NUMBER PROPERTIES – 1

Natural numbers

The counting numbers {1, 2, 3, …} are called natural numbers.

Whole numbers

The counting numbers as well as 0 {0, 1, 2, 3, …} are called whole numbers.

Integers

Positive and negative counting numbers, as well as zero {…, –2, –1, 0, 1, 2,…} are called Integers.

Rational numbers

The numbers that can be expressed as a **ratio** of an integer to a non-zero integer. Note that all integers are rational, but the converse is not true.

The above is not the entire list of Number system. But the knowledge of the above numbers is sufficient for an aptitude test.

Fractions and Decimals

Every fractional number can be represented in the form of decimals.

Example : $\frac{1}{2}$ = 0.5 and $\frac{1}{3}$ = 0.333...

Based on the decimal form, Rational Numbers are classified into recurring and non-recurring rational. $\frac{1}{3}$ is an example of recurring rational as the number 3 repeats without an end. And $\frac{1}{2}$ is an example of non-recurring rational as there are only finite number of terms after the decimal point.

Conversion of Recurring rational from Decimal to Fractional form

Application 1

Convert 3.67575757... into fractional form.

Solution

Let x = 3.67575757...

$\Rightarrow$ 10x = 36.7575... and 1000x = 3675.7575...

$\Rightarrow$ 990x = 3639 or x = $\frac{3639}{990}$

Divisibility Criteria

The following table is a ready reckoner for determining whether a given number *n* is divisible by a number *k*.

k	*n* is divisible by *k* if
2	the last digit of *n* is divisible by 2,
3	the sum of the digits of *n* is a multiple of 3,
4	the number formed by the last two digits is divisible by 4,
5	the last digit is 0 or 5,
6	*n* is divisible by both 2 and 3,
8	the number formed by the last three digits is divisible by 8,
9	the sum of the digits of *n* is a multiple of 9,
10	the last digit is 0,
11	the difference between the sum of digits in the odd positions and the sum of digits in the even positions is 0 or a multiple of 11.

Factors and Multiples

If *a* divides *b* without any remainder, then *a* is called as one of the factors of *b* and *b* as one of the multiples of *a*.

Prime factorization

The expression of a number as a product of prime factors is called the prime factorization of it.

HCF (Highest common factor) or GCD (Greatest common divisor)

The HCF of two or more than two numbers is the greatest number that divides them without any remainder.

LCM (Least common multiple)

The LCM of two or more than two numbers is least number which is exactly divisible by each one of the given numbers is called their LCM.

Product of two numbers = Product of their HCF and LCM

HCF and LCM of a fraction:

$$\text{HCF} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$$

$$\text{LCM} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$

Number of factors of a number

If $N = p^{\alpha}q^{\beta}\cdot\ldots\cdot r^{\gamma}$

Where, *p, q, ..., r* are prime, while $\alpha, \beta, \ldots, \gamma$ are positive integers.

Then, the number factors of N = $(\alpha + 1)(\beta + 1)\ldots(\gamma + 1)$

Digit in the Units Position

The method to find the digit in the Units position of a number *n* raised to a power *k* is given below in a step-by-step algorithmic style.

If the digit in the units position of *n* is 0, 1, 5 or 6, the required digit is 0, 1, 5 or 6 as the case may be irrespective of the value of *k*.

If the digit in the units position is 2, 3, 7 or 8, divide *k* by 4 and get the remainder, say *r*. If r>0 then the required digit is the units digit of 2^r, 3^r, 7^r or 8^r as the case may be and if r = 0, the required digits will be 6,1,1,6 respectively.

If the digit in the units position is 4 or 9 and *k* is odd, the required units digit is 4 or 9 as the case may be, and if k is even the answer is 6 for 4 and 1 for 9.

Application 2

Determine the digit in the units position of $13^{23} \times 27^{37} \times 19^{45}$.

Solution

13 ends in 3. So, divide the power 23 by 4 to get the remainder 3. 3^3 ends in 7 and hence 13^{23} ends in 7.

27 ends in 7. So, divide the power 37 by 4 to get the remainder 1. 7^1 ends in 7 and hence 27^{37} ends in 7.

19 ends in 9. So, divide the power 45 by 2 to get the remainder 1. 9^1 ends in 9 and hence 19^{45} ends in 9.

Now, 7 × 7 × 9 ends in 1 and so $13^{23} \times 27^{37} \times 19^{45}$ ends in 1. [Answer]

Maximum Power of a Factor in Factorial

The method to find the maximum power of a prime factor, say *p*, in a factorial, say *n*, consists of

- divide *n* by *p* and get the quotient, say q_1.
- divide q_1 by *p* and get the quotient, say q_2.
- divide q_2 by *p* and get the quotient, say q_3.
- continue the above process till the quotient obtained is less than *p*.

The required power is the sum of the successive quotients.

Application 3

What is the maximum power of 7 in 1024!?

Solution

7)1024

7) 146

7) 20

2

Since 2 < 7, the process is stopped and the maximum power of 7 = 146 + 20 + 2 = 168. [Answer]

Irrational Numbers

To simplify an irrational number (or surd) with an irrational denominator, multiply both the numerator and the denominator by the conjugate of the denominator.

SESSION – 1

1. Write 0.0512512... as a fraction in its lowest terms.
   (a) $\frac{512}{9990}$ (b) $\frac{256}{4995}$ (c) $\frac{512}{9900}$ (d) $\frac{128}{2475}$

2. The number 43xy20xy is exactly divisible by 12. How many two digit numbers as xy are possible?
   (a) 4 (b) 6
   (c) 8 (d) Cannot be determined

3. Let N = 5ab42ab. If N is exactly divisible by 180, then the sum of the digits in N is
   (a) 18 (b) 24 (c) 27 (d) 36

4. The HCF and LCM of two numbers are 73 and 2555 respectively. If one number is 365, then the other number is
   (a) 73 (b) 146 (c) 438 (d) 511

5. Which of the following is the largest four digit number that can be added to 5793 so that the sum is divisible by each of 12, 14, 33 and 42?
   (a) 9854 (b) 9924 (c) 9963 (d) 9915

6. Find the greatest number of five digits which when divided by 10, 13, 15 and 26 gives remainders 4, 7, 9 and 20 respectively.
   (a) 99854 (b) 99834 (c) 99844 (d)99914

7. How many pairs of positive integers a, b exist such that HCF (a, b) + LCM (a, b) = 77?
   (a) 10 (b) 8 (c) 7 (d) 6

8. The product of the two positive integers multiplied with the square of its HCF is 240. How many such pairs exist?
   (a) 9 (b) 7 (c) 5 (d) None of these

9. If a two digit number pq has 3 divisors, then how many divisors does the number pqpq have?
   (a) 3 (b) 6 (c) 9 (d) 12

10. Find the number of divisors of 1400 which are not perfect squares.
    (a) 24 (b) 20 (c) 18 (d) 16

11. Find the number of even divisors of 2016.
    (a) 20 (b) 22 (c) 30 (d) 24

12. Find the sum of divisors of 2475.
    (a) 2418 (b) 3627
    (c) 4836 (d) Cannot be determined

13. How many divisors of $2^7x3^3x11^2$ are perfect squares?
    (a) 15 (b) 16 (c) 17 (d) 18

14. The numbers A, B, C and D have 10, 15, 27 and 49 divisors respectively. Which of these could be a perfect cube?
    (a) A and B (b) B and C (c) C and D (d) A and D

15. What is the unit's digit of $127^{127!}$?
    (a) 7 (b) 9 (c) 3 (d) 1

16. Find the last non zero digit of the number 80^{1230}.
    (a) 8 (b) 4 (c) 2 (d) 6

17. What is the unit's digit of $52^{52^{52}}$?
    (a) 2 (b) 4 (c) 8 (d) 6

18. Find the number of trailing zeros in 229!
    (a) 50 (b) 55 (c) 60 (d) 65

19. Find the least number n such that no factorial has n trailing zeros or n+1 trailing zeros.
    (a) 29 (b) 30 (c) 49 (d) 50

20. Find the greatest value of n such that 180^n exactly divides 128!
    (a) 30 (b) 15
    (c) 31 (d) such n does not exist

NUMBER PROPERTIES – 2

Remainders

If you divide a number 'a' by another number 'b' which is not a factor of 'a', two remainders are possible. One remainder is positive and the other is negative.

Example

$\frac{8}{5}=\frac{5+3}{5}$, here the remainder is +3 and it is called the positive remainder.

$\frac{8}{5}=\frac{10-2}{5}$, here the remainder is –2 and it is called the negative remainder.

Useful Properties

1. $R\left[\frac{a+b}{d}\right]=R\left[\frac{a}{d}\right]+R\left[\frac{b}{d}\right]$
2. $R\left[\frac{ab}{d}\right]=R\left[\frac{a}{d}\right]R\left[\frac{b}{d}\right]$
3. $R\left[\frac{a^n}{d}\right]=R\left[\frac{a}{d}\right]^n$

Application 1

What is the remainder when 1973 × 1974 × 1975 is divided by 17?

Solution

$$R\left[\frac{1973\times1974\times1975}{17}\right]=R\left[\frac{1973}{17}\right]\times R\left[\frac{1974}{17}\right]\times R\left[\frac{1975}{17}\right]$$

= 1 × 2 × 3 = 6 (using Property 2)

Application 2

What is the remainder when 3^{41} is divided by 4?

Solution

$R\left[\frac{3^{41}}{4}\right]=R\left[\frac{3}{4}\right]^{41}$ (Using Property 3)

$= (-1)^{41} = -1$

∴The positive remainder is 3 (= 4 – 1).

Application 3

What is the remainder when 20^{84} is divided by 9?

Solution

$R\left[\frac{20^{84}}{9}\right]=R\left[\frac{20}{9}\right]^{84}$ (Using Property 3)

$=R\left[\frac{2^{84}}{9}\right]$

Now we go for the cyclicity of powers of 2 when divided by 9.

$R\left[\frac{2}{9}\right]$ = 2

$R\left[\frac{2^2}{9}\right]$ = 4

$R\left[\frac{2^3}{9}\right]$ = 8

$R\left[\frac{2^4}{9}\right]$ = 7

$R\left[\frac{2^5}{9}\right]$ = 5

$R\left[\frac{2^6}{9}\right]$ = 1

Here the cyclicity is 6. On dividing the exponent by 6, we get the remainder as 0.

So the required remainder is $R\left[\frac{2^0}{9}\right]$ = 1

Cancellation Rule

The remainder when 18 divided by 16 is 2.

We know that, $\frac{18}{16}=\frac{9}{8}$ (cancelling the common factor 2 from the numerator and denominator)

The remainder when 9 is divided by 8 is 1.

By multiplying the cancelled factor 2 with the remainder 1, we will get the actual remainder 2.

Application 4

What is the remainder when 3^{45} is divided by 324?

