First Technical Session : General Theory of Stability of Slopes

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Session 1/1

THE USE OF THE SLIP CIRCLE IN THE STABILITY ANALYSIS OF SLOPES

by

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INTRODUCTION

Errors may be introduced into the estimate of stability not only by the use of approximate methods of stability analysis, but also by the use of sampling and testing methods which do not reproduce sufficiently accurately the soil conditions and state of stress in the natural ground or compacted fill under consideration. Unless equal attention is paid to each factor, an elaborate mathematical treatment may lead to a fictitious impression of accuracy.

However, in a number of cases the uniformity of the soil conditions or the importance of the problem will justify a more accurate analysis, particularly if this is coupled with field measurements of pore pressure, which is the factor most difficult to assess from laboratory data alone. Two classes of problem of particular note in this respect are :

(i) The design of water-retaining structures, such as earth dams and embankments, where failure could have catastrophic results, but where an over-conservative design may be very costly.

(ii) The examination of the long-term stability of cuts and natural slopes where large scale earth movements may involve engineering works and buildings.

THE USE OF LIMIT DESIGN METHODS

It has been shown elsewhere by a relaxation analysis of a typical earth dam (Bishop, 1952) that, even assuming idealized elastic properties for the soil, local overstress will occur when the factor of safety (by a slip circle method) lies below a value of about 1·8. As the majority of stability problems occur in slopes and dams having lower factors of safety than this, a state of plastic equilibrium must be considered to exist throughout at least part of the slope.

Under these conditions a quantitative estimate of the factor of safety can be obtained by examining the conditions of equilibrium when incipient failure is postulated, and comparing the strength necessary to maintain limiting equilibrium with the available strength of the soil. The factor of safety (F) is thus defined as the ratio of the available shear strength of the soil to that required to maintain equilibrium. The shear strength mobilized is, therefore, equal to s, where :

$$s = \frac{1}{F}\left\{ c' + (\sigma_n - u) \tan \phi' \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

where c' denotes cohesion, ϕ' denotes angle of shearing resistance, } in terms of effective stress.*
σ_n denotes total normal stress,
u denotes pore pressure.

* Measured either in undrained or consolidated-undrained tests with pore-pressure measurement, or in drained tests carried out sufficiently slowly to ensure zero excess pore pressure.

Failure along a continuous rupture surface is usually assumed, but, as the shape and position of this surface is influenced by the distribution of pore pressure and the variation of the shear parameters within the slope, a generalized analytical solution is not possible and a numerical solution is required in each individual case. The rigorous determination of the shape of the most critical surface presents some difficulty (see, for example, Coenen, 1948), and in practice a simplified shape, usually a circular arc, is adopted, and the problem is assumed to be one of plane strain.

MECHANICS OF THE CIRCULAR ARC ANALYSIS

In order to examine the equilibrium of the mass of soil above the slip surface it follows from equation (1) that it is necessary to know the value of the normal stress at each point on this surface, as well as the magnitude of the pore pressure. It is possible to estimate the value of the normal stress by following the method developed by Fellenius (1927, 1936), in which the conditions for the statical equilibrium of the slice of soil lying vertically above each element of the sliding surface are fully satisfied.

A complete graphical analysis of this type is most laborious, and this may result in the examination of an insufficient number of trial surfaces to locate the most critical one. Several simplified procedures have been developed using the friction circle method (Taylor, 1937, 1948; Fröhlich, 1951), in which an assumption is made about the distribution of normal stress, but their use is limited to cases in which ϕ' is constant over the whole of the failure surface. More generally the " slices " method is used, with a simplifying assumption about the effect of the forces between the slices.

