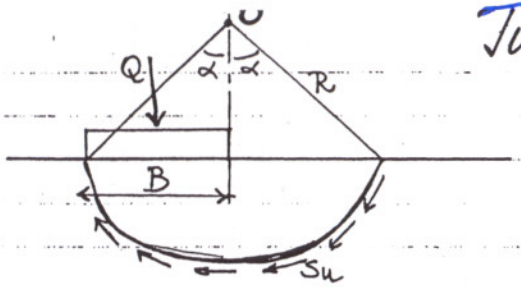
# Tutorial Sheet 1: Solutions

**1**




Note: $\sin\alpha = \frac{B}{R} \Rightarrow R = \frac{B}{\sin\alpha}$ ; $la = \frac{R \cdot \pi \cdot 2\alpha}{\pi}$

Taking moments about '0':

$$Q \cdot \frac{B}{2} = Su \cdot la \cdot R$$

$$= Su \cdot \frac{B}{\sin\alpha} \cdot \pi \cdot \frac{2\alpha}{\pi} \cdot \frac{B}{\sin\alpha}$$

$$Q = \frac{4 \cdot Su\, B\alpha}{\sin^2\alpha}$$

Critical value of $\alpha$ is that which produces the minimum value of Q. This occurs when $\frac{\partial Q}{\partial \alpha} = 0$.

$$\frac{\partial Q}{\partial \alpha} = 4\,Su\,B \left[\frac{\sin^2\alpha - \alpha \cdot 2\sin\alpha \cdot \cos\alpha}{\sin^4\alpha}\right]$$

$$= 4\,Su\,B \left[\frac{1}{\sin^2\alpha} - \frac{2\alpha \cdot \cos\alpha}{\sin^3\alpha}\right]$$

$$= \frac{4\,Su\,B}{\sin^2\alpha}\left[1 - \frac{2\alpha}{\tan\alpha}\right]$$

which $= 0$ if $2\alpha = \tan\alpha$

This can be solved by drawing a graph of $\alpha$ vs. $\tan\alpha$, or by trial and error. The equation is satisfied if:

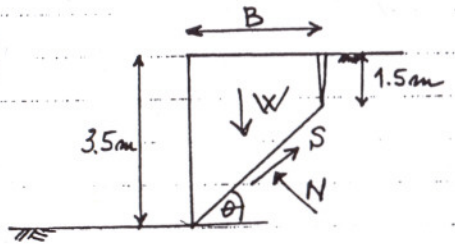
$$\boxed{\alpha = 66.7^\circ}$$

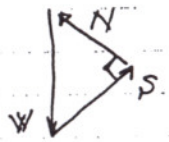
Substituting back into expression for Q:

$$Q = \frac{4 \cdot Su \cdot B \cdot (66.7 \cdot \frac{\pi}{180})}{\sin^2 66.7} = \underline{\underline{5.52\,Su\,B}}$$

**2**

a)




$$B = \frac{2}{\tan\theta} \; ; \quad S = Su \cdot \frac{2}{\sin\theta} \; ; \quad W = \frac{1.5+3.5}{2} \times B \times 19 = \frac{95}{\tan\theta}$$

Resolving parallel to failure surface:

$$S = W \cdot \sin\theta$$

$$Su = \frac{\sin\theta}{2} \cdot \frac{95}{\tan\theta} \cdot \sin\theta = 47.5 \sin\theta\cos\theta$$

$$= 23.75 \sin(2\theta)$$

Critical value of $\theta$ is for $\frac{\partial Su}{\partial \theta} = 0$

$$\frac{\partial Su}{\partial \theta} = 23.75 \cdot 2 \cdot \cos(2\theta) = 47.5\cos(2\theta)$$

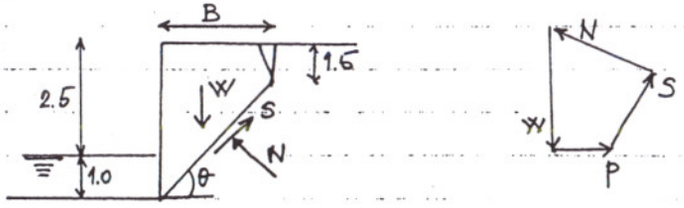
which $= 0$ when $\cos(2\theta) = 0 \Rightarrow 2\theta = \frac{\pi}{2}$

$$\boxed{\theta = \frac{\pi}{4}}$$

Substituting $\theta$ into expression for $S_u$ gives:

$$S_u = 23.75 \text{ kPa}$$

6)







As before: $S = S_u \frac{2}{\sin\theta}$ ; $W = \frac{95}{\tan\theta}$

$$P = \frac{1}{2} 1.0 \times \gamma_w \cdot 1.0 = \frac{1}{2} \cdot \gamma_w = 4.905$$

Resolving parallel to failure surface:

$$S + P\cos\theta = W \cdot \sin\theta$$

$$S_u = \frac{\sin\theta}{2}\left[\frac{95}{\tan\theta} \cdot \sin\theta - 4.905\cos\theta\right]$$

$$= \frac{95 - 4.905}{2} \sin\theta \cdot \cos\theta = 22.524 \sin(2\theta)$$

Critical $\theta$ when $\frac{\partial S_u}{\partial \theta} = 0$

$$\Rightarrow \quad \theta = \frac{\pi}{4}$$

$$\underline{S_u = 22.52 \text{ kPa}}$$