Solution

$R\left[\frac{3^{45}}{324}\right]=R\left[\frac{3^{45}}{3^4\times4}\right]$

$R\left[\frac{3^{41}}{4}\right]$ = 3 (by Application 2)

So the required remainder is $3 \times 3^4 = 243$

Fermat's Theorem

If 'p' is a prime number and 'a' natural number co-prime to 'p', then $R\left[\frac{a^{p-1}}{p}\right]$ = 1.

Application 5

What is the remainder when 3^{74} is divided by 73?

Solution

73 is a prime number and 3 is co-prime to 73.

By Fermat's theorem, $R\left[\frac{3^{72}}{73}\right]=1$

$R\left[\frac{3^{74}}{73}\right]=R\left[\frac{3^{72}}{73}\right]\times R\left[\frac{3^2}{73}\right]$ = 1 × 9 = 9

Wilson's Theorem

If 'p' is a prime number, then (p – 1)! + 1 is a multiple of p.

In other words, $R\left[\frac{(p-1)!}{p}\right]$ = p – 1, if p is a prime number.

Application 6

What is the remainder when 42! is divided by 43?

Solution

Since 43 is a prime number, by Wilson's theorem $R\left[\frac{42!}{43}\right] = 42$

Other Important Results

1. $a^n + b^n$ is divisible by (a + b) when n is odd.
2. $a^n - b^n$ is divisible by (a + b) when n is even.
3. $a^n - b^n$ is divisible by (a – b) for all the values of n.

Finding last two digits

Numbers ending with 1

Application 7

What is the last two digits of 81^{37}?

Solution

The unit digit is 1. The tens digit is the unit digit of the product of 8 and 7 (**8**1^{37}).

∴ Last two digits of 81^{37} = 61

Numbers ending with 3, 7 and 9

What is the special with the numbers 3, 7 and 9? We get 1 as the unit digit by raising these numbers to some power.

Example: $3^4 = 81$

Application 8

What is the last two digits of 3^{50}?

Solution

$3^{50} = (3^4)^{12} \times 3^2 = 81^{12} \times 09$

∴ Last two digits of $3^{50} = 61 \times 09 = 49$

Numbers ending with 5

Application 9

What is the last two digits of 35^{13}?

Solution

$35^{13} = (35^6)^2 \times 35$

We know that, unit digit of 35^6 is 5. If we raise a number ending with 5 to power 2, we get 25 as last two digits.

∴ Last two digits of $35^{13} = 25 \times 35 = 75$

There are only two possibilities for last two digits in this case: 25 and 75. If the product of unit digit of exponent and tens digit of the number is odd, then the last two digits are 75. Otherwise, it is 25.

In the above example, 3 x 3 = 9 (**3**5^{1}**3**) which is odd. So the last two digits are 75.

Numbers ending with 2

There is only one two digit even number which always ends in itself raised to any power i.e., 76.

And 2 raised to the power of 20 ends in 76. Let's use this in our next example.

Application 10

What is the last two digits of 2^{137}?

Solution

$2^{137} = (2^{20})^6 \times 2^{10} \times 2^7$

∴ Last two digits of $2^{137} = 76 \times 24 \times 28 = 72$

Numbers ending with 4, 6 and 8

Whatever the number is, we are going to consider only the last two digits along with its power to find the last two digits. Any two digit number ending with 4, 6 and 8 can be written as the product of numbers ending with 2 and an odd number or as a power of 2. Using this, the last two digits can be easily found.

Application 11

What is the last two digits of 14^{137}?

Solution

$14^{137} = 2^{137} \times 7^{137}$

Last two digits of 2^{137} = 72 (by application 10)

$7^{137} = (7^4)^{34} \times 7$

Last two digits of $7^{137} = 01 \times 07 = 07$

∴ Last two digits of $14^{137} = 72 \times 7 = 04$

Important tip: Last two digits of 37^2 is same as the last two digits of 13^2 as 37 is 13 less than 50. Also last two digits of 63^2 is same as the last two digits of 13^2 as 63 is 13 more than 50. The reason behind this is the formula of $(a + b)^2$ and $(a - b)^2$. Even 100 can be considered as the base instead of 50, when the numbers are closer to 100.

SESSION – 2

1. A number N gives a remainder of 6 when divided by D and a remainder of 19 when divided by 3D. What will be the remainder when 2N is divided by D?
   (a) 6 (b) 12 (c) 19 (d) 24

2. What is the remainder when $6^{90} - 5^{90}$ is divided by 91?
   (a) 0 (b) 1 (c) 89 (d) 90

3. Find the remainder when $3^{5555} + 5^{3333}$ is divided by 23.
   (a) 18 (b) 7 (c) 0 (d) None of these

4. What is the remainder when $23^{24^{25}}$ is divided by 7?
   (a) 1 (b) 2 (c) 4 (d) 6

5. What is the remainder when 35^{182} is divided by 37?
   (a) 15 (b) 11 (c) 8 (d) 4

6. Find the remainder when the number 3451 + 3452 +... + 3794 is divided by 345.
   (a) 340 (b) 175 (c) 53 (d) None of these

7. How many integers are there from 1 to 500 that leaves a remainder of 2 on division by 5 and a remainder of 5 on division by 7?
(a) 12 (b) 13 (c) 14 (d) 15

8. A number when divided by 33 leaves a remainder 10. The same number when divided by 22 leaves a remainder n. How many values can n take?
(a) 1 (b) 2 (c) 3 (d) 4

9. Find the remainder when 2^{64} is divided by 160.
(a) 36 (b) 66 (c) 96 (d) None of these

10. What is the last two digits in the expression 91 × 92 × 93 × … × 98?
(a) 80 (b) 30 (c) 40 (d) 50

11. Find the last two digits in $31^{77} \times 75^{84}$.
(a) 25 (b) 55 (c) 75 (d) 95

12. Find the last two digits in 59^{27}.
(a) 19 (b) 29 (c) 39 (d) 49

13. Find the last two digits in 78^{35}.
(a) 18 (b) 32 (c) 58 (d) 72

14. What is the last two digits of 16^{26}?
(a) 16 (b) 26 (c) 06 (d) None of these

15. What is the last two digits in the expression $31^{15} - 24^{15}$?
(a) 25 (b) 75 (c) 27 (d) 57

16. How many divisors of 140^5 will have at least one zero at its end?
(a) 225 (b) 300 (c) 350 (d) 425

17. Let K be the largest number with exactly three factors that divide 40! How many factors does K – 1 have?
(a) 16 (b) 24 (c) 32 (d) 40

18. How many factors does 6N have, if N is a number such that 2N has 28 factors and 3N has 30 factors?
(a) 35 (b) 32 (c) 28 (d) None of these

19. What is the unit digit in the expression $38^{38^{38}}$?
(a) 8 (b) 4 (c) 2 (d) 6

20. What is the last non zero digit in 25!?
(a) 9 (b) 4 (c) 2 (d) 1

AVERAGE

The average is a measure of central tendency of a set of numbers.

The average which is also known as the Arithmetic mean of a set of numbers can also be defined as the number by which we can replace each and every member of the set without changing the total of the set of numbers.

For example, the average of 4 numbers 12, 13, 17 and 18 is

$$\frac{12+13+17+18}{4}=\frac{60}{4}=15$$

WEIGHTED AVERAGE

When we have two or more groups whose individual averages are known, then to find the combined average of all the elements of all the groups, we use weighted average.

If we have k groups with averages $A_1, A_2 \ldots A_k$ and having $n_1, n_2, \ldots n_k$ elements, the weighted average

$$=\frac{n_1A_1+n_2A_2+\ldots+n_kA_k}{n_1+n_2+\ldots+n_k}$$

AGES AND AVERAGES

If the average age of a group of persons is *x* years today then after 'n' years, their average age will be (x + n) years. Also 'n' years ago, their average age would have been (x – n) years.

AVERAGE SPEED

If a man covers a distance at *x* km/h and an equal distance at y km/h, then the average speed during the whole journey is $\frac{2xy}{x+2}$ km / h.

Application 1

Find the average of all prime numbers between 30 and 50.

Solution

The prime numbers between 30 and 50 are 31, 37, 41, 43 and 47.

$$\text{Required average} = \frac{31+37+41+43+47}{5} = \frac{199}{5} = 39.8$$

AVERAGE OF THE FIRST 'N' NATURAL NUMBERS

The first 'n' natural numbers are 1, 2, 3, 4, …, n.

$$\text{Average} = \frac{1+2+3+4+\ldots+n}{n} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

AVERAGE OF THE SQUARES OF THE FIRST 'N' NATURAL NUMBERS

$$\text{Average} = \frac{1^2+2^2+3^2+\ldots+n^2}{n} = \frac{n(n+1)(2n+1)}{6n}$$

$$= \frac{(n+1)(2n+1)}{6}$$

MEDIAN

Median is a measure of central tendency which corresponds to the middle value of the set of quantities. If the quantities are arranged in the decreasing or increasing order of magnitude, the middle value in this arrangement is called the median.

The median is calculated as follows.

(i) Arrange the data in ascending or descending order of magnitude. Let the total number of quantities be 'n'.

(ii) If n is odd, then the median is the value of the $\left(\frac{n+1}{2}\right)^{th}$ observation.

(iii) If n is even, the median is the average of the $\frac{n}{2}^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observation.

MODE

Mode is the frequently occurred variable in a set of quantities.

Any given set of quantities can have more than 1 mode also.

Application 2

Find the median and mode of 15, 6, 16, 8, 21, 21, 9, 18, 25.

Solution

Arranging in ascending order: 6, 8, 9, 15, 16, 18, 21, 21, 25.

Since there are 9 items, median is the value of the $\frac{9+1}{2}=5^{th}$ item

$\therefore$ Median = 16

The quantity 21 occurs twice (most number of times).

$\therefore$ Mode = 21

Application 3

Find the average of the scores of the tests taken by Ram given that his scores were 78, 82, 84, 86, 87, 89, 93 and 95.

Solution

$$\text{Average score} = \frac{\text{Total scores in all the tests}}{\text{Number of tests taken}}$$

$$= \frac{78+82+84+86+87+89+93+95}{8}$$

$$= \frac{694}{8} = 86.75$$

Application 4

The average of a batsman after 25 innings was 56 runs per innings. If after the 26^{th} inning, his average increased by 2 runs, then what was his score in the 26^{th} inning?

Solution

Total runs after 25 innings $= 25 \times 56$

Total runs after 26 innings $= 26 \times 58$

$\therefore$ Runs in the 26^{th} inning $= 26 \times 58 - 25 \times 56$

$= 1508 - 1400 = 108$

Application 5

The average marks of a group of 20 students on a test is reduced by 4 when the topper who scored 90 marks is replaced by a new student. How many marks did the new student have?

Solution

Let the initial average be x.

$\Rightarrow$ Initial total = 20 x

New average $= x - 4 \Rightarrow$ New total $= 20(x-4) = 20x - 80$

The reduction of 80 is created by the replacement.

Hence the new student has 80 marks less than 90 or 10 marks.