The significance of this assumption may be examined by considering the equilibrium of the mass of soil (of unit thickness) bounded by the circular arc ABCD, of radius R and centre at O (Fig. 1(a)). In the case where no external forces act on the surface of the slope,

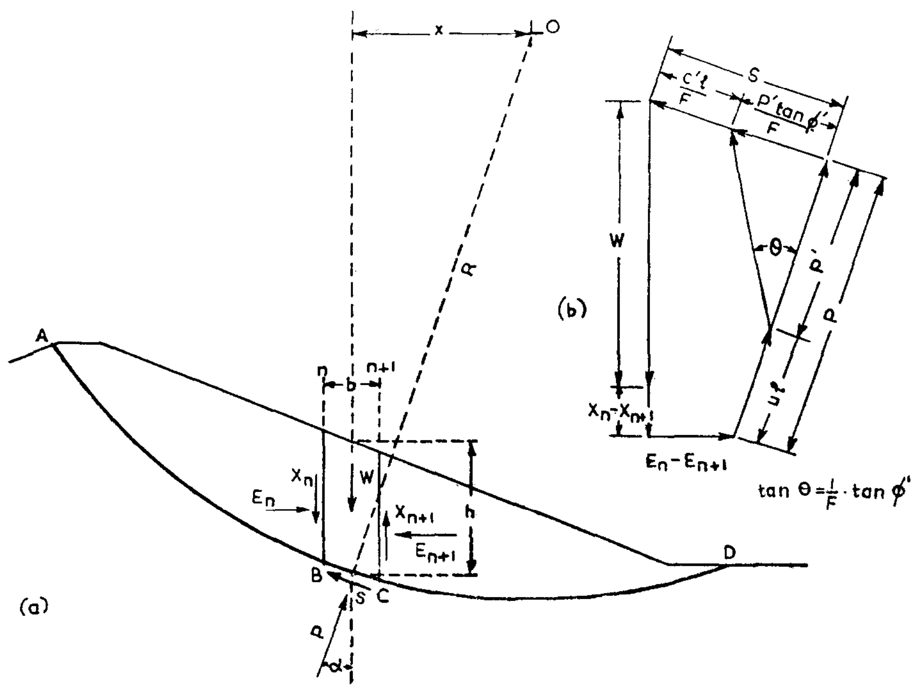

Fig. 1. Forces in the slices method

equilibrium must exist between the weight of the soil above ABCD and the resultant of the total forces acting on ABCD.

Let E_n, E_{n+1} denote the resultants of the total horizontal forces on the sections n and $n + 1$ respectively,

and	X_n, X_{n+1}	,,	the vertical shear forces,
	W	,,	the total weight of the slice of soil,
	P	,,	the total normal force acting on its base,
	S	,,	the shear force acting on its base,
	h	,,	the height of the element,
	b	,,	the breadth of the element,
	l	,,	the length BC,
	α	,,	the angle between BC and the horizontal,
	x	,,	the horizontal distance of the slice from the centre of rotation.

The total normal stress is σ_n, where

$$\sigma_n = \frac{P}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence, from equation (1), the magnitude of the shear strength mobilized to satisfy the conditions of limiting equilibrium is s where :

$$s = \frac{1}{F}\left\{ c' + \left(\frac{P}{l} - u\right) \tan \phi' \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The shear force S acting on the base of the slice is equal to sl, and thus, equating the moment about O of the weight of soil within ABCD with the moment of the external forces acting on the sliding surface, we obtain :

$$\Sigma Wx = \Sigma SR = \Sigma slR \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

It follows, therefore, from equation (3) that :

$$F = \frac{R}{\Sigma Wx} \cdot \Sigma\Big[c'l + (P - ul) \tan \phi'\Big] \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

From the equilibrium of the soil in the slice above BC, we obtain P, by resolving in a direction normal to the slip surface :

$$P = (W + X_n - X_{n+1}) \cos \alpha - (E_n - E_{n+1}) \sin \alpha \quad . \quad . \quad . \quad . \quad (6)$$

The expression for F thus becomes :

$$F = \frac{R}{\Sigma Wx} \cdot \Sigma\Big[c'l + \tan \phi' . (W \cos \alpha - ul) + \tan \phi' . \Big\{(X_n - X_{n+1}) \cos \alpha - (E_n - E_{n+1}) \sin \alpha\Big\}\Big] \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Since there are no external forces on the face of the slope, it follows that :

$$\Sigma(X_n - X_{n+1}) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8a)$$

$$\Sigma(E_n - E_{n+1}) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8b)$$

