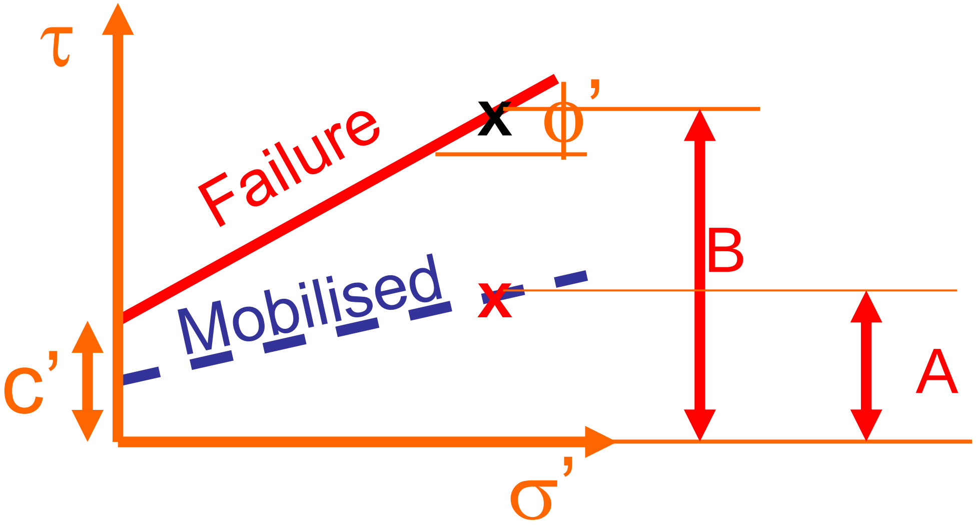
Factor of Safety



Effective Stress Analysis

$$F = B/A$$

Shear strength at failure

$B = c' + \sigma' \tan \phi'$

Mobilised Shear strength

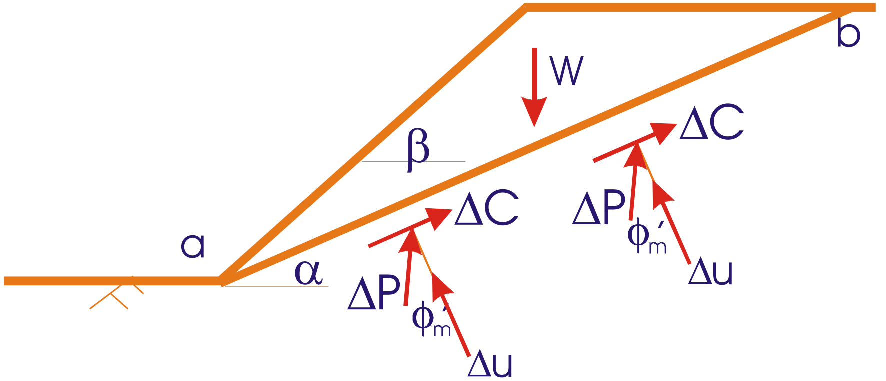
$A = c'_m + \sigma' \tan \phi'_m$

$$\boxed{\begin{array}{l} c'_m = c'/F \\ \tan \phi'_m = \tan \phi'/F \end{array}}$$

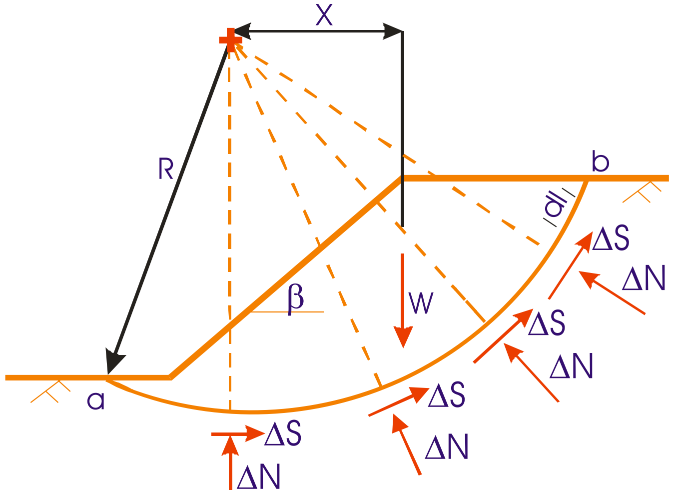
Statically determinate problems

- For planar slip surfaces, the resultant direction of **$\sum \Delta P$** is known

$$\Delta S = \Delta C + \Delta P \sin\phi'_m$$



- For total stress analysis using circular slip surfaces the resultant direction of **$\sum \Delta P$**, passes through the centre of the slip surface



Effective Stress analysis

ΔC forces act circumferentially at a constant radius R

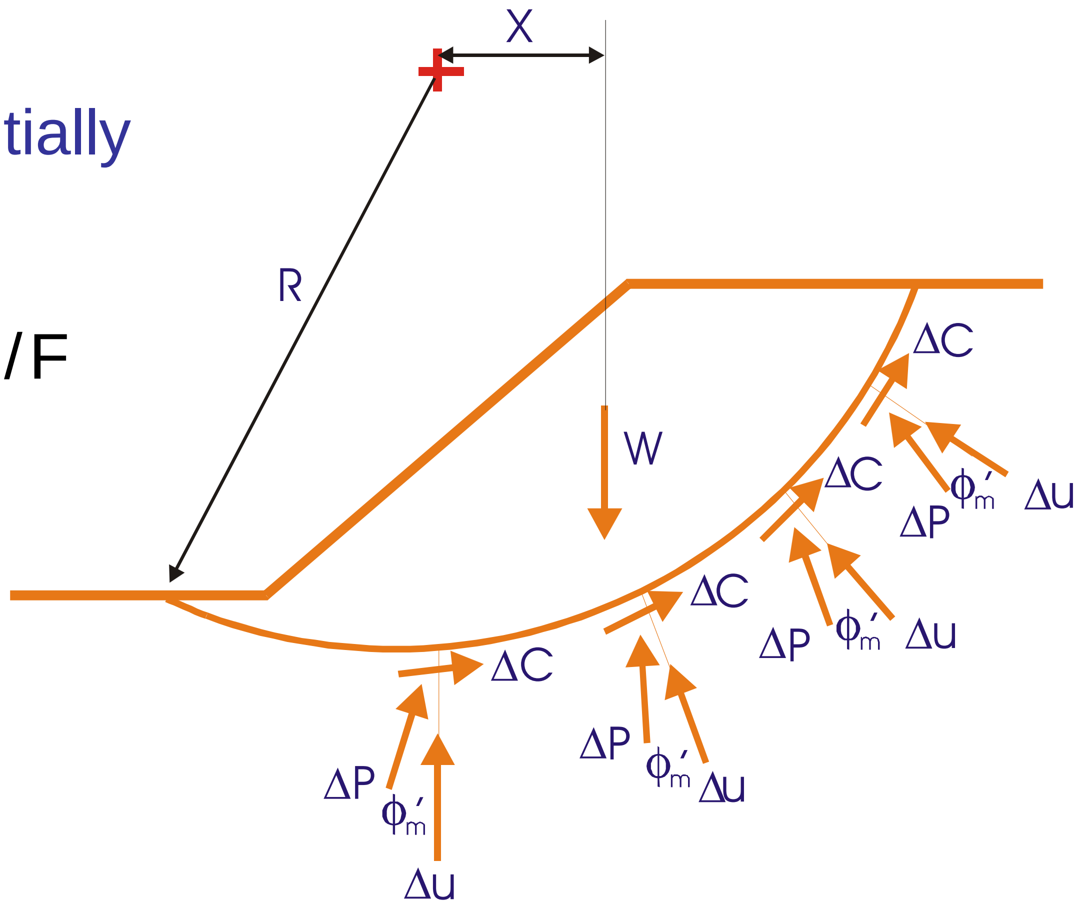
$$M_{\Delta C} = R\sum \Delta C = R\sum c'\,dl/F$$

$$\Delta P = dl\left(\frac{\sigma_n - u}{\cos\phi'_m}\right)$$

$$\phi'_m = \arctan\left(\frac{\tan\phi'}{F}\right)$$

$$\Delta S = \Delta C + \Delta P\sin\phi'_m$$

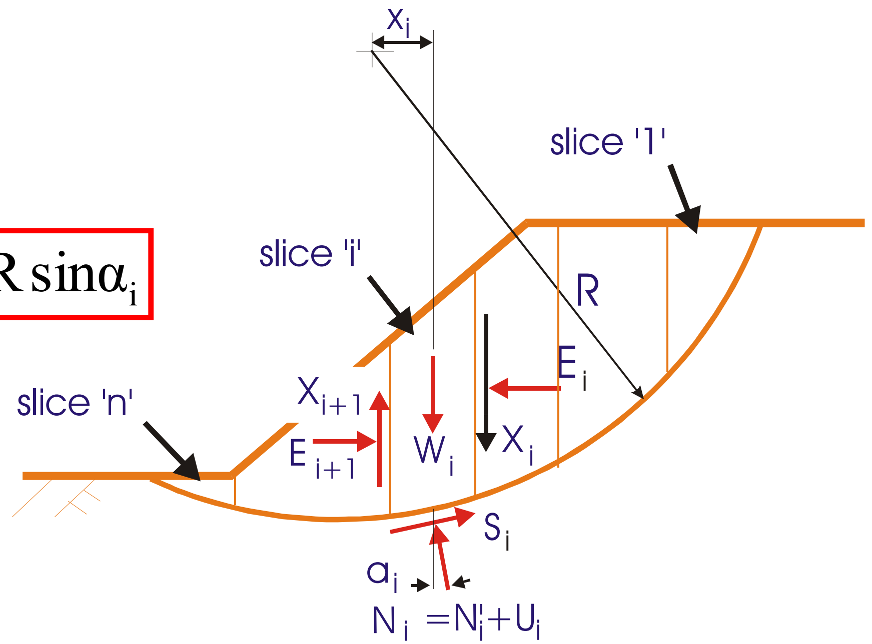
Magnitude, direction and line of action of the resultant force **$\sum\Delta$P** is unknown



The problem is statically <u>indeterminate</u>

Method of Slices

$$x_i = R\sin\alpha_i$$

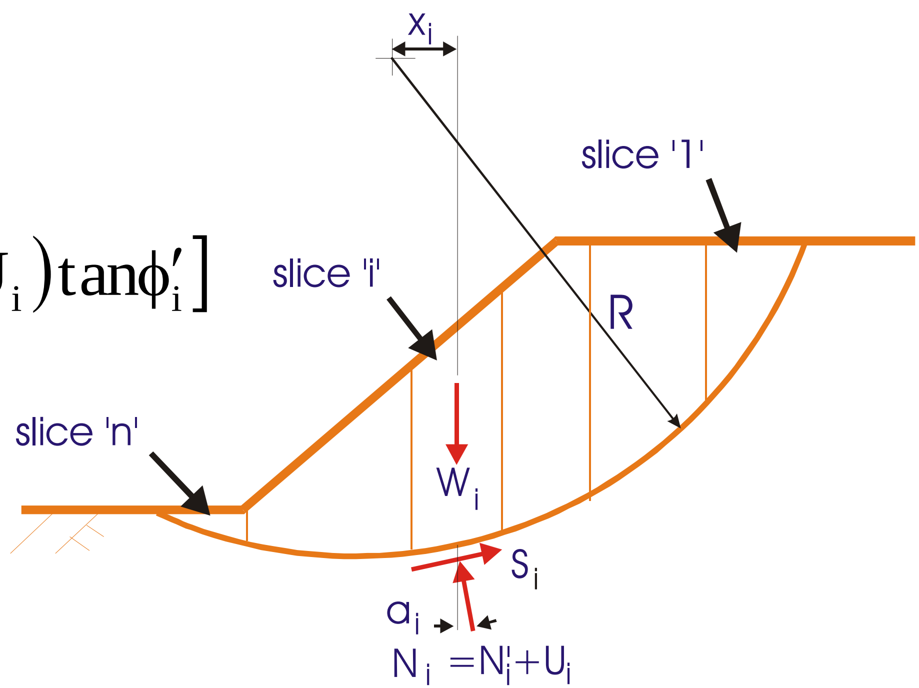


- This is essentially a numerical approach in which the sliding soil mass is divided into 'n' slices
- Equilibrium is then applied to each slice in turn
- The method is formulated so that the factor of safety is calculated for a known slope geometry