SESSION – 3

1. The average weight of 5 men is decreased by 3 kg when one of them weighing 150 kg is replaced by another person. Find the weight of the new person.
   (a) 165 kg (b) 135 kg (c) 138 kg (d) 162 kg

2. A train travels 8 km in the first quarter of an hour, 6 km in the second quarter and 40 km in the third quarter. Find the average speed of the train per hour over the entire journey.
   (a) 72 km/h (b) 82 km/h
   (c) 77.33 km/h (d) 78.5 km h

3. The average of the salaries of 17 teachers is Rs.45000. 3 teachers went out and the average is dropped by Rs.2500. What is the sum of the salaries of the 3 teachers who left?
   (a) Rs.173000 (b) Rs.176000
   (c) Rs.170000 (d) Rs.85000

4. The average of 71 results is 48. If the average of the first 59 results is 46 and that of the last 11 is 52, find the 60^{th} result.
   (a) 132 (b) 122 (c) 134 (d) 128

5. The average marks of 3 students A, B, C is 48. When another student D joins the group, the average becomes 46 marks. If another student E who has 3 marks more than D, joins the group, the average of the 4 students B, C, D, E becomes 45 marks. How many marks did A get in the exam?
   (a) 47 (b) 43 (c) 40 (d) 53

6. Find the average of the first 5 multiples of 7.
   (a) 20 (b) 21 (c) 28 (d) 30

7. The average age of a group of men is increased by 5 years when a person aged 18 years is replaced by a new person of age 38 years. Find the number of men in the group.
   (a) 3 (b) 4 (c) 5 (d) 6

8. A person travels three equal distances at a speed of x km/h, y km/h and z km/h respectively. Find the average speed during the whole journey.

(a) $xyz/(xy+yz+zx)$ (b) $\frac{xy+yz+zx}{xyz}$

(c) $3xyz/(xy+yz+zx)$ (d) None of these

9. A school has only four classes that contain 10, 20, 30, 40, students respectively. The pass percentage of these classes are 20%, 30%, 60% and 100% respectively. Find the pass percent of the entire school.
(a) 56% (b) 76% (c) 34% (d) 66%

10. The mode of the set of numbers 1, 3, 6, 9, 3, 6, 3, 12, 15, 3, 9, 1, 9, 6, 9, 3, 6, 3, 12, 3 is
(a) 3 (b) 1 (c) 6 (d) 9 (e) 12

11. Some friends have planned to go to a picture by bus for which the charge for all is Rs.720. If the number of persons increased by 4, then the charge per person will decrease by Rs.6, when the total charges remains the same. Find the number of persons.
(a) 20 (b) 24 (c) 18 (d) 30

Directions for Q12 to Q14: Answer the questions based on the following information.

At the end of a soccer season, every player had a prime number of goals and the average of the 11 players was also a prime number. No player's individual tally was the same as of anyone else's or as the average. Nobody has scored more than 45 goals.

12. What was the average of their goals scored?
(a) 28 (b) 23 (c) 29 (d) 31

13. What was the maximum goals scored by a single person?
(a) 43 (b) 38 (c) 27 (d) 34

14. What was the minimum number of goals scored by a single person?
(a) 6 (b) 4 (c) 5 (d) 9

15. The average weight of 3 men A, B and C is 84 kg. Another man D joins the group and the average now becomes 80 kg. If another man E whose weight is 3 kg more than that of D replaces A, then the average weight of B, C, D and E becomes 79 kg. The weight of A is (in kg)
(a) 85 (b) 80 (c) 75 (d) 72

16. A man whose bowling average is 12.4, takes 5 wickets for 26 runs and thereby decreases his average by 0.4. The number of wickets taken by him before his last match was
(a) 96 (b) 85 (c) 83 (d) 95

17. A sequence of seven consecutive integers is given. The average of the first five integers is n. The average of all the seven integers is
(a) n
(b) n + 1
(c) n + 2
(d) Kn, where K is a function of n

18. The average of 20 numbers is zero. Of them, at the most, how many may be greater than zero?
(a) 20 (b) 11 (c) 10 (d) 19

19. If the average of a set of observations $x_1, x_2 \ldots x_{10}$ is 20, the average of $x_1+4, x_2+8, x_3+12, \ldots x_{10}+40$ is
(a) 34 (b) 42 (c) 38 (d) 40

20. The average salary of male employees in a firm was Rs.5200 and that of females was Rs.4200. The average salary of all the employees was Rs.5000. Find the percentage of male employees.
(a) 80 (b) 60 (c) 40 (d) 20

SESSION – 4

1. In a family of 8 males and a few ladies, the average monthly consumption of grain per head is 10.8 kg. If the average monthly consumption per head be 15 kg in the case of males and 6 kg in the case of females, find the number of females in the family.
(a) 8 (b) 7 (c) 9 (d) 15

2. The average weight of 40 teachers of a school is 80 kg. If however the weight of the principal be included, the average decreases by 1 kg. What is the weight of the principal?
(a) 109 kg (b) 29 kg
(c) 39 kg (d) None of these

3. The captain of a Cricket team of 11 members is 26 years old and the wicket keeper is 3 years older. If the ages of these two are excluded, the average age of the remaining players is one year less than the average age of the whole team. Find out the average age of the team.
(a) 23 years (b) 20 years
(c) 21 years (d) 24 years

4. The average of 50 observations was 36. It was found later that an observation 48 was wrongly taken as 23. Find the corrected new average.
(a) 35.2 (b) 36.1 (c) 36.5 (d) 39.1

5. A library has an average of 510 visitors on Sundays and 240 on other days.
Find the average number of visitors per day in a month of 30 days beginning with a Sunday.
(a) 250 (b) 276 (c) 280 (d) 285

6. Typist A can type a sheet in 6 minutes, typist B in 7 minutes and typist C in 9 minutes. Find the average number of sheets typed per hour per typist for all three types.
(a) $\frac{265}{33}$ (b) $\frac{530}{63}$ (c) $\frac{655}{93}$ (d) $\frac{530}{33}$

7. If the average of 5 positive integers is 40 and the difference between the largest and the smallest of these 5 numbers is 10. Find the maximum value possible for the largest of these 5 integers.
(a) 50 (b) 52 (c) 49 (d) 48

8. The average age of 35 students in a class is 16 years. The average age of 21 students is 14 years. What is the average age of remaining 14 students?
(a) 15 years (b) 17 years
(c) 18 years (d) 19 years

9. The average of 5 consecutive numbers is 'n'. The next 2 numbers are also included, find the resultant average.
(a) Remains the same (b) Increases by 1
(c) Increases by 1.4 (d) Increases by 2

10. A group of 20 friends went to a hotel. 16 of them paid Rs.50 each and the remaining 4 paid Rs.10, Rs.15, Rs.25 and Rs.30 more than the average amount paid by all. Find the total bill.
(a) Rs.1040 (b) Rs.1180 (c) Rs.1100 (d) Rs.1200

11. In a cricket team, the average age of eleven players is 28 years. Out of these, the average ages of three groups of three players each are 25 years, 28 years, and 30 years respectively. If in these groups, the captain and the youngest player are not included and the captain is eleven years older than the youngest player, what is the age of the captain?
(a) 35 (b) 28 (c) 21 (d) 30

12. There are 27 students in a class whose average weight is 36 kg. Among these 27 students, 15 are boys and 12 are girls. The average weight of 15 boys is 6.75 kg more than the average weight of 12 girls. What is the average weight of 12 girls (in kg)?
(a) 41.25 (b) 37.50
(c) 26.65 (d) 32.25

13. Stan was asked to calculate the arithmetic mean of ten positive integers each of which has two digits. By mistake he interchanged the digits of one of these ten numbers. As a result his answer was 1.8 more than what it should have been. The difference between the digits of the number that was wrongly used is
(a) 1 (b) 2
(c) 3 (d) Cannot be determined

14. The average of five different positive numbers is 25. x is the decrease in the average when the smallest number among them is replaced with 0. What can be said about x?
(a) x < 5 (b) x > 5
(c) x = 5 (d) None of these

15. Natural numbers 1 to 25 (both inclusive) are split into 5 groups of 5 numbers each. The medians of these 5 groups are A, B, C, D and E. If the average of these medians is m, what are the smallest and the largest values m can take?
(a) 8, 16 (b) 10, 1 (c) 9, 17 (d) 7, 15

16. Consider a class of 40 students whose average weight is 40 kg. 'm' new students join this class whose average weight is 'n' kg. If it is known that m + n = 50, what is the maximum possible average weight of the class now?
(a) 40.18 kg (b) 40.56 kg
(c) 40.67 kg (d) 40.49 kg

17. Visitors to a show were charged Rs.15.00 each on the first day, Rs.7.50 on the second day and Rs.2.50 on the third day. The total attendance for the three days was in the ratio 2:5:13 respectively. Find the average charge per person for the whole show.
(a) Rs.7 (b) Rs.5 (c) Rs.9 (d) Rs.11

18. There were 35 students in a hostel. If the number of students is increased by 7, the expenses of the mess were increased by Rs.42 per day, while the average expenditure per head diminished by Re.1. The original expenditure of the mess was
(a) Rs.400 (b) Rs.420 (c) Rs.432 (d) Rs.442

19. The average age of 4 players is 18.5 years. If the age of the coach is also included, the average age increases by 20 %. The age of coach is
(a) 28 years (b) 31 years
(c) 34 years (d) 37 years

20. The monthly sales for the first 11 months of the year of a certain salesman were Rs.12,000. But due to the illness during the last month, the average sales for the whole year came down to Rs.11375. Find the value of the sale during the last month.
(a) Rs.4,500 (b) Rs.6,000
(c) Rs.10,000 (d) Rs.8,000

PROGRESSIONS

A set of quantities are said to be in Arithmetic progression, if they increase or decrease by a common difference. It is denoted by AP.

If 'a' is the first term and 'd' is the common difference of an arithmetic progression, it is given by a, a + d, a + 2d, … a + nd, …

The common difference is obtained by subtracting any term of the series from the next term. The r^{th} term of an arithmetic sequence with first term a and common difference is d is given by $t_r = a + (r-1)d$.

For example, 3, 7, 11, 15, 19, … is an arithmetic sequence.

10^{th} term $= t_{10} = 3 + (10-1)4 = 3 + 36 = 39.$

Sum to 'n' terms of an Arithmetic Progression

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

If a is the first term and L is the last term or the n^{th} term, then

$$S_n = \frac{n(a+L)}{2}$$

Arithmetic mean between any 2 quantities

If a, b are any 2 quantities, Arithmetic mean $= \frac{a+b}{2}$

GEOMETRIC PROGRESSIONS

Quantities are said to be in Geometric Progression. When they increase or decrease by a constant factor. The constant factor is called the common ratio. The general Geometric sequence whose first term is 'a' and common ratio is 'r' is given by $a, ar, ar^2, ..., ar^{n-1}$.

Geometric Progression is denoted by G.P. The n^{th} term of a G.P. with first term 'a' and common ratio 'r' $= ar^{n-1}$.