However, except in the case where ϕ' is constant along the slip surface and α is also constant (*i.e.*, a plane slip surface), the terms in equation (7) containing X_n and E_n do not disappear. A simplified form of analysis, suggested by Krey (1926) and Terzaghi (1929) and also presented by May (1936) as a graphical method, implies that the sum of these terms

$$\Sigma \tan \phi' . \Big\{(X_n - X_{n+1}) \cos \alpha - (E_n - E_{n+1}) \sin \alpha\Big\}$$

may be neglected without serious loss in accuracy. This is the method at present used, for example, by the U.S. Bureau of Reclamation (Daehn and Hilf, 1951).

Putting $x = R \sin \alpha$, the simplified form may be written :

$$F = \frac{1}{\Sigma W \sin \alpha} \cdot \Sigma \Big[c'l + \tan \phi' . (W \cos \alpha - ul) \Big] \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In earth dam design the construction pore pressures are often expressed as a function of the total weight of the column of soil above the point considered, i.e.

$$u = \bar{B} \left(\frac{W}{b} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where $\bar{B}$ is a soil parameter based either on field data or laboratory tests.*

In this case, putting $l = b \sec \alpha$, the expression for factor of safety can be further simplified to :

$$F = \frac{1}{\Sigma W \sin \alpha} \cdot \Sigma \Big[c'l + \tan \phi' . W(\cos \alpha - \bar{B} \sec \alpha) \Big] \quad . \quad . \quad . \quad . \quad (11)$$

This expression permits the rapid and direct computation of the value of F which is necessary if sufficient trial circles are to be used to locate the most critical surface. However, as can be seen from the examples quoted later, the values of F are, in general, found to be conservative, and may lead to uneconomical design. This is especially marked where conditions permit deep slip circles round which the variation in α is large.

To derive a method of analysis which largely avoids this error it is convenient to return to equation (5). If we denote the effective normal force $(P - ul)$ by P' (see Fig. 1(b)), and resolve the forces on the slice vertically, then we obtain, on re-arranging :

$$P' = \frac{W + X_n - X_{n+1} - l \left(u \cos \alpha + \frac{c'}{F} \sin \alpha \right)}{\cos \alpha + \frac{\tan \phi' . \sin \alpha}{F}} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Substituting in equation (5) and putting $l = b \sec \alpha$ and $x = R \sin \alpha$, an expression for the factor of safety is obtained :

$$F = \frac{1}{\Sigma W \sin \alpha} \cdot \Sigma \left[\left\{ c'b + \tan \phi' . (W(1 - \bar{B}) + (X_n - X_{n+1})) \right\} . \frac{\sec \alpha}{1 + \frac{\tan \phi' . \tan \alpha}{F}} \right] \quad . \quad (13)$$

The values of $(X_n - X_{n+1})$ used in this expression are found by successive approximation, and must satisfy the conditions given in equations (8). In addition, the positions of the lines of thrust between the slices should be reasonable, and no unbalanced moment should be implied in any slice. The factor of safety against sliding on the vertical sections should also be satisfactory, though since the slip surface assumed is only an approximation to the actual one, overstress may be implied in the adjacent soil.

The condition $\Sigma(X_n - X_{n+1}) = 0$ in equation (8a) can be satisfied directly by selecting appropriate values of X_n, etc. The corresponding sum $\Sigma(E_n - E_{n+1})$ can be readily computed in terms of the expression used in equation (13). Resolving the forces on a slice tangentially, we obtain the expression :

$$(W + X_n - X_{n+1}) \sin \alpha + (E_n - E_{n+1}) \cos \alpha = S$$

or

$$(E_n - E_{n+1}) = S \sec \alpha - (W + X_n - X_{n+1}) \tan \alpha \quad . \quad . \quad . \quad (14)$$

* In practice the value of B will vary along the slip surface, though for preliminary design purposes it is convenient to use a constant average value throughout the impervious zone.