Conventional Method of Slices

Resisting moment

$$M_R = R\sum S_i = \frac{R}{F}\sum\left[c_i' l_i + \left(N_i - U_i\right)\tan\phi_i'\right]$$



Disturbing moment

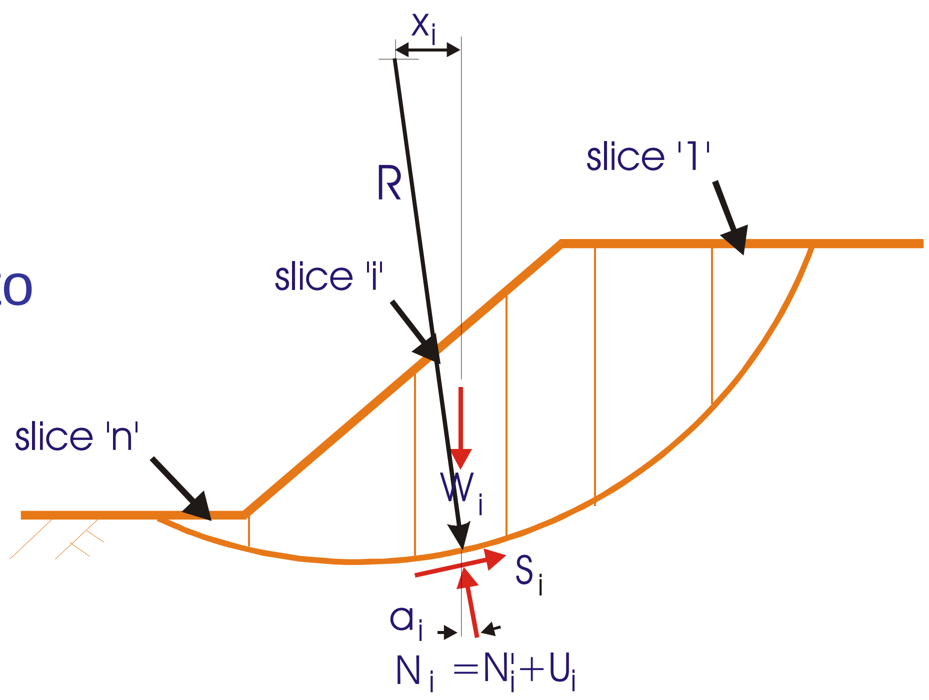
$$M_D = \sum W_i\, x_i$$

Equating the above two equations and rearranging an expression for the FoS is obtained:

$$F = R\,\frac{\sum\left[c_i' l_i + \left(N_i - U_i\right)\tan\phi_i'\right]}{\sum W_i\, x_i}$$

Conventional Method of Slices

$$F = R\,\frac{\sum\left[c_i' l_i + \left(N_i - U_i\right)\tan\phi_i'\right]}{\sum W_i x_i}$$



Resolving forces perpendicular to base of slice

$$N_i = W_i \cos\alpha_i$$

and noting that $x_i = R\sin\alpha_i$

$$\boxed{F = \frac{\sum\left[c_i' l_i + \left(W\cos\alpha_i - U_i\right)\tan\phi_i'\right]}{\sum W\sin\alpha_i}}$$

Bishop Simplified Method of Slices

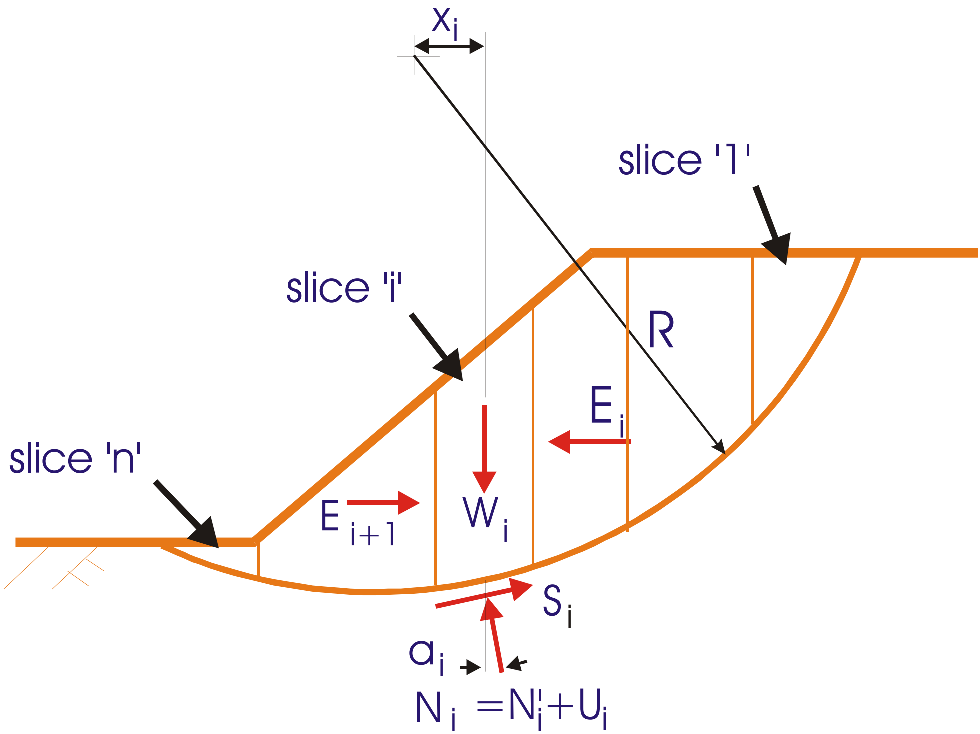
The equation for the factor of safety is derived in a similar way to that for the conventional method

$$F = R\,\frac{\sum\left[c'_i\, l_i + \left(N_i - U_i\right)\tan\phi'_i\right]}{\sum W_i\, x_i}$$

Resolving forces vertically:

$$N_i \cos\alpha + S_i \sin\alpha_i - W_i = 0$$

$$N_i = \frac{W_i - \dfrac{\sin\alpha_i}{F}\left(c'_i\, l_i - U_i \tan\phi'_i\right)}{\cos\alpha_i + \dfrac{\sin\alpha_i \tan\phi'_{ii}}{F}}$$



Bishop Simplified Method of Slices

$$F = \frac{1}{\sum W_i \sin\alpha_i} \sum \frac{c_i' \, l_i \cos\alpha_i + \left(W_i - U_i \cos\alpha_i\right)\tan\phi_i'}{\cos\alpha_i + \dfrac{\sin\alpha_i \tan\phi_i'}{F}}$$

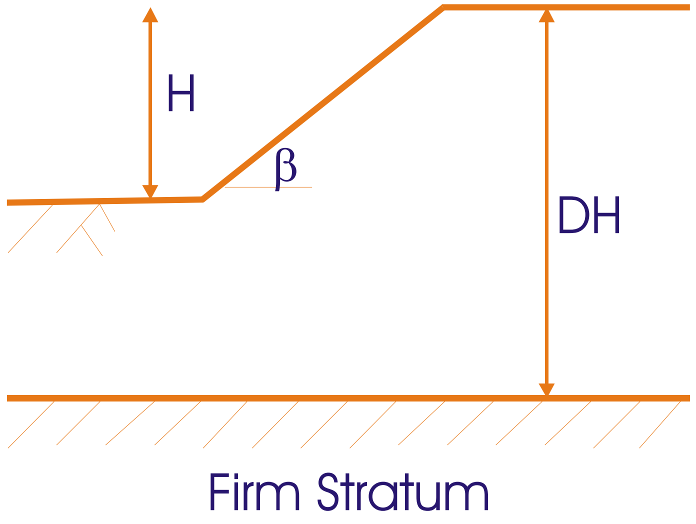
- An initial value of F is assumed and used to evaluate the right hand side of the above equation and thus determine a new value of F
- The procedure is then repeated with this new value of F
- This continues until successive changes in the value of F are small.

Stability Charts for undrained analyses

Taylor (1948)- based on Circular Arc method

- Su is constant with depth
- There are no tension cracks
- No water pressures act on the slope

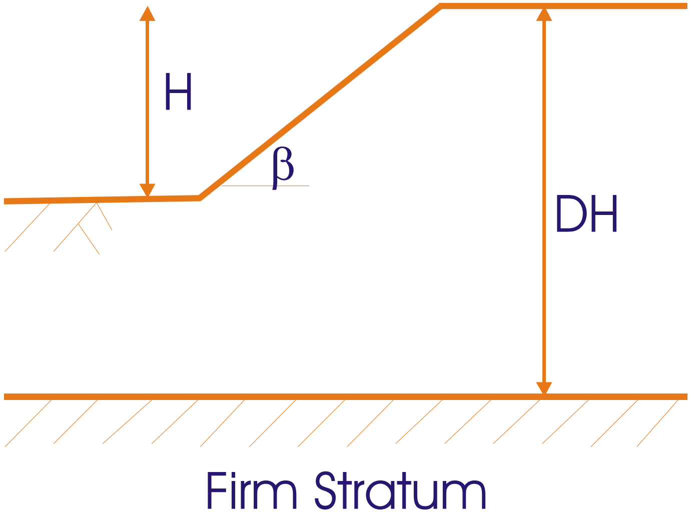
D: ratio of the depth of the homogenous layer to the height of the slope, H



Stability Charts for undrained analyses

Independent variables:

- Height H
- Slope angle β
- Mobilised strength Su/F
- Unit weight γ
- Depth factor D



Dimensionless Stability number

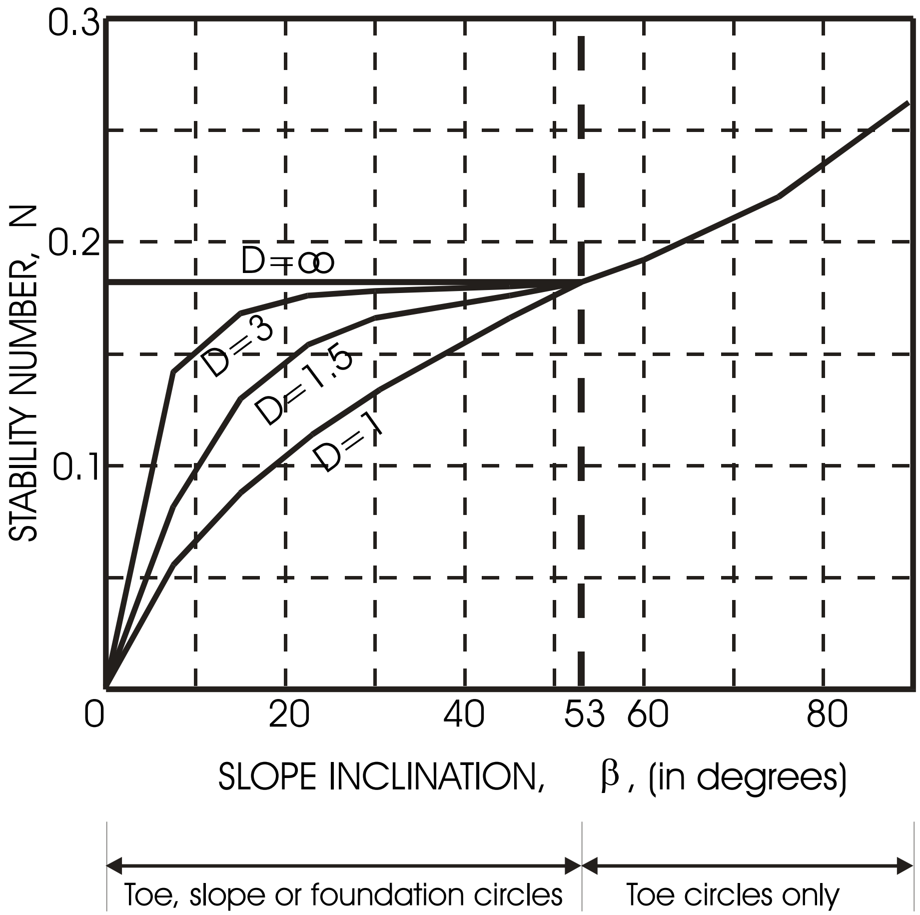
$$N = \frac{S_u}{F \gamma H} = f(\beta, D)$$