GEOMETRIC MEAN

When 3 quantities a, b, c are in G.P., then

$$\frac{b}{a} = \frac{c}{b} \Rightarrow b^2 = ac \Rightarrow b = \sqrt{ac}$$

$\sqrt{ac}$ is called the Geometric mean between a and c. For example, the Geometric mean of 3 and 12 is $\sqrt{3\times 12} = \sqrt{36} = 6$.

'n' Geometric means between 2 given quantities

Let a, b be the given quantities and $G_1, G_2 ... G_n$ be the Geometric means between 'a' and 'b'.

$\therefore$ $a, G_1, G_2 ..., G_n, b$ are in GP.

$$b = (n+2)^{th} \text{ term} = ar^{n+1}$$

$$\Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

Hence the Geometric means are

$$a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}, a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}, ... a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

Sum to 'n' terms of a Geometric Progression

If 'a' is the first term, 'r' the common ratio and S_n the sum to n terms, then

$$S_n = \frac{a\left(1-r^n\right)}{r} \text{ if } r < 1$$

$$= \frac{a\left(r^n - 1\right)}{r-1} \text{ if } r > 1$$

Sum of an infinite G.P. when r < 1

$$S_n = \frac{a}{1-r}$$

For example, $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ... = \frac{1}{1-\frac{1}{2}} = 2$

SMART Tip

The sum of the term numbers which exhibit equal sums is constant for a given AP.

$S_{12} = S_{18}$ in an AP, then $S_{11} = S_{19}, S_{10} = S_{20} .... S_{30} = S_0 = 0$.

HARMONIC PROGRESSIONS

A set of quantities is said to form Harmonic Progression if the reciprocals of the quantities form an A.P.

In general, if a, b, c, d are in A.P. then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}$ are in Harmonic Progression.

Harmonic mean between two numbers

If a, b are two quantities and then Harmonic mean is H,

$$\frac{1}{H} - \frac{1}{a} = \frac{1}{b} - \frac{1}{H}$$

$$\Rightarrow \frac{2}{H} = \frac{1}{b} - \frac{1}{a}$$

$$\Rightarrow H = \frac{2ab}{a+b}$$

RELATIONSHIPS BETWEEN THE PROGRESSIONS

If A, G, H are the Arithmetic, Geometric and Harmonic means between 2 quantities 'a' and 'b'

$$A = \frac{a+b}{2}, \quad G = \sqrt{ab}, \quad H = \frac{2ab}{a+b}$$

$$AH = G^2$$

Also $A > G > H$

SMART Tip

1. Whenever 3 terms of an A.P. to be assumed, consider the terms as a – d, a, a + d to avoid complicated calculations.
2. In the same way, 3 terms of a G.P. can be assumed as $\frac{a}{r}, a, ar$.

Application 1

Find the sum of all numbers divisible by 6 in between 100 and 400.

Solution

Here $t_1 = 102$, last term = 396 = t_n.

Let 'n' be the number of terms.

$$a+(n-1)d=t_n$$

$$\Rightarrow 102+(n-1)\times 6=396$$

$$\Rightarrow (n-1)\times 6=396-102=294$$

$$\Rightarrow n-1=\frac{294}{6}=49$$

$$\Rightarrow n=50$$

$$\therefore S_{50}=\frac{50(102+396)}{2}=\frac{50\times 498}{2}=12450$$

Application 2

Find t_{18} and S_{18} for the following series 2, 8, 32, ...

Solution

2, 8, 32, ... is a G.P. with first term 2 and common ratio 4.

$$t_{18}=ar^{17}=2\times 4^{17}=2^{35}$$

$$S_{18}=\frac{a(r^n-1)}{r-1}=\frac{2(4^{18}-1)}{4-1}=\frac{2}{3}(4^{18}-1)$$

Application 3

The 50^{th} and 60^{th} terms of an Arithmetic Progression are 100 and 120 respectively. Find the 10^{th} term.

Solution

Let 'a' be the first term and 'd' be the common difference.

$t_{50} = a + 49d = 100$ ... (i)

$t_{60} = a + 59d = 120$ ... (ii)

$(ii) - (i) \rightarrow 10d = 20 \Rightarrow d = 2$

Substituting in (i)

$a + 98 = 100 \Rightarrow a = 2$

$t_{10} = a + 9d = 2 + 9\times 2 = 2 + 18 = 20$

SMART Tip

1. Questions in H.P. can be solved by inverting the terms to form an A.P. and then use the corresponding properties.
2. To solve certain complicated problems, the option based approach is the test. This will end up in saving a lot of time.
3. For simplification of numbers, number properties can be made use of.

Application 4

The sum to infinity of the terms of a G.P. is 120. The sum to infinity of the squares of the terms of the same G.P. is 2880. Find the series.

Solution

Let the first term and the common ratio of the G.P. be 'a' and 'r' respectively.

Sum to infinity $= \frac{a}{1-r} = 120$ ... (i)

The squares of the terms are, $a^2, a^2r^2, a^4r^4, \ldots$

First term $= a^2$ and common ratio $= r^2$.

$\therefore$ Sum of the squares $= \frac{a^2}{1-r^2} = 2880$ ... (ii)

(ii) ÷ (i) $\rightarrow \frac{a}{1+r} = 24$ ... (iii)

(i) ÷ (iii) $\rightarrow \frac{1+r}{1-r} = \frac{120}{24} = 5$

$$\Rightarrow 1+r = 5-5r \Rightarrow 6r = 4 \Rightarrow r = \frac{2}{3}$$

Substituting $r = \frac{2}{3}$ in (iii), $\frac{a}{1+\frac{2}{3}} = 24$

$\Rightarrow 3a = 120 \Rightarrow a = 40$

Hence the series is $40, \frac{80}{3}, \frac{160}{9}, \ldots$

Application 5

If there are six Arithmetic means between 5 and 33, find the common difference.

Solution

If there are six arithmetic means between 5 and 33, 33 is the 8^{th} term of the A.P. with first term = 5.

$\Rightarrow 5 + 7d = 33 \Rightarrow 7d = 28 \Rightarrow d = \frac{28}{7} = 4.$

Application 6

Find the harmonic mean of $\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5}$.

Solution

The terms $=\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5}$

Harmonic mean

$$=\frac{4}{\left(\frac{1}{1/2}+\frac{1}{1/3}+\frac{1}{1/4}+\frac{1}{1/5}\right)}=\frac{4}{2+3+4+5}=\frac{4}{14}=\frac{2}{7}$$

Application 7

If a, b, c are in G.P., then log a, log b, log c are in _____.

Solution

a, b, c are in G.P. $\Rightarrow b^2=ac$

$\Rightarrow 2\log b=\log a+\log c$

$\Rightarrow \log b-\log a=\log c-\log b$

$\Rightarrow$ a, b, c are in A.P.

SESSION – 5

1. How many terms are there in the A.P. 20, 25, 30, ... 130.
   (a) 22 (b) 23 (c) 21 (d) 24

2. The sum of numbers between 100 and 500 which are divisible by 6 is
   (a) 67 (b) 498 (c) 2010 (d) 20100

3. If the first, second and the last terms of an A.P. are a, b, c respectively, find the number of terms.
   (a) $\frac{1}{2}(a+b+c)$ (b) $\frac{1}{2}(b+c-2a)$
   (c) $\frac{b+c-2a}{b-a}$ (d) None of these

4. If in an A.P. the ratio of the sum of m terms to the sum of n terms is $m^2:n^2$, then if a is the first term and d the common difference, then
   (a) a = 2d (b) a = d
   (c) d = 2a (d) None of these

5. If x, 2x + 2, 3x + 3 are in G.P., the fourth term is
   (a) 27 (b) – 27 (c) 13.5 (d) – 13.5

6. The sum of an infinite G.P. is 23 and the sum of the squares of infinite terms is 69. Find the first term.
   (a) $\frac{69}{13}$ (b) $\frac{10}{3}$ (c) $\frac{9}{10}$ (d) $\frac{13}{69}$

7. If α,β are the roots of $x^2-3x+a=0$, and γ,δ are the roots of $x^2-12x+b=0$ and $\alpha,\beta,\gamma,\delta$ form an increasing G.P. then
   (a) a = 3, b = 12 (b) a = 12, b = 3
   (c) a = 2, b = 32 (d) a = 4, b = 16

8. The sum of the first three terms of an increasing G.P. is 13 and their product is 27. Find the sum of the first 5 terms.
   (a) 121 (b) $\frac{40}{3}$ (c) 243 (d) 363

9. If $a^{1/x}=b^{1/y}=c^{1/z}$ and a, b, c are in G.P. then x, y, z are in
   (a) A.P. (b) G.P.
   (c) H.P. (d) None of these

10. The Harmonic mean of 2 numbers is 4 and the Arithmetic and Geometric means satisfy the relation $2A+G^2=27$. Find the numbers.
    (a) 6, 3 (b) 5, 4
    (c) 5, – 2.5 (d) – 3, 1

11. If H is the harmonic mean between P and Q the value of $\frac{H}{P}+\frac{H}{Q}$ is
    (a) 2 (b) PQ(P + Q)
    (c) $\frac{P+Q}{PQ}$ (d) None of these

12. The relationship between a, b, c, d when a, b, c are in A.P. and b, c, d are in H.P. is
    (a) ab = cd (b) ac = bd
    (c) ad = bc (d) None of these

13. The sum of the infinite series of a G.P. is 4 and the sum of the cubes of these terms is 192. Find the common ratio.
    (a) 2 (b) – 2
    (c) $\frac{1}{2}$ (d) None of these

14. If the sum to n terms of an A.P. is $3n^2+5n$, while $t_m=164$, find the value of m.
    (a) 25 (b) 26 (c) 27 (d) 28

15. If the harmonic mean and geometric mean of 2 positive numbers are in the ratio 4:5, then the 2 numbers are in the ratio
    (a) 4:1 (b) 3:1 (c) 2:1 (d) 3:2

16. The harmonic mean between 2 numbers is $\frac{16}{5}$, their A.M. is A and G.M. is G. If $2A+G^2=26$, the numbers are
    (a) 6, 8 (b) 4, 8 (c) 2, 8 (d) 1, 8

17. The sum of the first 4 terms of an A.P. is 28 and the sum of the first 8 terms of the same A.P. is 88. Find the sum of the first 16 terms of the A.P.
(a) 346 (b) 340 (c) 304 (d) 268

18. If x be the first term, y be the n^{th} term and p be the product of n terms of a G.P. Find the value of p^2.
(a) $(xy)^{n-1}$ (b) $(xy)^n$
(c) $(xy)^{1-n}$ (d) $(xy)^{n/2}$

19. If a, b, c are in A.P., b, c, d are in G.P. and c, d, e are in H.P., then a, c, e are in
(a) A.P. (b) G.P.
(c) H.P. (d) None of these

20. If a, b, c are in A.P. and a, c – b, b – a are in G.P. ($a \neq b \neq c$) then a:b:c is
(a) 1:3:5 (b) 1:2:4
(c) 1:2:3 (d) None of these

SESSION – 6

1. a, b, c is a geometric progression (a, b, c are real numbers). If $a + b + c = 26$ and $a^2 + b^2 + c^2 = 364$, find the value of b.
(a) 5 (b) 4 (c) 6 (d) 4.75

