Tutorial sheet 2 - Solutions

(1) From notes:

$$\sigma_{xa}' = \sigma_z' \cdot \tan^2(45-\varphi'/2) - 2c'\tan(45-\varphi'/2)$$

$$\sigma_{xp}' = \sigma_z' \cdot \tan^2(45+\varphi'/2) + 2c'\tan(45+\varphi'/2)$$

Top soil layer: $c'=0$, $\varphi'=35°$

$\Rightarrow$ $\sigma_{xa}' = 0.271\,\sigma_z'$; $\sigma_{xp}' = 3.6902\,\sigma_z'$

Bottom soil layer: $c'=5$, $\varphi'=20°$

$\Rightarrow$ $\sigma_{xa}' = 0.4903\,\sigma_z' - 7.002$

$\sigma_{xp}' = 2.0396\,\sigma_z' + 14.281$

a) zero pore water pressure:

Top layer:

$\sigma_z' = 18\cdot z$

$\sigma_{xa}' = 0.271\cdot 18z = 4.878z$

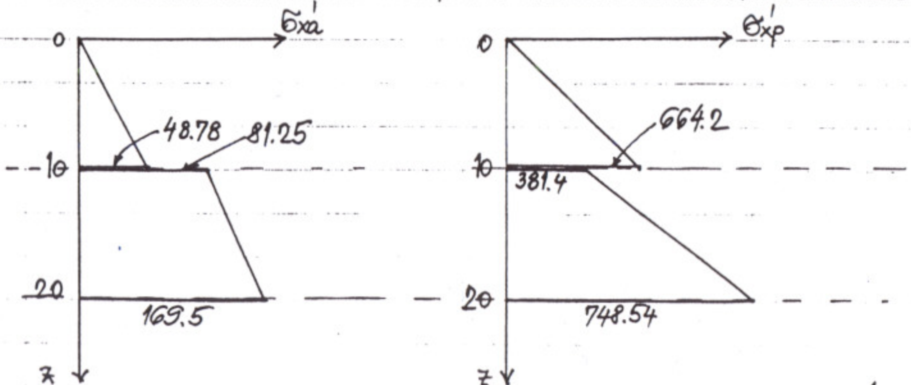
$\sigma_{xp}' = 3.6902\cdot 18z = 66.4236z$

Bottom layer:

$\sigma_z' = 18\cdot z$

$\sigma_{xa}' = 8.8254z - 7.002$

$\sigma_{xp}' = 36.7128z + 14.281$



b) a static ground water table 2.0m below ground surface

Top layer: $z < 2.0\,m \Rightarrow$ $\sigma_z' = 18z$

$\sigma_{xa}' = 4.878z$

$\sigma_{xp}' = 66.4236z$

$z > 2.0\,m \Rightarrow$ $\sigma_z' = 2.0\times 18 + (20-9.81)(z-2)$

$= 36 + 10.2(z-2)$

$\sigma_{xa}' = 2.764\cdot z + 4.2276$

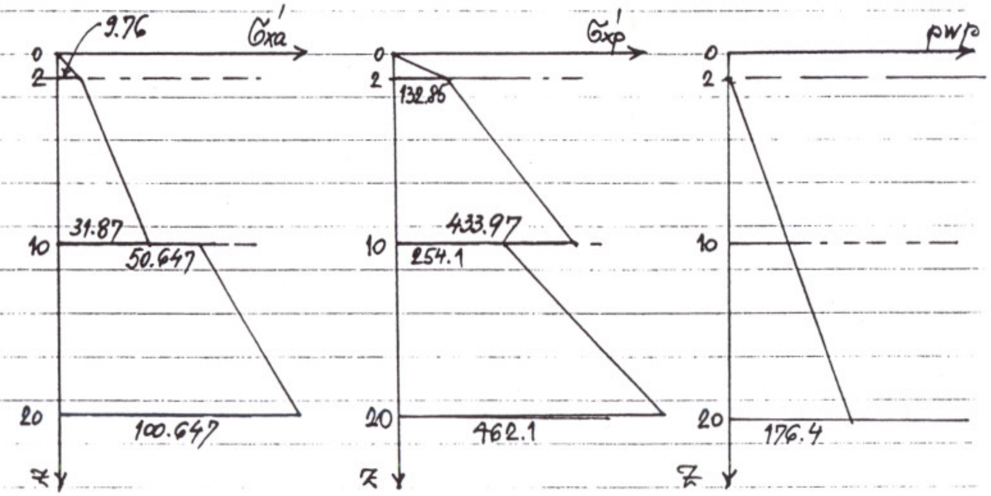
$\sigma_{xp}' = 37.64z + 57.567$

Bottom layer:

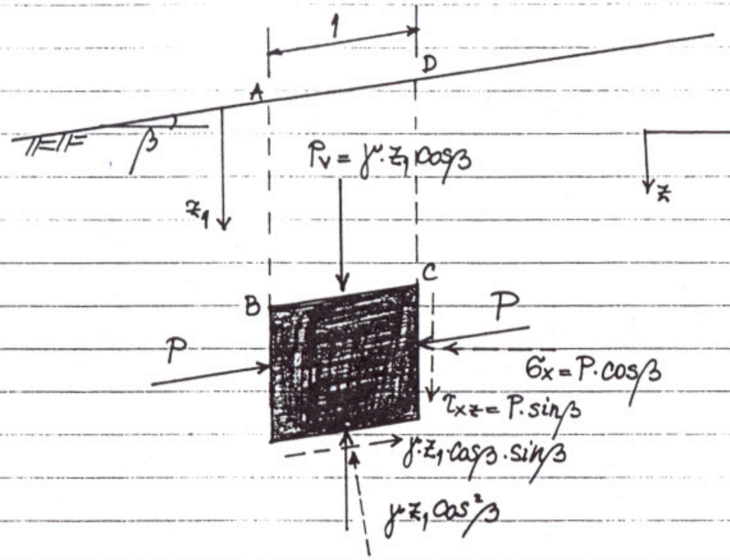
$\sigma_z' = 36 + 10.2(z-2)$

$\sigma_{xa}' = 5.0z + 0.647$

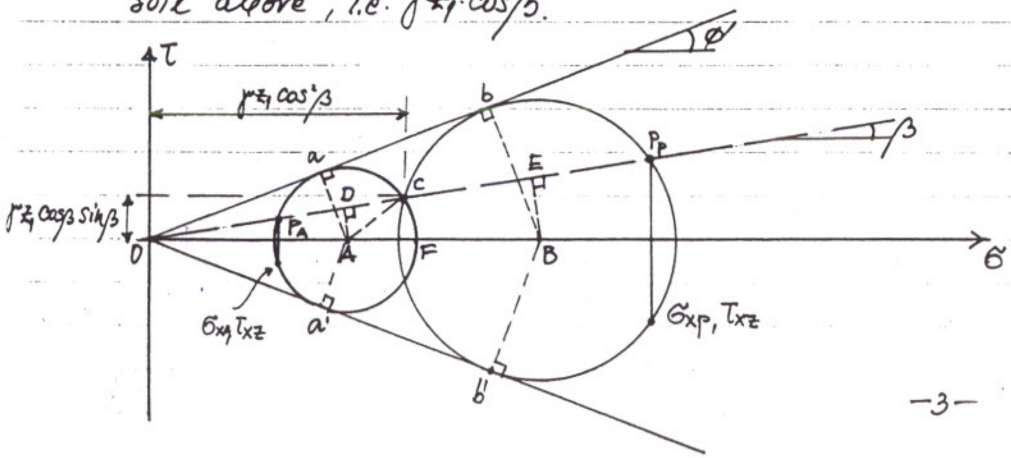
$\sigma_{xp}' = 20.8z + 46.099$



(2) Active and passive Rankine states in a semi-infinite cohesionless soil mass with inclined surface



Consider a shaded element of soil at depth z_1. Since, at any constant depth below the surface, all such elements are similar, the forces on the vertical surfaces AB and CD are equal and parallel to the ground surface. The resulting stress P_v on the inclined surface of the element is then vertical and equal to the weight of soil above, i.e. $\gamma z_1 \cdot \cos\beta$.



Active case

$$\frac{\sigma_{xA}}{\gamma z_1 \cos^2\beta} = \frac{P_a \cos\beta}{\gamma z_1 \cos^2\beta} = \frac{OP_A}{OC} = \frac{OD - DP_A}{OD + DC}$$

$$OD = OA \cdot \cos\beta$$
$$DP_A = DC = \left(AC^2 - AD^2\right)^{\frac{1}{2}}$$
$$AC = Aa = OA \cdot \sin\varphi'$$
$$AD = OA \cdot \sin\beta$$

$$DC = OA\left(\sin^2\varphi' - \sin^2\beta\right)^{\frac{1}{2}}$$

Noting: $\cos^2\varphi' + \sin^2\varphi' = 1$
$\cos^2\beta + \sin^2\beta = 1$

$$DC = OA\left(\cos^2\beta - \cos^2\varphi'\right)^{\frac{1}{2}}$$

$$P_a = \gamma z_1 \cos\beta \, \frac{\cos\beta - \left(\cos^2\beta - \cos^2\varphi'\right)^{\frac{1}{2}}}{\cos\beta + \left(\cos^2\beta - \cos^2\varphi'\right)^{\frac{1}{2}}}$$

$\sigma_{xA} = P_a \cdot \cos\beta$; $\tau_{xz} = P_a \sin\beta$

The planes of max stress obliquity are inclined at:

$$\pm\alpha = \frac{1}{2} a\hat{A}F = \frac{1}{2}\left(\pi - \frac{\pi}{2} + \varphi'\right)$$
$$= \pm\left(\frac{\pi}{4} + \frac{\varphi'}{2}\right)$$ to the plane on which σ_1 acts.

Passive case

$$\frac{\sigma_{xp}}{\gamma z_1 \cos^2\beta} = \frac{P_p \cos\beta}{\gamma z_1 \cos^2\beta} = \frac{OP_p}{OC} = \frac{OE + EP_p}{OE - EC}$$

$$OE = OB \cdot \cos\beta$$
$$EP_p = EC = \left(BC^2 - EB^2\right)^{\frac{1}{2}}$$
$$BC = Bb = OB \cdot \sin\varphi'$$
$$EB = OB \cdot \sin\beta$$

$$EC = OB\left(\sin^2\varphi' - \sin^2\beta\right)^{\frac{1}{2}}$$

$$EC = OB\left(\cos^2\beta - \cos^2\varphi'\right)^{\frac{1}{2}}$$

$$P_p = \gamma z_1 \cos\beta \, \frac{\cos\beta + \left(\cos^2\beta - \cos^2\varphi'\right)^{\frac{1}{2}}}{\cos\beta - \left(\cos^2\beta - \cos^2\varphi'\right)^{\frac{1}{2}}}$$

$\sigma_{xp} = P_p \cos\beta$; $\tau_{xz} = P_p \cdot \sin\beta$

The planes of max stress obliquity are inclined at:

$$\pm\alpha = \frac{1}{2}\left(\frac{\pi}{2} - \varphi\right) = \pm\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)$$ to the plane on which σ_3 acts.