Stability Charts for undrained analyses

$\beta > 53^o$, failure occurs along a toe circle

$\beta < 53^o$, the type of failure depends on the value of the depth factor D

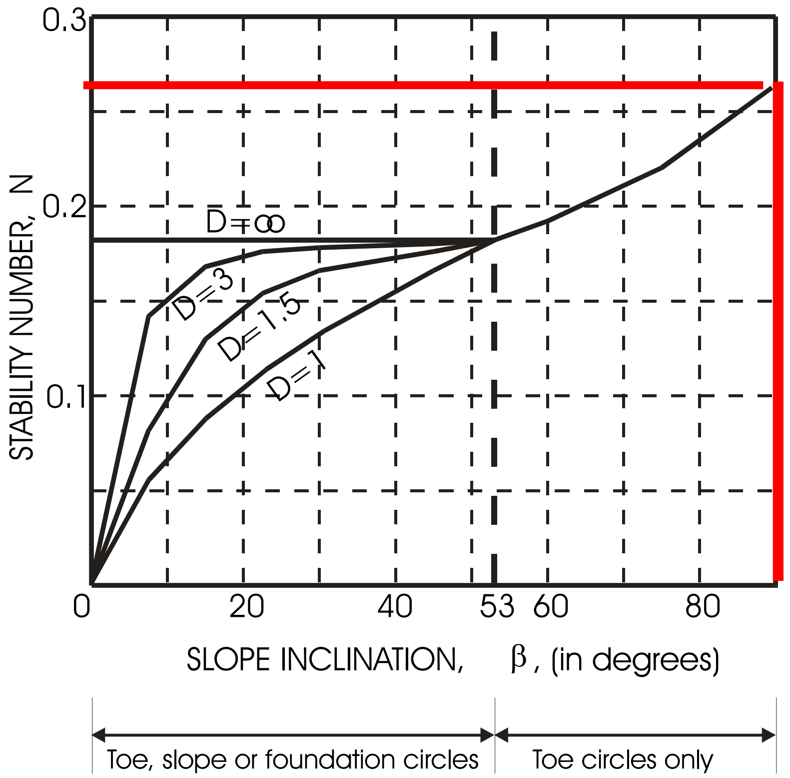
$$N = \frac{S_u}{F \gamma H}$$



Stability Charts for undrained analyses

Vertical cut:
N=0.261, H=3.83 Su/γ
This is also an upper bound solution
A value of H=4 Su/γ is obtained using a planar slip surface

Tension cracks may be considered by artificially reducing the value of Su by 10-15%



Stability Charts for drained analyses

- In all analysis presented so far it is assumed that the distribution of pore water pressure in the slope is known
- It is computationally convenient to assume that the pore pressures are linearly related to the total overburden stress:

$$u = r_u.\gamma.z$$

Where z is the vertical depth below the ground surface and r_u is a constant for the whole slope

Stability Charts for drained analyses

- For "simple" slopes where:
  - soil conditions are uniform
  - slope angle is constant
  - there are no tension cracks

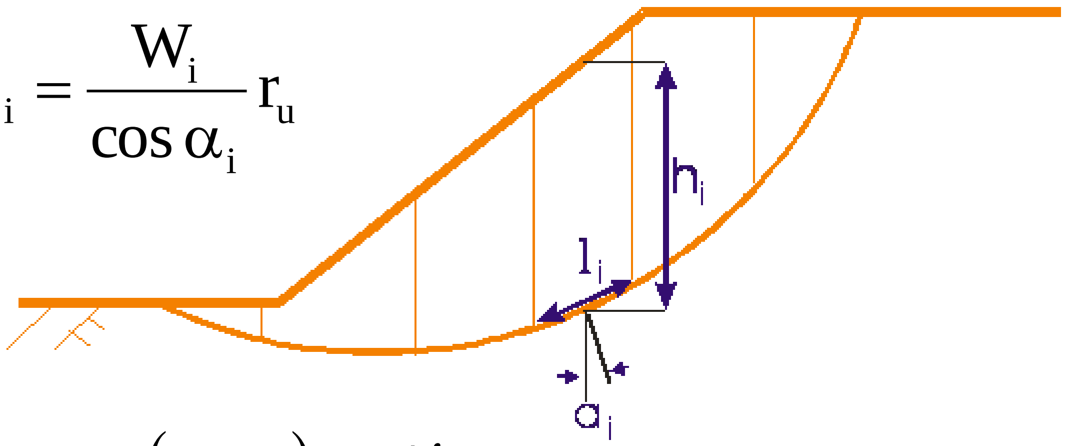
Parametric studies based on the method of slices have been performed and the results presented in terms of charts and graphs

Stability Charts for drained analyses

Bishop Simplified Method of Slices:

$$F = \frac{1}{\sum W_i \sin\alpha_i} \sum \frac{c_i' \, l_i \cos\alpha_i + \left(W_i - U_i \cos\alpha_i\right)\tan\phi_i'}{\cos\alpha_i + \dfrac{\sin\alpha_i \tan\phi_i'}{F}}$$

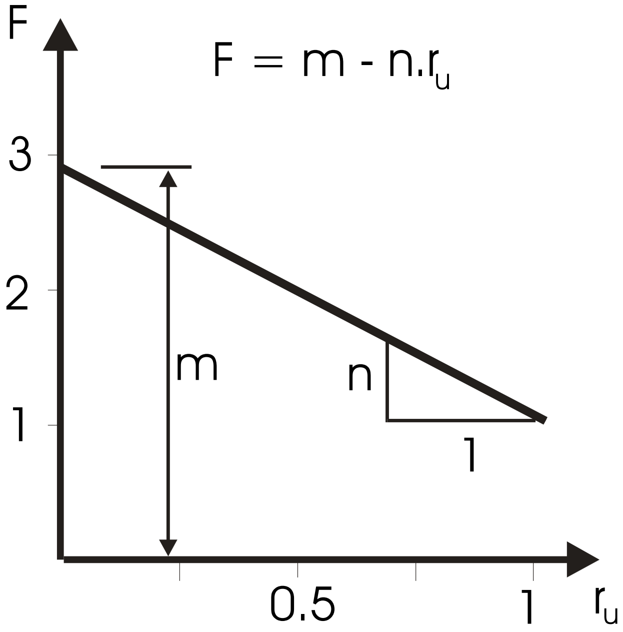
$$\left.\begin{aligned} U_i &= u_i l_i = l_i \, r_u \, \gamma \, h_i \\ W_i &= \gamma \, h_i \, l_i \cos\alpha_i \end{aligned}\right\} \qquad U_i = \frac{W_i}{\cos\alpha_i} r_u$$



$$F = \frac{1}{\sum W_i \sin\alpha_i} \sum \frac{c_i' \, l_i \cos\alpha_i + W_i \left(1 - r_u\right)\tan\phi_i'}{\cos\alpha_i + \dfrac{\sin\alpha_i \tan\phi_i'}{F}}$$

Stability Charts for drained analyses

$$F = \frac{1}{\sum W_i \sin\alpha_i} \sum \frac{c_i' \, l_i \cos\alpha_i + W_i \left(1 - r_u\right) \tan\phi_i'}{\cos\alpha_i + \dfrac{\sin\alpha_i \tan\phi_i'}{F}}$$