2. Let the n^{th} term of AP be defined as t_n, and sum up to 'n' terms be defined as S_n. If $|t_8| = |t_{16}|$ and t_3 is not equal to t_7, what is S_{23}?
(a) 23 ($t_6 - t_8$) (b) 0
(c) 23 t_{11} (d) Cannot be determined

3. The sum of 2n terms of A.P. {1, 5, 9, 13.....} is greater than sum of n terms of A.P. {56, 58, 60.....}. What is the smallest value n can take?
(a) 8 (b) 9 (c) 10 (d) 6

4. Consider a, b, c are in a G.P. such that $|a + b + c| = 15$. The median of these three terms is a and b = 10. If a > c, what is the product of the first 4 terms of this G.P.?
(a) 40000 (b) 80000 (c) 32000 (d) 48000

5. Second term of a GP is 1000 and the common ratio is r where r is a natural number. P_n is the product of n terms of this GP. $P_6 > P_5$ and $P_6 > P_7$, what is the sum of all possible values of n?
(a) 4 (b) 9 (c) 5 (d) 13

6. Among four numbers, first three are in G.P and the last three are in A.P whose common difference is 6. If the first and the last terms are the same, then first number is
(a) 2 (b) 4 (c) 6 (d) 8

7. The set of natural numbers is divided as groups as: (1) (2, 3, 4) (5, 6, 7, 8, 9)..... then the sum of numbers in the n^{th} group is
(a) $(n + 1)^3 - n^3$ (b) $(n + 1)^3 - (n + 2)^3$
(c) $(n - 1)^3 + n^3$ (d) None of these

8. If a, b, c are in G.P and x and y are A.M of a, b and b, c respectively, then a/x + c/y is equal to
(a) 0 (b) 1 (c) 2 (d) 1 /2

9. If the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in A.P, then the common difference is
(a) ±1 (b) ±2 (c) ±3 (d) ±4

10. If there are 'm' A.M's between 1 and 31 and the ratio of 7^{th} and $(m - 1)^{th}$ means is 5:9, the value of m is
(a) 12 (b) 13 (c) 14 (d) 15

11. After knee surgery, your trainer tells you to return to your jogging program slowly. He suggests jogging for 12 minutes each day for the first week. Each week thereafter, he suggests that you increase that time by 6 minutes per day. How many weeks will it be before you are up to jogging 60 minutes per day?
(a) 8 weeks (b) 9 weeks
(c) 7 weeks (d) None of these

12. The sum of the interior angles of a triangle is 180°, of a quadrilateral is 360° and of a pentagon is 540°. Assuming this pattern continues, find the sum of the interior angles of a dodecagon (12 sides).
(a) 3600° (b) 1980° (c) 1800° (d) 3420°

13. A mine worker discovers an ore sample containing 500 mg of radioactive material. It is discovered that the radioactive material has a half life of 1 day. Find the amount of radioactive material remaining in the sample at the beginning of the 7^{th} day.
(a) 15.63 mg (b) 7.815 mg
(c) 31.26 mg (d) 3.9075 mg

14. You visit the Grand Canyon and drop a penny off the edge of a cliff. The distance the penny will fall is 16 feet the first second, 48 feet the next second, 80 feet the third second, and so on in an arithmetic sequence. What is the total distance the object will fall in 6 seconds?
(a) 576 feet (b) 675 feet
(c) 570 feet (d) 729 feet

15. Solve the following English nursery rhyme:
As I was going to St.Ives, I met a man with seven wives. Each wife had seven sacks. Each sack had seven cats. Each cat had seven kits. A man, wives, sacks, cats and kits – how many people did I meet?
(a) 2401 (b) 2400 (c) 2800 (d) 2801

16. A certain city has projected population of 1,86,624 in 10 years from now. If its population increases geometrically at the rate of 20% every two years, what is the population right now?
(a) 90000 (b) 36000
(c) 18000 (d) None of these

17. Hari bought a new car and is slowly breaking it in by increasing his mileage by a constant number of miles each week. How many miles will he have to drive the car in the 11th week if he drove it 280 miles in the third week and 400 miles in the sixth week?
(a) 560 (b) 680 (c) 720 (d) 600

18. Nandhini wants to host a party. She invites 3 friends and told them to invite 3 of their friends. The 3 friends do invite 3 others and asks each to invite 3 more people. The invitation process goes on for 5 generations of invitations. Including herself, how many people can Nandhini expect at her party?
(a) 120 (b) 124 (c) 121 (d) 240

19. In a library there are certain number of books and they are stacked in layers on a shelf. Each layer of books has one less than the layer below it, if there are 20 books on the bottom layer and one on the top layer, how many books are stacked on the shelf?
(a) 210 (b) 120 (c) 420 (d) 105

20. A piece of equipment cost a certain factory Rs.600,000. If it depreciates in value, 15% the first year, 13.5 % the next year, 12% the third year and so on, what will be its value at the end of 10 years, all percentages applying to the original cost?
(a) Rs.2,00,000 (b) Rs.1,05,000
(c) Rs.4,05,000 (d) Rs.6,50,000

PERCENTAGE

Percentages are very important as most of the chapters in Commercial Arithmetic use percentages. This also helps to develop calculation skills. Percent literally means 'for every hundred'. The basic utility of percentage arises from the fact that it is one of the most powerful tools for comparison of numerical data and information. It is also one of the simplest tools for comparison.

Percentage represents a fraction with denominator 100. Percent implies for every 100 and is denoted by the symbol %. To write a fraction or a decimal as a percentage, convert it to an equivalent fraction with denominator 100.

For example if a student scored 30 marks out of 50, the percentage of marks = $\frac{30}{50}\times 100\% = 60\%$

$\frac{1}{5} = \frac{20}{100} = 20\%$

$0.75 = \frac{75}{100} = 75\%$

Suppose a man invests 5% of his income in shares. This means he invests Rs.5 for every Rs.100 of his income in shares.

CONVERSION OF A PERCENTAGE AS A FRACTION

To express x% as a fraction, we have x% = $\frac{x}{100}$

Thus, $20\% = \frac{20}{100} = \frac{1}{5}$, $45\% = \frac{45}{100} = \frac{9}{20}$

CONVERSION OF A FRACTION AS A PERCENTAGE

To express $\frac{a}{b}$ as a percent, we have $\frac{a}{b} = \left(\frac{a}{b}\times 100\right)\%$

Thus, $\frac{1}{4} = \left(\frac{1}{4}\times 100\right)\% = 25\%$, $\frac{3}{8} = \left(\frac{3}{8}\times 100\right)\% = 37.5\%$

Suppose A invests 15% of his income in shares and his income is Rs.50,000, the amount invested in shares = 15% of Rs.50,000

$= \frac{15}{100}\times \text{Rs.}50{,}000 = \text{Rs.}7500$

Example 1

Express $6\frac{3}{4}$ as rate percent.

Solution

$6\frac{3}{4} = \frac{27}{4} = \left(\frac{27}{4}\times 100\right)\% = \left(\frac{2700}{4}\right)\% = 675\%$

Example 2

Evaluate 28% of 450 + 45% of 280.

Solution

28% of 450 = $\frac{28}{100}\times 450 = 126$

45% of 280 = $\frac{45}{100}\times 280 = 126$

$\therefore$ 28% of 450 + 45% of 280 = 126 + 126 = 252

Example 3

88% of 370 + 24% of 210 – x = 118, find x.

Solution

88% of 370 = $\frac{88}{100}\times 370 = 325.6$

$24\%\text{ of }210 = \frac{24}{100}\times 210 = 50.4$

$\therefore 325.6 + 50.4 - x = 118$

$\Rightarrow x = 325.6 + 50.4 - 118 = 376 - 118 = 258$

Example 4

270 candidates appeared for the exam out of which 252 passed. Find the pass percentage.

Solution

$\left(\frac{252}{270}\times 100\right)\% = \frac{280}{3}\% = 93\frac{1}{3}\%$

PERCENTAGE INCREASE AND DECREASE

Percentages are often used to indicate the changes in the quantity.

$$\text{Percentage change} = \frac{\text{Final quantity} - \text{Initial quantity}}{\text{Initial quantity}}$$

For example, if the cost of an article changes from Rs.10 to Rs.12, the percentage change is given by $\frac{12-10}{10}\times 100\% = 20\%$

If a quantity increases by x%, then its value gets multiplied by $\frac{100+x}{100}$.

Similarly if a quantity decreases by x%, then its value gets multiplied by $\frac{100-x}{100}$.

Example 5

The population of a town has increased from 60,000 to 65000. Find the increase percent.

Solution

$$\text{Percentage increase} = \frac{65000-60000}{60000}\times 100\%$$

$$= \frac{5000}{60000}\times 100\% = \frac{25}{3} = 8\frac{1}{3}\%$$

$\therefore$ The increase percent is $8\frac{1}{3}\%$.

Example 6

A man loses 12.5% of his money and after spending 70% of the remainder, he is left with Rs.210. How much he had at first?

Solution

Let the amount he had at first be Rs.x.

$\therefore$ Amount after losing $12\frac{1}{2}\%$ = $\text{Rs.}\frac{100-12.5}{100}\times x$

$= \text{Rs.}\frac{87.5}{100}\times x = \text{Rs.}\frac{7}{8}x$

Amount after spending 70% $= \text{Rs.}\frac{100-70}{100}\times\frac{7}{8}x$

$= \text{Rs.}\frac{3}{10}\times\frac{7}{8}x = \text{Rs.}\frac{21}{80}x$

Now, $\frac{21}{80}x = 210$

$\Rightarrow x = \frac{210\times 80}{21} = \text{Rs.}800$

The following Properties are Helpful to Solve Problems.

(1) Given an amount, if x% of the amount is taken by the first, y% of the remaining is taken by the second and z% of the remaining is taken by the third, the amount left is A, then the given amount

$$= A\times\left(\frac{100}{100-x}\right)\times\left(\frac{100}{100-y}\right)\times\left(\frac{100}{100-z}\right)$$

(2) Given an amount, if x% of the amount is added, then y% of the increased quantity is added and then z% of the increased quantity is added remaining quantity is A, the initial amount is given by

$$A\times\left(\frac{100}{100+x}\right)\times\left(\frac{100}{100+y}\right)\times\left(\frac{100}{100+z}\right)$$

Example 7

After deducting 10% from a certain sum, and then 20% from the remainder and then 10% from the remainder, there is Rs.3240 left. Find the original sum.

Solution

Let the original sum be Rs.x.

$x = 3240\times\frac{100}{90}\times\frac{100}{80}\times\frac{100}{90}$

$= \text{Rs.}\frac{3240\times 1000000}{648000} = \text{Rs.}5000$

REDUCTION IN CONSUMPTION

(1) If the price of a commodity increases by r% then the reduction in consumption without increasing the expenditure is $= \left(\frac{r}{100+r}\times 100\right)\%$

(2) If the price of a commodity decreases by r%, then increase in consumption without decreasing the expenditure $= \left(\frac{r}{100-r}\times 100\right)\%$

Example 8

If the price of petrol is increased by 30%, by how much percent a car owner must reduce his consumption in order to maintain the same budget.