Now, if equation (13) is written :

$$F = \frac{1}{\Sigma W \sin \alpha} \cdot \Sigma[m] \quad \ldots \ldots \ldots \quad (15)$$

then

$$S = \frac{m}{F} \quad \ldots \ldots \ldots \ldots \quad (16)$$

and hence

$$\Sigma(E_n - E_{n+1}) = \Sigma\left[\frac{m}{F} \sec \alpha - (W + X_n - X_{n+1}) \tan \alpha\right] \quad \ldots \quad (17)$$

The X values must, therefore, also satisfy the condition that :

$$\Sigma\left[\frac{m}{F} \sec \alpha - (W + X_n - X_{n+1}) \tan \alpha\right] = 0 \quad \ldots \ldots \quad (18)$$

In practice, an initial value of F is obtained by solving equation (13) on the assumption that $(X_n - X_{n+1}) = 0$ throughout. By suitably tabulating the variables a solution can be obtained after using two or three trial values of F. The use of $(X_n - X_{n+1}) = 0$ satisfies equation (8*a*), but not equation (18). Values of $(X_n - X_{n+1})$ are then introduced in order to satisfy equation (18) also. These values can then be finally adjusted, either graphically or analytically, until the equilibrium conditions are fully satisfied for each slice.

From the practical point of view it is of interest to note that, although there are a number of different distributions of $(X_n - X_{n+1})$ which satisfy equation (18), the corresponding variations in the value of F are found to be insignificant (less than 1% in a typical case).

It should also be noted that, since the error in the simplified method given in equations (9) and (11) varies with the central angle of the arc, it will lead to a different location for the most critical circle than that given by the more rigorous method. It will, therefore, generally be necessary to examine a number of trial circles by this latter method.

PARTIALLY SUBMERGED SLOPES

The total disturbing moment is now that of the soil above ABCD, less the moment about O of the water pressure acting on DLM (Fig. 2 (*a*)). If we imagine a section of water bounded by a free surface at MN and outlined by NDLM, and similarly take moments about O, the normal forces on the arc ND all pass through O and the moment of the water pressure on DLM is therefore equal to the moment of the mass of water NDLM about O.

Since the weight of a mass of saturated soil less the weight of water occupying the same volume is equal to its submerged weight, the resultant disturbing moment due to the mass of soil above ABCD and to the water pressure on DLM is given by using the bulk density of the soil above the level of the external free water surface and the submerged density below.

It should be noted that the boundary MN implies nothing about the magnitude of the pore pressures inside the slope and is only used to obtain the statically equivalent disturbing moment.

The disturbing moment is therefore $\Sigma(W_1 + W_2)x$ . . . . (19)

where W_1 denotes full weight of the soil in the slice above MN,

W_2 ,, submerged weight of soil in the part of the slice below MN.

It is also convenient to obtain the expression for the effective normal force P' in terms of W_1 and W_2. It can be seen from Fig. 2 (*b*) and (*c*) that, resolving vertically as before, it follows that :

$$P' = \frac{W_1 + W_2 + X_n - X_{n+1} - l\left(u_s \cos \alpha + \frac{c'}{F} \sin \alpha\right)}{\cos \alpha + \frac{\tan \phi' . \sin \alpha}{F}} \quad \ldots \ldots \quad (20)$$