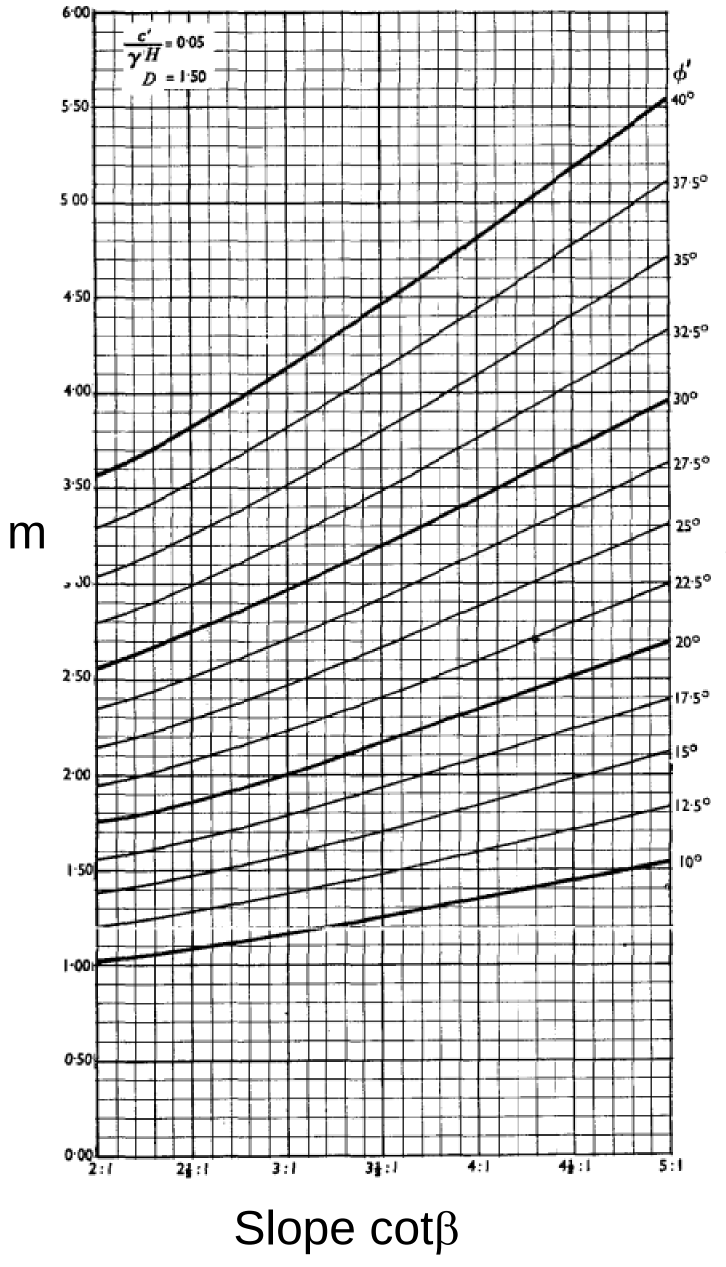
$$\boxed{F = m - n\, r_u}$$

where m and n are given in terms of ϕ’, D, β and $c'/(\gamma H)$

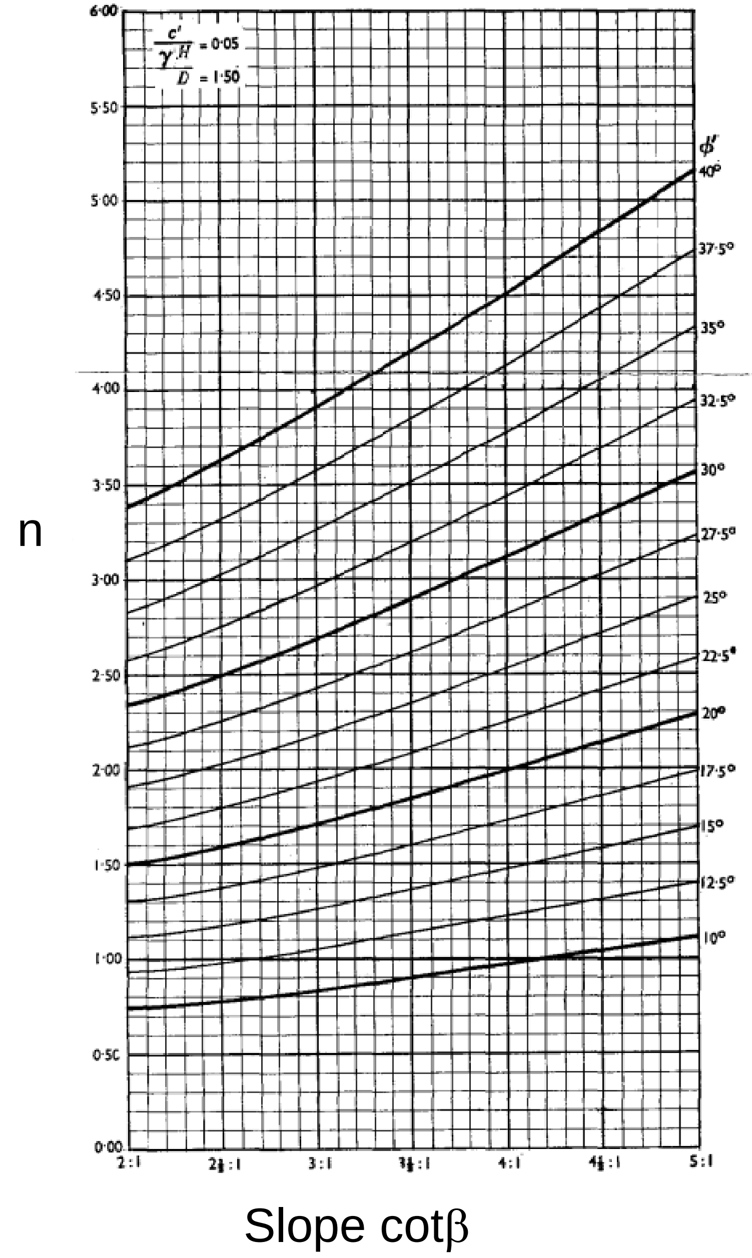
$$F = m - n\, r_u$$

Stability coefficients m and n for $\frac{c'}{\gamma H} = 0{\cdot}05$ and $D = 1{\cdot}25$

ϕ'	Slope 2 : 1		Slope 3 : 1		Slope 4 : 1		Slope 5 : 1	
	m	n	m	n	m	n	m	n
10·0	0·919	0·633	1·119	0·766	1·344	0·886	1·594	1·042
12·5	1·065	0·792	1·294	0·941	1·563	1·112	1·850	1·300
15·0	1·211	0·950	1·471	1·119	1·782	1·338	2·109	1·562
17·5	1·359	1·108	1·650	1·303	2·004	1·567	2·373	1·831
20·0	1·509	1·266	1·834	1·493	2·230	1·799	2·643	2·107
22·5	1·663	1·428	2·024	1·690	2·463	2·038	2·921	2·392
25·0	1·822	1·595	2·222	1·897	2·705	2·287	3·211	2·690
27·5	1·988	1·769	2·428	2·113	2·957	2·546	3·513	2·999
30·0	2·161	1·950	2·645	2·342	3·221	2·819	3·829	3·324
32·5	2·343	2·141	2·873	2·583	3·500	3·107	4·161	3·665
35·0	2·535	2·344	3·114	2·839	3·795	3·413	4·511	4·025
37·5	2·738	2·560	3·370	3·111	4·109	3·740	4·881	4·405
40·0	2·953	2·791	3·642	3·400	4·442	4·090	5·273	4·806



$\frac{c'}{\gamma H} = 0.05$
$D = 1.50$
ϕ'
40°
37.5°
35°
32.5°
30°
27.5°
25°
22.5°
20°
17.5°
15°
12.5°
10°
6.00
5.50
5.00
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
2:1
2½:1
3:1
3½:1
4:1
4½:1
5:1
m
Slope cotβ

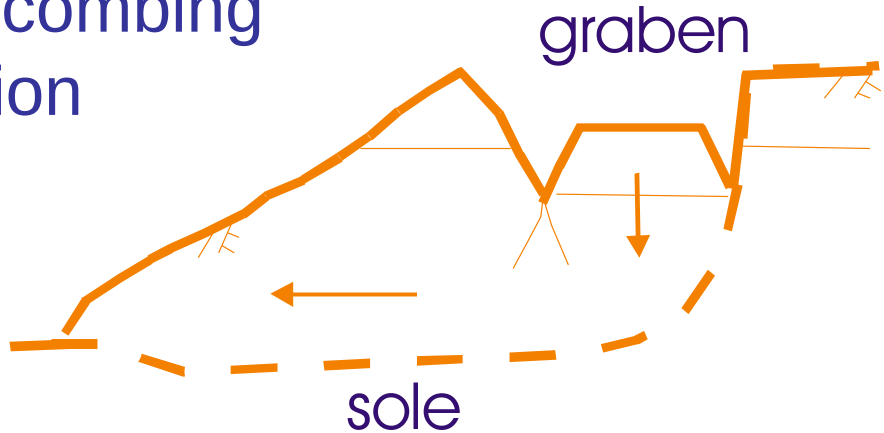


$\frac{c'}{\gamma H} = 0.05$
$D = 1.50$
ϕ'
40°
37.5°
35°
32.5°
30°
27.5°
25°
22.5°
20°
17.5°
15°
12.5°
10°
6.00
5.50
5.00
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
2:1
2½:1
3:1
3½:1
4:1
4½:1
5:1
n
Slope cotβ

Compound Movements

Compound movements

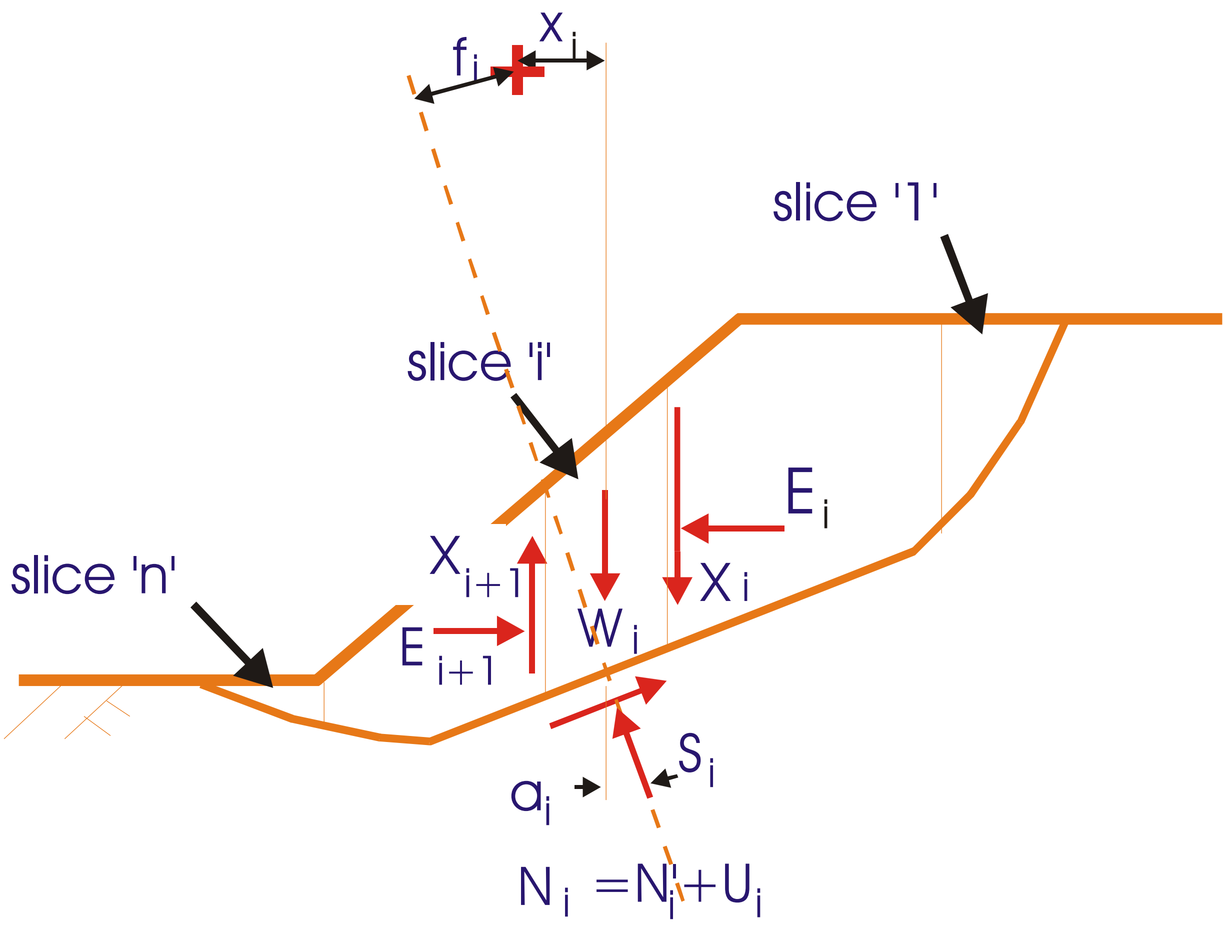
Characterise Compound Slides combing *translational* and *rotational* motion



Applications:

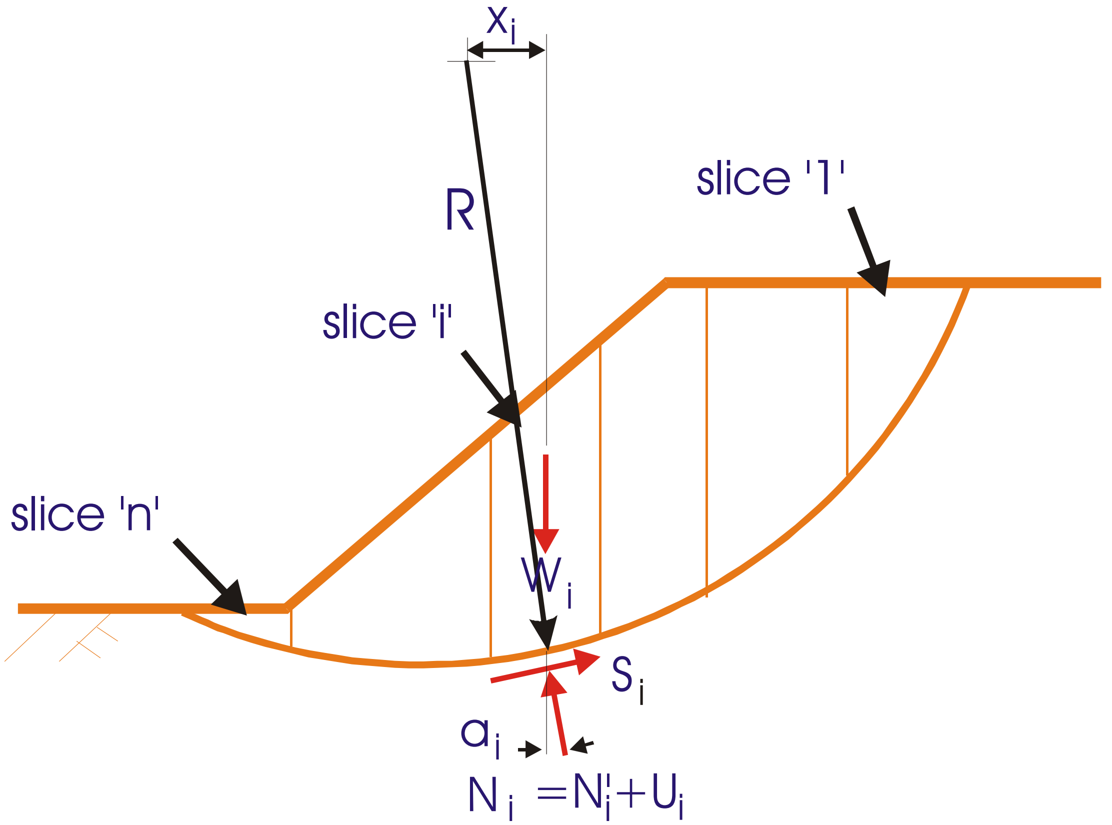
- Non-homogeneous soil conditions
- The slip surface is often quite complex following zones or layers of relatively weak soil or weak interfaces between soil and other materials (i.e. Geo-synthetics).