Solution

Decrease in consumption $= \left(\frac{r}{100+r} \times 100\right)\%$

$$= \frac{30}{130} \times 100\% = \frac{30}{13}\% = 23\frac{1}{13}\%$$

Example 9

The price of a commodity decreases by 20%. By what percentage should the quantity be increased so as to keep the revenue constant?

Solution

Increase in quantity $= \frac{20}{100-20} \times 100\% = \frac{20}{80} \times 100\% = 25\%$

Example 10

The number of seats in a cinema hall is increased by 25%. The price of a ticket also is increased by 10%. What is the effect on the revenue collected?

Solution

Since there is increase in seats as well as increase in the price of tickets,

Percentage effect

$$= 25 + 10 - \frac{25 \times (-10)}{100} = 35 + 2.5 \quad = 37.5\%$$

Example 11

The salary of a person was reduced by 10%. By what percent should the reduced salary be raised so as to bring it at par with his original salary?

Solution

Let the original salary be Rs.100.

New salary = Rs.100 – 10% of Rs.100

$$= \text{Rs}(100-10) = \text{Rs}.90$$

$\therefore$ Increase on 90 = 10

$\Rightarrow$ Increase on 100 = $\frac{10}{90} \times 100\% = \frac{100}{9}\% = 11\frac{1}{9}\%$

RESULTS ON POPULATION

If the population of a town is P and it increases at the rate of r% per annum then, (i) Population after 'n' years = $P\left(1+\frac{r}{100}\right)^n$

(ii) Population 'n' years ago $= \frac{P}{\left(1+\frac{r}{100}\right)^n}$

Results on Depreciation

Let the present value of a machine be P.

If it depreciates at the rate of r% per annum then

(i) Value of the machine after n years $= P\left(1-\frac{r}{100}\right)^n$

(ii) Value of the machine n years ago $= \frac{P}{\left(1-\frac{r}{100}\right)^n}$

Example 12

A district has 64000 inhabitants. If the population increases at the rate of $2\frac{1}{2}\%$ per annum, find the number of inhabitants at the end of 3 years.

Solution

Number of inhabitants = 64000

Increase percent = $2\frac{1}{2}\%$

$\therefore$ Number of inhabitants after 3 years

$$= 64000\left(1+\frac{5}{200}\right)^3 = 64000 \times \left(\frac{41}{40}\right)^3 = (41)^3 = 68921$$

Example 13

Depreciation applicable to an equipment is 20%. The value of the equipment 3 years from now will be less by what percent?

Solution

Let the present value be Rs.100.

Value after 3 years = Rs.100 $\left(1-\frac{20}{100}\right)^3$

$$= \text{Rs}.100 \times \left(\frac{4}{5}\right)^3 = \text{Rs}.\frac{6400}{125}$$

$$= \text{Rs}.51.20$$

$\therefore$ Reduction in value = Rs.100 – Rs.51.20 = Rs.48.80

$\therefore$ Reduction percent $= \frac{48.80}{100} \times 100\% = 48.8\%$

Example 14

During 1 year, the population of a town increased by 5% and during the next year, the population decreased by 5%. If the total population is 9975 at the end of the second year, what was the population in the beginning of the first year?

Solution

Population in the beginning of the first year

$$= \frac{9975}{\left(1+\frac{5}{100}\right)\left(1-\frac{5}{100}\right)} = 9975 \times \frac{20}{21} \times \frac{20}{19}$$

$$= 25 \times 20 \times 20 = 10000$$

SMART Tip

Increase and Subsequent Decrease of x%:

- If there is increase of x% and subsequently x% decrease then there is always loss / decrease in the condition.

PERCENTAGE RELATIONSHIP

(1) If first value is r% more than the second value, then the second is $\left(\frac{r}{100+r} \times 100\right)\%$ less than the first value.

(2) If the first value is r% less than the second value, then the second value is $\left(\frac{r}{100-r} \times 100\right)\%$ more than the first value.

Example 15

If A's salary is 25% more than B, find how much percent B's salary is less than that of A.

Solution

Let D's salary be Rs.100 per month.

$\therefore$ A's salary = Rs.125 per month

B's salary is less than A's salary by $\left(\frac{25}{100+25} \times 100\right)\%$

$$= \frac{25}{125} \times 100 = \frac{100}{5} = 20\%$$

SMART Tip

A is x% Less than (or More than) B:

- If A is x% more than B, then B is less than by A $\frac{x}{(100+x)} \times 100$.
- If A is x% less than B, then B is more than by A $\frac{x}{(100-x)} \times 100$.

Example 16

Express 20 as a percentage of 80.

Solution

If one is 80, the other is 20.

$\therefore$ If one is 100, the other is $\frac{20}{80} \times 100 = 25$

$\therefore$ 20 is 25% of 80

Example 17

Express 250 as a percentage of 50.

Solution

Let x% of 50 = 250

$$\Rightarrow \frac{x}{100} \times 50 = 250 = x = \frac{25000}{50} = 500$$

$\therefore$ 250 is 500% of 50

SESSION – 7

1. What is 20% of 25% of $\frac{1}{10}$ of 4000?
   (a) 10 (b) 20 (c) 40
   (d) 60 (e) None of these

2. 50% of a number is 500 more than 75% of the same number, what is 20% of that number?
   (a) 400 (b) 800 (c) 600
   (d) 500 (e) None of these

3. If the numerator of fraction is increased by 150% and the denominator of the fraction is increased by 200%, the resultant fraction is $\frac{5}{21}$. What is the original fraction?
   (a) $\frac{3}{7}$ (b) $\frac{2}{14}$ (c) $\frac{3}{8}$
   (d) $\frac{2}{6}$ (e) None of these

4. Successive discounts of 20% and 15% are equivalent to a single discount of
   (a) 68% (b) 65% (c) 35%
   (d) 32% (e) None of these

5. If 60% of a number is equal to one third of another number, what is the ratio of the first number to the second?
(a) 9:5 (b) 5:9 (c) 6:1
(d) 1:6 (e) None of these

6. The price of an article is raised by 25% and then two successive discounts of 10% each are allowed. Ultimately the price of the article is
(a) increased by 2% (b) remains the same
(c) decreased by 1.25% (d) increased by 1.25%
(e) None of these

7. A test consists of 120 questions carrying 1 mark each, ravi answered 90% of the first 60 questions correctly. What percent of the other 60 questions does he need to answer correctly to score 80% on the entire exam?
(a) 80 (b) 70 (c) 50
(d) 60 (e) None of these

8. Radha spends 40% of her salary on food, 20% on house rent, 10% on entertainment and 10% on conveyance. If her savings at the end of a month are Rs.1500, then her salary per month. (in Rs) is
(a) 8000 (b) 7500 (c) 6000
(d) 10000 (e) None of these

9. 150 % of 500 + ? of 700 = 960
(a) 12% (b) 18% (c) 20%
(d) 30% (e) None of these

10. The income of a company increases 20 % per annum. If its income is Rs.2664000 in the year 2009, what was its income in the year 2007 (in Rs.)?
(a) 2850000 (b) 2121000 (c) 1855000
(d) 2220000 (e) None of these

11. (47% of 2750) – (42% of 2990) = ?
(a) 35.7 (b) 37.7 (c) 36.7
(d) 46.7 (e) None of these

12. The reduction of 25% in the price of salt enabled a purchaser to obtain 4 kg more for Rs.160. The reduced price of salt per kg is
(a) Rs.2 per kg (b) Rs.5 per kg
(c) Rs.4 per kg (d) Rs.10 per kg
(e) None of these

13. What is the percentage is equal to $5\frac{1}{4}$?
(a) 525% (b) 425% (c) 625%
(d) 715% (e) None of these

14. What percent of $\frac{2}{7}$ is $\frac{1}{35}$?
(a) 15% (b) 18% (c) 10%
(d) 12% (e) None of these

15. The greatest of $16\frac{2}{3}\%$, $6\frac{2}{3}\%$, 0.3, 0.01 is
(a) $16\frac{2}{3}\%$ (b) $6\frac{2}{3}\%$ (c) 0.3
(d) 0.01 (e) None of these

16. Length of the rectangle is increased by 30%. To maintain the same area the breadth will have to be decreased by
(a) $23\frac{1}{13}\%$ (b) $76\frac{12}{13}\%$ (c) 30%
(d) 15% (e) None of these

17. Salary of a person is first increased by 20% and then it is decreased by 20%. Change in his salary is
(a) 4% increased
(b) 4% decreased
(c) 8% decreased
(d) neither increased or decreased
(e) None of these

18. A reduction of 20% in the price of rice enables a person to buy 3.5 kg more rice for Rs.385. The original price of rice per kg is
(a) Rs.20 (b) Rs.22.50 (c) Rs.25
(d) Rs.27.50 (e) None of these

19. Difference between two numbers is 1660. If $6\frac{1}{2}\%$ of one number is $8\frac{1}{2}\%$ of the other number. The smaller number is
(a) 7055 (b) 5395 (c) 3735
(d) 2075 (e) None of these

20. A person spends 80% of his income. If his income & expenditure are increased by 40% and 20% respectively then what is the % increase in saving?
(a) 120% (b) 125% (c) 130% (d) 110%

SESSION – 8

1. An ore contains 25% of an alloy that has 90% iron. In the remaining 75% of the ore, there is no iron. How many kilograms of the ore are needed to obtain 45 kg of pure iron?
(a) 200 kg (b) 250 kg
(c) 300 kg (d) 266.66 kg

2. The population of a village is 1,00,000. The increase rate per annum is 10%. Find the population at the starting of the fourth year.
(a) 1,33,100 (b) 1,21,000
(c) 1,33,000 (d) None of these

3. The price of rice falls by 20%. How much rice can be bought now with the money that was sufficient to buy 20 kg of rice previously?
(a) 5 kg (b) 15 kg (c) 25 kg (d) 30 kg

4. Of the adult population in town A, 45% of men and 25% of women are married. What percentage of the total population of adults is married (assume that no man marries more than one woman and vice versa)?
(a) 33.33% (b) 31.1%
(c) 32.14% (d) None of these

5. A manufacturer of detergent cakes increased the price of his product as a result of which there was a decrease of 20% in its sales. If the sales turnover increases by 20% due to this change, what was the percentage increase in the price?
(a) 20% (b) 36% (c) 40% (d) 50%

6. In the university examination last year, Rajesh scored 65% in English and 82% in History. What is the minimum percentage he should score in Sociology, which is out of 50 marks (English and History were for 100 marks each), if he aims at getting 78% overall?
(a) 94% (b) 92% (c) 98% (d) 96%

7. 40% marks are essential to pass an examination. A obtains 10% marks less than the pass marks and B obtains 11.11% marks less than A. What percent less than the sum of A's and B's marks should C obtain to pass the exam?
(a) 40% (b) $41\frac{3}{17}\%$ (c) 28% (d) $41\frac{5}{17}\%$

