separately.

Part 1. Evaluating $\Phi(a_2)$ when $a_2 = b$ yields

$$\Phi(b) = \frac{2b - pb^2 - 1}{2(1-pb)},$$

and it is straightforward to verify that the numerator of this expression is negative if and only if $b > \theta^*$.

Part 2. Evaluating $\Phi(a_2)$ when $a_2 = 1$ yields

$$\Phi(1) = \frac{2p - 2 + 2b - pb + 2\sqrt{(1-p)(1-b)(1-b+2pb)}}{2p}.$$

It is sufficient to show that the numerator of this expression is positive. This is equivalent to establishing that

$$2\sqrt{(1-p)(1-b)(1-b+2pb)} > -2p + 2 - 2b + pb. \tag{A1}$$

Notice that the left-hand side of equation (A1) is always positive.

When $b \geq (2-2p)/(2-p)$, then the right-hand side of (A1) is negative; hence the required inequality holds. When $b < (2-2p)/(2-p)$, then squaring both sides of (A1) is permissible, since both the left-hand and right-hand sides are positive. After some simplification, observe that the required inequality is

$$\Gamma \equiv 4p + 4pb - 4b - 4 - 7pb^2 + 8b^2 < 0.$$

We establish that Γ is negative for all $b < (2-2p)/(2-p)$. First, notice that $\partial^2\Gamma/\partial b^2 = 16 - 14p > 0$, so Γ is strictly convex. As a consequence, we need only establish that $\Gamma < 0$ at $b = 0$ and $b = (2-2p)/(2-p)$. Observe that

$$\Gamma\,|_{b=0} = 4p - 4 < 0$$

and

$$\Gamma\,|_{b=\frac{2-2p}{2-p}} = -4p\frac{(1-p)(5-4p)}{(2-p)^2} < 0.$$

Part 3. To establish that Φ is strictly increasing in a_2, it is sufficient to show that $dy_1/da_2 < 1$ and $dy_2/da_2 < 1$. First observe that

$$\frac{dy_1}{da_2} = \frac{1}{p}\left(1 - \frac{(1-p)(a_2+pb-b)}{\sqrt{((1-p)(a_2-b)(a_2-b+2pb))}}\right).$$

It is sufficient to show that

$$1 - \frac{(1-p)(a_2+pb-b)}{\sqrt{((1-p)(a_2-b)(a_2-b+2pb))}} < p,$$

or equivalently,

$$Z \equiv 2a_2b - b^2 - 2pa_2b - a_2^2 + pb^2 < 0.$$

Observe that Z is strictly concave in a_2 and attains a unique global maximum at $a_2 = b(1-p)$. After evaluating Z at $a_2 = b(1-p)$, we find that $Z = -pb^2(1-p) < 0$. Hence, we conclude that $dy_1/da_2 < 1$. Second, observe that

$$\frac{dy_2}{da_2} = \frac{(a_2-1-b)^2 + pb(2pb-3b+2a_2-2)}{2(-1+a_2-b+pb)^2}.$$

Hence, we must show that

$$(a_2-1-b)^2 + pb(2pb-3b+2a_2-2) < 2(-1+a_2-b+pb)^2,$$

or equivalently,

$$\zeta \equiv -a_2^2 + 2a_2 + 2a_2b - 1 - 2b - b^2 + pb^2 - 2pa_2b + 2pb < 0.$$

Now, observe that ζ is strictly concave in a_2 and attains a global maximum $a_2 = 1+b(1-p)$. Evaluating ζ at $a_2 = 1+b(1-p)$ yields $\zeta = -pb^2(1-p) < 0$. We therefore conclude that $dy_2/da_2 < 1$. *Q.E.D.*

We are now in a position to prove Proposition 4. By construction, an equilibrium need only satisfy the condition that $\Phi(a_2) = 0$ for some $a_2 \in (b, 1)$. When $b < \theta^*$, Lemma A1 implies that there is a unique a_2 in this interval satisfying $\Phi(a_2) = 0$. Hence, an equilibrium exists and all other equilibria are economically equivalent in that they must induce the same $Y_\beta(\theta)$ mapping. When $b \geq \theta^*$, Lemma A1 implies that there is no value of a_2 in this interval satisfying $\Phi(a_2) = 0$. Hence there is no size 2 semiresponsive equilibrium for these parameters. This completes the proof. *Q.E.D.*