Method of Slices for Compound movements



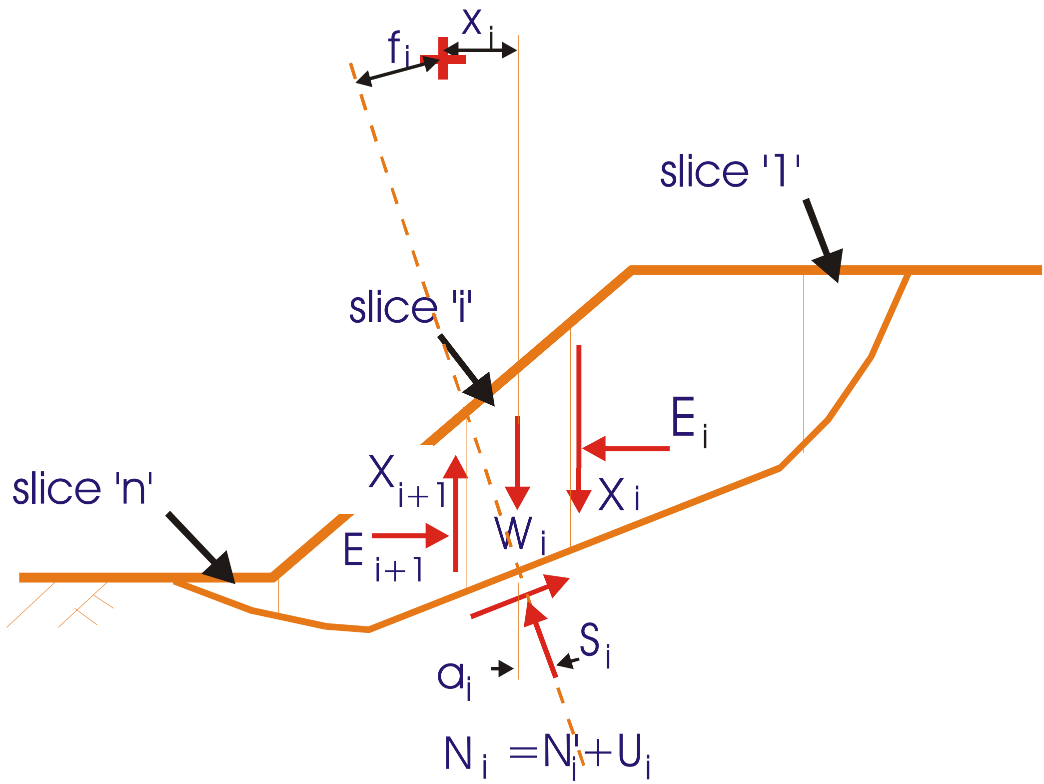
For purely rotational movements

$$\boxed{x_i = R\sin\alpha_i}$$



Method of Slices for compound movements

- The lever arm, f_i, is now a <u>variable</u>



Method of Slices

- Force Equilibrium (only) Procedures
- Moment and Force (partly) Equilibrium Procedures
- Complete Equilibrium (rigorous??) Procedures

- Conventional Method
- Bishop's Simplified
- Janbu's Simplified
- Morgestern & Price
- Spenser's
- ….many others!

Available tools:

- Hand calculations
- Stability graphs
- Spreadsheets
- LE computer programs

Conventional Method

Rotational Movements:

- Moment equilibrium
- Force equilibrium perpendicularly to the slip surface

Non-Rotational Movements:

- Force equilibrium parallel and perpendicularly to the slip surface

$$\boxed{F = \frac{\sum\left[c'_i\, l_i + \left(W\cos\alpha_i - U_i\right)\tan\phi'_i\right]}{\sum W\sin\alpha_i}}$$

Janbu's Simplified Method

<u>Assumptions</u>: Number

Neglects inter-slice shear forces (i.e. X=0):

•Magnitude of inter-slice force, X (X=0) n-1

The normal force, N acts through the midpoint of the base of the slice:

•Point of application (i.e. line of action) of N n

Total number of assumptions: ∑= 2n-1

<u>Procedure</u>:

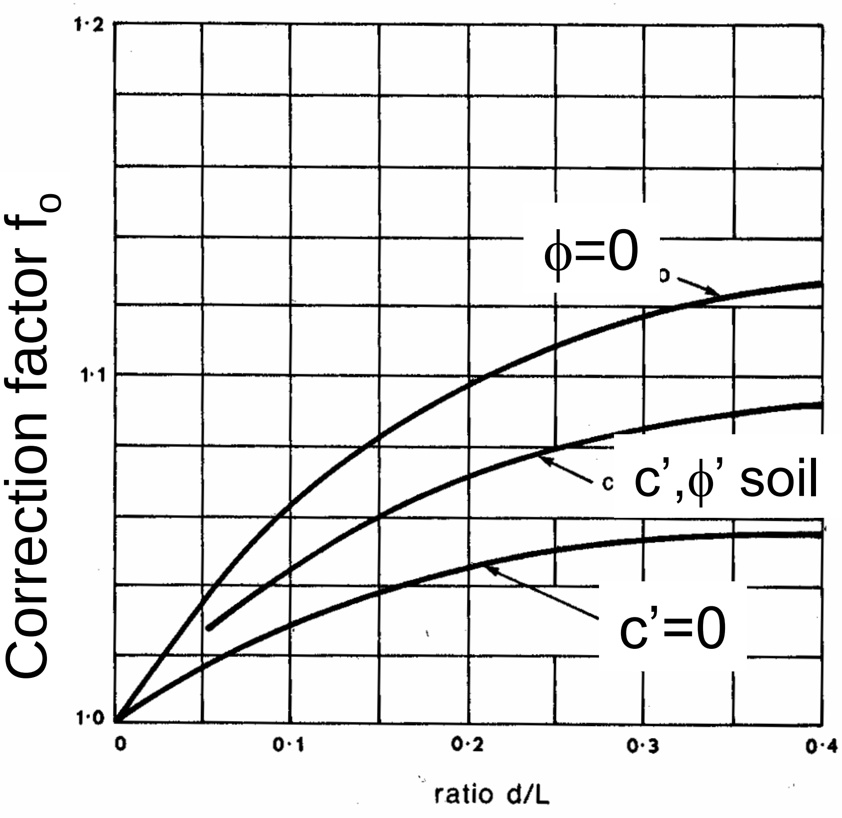
- Horizontal and vertical equilibrium
- Moment equilibrium is not considered

$$F = \frac{1}{\sum W \tan\alpha} \sum \left\{ c_i' l_i + \left(W - U_i \right) \tan\phi_i' \right\} \left\{ \frac{\sec^2\alpha_i}{1 + \tan\alpha_i \dfrac{\tan\phi_i'}{F}} \right\}$$

Janbu's Simplified Method

$$F = \frac{1}{\sum W \tan\alpha} \sum \left\{ c_i' l_i + (W - U_i) \tan\phi_i' \right\} \left\{ \frac{\sec^2\alpha_i}{1 + \tan\alpha_i \dfrac{\tan\phi_i'}{F}} \right\}$$

- In most cases underestimates the FoS
- Empirical correction factor is applied (based on 30-40 cases) to the *converged* FoS



Janbu's Generalised Procedure of Slices

<u>Assumptions</u>: Number

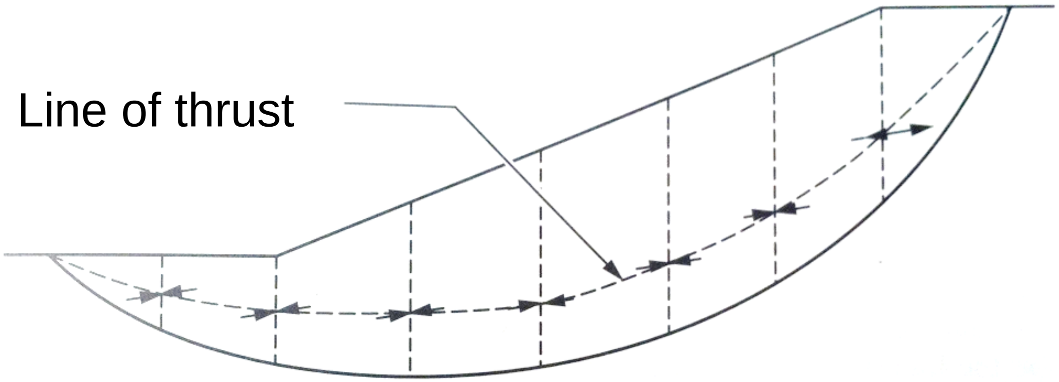
The point of application of the horizontal interslice force (E) is assumed :

- A trust line is specified across the slope n-1

The normal force, N acts through the midpoint of the base of the slice:

- Point of application (i.e. line of action) of N n

Total number of assumptions: $\sum = 2n-1$

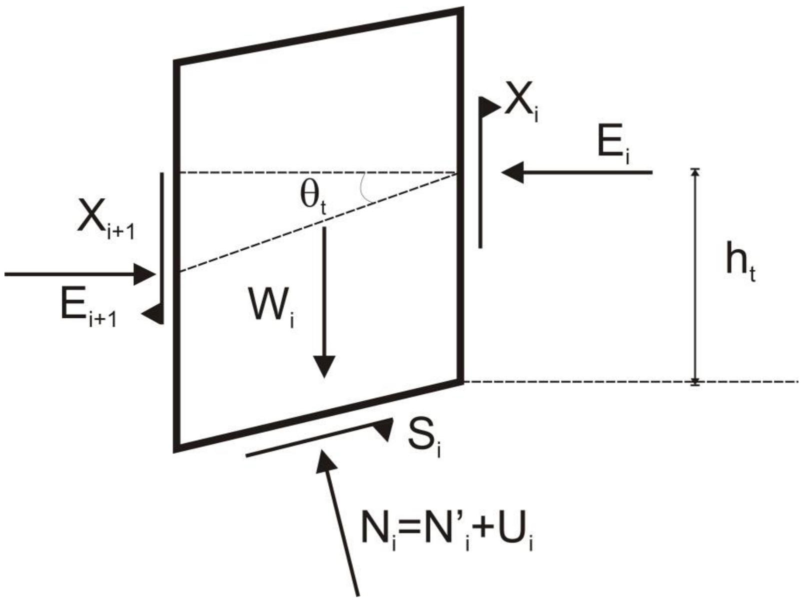


Janbu's Generalised Procedure of Slices

- Moment Equilibrium about the centre of the base for an individual slice of infinitesimal width dx:

$$X_i = -E_i \tan\theta_t + h_t \frac{dE}{dx}$$

$$X_i = -E_i \tan\theta_t + h_t \frac{E_i - E_{i+1}}{\Delta x}$$



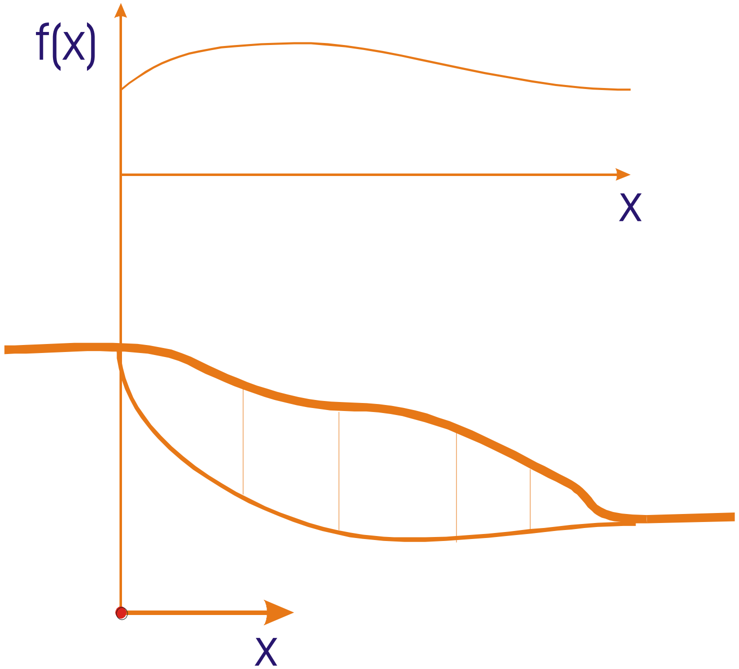
- Force equilibrium in the horizontal and vertical directions.
- Iterative procedure, in which X is assumed to be zero for the first iteration