8. Jack goes to a shop to buy a fastrack watch costing Rs.2,916. The rate of sales tax is 8% and the final value is rounded off to the next higher integer. He tells the shopkeeper to reduce the price of the watch so that he has to pay Rs.2,916 inclusive of sales tax. Find the reduction needed in the price of watch.
(a) Rs.180 (b) Rs.216 (c) Rs.200 (d) Rs.210

9. The population of towns A and B is in the ratio of 1:4. For the next 2 years, the population of A would increase and that of B would decrease by the same percentage every year. After 2 years, their population became equal. What is the percentage change in the population?
(a) 33.33% (b) 66.67%
(c) 25% (d) Not possible

10. The owner of a flower shop follows a particular pattern for his business. During a period of inflation, he raises his price by P% and during a slowdown, he decreases his price by P%. After a year in which there was inflation first, followed by a slowdown, the cost of a red-rose bouquet decreases by Rs.162. After another year, in which there was inflation followed by a slowdown, the cost of this bouquet reduced by a further Rs.147.42. What was the original price of the red-rose bouquet?
(a) Rs.1,500 (b) Rs.1,000
(c) Rs.1,800 (d) Rs.1,111

11. Consider a big cube formed of 64 smaller cubes of which 40 are colored. The smaller cubes are arranged such that the exposure of the colored cubes to the outside is minimized. What is the percentage of the exposed area that is colored?
(a) 62.5% (b) 50% (c) 41.6% (d) 57.1%

12. Mr. X is a computer programmer. He is assigned three jobs for which time allotted is in the ratio of 5:4:2 (jobs need to be done individually). But due to some technical issue, 10% of the time allotted for each job gets wasted. Thereafter, owing to the lack of interest, he invests only 40%, 30%, 20% of the hours of what was actually allotted to do the three jobs individually. Find how much percentage of the total time allotted is the time invested by X?
(a) 38.33% (b) 39.45% (c) 36.66% (d) 32.72%

13. Australia scored a total of x runs in 50 overs. India tied the scores in 20% less overs. If India's average run rate had been 33.33% higher the scores would have been tied 10 overs earlier. Find how many runs were scored by Australia?
(a) 200 (b) 250
(c) 275 (d)Cannot be determined

14. A motorist uses 24% of his fuel in covering the first 20% of his total journey (in city driving conditions). He knows that he has to cover another 25% of his total journey in city driving conditions. What should be the minimum percentage increase (approx.) in the fuel efficiency for non-city driving over the city driving, so that he is just able to cover his entire journey without having to refuel?
(a) 39.2% (b) 43.5% (c) 45.6% (d) 41.2%

15. Two vessels contain equal quantities of 40% alcohol. A changed the concentration of the first vessel to 50% by adding extra quantity of pure alcohol. B changed the concentration of the second vessel to 50% by replacing a certain quantity of the solution with pure alcohol. By what percentage is the quantity of alcohol added by A more than that replaced by B?
(a) 20% (b) 25% (c) 40% (d) 50%

Directions for Q16 to Q20: Read the following and answer the questions that follow.

Five friends - A, B, C, D and E went on a shopping trip. Before shopping, the first person had Rs.400, the second person had 25% less than first person, the third person and the fourth person had 50% less than the first person and the fifth person had 66.66% less than the second person. While shopping they did not lend or borrow from each other. After the shopping was over, it was observed that they were left with Rs.165, Rs.95, Rs.70, Rs.40 and Rs.10, not necessarily in that order. We don't know who is the first or the second or onwards. Following is known about the money they started with, they spent, or they were left with.

- A started with more money than D.
- B spent Rs.15 more than C.
- E started with more money than just one another person of the group.
- A spent the most but did not end with the least.
- C started with 66.66% of the money that B started with.
- D spent the least and ended with more than A and C.
- E spent Rs.35.

16. Who ended with the maximum amount of money?
(a) A (b) E (c) C (d) B

17. How much money did A spend?
(a) Rs.205 (b) Rs.190 (c) Rs.35 (d) Rs.360

18. In ascending order of spending, E would rank at which position?
(a) 1 (b) 2 (c) 3 (d) 5

19. Who ended with Rs.40?
(a) A (b) B (c) C (d) D

20. Which two persons started with the same amount?
(a) A & B (b) A & C (c) C & E (d) D & E

RATIO AND PROPORTIONS

A ratio is a relation between two or more quantities normally of same kind. The ratio a to b is written as a:b and is also expressed as a fraction a/b (b ≠ 0). While considering homogeneous quantities, the ratio is also expressed as percentage.

If two quantities are in the ratio a:b, it means that if the first quantity is ax, then the second quantity is bx. A and B are in the ratio a:b ⇒ as a proportion of the total, A is a/a + b and B is b/a + b.

Ratio is a mere number and to find the ratio of two quantities, they must be expressed in the same units.

If the two ratios are equal, then the four quantities are in proportion. Thus if a:b is equal to c:d, then a, b, c, d are in proportion and the proportion is written as a:b::c:d, where a and d are called the extremes and b and c the means. Thus if a:b::c:d, then ad = bc.

Proportionality: The values of two quantities may not be independent. For example, the volume of a cube is proportional to its side and vice versa. Two mutually dependent quantities are proportional if the ratio of their values remains constant.

If a/b = c/d, then each term of the ratio a/b and c/d is called proportional. a, b, c and d are respectively the first, second, third and fourth proportionals. Here, a and d are known as extremes and b, c are known as means.

Direct Proportion: If two quantities are so related that an increase or decrease in one quantity produces a proportional increase or decrease in the other, then the two quantities are said to be directly proportional to each other, e.g. the weight of solid material is proportional to its volume.

Direction Variation: If A is said to vary directly as B, then as A increases, B also increases but not proportionally. The variation is denoted by $A \propto B$ or $A = kB$, where k is a constant.

For example, the total cost of production is directly related to the number of items produced.

Here, the variation is direct but not proportional.

Inverse Proportion: If two quantities are so related that an increase in one of them produces a proportional decrease in the other or vice versa, then the two quantities are said to be inversely proportional to each other, e.g. the time a train takes to travel a certain distance is inversely proportional to its speed.

Inverse Variation: If A is inversely related to or varies inversely as B increases, A decreases but not proportionally. This relation can be expressed mathematically as $A \propto \frac{1}{B} \rightarrow A = k \times \frac{1}{B}$ where k is constant. Here, the variation is inverse but not proportional.

Application 1

If all the members of a team are either juniors or seniors and if the ratio of the juniors to seniors in the team is 3:5, what percent of the team members are seniors?
(a) 50% (b) 60% (c) 62.5% (d) 65%

Ans: [c]

Solution

Ratio of juniors to seniors = 3:5 (i.e. 5 out of 8 members are seniors)

Seniors as a percent of the team = 5/8 × 100 = 500/8 = 62.5

Percentage of seniors = 62.5%

Application 2

A club had 3 boys and 5 girls. During a membership drive, same number of boys and girls joined the club. How many members does the club have now if the ratio of boys to girls is 3:4?
(a) 12 (b) 14 (c) 16 (d) 15

Ans: [b]

Solution

Number of boys = 3

Number of girls = 5

Let the number of boys and girls who joined during the membership drive be *x* each.

The increased number of boys = 3 + *x*

The increased number of girls = 5 + *x*

$$\frac{3+x}{5+x}=\frac{3}{4}$$

$$\Rightarrow 4(3+x)=3(5+x)$$

$$\Rightarrow 12+4x=15+3x$$

$$\Rightarrow x=3$$

The number of boys now = 3 + 3 = 6 and the number of girls

= 5 + 3 = 8

The club now has 6 + 8 = 14 members

Application 3

If two numbers are in the ratio 6:13 and their least common multiple is 312, what is the sum of the numbers?
(a) 60 (b) 76 (c) 65 (d) 75

Ans: [b]

Solution

Let the two numbers be 6*x* and 13*x*.

LCM of the numbers = 78*x* = 312

$$x=\frac{312}{78}=4$$

The two numbers are 6 × 4 = 24 and 13 × 4 = 52. The sum of the two numbers = 24 + 52 = 76.

Application 4

The sum of the salaries of John and Mary is Rs.12000 per month. John spends 80% of his salary and Mary spends 70% of her salary. If their savings are in the ratio 4:3, what is the salary of John?
(a) Rs.8000 (b) Rs.8500 (c) Rs.6500 (d) Rs.7850

Ans: [a]

Solution

Let John's salary be Rs.X and Mary's salary be Rs.(12000 – X).

John's spending is 80% and his saving is 20% ⇒ X × 20/100 = 0.2X

Mary's spending is 70% and her saving is 30% ⇒ $\frac{30}{100}$ (12000 – X)

The ratio of John's saving to Mary's saving = 4:3

→ 0.2X : 0.3(12000 – X) = 4:3

→ 3(2X) = 4 × 3(12000 – X)

→ 6X = 144000 – 12X

→ 18X = 144000 ⇒ X = 8000

Salary of John = Rs.8000

Application 5

Anand, Krish and Selva have invested a total of Rs.1,20,000 in a business in the ratio 4:5:6. At the end of one year, the profit earned was Rs.12,000. What is the profit share of Krish?
(a) Rs.3000 (b) Rs.4000 (c) Rs.5000 (d) Rs.6000

Ans: [b]

Solution

The investment ratio of Anand : Krish : Selva

4 : 5 : 6

Proportion of Krish = $\frac{5}{15}$

Total Profit = Rs.12,000

∴ Profit share of Krish = $\frac{5}{15}$ × Rs.12000 = Rs.4000

Application 6

Find the number that must be subtracted from the terms of the ratio 13:17 to make it equal to 5:9.
(a) 7 (b) 9 (c) 8 (d) 5

Ans: [c]

Solution

Let the two numbers to be subtracted be *x*.

Then $\frac{13-x}{17-x}=\frac{5}{9}$

→ 119 – 9*x* = 85 – 5*x*

→ *x* = 8

Application 7

The incomes of A and B are in the ratio 7:2 and their expenditures are in the ratio 4:1. If each saves Rs.1000, find their expenditures.
(a) Rs.20000, Rs.5000 (b) Rs.22000, Rs.7000
(c) Rs.18000, Rs.5000 (d) Rs.19000, Rs.6000

Ans: [a]

Solution

Let the incomes of A and B be 7*x* + 2*x* and their expenditures 4*y* & *y* respectively.

