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Hyperbolic Systems of Conservation Laws and the Mathematical Theory of Shock Waves

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Preface

The mathematical theory of hyperbolic systems of conservation laws and the theory of shock waves presented in these lectures were started by Eberhardt Hopf in 1950, followed in a series of studies by Olga Oleinik, the author, and many others. In 1965, James Glimm introduced a number of strikingly new ideas, the possibilities of which are explored.

In addition to the mathematical work reported here there is a great deal of engineering lore about shock waves; much of that literature up to 1948 is reported in *Supersonic Flow and Shock Waves* by Courant and Friedrichs. Subsequent work, especially in the sixties, relies on a great deal of computation.

A series of lectures, along the lines of these notes, was delivered at a Regional Conference held at the University of California at Los Angeles in September, 1971, arranged by the Conference Board of Mathematical Sciences, and sponsored by the National Science Foundation. The notes themselves are based on lectures delivered at Oregon State University in the summer of 1970, and at Stanford University, summer of 1971. To all these institutions, my thanks, and my thanks also to the Atomic Energy Commission, for its generous support over a number of years of my research on hyperbolic conservation laws. I express my gratitude to Julian Cole and Victor Barcilon, organizers of the regional conference, for bringing together a very stimulating group of people.

New York PETER D. LAX

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