Morgenstern & Price Method

Assumptions: Number

The inter-slice forces E, X are simply related to one another:

- The ratio of inter-slice forces is taken as $X/E = \lambda f(x)$ n-1



Morgenstern & Price Method

<u>Assumptions</u>: Number

The inter-slice forces E, X are simply related to one another:

- The ratio of inter-slice forces is taken as $X/E' = \lambda f(x)$ n-1

The normal force, N acts through the mid point of the base of the slice :

- Point of application (i.e. line of action) of N n

Total number of assumptions: $\sum$= 2n-1

However λ is derived from the calculation procedure and is actually introduced to balance the extra assumption

There are exactly the same number of unknowns as equations

Summary & Comparison of LE Methods

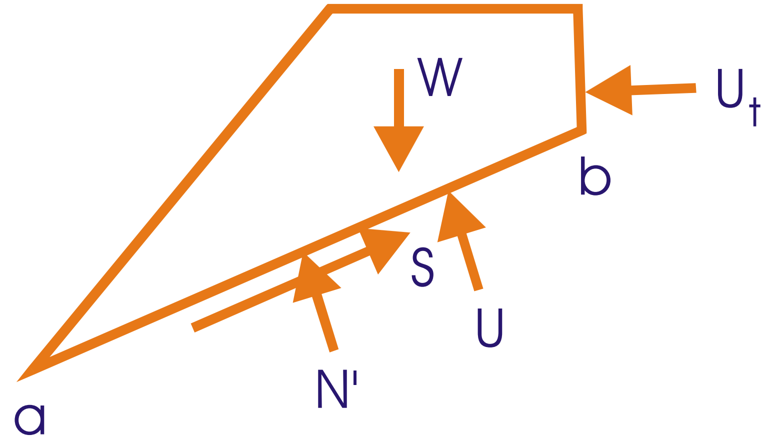
Summary of LE Methods

PLANAR SURFACES

UNKNOWNS:

- FoS or geometry or soil strength
- Magnitude of N’
- Line of action of N’
- Magnitude of S

4 equations + 4 unknowns= solvable



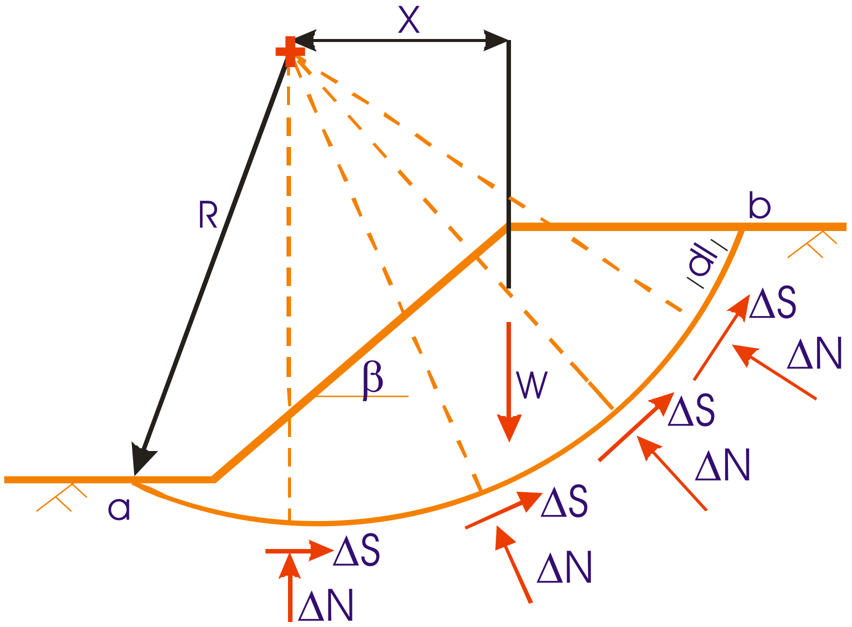
Summary of LE Methods

ROTATIONAL SLIDES

Total Stress Analysis

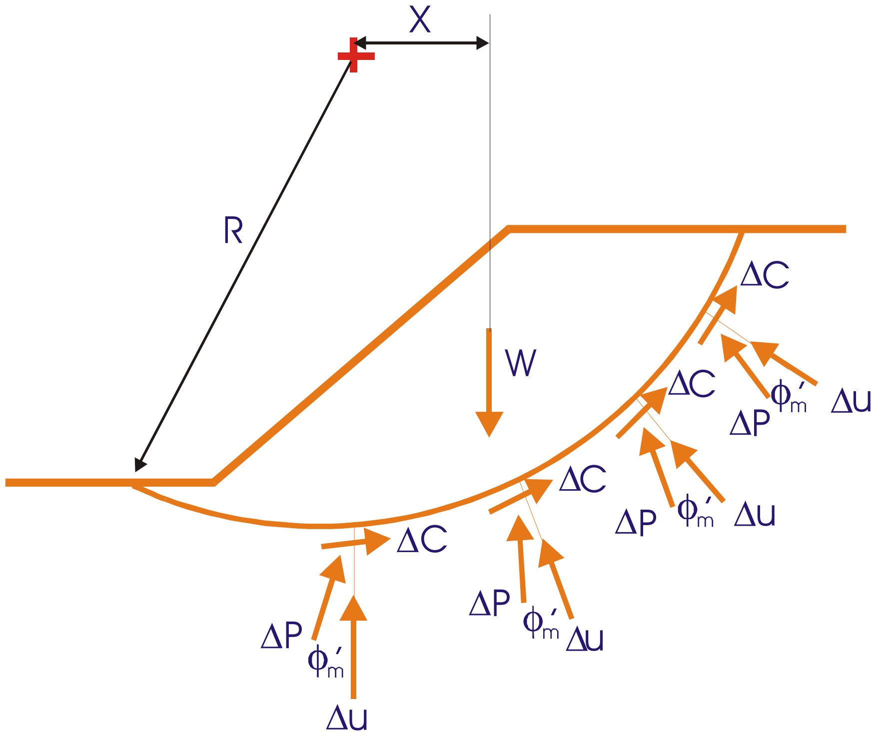
UNKNOWNS:

- FoS or geometry or soil strength
- Magnitude of N
- Direction of N
- Magnitude of S



4 equations + 4 unknowns= <u>solvable</u>

Summary of LE Methods



ROTATIONAL SLIDES

Effective Stress Analysis

UNKNOWNS:

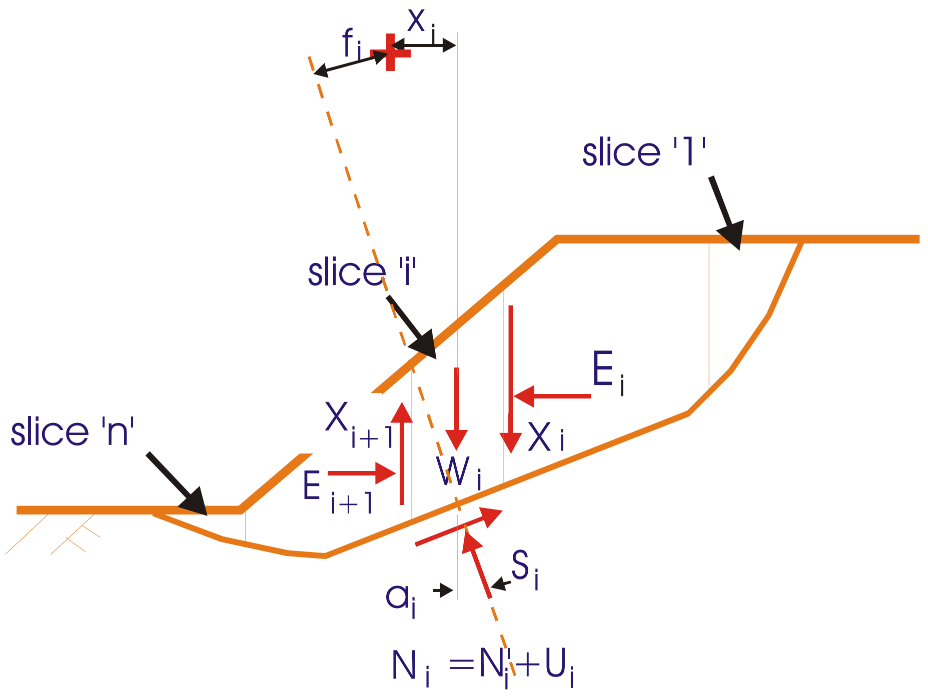
- FoS or geometry or soil strength
- Magnitude of P
- Direction of P
- Line of action of P
- Magnitude of C

4 equations + 5 unknowns= <u>indeterminate</u>

Summary of LE Methods

COMPOUND SLIDES
Effective Stress Analysis
UNKNOWNS:

- FoS or geometry or soil strength
- Magnitude of P
- Direction of P
- Line of action of P
- Magnitude of C



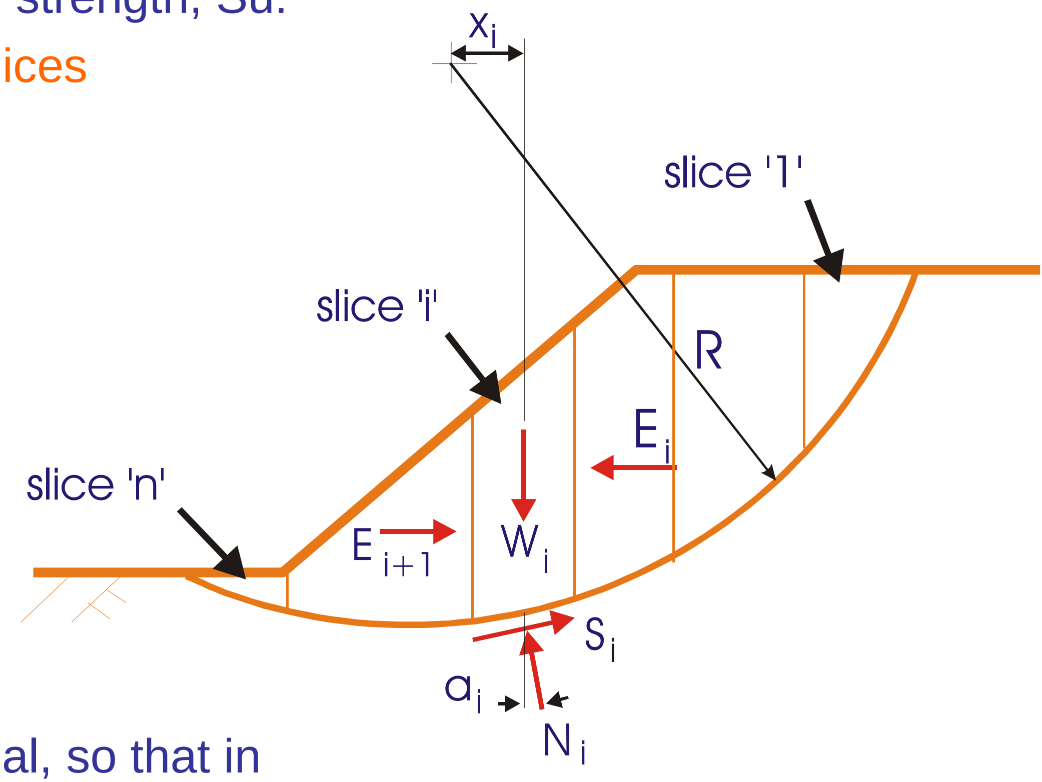
4 equations + 5 unknowns= indeterminate

Comparison of Methods of Slices

Factor of safety terms of the undrained strength, Su:

1. Using the Conventional method of slices
2. Using Bishop's Simplified method

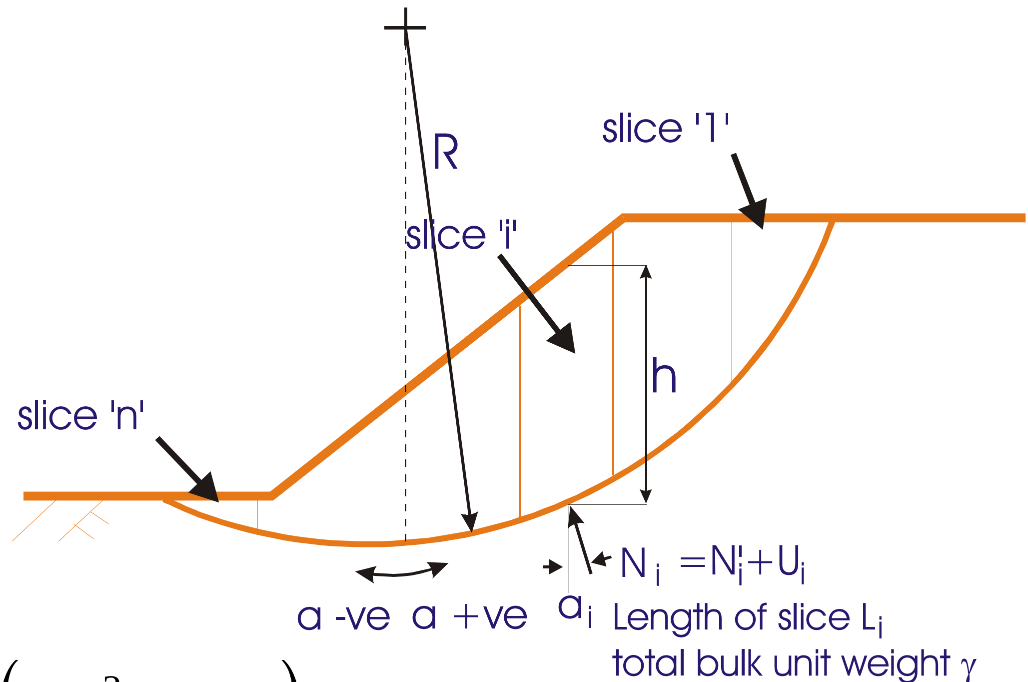
$$F = \frac{1}{\Sigma W_i \sin \alpha_i} \Sigma[S_u l_i]$$



Soil is now purely cohesive, not frictional, so that in neither the Conventional or Bishop's method is needed to consider N′

Conventional Method

$$\boxed{F = \frac{\sum\left[c'_i\, l_i + \left(W\cos\alpha_i - U_i\right)\tan\phi'_i\right]}{\sum W\sin\alpha_i}}$$



$$N' = W\cos\alpha - U$$

$$W = \gamma\, h\, L\cos\alpha$$

$$r_u = \frac{u}{\gamma\, h}$$

$$N' = \gamma\, h\, L\cos^2\alpha - r_u\,\gamma\, h\, L = \gamma\, h\, L\left(\cos^2\alpha - r_u\right)$$

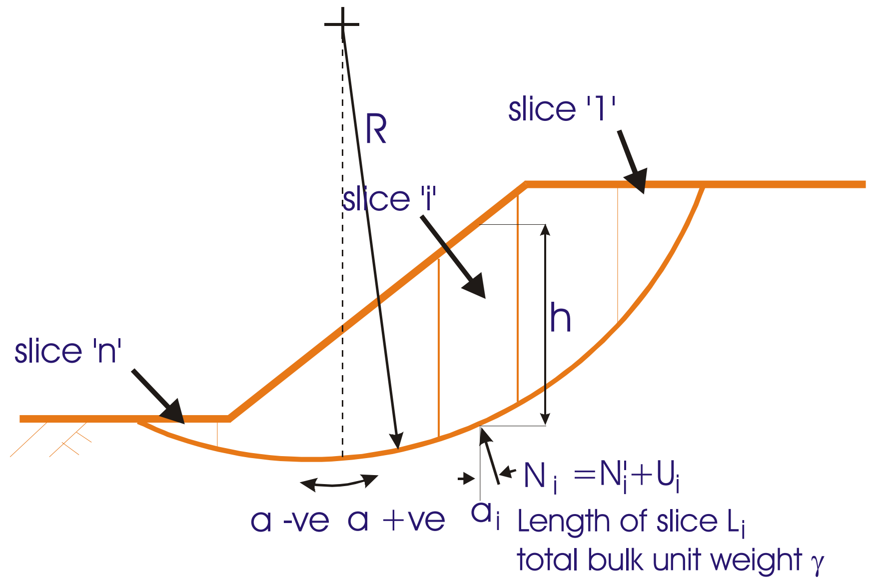
For $r_u > \cos^2\alpha$ N' **Becomes negative!**

e.g. r_u = 0.4 and α =-50˚

Investigation Task:
Comparison of Methods of Slices

Bishop Simplified

$$N_i = \frac{W_i - \dfrac{\sin\alpha_i}{F}\left(c_i' \, l_i - U_i \tan\phi_i'\right)}{\cos\alpha_i + \dfrac{\sin\alpha_i \tan\phi_{ii}'}{F}}$$



For c'=0

$$N' = \gamma h L (1 - r_u) \frac{\sec a}{1 + \dfrac{\tan\alpha \tan\phi'}{F}}$$

$$r_u = \frac{u}{\gamma h} \qquad W = \gamma h L \cos\alpha$$

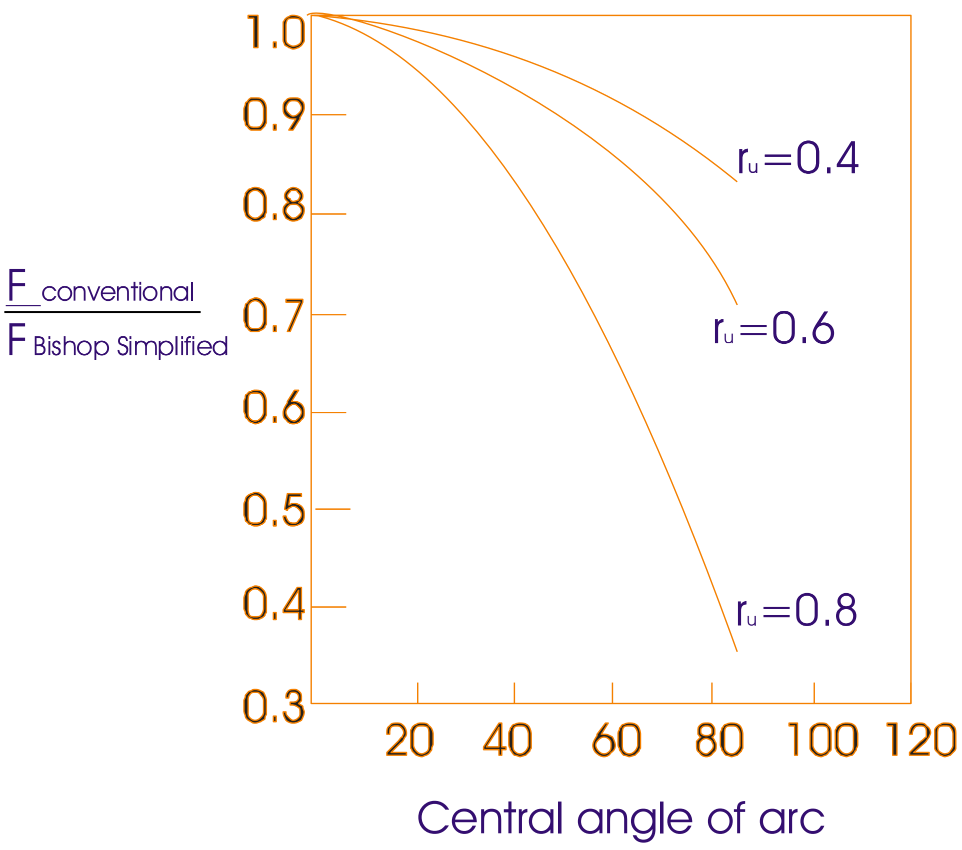
For $\alpha = -50^{\circ}$
To get N'<0

$$\frac{\tan\phi'}{F} > 0.84 \qquad \phi' > 40^{\circ}$$

e.g. for F=1

So uplift problem is less severe than with Conventional method

Investigation Task:
Comparison of Methods of Slices

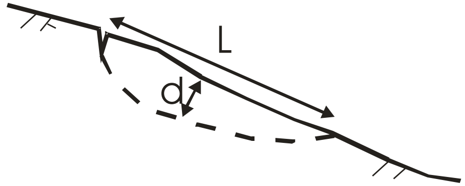


After Bishop (1955)

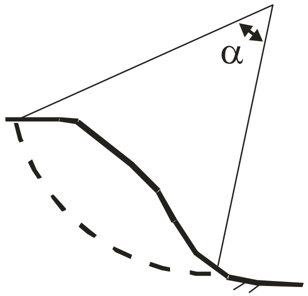
Comparison of Methods of Slices

Rotational slides

LANDSLIDE	SHAPE OF CROSS-SECTION	FACTOR OF SAFETY	
		Conventional	Bishop Simplified*
Northolt	$d/L=0.14;\ \alpha=64^{\circ}$	0.94	1.0
Drammen	$d/L=0.19;\ \alpha=82^{\circ}$	0.79	1.0
Lodalen	$d/L=0.20;\ \alpha=85^{\circ}$	0.79	1.0



Non-circular



circular

Comparison of Methods of Slices

Compound slides

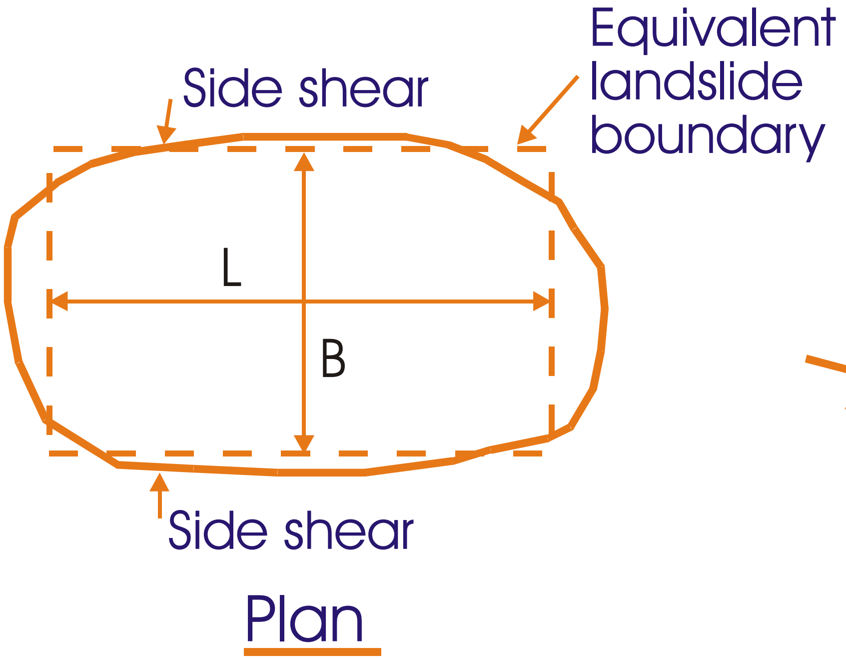
LANDSLIDE	SHAPE OF CROSS_SECTION	FACTOR OF SAFETY		
		Conventional	Janbu	Morgenstern & Price*
Walton's Wood	d/L=0.06	0.98	1.03	1.0
Guildford	d/L=0.09	0.97	1.00	1.0
Sudbury Hill	d/L=0.11	0.96	0.95	1.0
Folkestone Warren	d/L=0.17	0.92	0.97	1.0

