$$\boxed{\frac{1}{\Delta x_i}\int\limits_{x_{i-\frac12}}^{x_{i+\frac12}} F_i(x)\,dx = \bar{f}_i(t)}$$

?

, где $F_i(x) = \frac{1}{2}\left(1+\gamma_i\,\mathrm{th}\left(\beta\left(\frac{x-x_{i-\frac12}}{\Delta x_i}-\tilde{x}_i\right)\right)\right)$

$$\int\limits_{x_{i-\frac12}}^{x_{i+\frac12}} 1+\gamma_i\,\mathrm{th}\left(\beta\left(\frac{x-x_{i-\frac12}}{\Delta x_i}-\tilde{x}_i\right)\right)dx = 2\Delta x_i\,\bar{f}_i$$

$$\int\limits_{x_{i-\frac12}}^{x_{i+\frac12}} dx = \Delta x_i \qquad \int\limits_{x_{i-\frac12}}^{x_{i+\frac12}} \mathrm{th}\left(\beta\left(\frac{x-x_{i-\frac12}}{\Delta x_i}-\tilde{x}_i\right)\right)dx = \frac{\Delta x_i}{\gamma_i}(2\bar{f}_i-1)$$

$$\int \mathrm{th}\,x\,dx = \int\frac{\mathrm{sh}\,x}{\mathrm{ch}\,x}dx = \int\frac{d\,\mathrm{ch}\,x}{\mathrm{ch}\,x} = \int\frac{dt}{t} = \ln|t|+C =$$

$$= \ln|\mathrm{ch}\,x|+C = \ln\mathrm{ch}\,x + C$$

$$\int\limits_a^b \mathrm{th}\,x\,dx = \ln\mathrm{ch}\,x\Big|_a^b = \ln\mathrm{ch}\,b - \ln\mathrm{ch}\,a =$$

$$= \ln\frac{\mathrm{ch}\,b}{\mathrm{ch}\,a}$$

$$\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} th\left(\beta\left(\frac{x - x_{i-\frac{1}{2}}}{\Delta x_i} - \tilde{x}_i\right)\right) dx =$$

$$= \frac{\Delta x_i}{\beta} \int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} th\left(\beta\left(\frac{x - x_{i-\frac{1}{2}}}{\Delta x_i} - \tilde{x}_i\right)\right) \cdot d\left(\beta \frac{x - x_{i-\frac{1}{2}}}{\Delta x_i} - \beta \tilde{x}_i\right) =$$

$$= \frac{\Delta x_i}{\beta} \ln \frac{ch\left(\beta\left(1 - \tilde{x}_i\right)\right)}{ch\left(\beta\left(-\tilde{x}_i\right)\right)} = \frac{\Delta x_i}{\gamma_i}\left(2\bar{\gamma}_i - 1\right)$$

$$\ln \frac{ch\left(\beta\left(1 - \tilde{x}_i\right)\right)}{ch\, \beta \tilde{x}_i} = \frac{\beta}{\gamma_i}\left(2\bar{\gamma}_i - 1\right) \quad \Big|^{\exp}$$

$$\frac{ch\beta\, ch\beta\tilde{x}_i - sh\beta\, sh\beta\tilde{x}_i}{ch\beta\tilde{x}_i} = \exp\left(\frac{\beta}{\gamma_i}\left(2\bar{\gamma}_i - 1\right)\right)$$

$$ch\beta - sh\beta\, th\beta\tilde{x}_i = \exp\left(\frac{\beta}{\gamma_i}\left(2\bar{\gamma}_i - 1\right)\right)$$

$$th\beta\tilde{x}_i = cth\beta - \frac{1}{sh\beta} \exp\left(\frac{\beta}{\gamma_i}\left(2\bar{\gamma}_i - 1\right)\right)$$

$$\left\{ arcth z = \frac{1}{2} \ln \frac{1+z}{1-z} \right\}$$

$$\hat{x}_i = \frac{1}{2\beta} \ln \underbrace{\left(\frac{1 + \frac{ch\beta - \exp\left(\frac{\beta}{\gamma_i}(2\tilde{r}_i - 1)\right)}{sh\beta}}{1 - \frac{ch\beta - \exp\left(\frac{\beta}{\gamma_i}(2\tilde{r}_i - 1)\right)}{sh\beta}} \right)}_{\text{Рассмотрим}}$$

$$]\ \frac{\beta}{\gamma_i}(2r_i - 1) = \alpha$$

$$\frac{1 + \frac{ch\beta - \exp\alpha}{sh\beta}}{1 - \frac{ch\beta - \exp\alpha}{sh\beta}} = \frac{sh\beta + ch\beta - \exp\alpha}{sh\beta - ch\beta + \exp\alpha} =$$

$$\boxed{\cancel{= \frac{\exp\beta - \exp\alpha}{-\exp-\beta + \exp\alpha}}} = \frac{sh\beta + ch\beta - sh\alpha - ch\alpha}{sh\beta - ch\beta + sh\alpha + ch\alpha} =$$

$$= \frac{2\, sh\frac{\beta-\alpha}{2}\, ch\frac{\beta+\alpha}{2} + 2\, sh\frac{\beta+\alpha}{2}\, sh\frac{\beta-\alpha}{2}}{2\, sh\frac{\beta+\alpha}{2}\, ch\frac{\beta-\alpha}{2} - 2\, sh\frac{\beta+\alpha}{2}\, sh\frac{\beta-\alpha}{2}} \ ⓩ$$

$$]\ \frac{\beta-\alpha}{2} = b \qquad \frac{\beta+\alpha}{2} = a$$

$$ⓩ\ \frac{sh\, b\, ch\, a + sh\, a\, sh\, b}{sh\, a\, ch\, b - sh\, a\, sh\, b} =$$

$$= \frac{sh\, b \exp a}{sh\, a \exp -b} = \frac{(\exp b - \exp -b)\exp a}{(\exp a - \exp -a)\exp -b} =$$

$$= \frac{\exp a \exp b - \exp a \exp -b}{\exp a \exp -b - \exp -a \exp -b} = \frac{\exp a \exp 2b - \exp a}{\exp a - \exp -a} =$$

$$= \frac{\exp 2b - 1}{1 - \exp -2a} = \frac{\exp(\beta - \alpha) - 1}{1 - \exp(-\beta - \alpha)}$$

$$\boxed{\tilde{x}_i} = \frac{1}{2\beta}\ln\left(\frac{\exp\left(\beta - \frac{\beta}{\gamma_i}(2\bar{f}_i - 1)\right) - 1}{1 - \exp\left(-\beta - \frac{\beta}{\gamma_i}(2\bar{f}_i - 1)\right)}\right) =$$

$$\boxed{= \frac{1}{2\beta}\ln\left(\frac{\exp\left(\frac{\beta}{\gamma_i}\left(1 + \gamma_i - 2\bar{f}_i\right)\right) - 1}{1 - \exp\left(\frac{\beta}{\gamma_i}\left(1 - \gamma_i - 2\bar{f}_i\right)\right)}\right)}$$