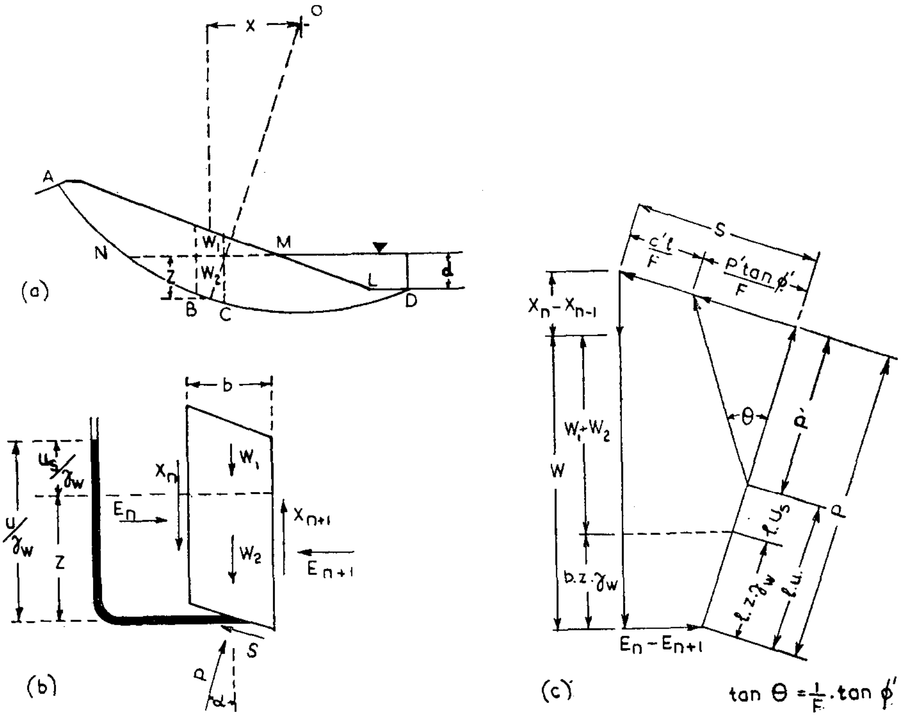

Fig. 2. Partially Submerged Slopes

where u_s is the pore pressure expressed as an excess over the hydrostatic pressure corresponding to the water level outside the slope; that is

$$u_s = u - \gamma_\omega . z \quad \ldots\ldots\ldots\ldots \quad (21)$$

where γ_ω denotes the density of water,
and z ,, the depth of slice below MN. (If no part of the slice is submerged, $u_s = u$.)

It follows, therefore, that for partial submergence the expression for factor of safety * becomes :

$$F = \frac{1}{\Sigma(W_1 + W_2)\sin\alpha} . \Sigma\left[\left\{c'b + \tan\phi'.(W_1 + W_2 - bu_s + \overline{X_n - X_{n+1}})\right\}\frac{\sec\alpha}{1 + \dfrac{\tan\phi'.\tan\alpha}{F}}\right] \quad \ldots\ldots \quad (22)$$

The forces between the slices now have to satisfy the conditions :

$$\Sigma(X_n - X_{n+1}) = 0 \quad \ldots\ldots\ldots\ldots \quad (23a)$$

$$\Sigma(E_n - E_{n+1}) = -\frac{1}{2} . \gamma_w d^2 \quad \ldots\ldots\ldots \quad (23b)$$

* A similar treatment of the simplified method leads to the expression

$$F = \frac{1}{\Sigma(W_1 + W_2)\sin\alpha}\Sigma\left[c'l + \tan\phi'.(\overline{W_1 + W_2}\cos\alpha - u_s b\sec\alpha)\right] \quad . \quad . \quad (22a)$$

This treatment of the pore pressure leads to consistent results throughout the full range of submergence (Bishop, 1952).

where d is the depth of water at the toe of the slip, which causes a horizontal water thrust on the vertical section with which the slip terminates.

To obtain an expression for $\Sigma(E_n - E_{n+1})$, we resolve tangentially as before :

$$(W_1 + W_2 + bz\gamma_\omega + X_n - X_{n+1}) \sin\alpha + (E_n - E_{n+1}) \cos\alpha = S$$

or $$(E_n - E_{n+1}) = S \sec\alpha - (W_1 + W_2 + X_n - X_{n+1}) \tan\alpha - \gamma_w zb \tan\alpha \quad . \quad . \quad (24)$$

Writing equation (22) as :

$$F = \frac{1}{\Sigma(W_1 + W_2) \sin\alpha} \, . \, \Sigma[m] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

then $$S = \frac{m}{F} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Hence

$$\Sigma(E_n - E_{n+1}) = \Sigma\left[\frac{m}{F} \sec\alpha - (W_1 + W_2 + X_n - X_{n+1}) \tan\alpha\right] - \Sigma\left[\gamma_\omega zb \tan\alpha\right] \quad . \quad (27)$$