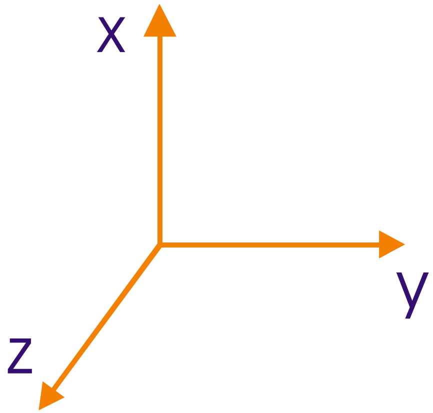
Comparison of Methods of Slices

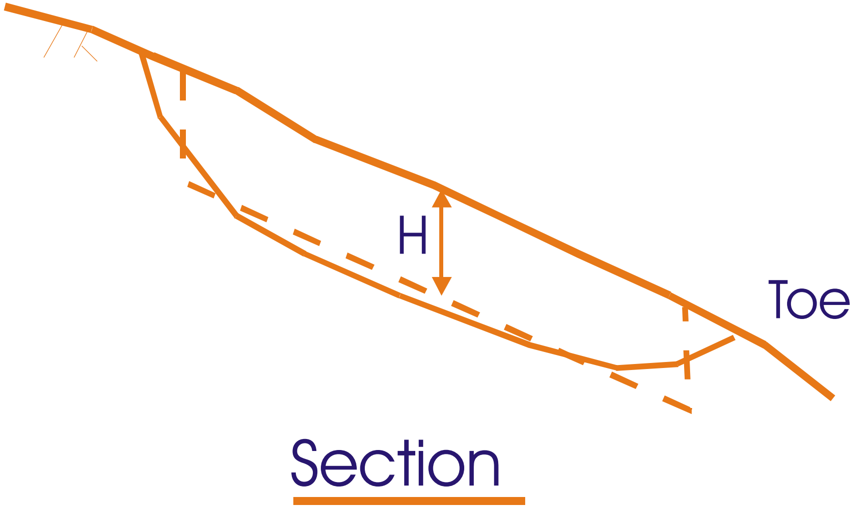
- FoS computed by force equilibrium methods is very sensitive to the assumed relationship between the interslice forces
- Complete equilibrium methods (like Morgenstern & Price and Spencer) yield similar values but they often suffer from numerical convergence problems
- Simple methods like the Conventional methods can give a first guess of initial FoS for more sophisticated methods

Three dimensional analysis

- Slopes of limited lateral extent

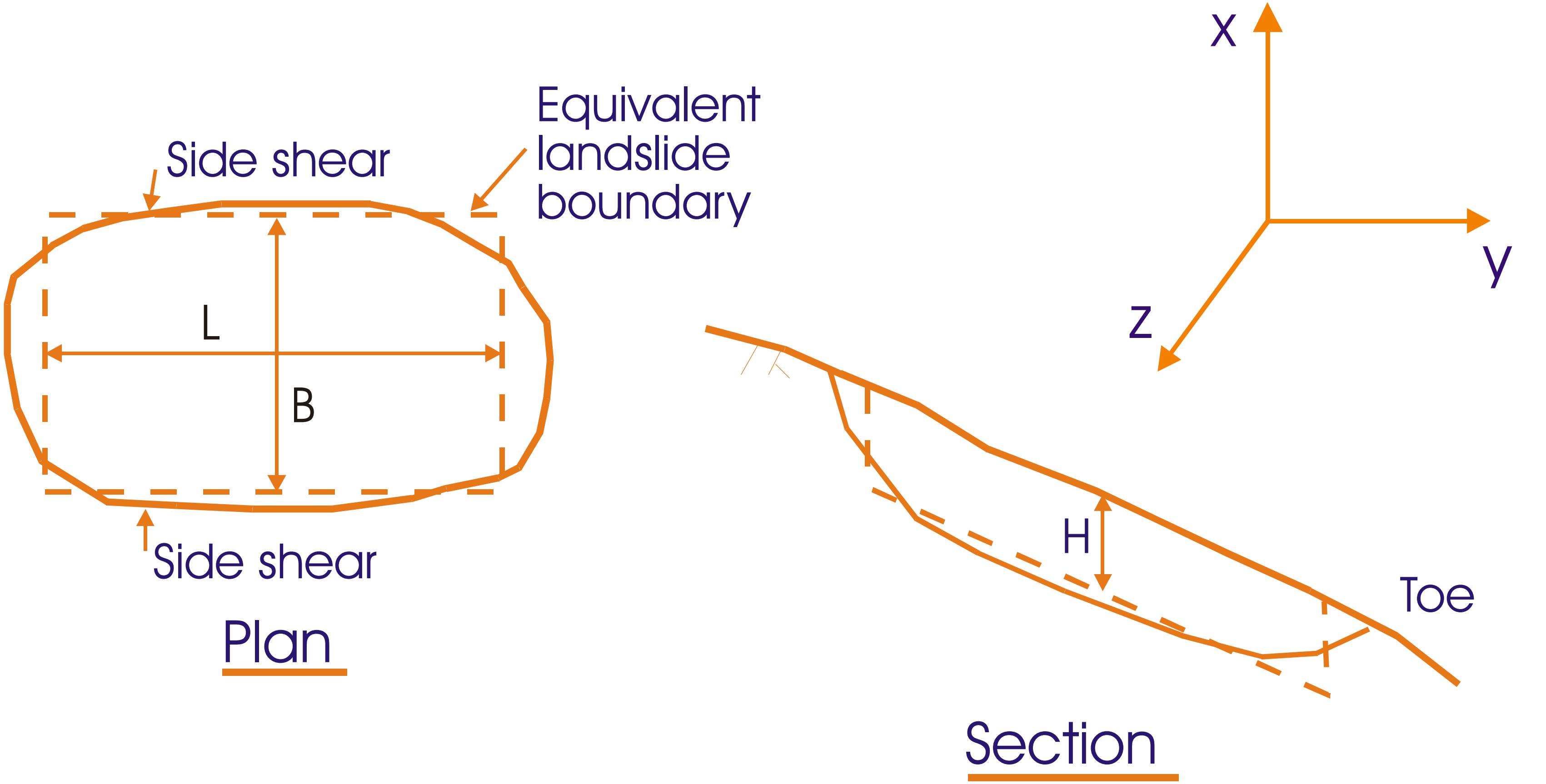






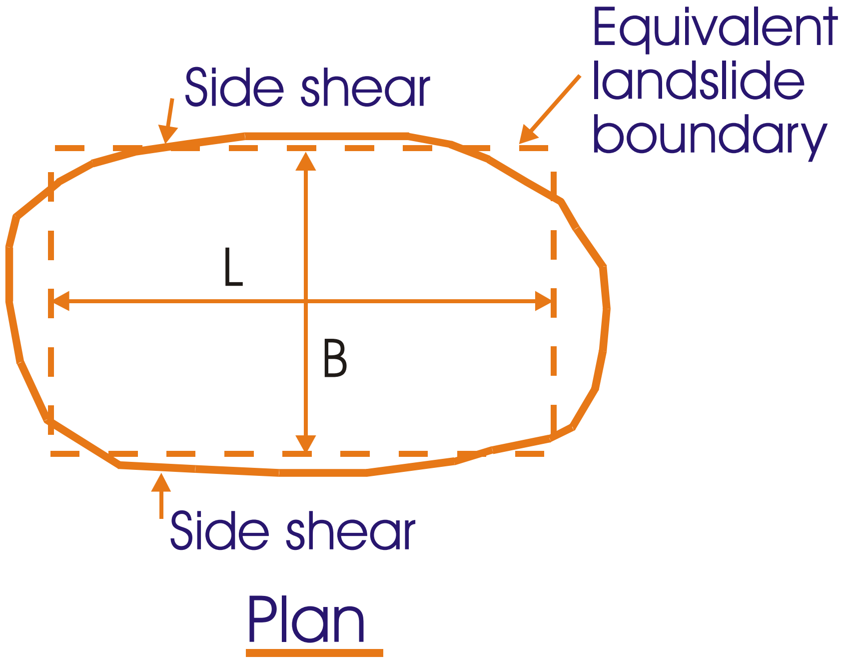
Three dimensional analysis

- Slopes of limited lateral extent
- Slopes that are curved in plan or contain corners



Three dimensional analysis

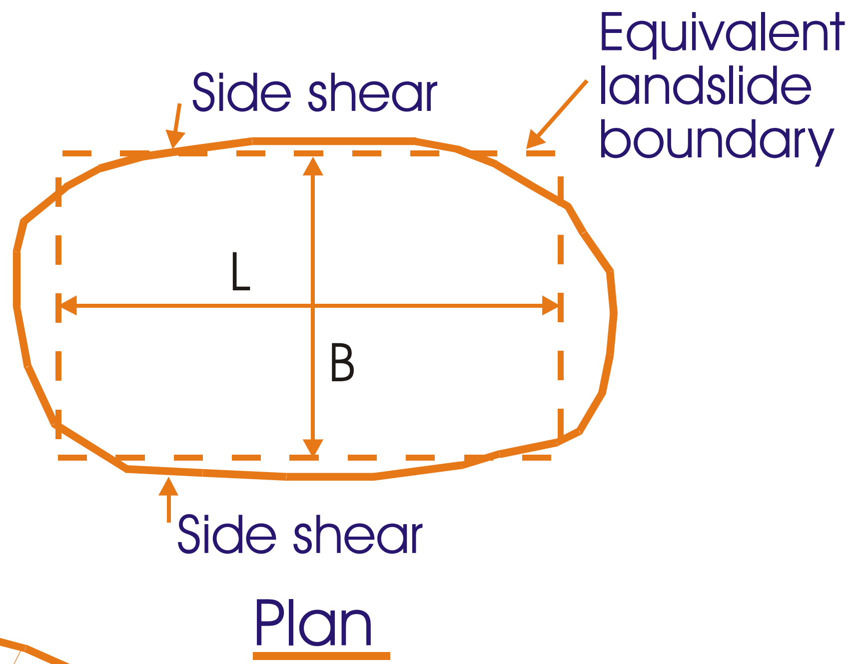
- Slopes of limited lateral extent
- Slopes that are curved in plan or contain corners
- Slopes that are subjected to load of limited extent at the top



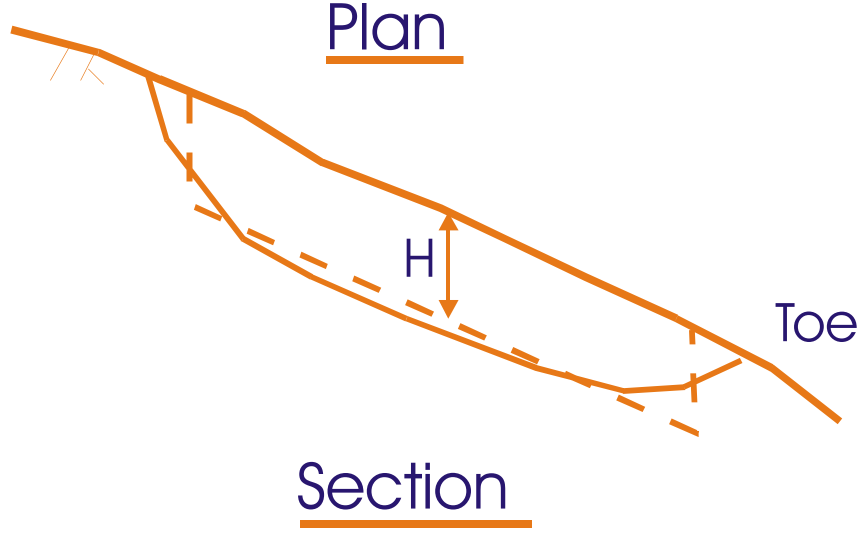
Three dimensional analysis

$$F_2 = \frac{\sum(\text{Resisting Moments or forces}, R_m)}{\sum(\text{Disturbing Moments or forces}, D)}$$

$$F_3 = \frac{\sum(R_m B + M_1 + M_2)}{\sum(D B)}$$



$F_3 \geq F_2$



However for back analysis 2D analysis predicts higher strength than 3D

Method of columns

- An extension of the method of slices
- Soil mass is subdivided in a number of columns, each with an approximate square cross section in plan view
- e.g. Bishop's simplified, Janbu's and Morgenstern & Price methods

Note that the effects of the adopted assumptions can be as large as the 3D effects themselves!