Proof of Proposition 5. We first show that when $b \geq (1/p)(1-\sqrt{1-p})$, the unique categorical ranking-system equilibrium is characterized by babbling. To see this, notice that for $N(p, b) = 1$, we require that $b > 2/(3+\sqrt{9-8p})$. Next, notice that for all p, $(1/p)(1-\sqrt{1-p}) > 2/(3+\sqrt{9-8p})$. To see this, we rewrite this condition as $(1-\sqrt{1-p})(3+\sqrt{9-8p}) - 2p > 0$. Since the left-hand side of the above expression is increasing in p, the left-hand side of the expression is zero when $p = 0$.

From Proposition 3, we know that a semiresponsive equilibrium exists for $b \geq (1/p)(1-\sqrt{1-p})$. It is trivial to show that the semiresponsive equilibrium is more informative than the categorical ranking system characterized by babbling. *Q.E.D.*

Proof of Proposition 6. More-favorable recommendations are issued more frequently than less-favorable recommendations if and only if for all $i = 1, 2, \ldots, N-1$, $a_{i+1}^0 - a_i^0 > a_i^0 - a_{i-1}^0$ and $a_{i+1}^b - a_i^b > a_i^b - a_{i-1}^b$. Suppose that the proposition does not hold. Consider the recommendations sent by analysts with aligned incentives; then there exists a k such that $a_{k+1}^0 - a_k^0 \leq a_k^0 - a_{k-1}^0$. Therefore, $a_{k+1}^0 + a_{k-1}^0 \leq 2a_k^0$.

Since y_i is the convex combination of the intervals of θ reported by analysts with aligned and misaligned incentives, it follows that $y_{k+1} < (a_{k+1}^0 + a_k^0)/2$ and $y_k < (a_k^0 + a_{k-1}^0)/2$. Thus, $y_{k+1} + y_k < (a_{k+1}^0 + 2a_k^0 + a_{k-1}^0)/2 \leq 2a_k^0$. However, the no-arbitrage condition $y_{k+1} - a_k^0 = a_k^0 - y_k$ implies that $y_{k+1} + y_k = 2a_k^0$. This yields a contradiction.

The claim of stochastic dominance follows from the fact that all intermediate recommendations have the same frequency and that the first and last cut points are shifted b distance to the left when $\beta = b$.

Finally, the claim that nonextreme recommendations are issued with the same frequency follows directly from the fact that $a_i^0 = a_i^b - b$ and θ is uniformly distributed. *Q.E.D.*

Proof of Proposition 7. We prove the proposition in two steps. We begin by proving the first part of the proposition that $y(m_1) - 1/2 < y(m_2) - 1/2 < 0 < y(m_3) - 1/2$. It has already been established that $a_1^0 < 1/3$ and $a_2^0 < 2/3$, which arises from the optimism in the analyst's report. Therefore, $y(m_1) \leq \frac{1}{6}$ and $y(m_2) \leq 1/2$. Since $a_2^0 - b > 0$, it follows that

$$y(m_3) = \pi(m_3)\left(\frac{a_2^0 + 1}{2}\right) + (1 - \pi(m_3))\left(\frac{a_2^0 - b + 1}{2}\right) > \frac{1}{2}.$$

We now prove the second part of the proposition that $|y(m_1) - 1/2| > |y(m_2) - 1/2|$ and $|y(m_1) - 1/2| > |y(m_3) - 1/2|$. Consider the first part of the claim: $y(m_1) < y(m_2)$ implies that $1/2 - y(m_1) > 1/2 - y(m_2)$. Consider the second part of the claim: since $a_2^0 < 2/3$ (from the optimism in the analyst's report), it follows that $y(m_3) \leq (a_2^0 + 1)/2 < 5/6$. This observation together with $y(m_1) \leq 1/6$ gives the result that $1/2 - y(m_1) > y(m_3) - 1/2$, or $1 > y(m_1) + y(m_3)$. *Q.E.D.*

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