Now $$\Sigma\left[\gamma_\omega zb \tan\alpha\right] = \frac{1}{2}\gamma_\omega \, . \, d^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

Thus equation (23b) is satisfied when

$$\Sigma\left[\frac{m}{F} \sec\alpha - (W_1 + W_2 + X_n - X_{n+1}) \tan\alpha\right] = 0 \quad . \quad . \quad . \quad . \quad (29)$$

It can be seen, therefore, that for partially submerged slopes an analogous method is obtained by using submerged densities for those parts of the slices which lie below the level of the external free water surface, and by expressing the pore pressures there as an excess above the hydrostatic pressure corresponding to this water level.

PRACTICAL APPLICATION

Only a limited number of stability analyses have so far been carried out in which both the simplified and the more rigorous methods have been used. From these it appears, however, that the major part of the gain in accuracy can be obtained by proceeding with the solution of equations (13) or (22) only as far as the starting value of F given by taking $(X_n - X_{n+1}) = 0$. This avoids the more time-consuming stages of the solution, and provides a convenient routine method in which the time required for the analysis of each trial circle is from one to two hours or about twice that required using the simplified methods.

This may be illustrated by the two following cases.

(i) *Pore pressures set up in a boulder clay fill during construction.*—The cross-section of the dam and the soil properties used in the analysis are given in Fig 3 (a). From laboratory tests and other data it is considered that an average pore pressure equal to 40% of the weight of the overlying soil might be expected, *i.e.* $\bar{B} = 0{\cdot}4$.

For this condition the simplified method indicates a factor of safety of 1·38. The value given by equation (13), with $(X_n - X_{n+1}) = 0$, is 1·53.

The two methods do not lead to the same critical circle. The circle having a factor of safety of 1·38 by the simplified method gives a value of 1·59 using equation (13) with $(X_n - X_{n+1}) = 0$. The introduction of trial values of $(X_n - X_{n+1})$ makes only a small further change in the factor of safety, a value of 1·60 or 1·61 being obtained depending on the assumed distribution of X_n.

The form of tabulation used for the routine solution with $(X_n - X_{n+1}) = 0$ is given in Table 1.

It is useful from the design point of view to know the influence of possible variations in construction pore pressure on the factor of safety, and for this purpose the factor of safety may be plotted directly against average pore pressure ratio as in Fig. 3 (b).

Material	Density lb/cu. ft	c' lb/sq. ft	ϕ'
Clay fill	145	350	37·5°

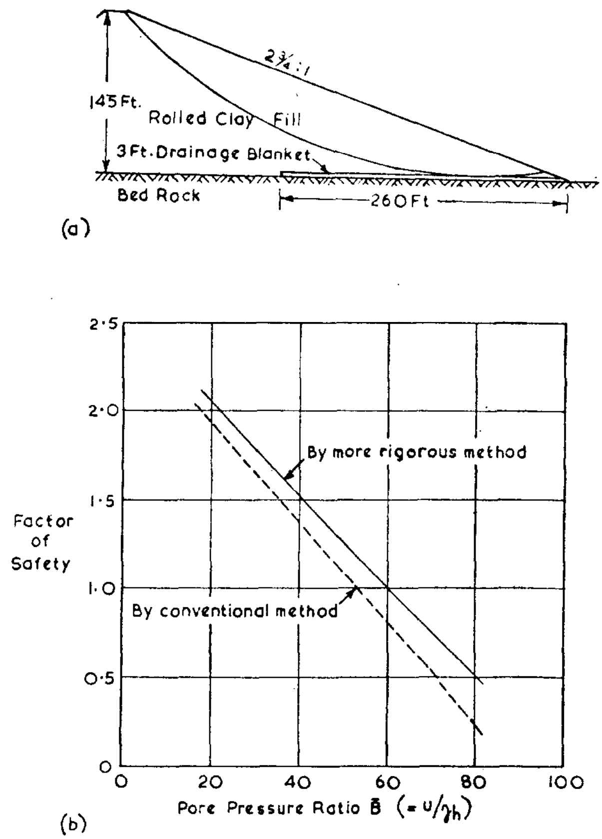

Fig. 3. End of construction case: relationship between factor of safety and pore pressure