∴ 7*x* – 4*y* = 1000 ... (1)

2*x* – *y* = 1000 ... (2)

Multiplying (1) by 2, 14*x* – 8*y* = 2000 ... (3)

and multiplying (2) by 7, 14*x* – 7*y* = 7000 ... (4)

Solving (3) and (4), *y* = 5000

∴ Expenditure of A = 4*y* = Rs.20,000 and that of B = Rs.5,000

SESSION – 9

1. A, B, C and D are four quantities of the same kind such that A:B = 3:4, B:C = 5:6, C:D = 11:9. What is the ratio of A to D?
(a) 1:3 (b) 15:72 (c) 55:72 (d) 15:216

2. The ratio of three numbers is 3:4:5 and the sum of their squares is 1250. The sum of the three numbers is
(a) 60 (b) 90 (c) 30 (d) 50

3. Two numbers are in the ratio 3:5. If each number is increased by 10, the ratio becomes 5:7. The numbers are
(a) 3, 5 (b) 7, 9 (c) 13, 22 (d) 15, 25

4. A, B and C play cricket. The ratio of A's runs to B's runs is 4:3 and the ratio of B's runs to C's is 3:6. What is the ratio of A's runs to C's runs?
(a) 2:1 (b) 2:3 (c) 3:2 (d) None of these

5. Three numbers A, B, C are in the ratio of 12:15:25. If the sum of the numbers is 312, find the ratio between the difference of B and A and the difference of C and B.
(a) 3:7 (b) 10:3 (c) 3:10 (d) None of these

6. The prices of a scooter and a television set are in the ratio 3:2. If the scooter costs Rs.6000 more than the television, then the price of television set is
(a) Rs.18,000 (b) Rs.12,000
(c) Rs.24,000 (d) Rs.28,000

7. Two numbers are in the ratio 4:9. If the larger number is 35 more than the smaller number, then the product of the numbers is
(a) 1764 (b) 1564 (c) 1864 (d) 1664

8. Rs.770 have been divided among A, B, and C such that A receives $2/9^{th}$ of what B and C together received. What is A' share?
(a) Rs.140 (b) Rs.154
(c) Rs.165 (d) Rs.170

9. The ratio of present ages of Sita and Gita is 4:3. If 4 years before, the ratio of their ages was 2:1, the present age of Sita is
(a) 8 years (b) 10 years
(c) 12 years (d) None of these

10. A person earns 15% on an investment but loses 10% on another. If the ratio of the two investments be 3:5, what is the gain or loss on the two investments taken together?
(a) 10% loss (b) $13\frac{1}{8}$% loss
(c) $13\frac{1}{8}$% gain (d) None of these

11. The ratio of milk and water in 108 litre of adulterated milk is 28:3. The amount of water which must be added to make the ratio 3:1 is (in litre)
(a) 14 (b) 3 (c) 4 (d) 10

12. 30 litres of mixture contains milk and water in the ratio 3:5. If 6 litres of this mixture are replaced by 6 litres of milk, the ratio of milk to water in the new mixture will become
(a) 1:2 (b) 1:1 (c) 2:3 (d) 7:3

13. In what ratio must 35% acid be mixed with 50% alcohol to get a mixture of 45% of acid strength?
(a) 1:3 (b) 1:2 (c) 2:3 (d) 3:2

14. One type of liquid contains 25% of milk, the other contains 30% of milk. A can is filled with 6 parts of first liquid and 4 parts of second liquid. Find % of milk in the mixture.
(a) 29% (b) 31% (c) 27% (d) 33%

15. A container contains 40 litres of milk. From this container 4 litres of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by the container?
(a) 29.16 litres (b) 26.45 litres
(c) 24 litres (d) 28.26 litres

16. In a 80 litres mixture of milk and water, they are in the ratio 3:1. If this ratio is to be 4:1, then the quantity of milk to be added is
(a) 15 litres (b) 20 litres
(c) 30 litres (d) 35 litres

17. 500 gm of sugar solution has 60% sugar in it. How much sugar should be added to make it 90% in the solution?
(a) 1500 gm (b) 1800 gm
(c) 1900 gm (d) 1400 gm

18. Three containers P, Q and R have volumes p, q and r respectively; and container P is full of water while the other two are empty. If from container P water is poured into container Q which becomes $1/3^{rd}$ full, and into container R which becomes half full, how much water is left in container P?
(a) p – q/2 – r/3 (b) (6p – 2q – 3r)/6
(c) (p – q – r)/6 (d) (5p – 3q – 2r)/6

19. Two vessels contain spirit of 0.6 and 0.8 concentrations. If three litres from the first vessel and four litres from the second vessel are mixed, what will be the ratio of the spirit and the water in the resultant solution?
(a) 13:7 (b) 5:2 (c) 6:5 (d) 7:17

20. How much water must be added to 60 litres of milk at 1.5 litres for Rs.20 so as to have a mixture worth Rs.10 $\frac{2}{3}$ a litre?
(a) 15 litres (b) 10 litres
(c) 12 litres (d) 18 litres

SESSION – 10

1. The ratio of the number of gents to the number of ladies in a group was 2:3. When 20 more gents joined the group, the ratio was reversed. Find the number of ladies in the group?
(a) 40 (b) 24 (c) 20 (d) 12

2. The cost of a book and the cost of a pen are in the ratio 3:2. If the cost of 10 books and 6 pens is Rs.63, the cost of a book is
(a) Rs.5.50 (b) Rs.6.50 (c) Rs.3.50 (d) Rs.4.50

3. Rs.432 is divided amongst three workers A, B and C such that 8 times A's share is equal to 12 times B's share and the latter in turn is equal to 6 times C's share. How much did A get?
(a) Rs.192 (b) Rs.133 (c) Rs.144 (d) Rs.128

4. A sum of Rs.53 is divided among A, B and C in such a way that A gets Rs.7 more than what B gets and B gets Rs.8 more than what C gets. The ratio of their shares is
(a) 16:9:18 (b) 25:18:10
(c) 18:25:10 (d) 15:8:30

5. Ajay, Aman, Suman and Geeta rented a house and agreed to share the rent as follows:
Ajay:Aman = 8:15, Aman:Suman = 5:8 and Suman:Geeta = 4:5. The part of rent paid by Suman will be
(a) $\frac{24}{77}$ (b) $\frac{12}{55}$ (c) $\frac{13}{66}$ (d) $\frac{17}{77}$

6. The annual income and expenditure of a man and his wife are in the ratios 5:3 and 3:1 respectively. If they decide to save equally and find a balance of Rs.40,000 at the end of the year, their incomes were
(a) Rs.50000, Rs.30000 (b) Rs.60000, Rs.40000
(c) Rs.30000, Rs.20000 (d) None of these

7. If the daily income of A, B and C are in the ratio of 2:5:11 and the income of B is Rs.180 more than that of A, then what is the ratio of the difference of income between A and B to that between B and C?
(a) 2:3 (b) 3:4 (c) 1:2 (d) 5:4

8. The height of a student is of the same proportion as the square of his age (between 5 and 17 years). What will be the height of the student after 7 years, if he is 4 feet tall at the age of 9 years?
(a) $5\frac{1}{3}$ft (b) $4\frac{2}{5}$ft (c) $2\frac{2}{3}$ft (d) $5\frac{2}{3}$ft

9. p, q and r are three positive numbers and $Q = \frac{p+q+r}{2}$. If (Q – p):(Q – q):(Q – r) = 2:5:7, what is the ratio of p, q and r?
(a) 12:5:9 (b) 12:9:5 (c) 12:9:7 (d) 12:7:9

10. A bus started its journey with old passengers to young passengers in the ratio 3:1. At the first stop, 16 passengers got down with old and young passengers in the same ratio, and 6 new young passengers got in. The ratio of old to young passengers then became 2:1. What was the total number of passengers in the bus when it started?
(a) 52 (b) 56 (c) 60 (d) 64

11. There are 90 litres Castrol and 150 litres CRB mobil oil. The price of Castrol is Rs.80 per litre and the price of CRB is Rs.75 per litre. Equal amounts of Castrol and CRB are taken out and then CRB is poured in the vessel of Castrol and Castrol is poured out in the vessel of CRB. Now the rate of both the mixtures is same. What is the amount of mobil oil taken out from each of the vessel?
(a) 55.25 litres (b) 56.25 litres
(c) 50.25 litres (d) None of these

12. Two vessels have equal volumes of pure alcohol and Pepsi. A bartender is mixing the drinks. He takes half the volume of the first vessel containing alcohol and transfers it to the second vessel containing Pepsi. He now transfers ½ of the resultant solution from the second vessel to the first. He repeats the process once more transferring always ½ of the resultant solution to the other vessel. Find the fractional volume of alcohol in the first container?
(a) $\frac{7}{16}$ (b) $\frac{11}{32}$
(c) $\frac{11}{21}$ (d) None of these

13. Three vessels whose capacities are in the ratio 2:3:5 are filled with mixtures of milk and water. The ratio of milk to water in each of the 3 vessels is respectively 4:3, 2:1 and 3:2. What is the ratio of milk to water in the resultant mixture obtained by adding a half of the 1st vessel, 2/3 of the 2nd vessel and 3/5 of the 3rd vessel?
(a) 389:241 (b) 241:389
(c) 240:390 (d) None of these

14. From a tank of petrol , which contains 200 litres of petrol, the seller replaces each time with kerosene when he sells 40 litres of petrol (or mixture). Every time he sells out only 40 litres of petrol (pure or impure). After replacing the petrol with kerosene 4th time, the total amount of kerosene in the mixture is
(a) 81.92 litres (b) 96 litres
(c) 118.08 litres (d) None of these

15. In what ratio must water be mixed with milk to gain $16\frac{2}{3}$% on selling the mixture at cost price?
(a) 1:6 (b) 6:1 (c) 2:3 (d) 4:3

16. A shopkeeper mixes two varieties of pulses to get a mixture of pulses. He uses 1 kg and 4 kg of pulses costing Rs.10 and Rs.20 per kg respectively. What is the cost of the resultant mixture (in Rs.per kg)?
(a) 11 (b) 15 (c) 17 (d) 25 (e) 18

17. Milk contained in a vessel capacity 72 litres is diluted by replacing it with water twice. After the replacement the ratio of milk to water is 25:11. Find the quantity of water added each time.
(a) 24 litres (b) 30 litres (c) 12 litres
(d) 42 litres (e) 10 litres

18. A teacher teaches two different classes having the same number of students. In one class the ratio of the number of students who passed to the number of students who failed is 3:4. In the other class, the same ratio is 4:5. The teacher wants to find the total passing percentage of all her students in both classes. The approximate value of this quantity is
(a) 41% (b) 44% (c) 47% (d) 50%

19. The alloy Cuzi contains copper and zinc in the ratio 3:4. The alloy Zial contains zinc and aluminium in the ratio 5:2. The alloy Alfie contains aluminium and iron in the ratio 1:3. Equal amounts of each alloy are melted together and recast into a new alloy. What is the ratio of the amount of copper to the amount of aluminium in this new alloy?
(a) 1:2 (b) 2:3 (c) 3:4 (d) 4:5

20. Mr.Dayal, a shopkeeper, bought two varieties of orange juice at Rs.50 per litre and Rs.42 per litre respectively. He mixed them in some proportion to get a drink he called 'Oranj-La', which he sold at Rs.54 a litre, thereby making a profit of 20%. How much of the Rs.50 variant of juice is present in 40 litres of Oranj-La?
(a) 15 litres (b) 20 litres (c) 30 litres
(d) 25 litres (e) 32 litres