(ii) *Pore pressures set up in a moraine fill during a partial rapid drawdown.*—The cross-section of the dam and the soil properties used in the analysis are given in Fig. 4. The basis of the method by which the draw-down pore pressures are calculated has been described by the author elsewhere (Bishop, 1952), and their values are indicated by a line representing stand-pipe levels above different points on the slip surface.

Using the average values of the soil properties (Case I), the simplified method (equation (22*a*)) gives a factor of safety of 1·50, and the more rigorous solution (equation (22), with $(X_n - X_{n+1}) = 0$) gives a value of 1·84.* As before, the two methods lead to different critical circles. The inclusion of trial values of $(X_n - X_{n+1})$ raises the value from 1·84 to 1·92, the value by the simplified method for this particular circle being 1·53. About 80% of the gain in accuracy in this case is thus achieved without introducing the X_n terms.

The form of tabulation for the routine solution in the case of partial submergence is given in Table 2.

CONCLUSION

Errors may be introduced into an estimate of stability by both the sampling and testing procedure, and in many cases an approximate method of stability analysis may be considered adequate. Where, however, considerable care is exercised at each stage, and, in particular, where field measurements of pore pressure are being used, the simplified " slices " method of analysis is not sufficiently accurate.

This is illustrated by the two examples given above. The error is likely to be of particular importance where deep slip circles are involved. This is illustrated by Fig. 5 in which the collected results of the two stability analyses are plotted in terms of the value of the central angle of the arc. It will be seen that the value of the factor of safety given by the simplified method, expressed as a percentage of that given by the more rigorous method [using $(X_n - X_{n+1}) = 0$], drops rapidly as the central angle of the arc increases. This effect is particularly marked for the higher values of excess pore pressure.

* For Case II the corresponding values are 1·14 and 1·48.

Material	Density—lb/cu. ft		Case I		Case II	
	Submerged	Above W.L.	c' lb/sq. ft	ϕ'	c' lb/sq. ft	ϕ'
Moraine	72	135	450	37°	0	40°
Rock	72	118	0	45°	0	40°

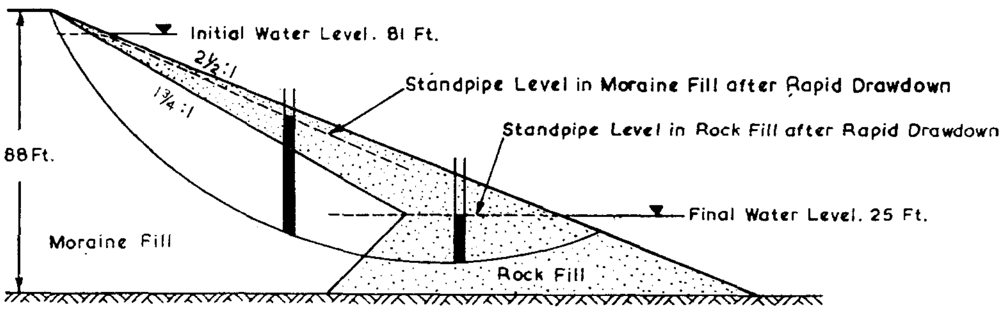

Fig. 4. Drawdown analysis of an upstream slope

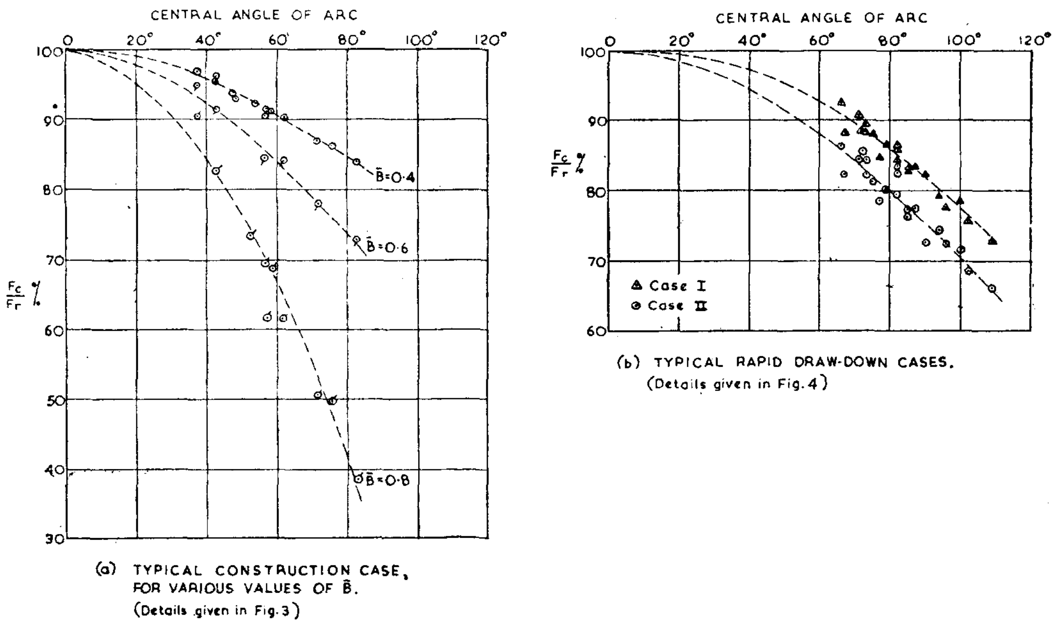

Fig. 5. Influence of central angle of arc on accuracy of conventional method

F_c is factor of safety by conventional method.

F_r is factor of safety by more rigorous method on same sliding surface.

Circle No. 1

Table 1

Slice No.	b ft	h ft	W lb. $\times 10^3$	α	$\sin\alpha$	(1) $W\sin\alpha$	(2) $c'b$	(3) $W(1-\bar{B})\tan\phi'$	(4) $(2)+(3)$	$\sec\alpha$	$\tan\alpha$	(5) $\dfrac{\sec\alpha}{1+\dfrac{\tan\phi'.\tan\alpha}{F}}$		(6) $(4)\times(5)$	
												$F=1{\cdot}4$	$F=1{\cdot}6$	$F=1{\cdot}4$	$F=1{\cdot}6$
1	50	23	167	42·2°	·672	112	17	77	94	1·350	·907	·902	·940	85	88
2	50	54	381	34·6°	·568	217	17	175	192	1·215	·690	·883	·912	169	175
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
8	40	11	64	−9·2°	−·160	−10	14	29	43	1·013	−·162	1·110	1·097	47	47
						777								1240	1262

$$F=\frac{1240}{777}=1{\cdot}60 \qquad F=\frac{1262}{777}=1{\cdot}62$$

Circle No.

Table 2

Slice No.	b ft.	h_r (rock fill)	h_m (moraine)	h_s (submerged)	W_r $\Big\}W_1$	W_m	W_2	(W_1+W_2) lb. $\times 10^3$	α	$\sin\alpha$	(1) $(W_1+W_2)\sin\alpha$	h_1 (excess standpipe ht.)	$u_s=\gamma_\omega\,.\,h_1$	$b\,.\,u_s$	$(W_1+W_2-b\,.\,u_s)$	(2) $(W_1+W_2-b\,.\,u_s)\tan\phi'$	(3) $c'\,.\,b$	(4) $(2)+(3)$	$\sec\alpha$	$\tan\alpha$	(5) $\dfrac{\sec\alpha}{1+\dfrac{\tan\phi'.\tan\alpha}{F}}$	(6) $(4)\times(5)$
1 2 etc.																						

$$F=\frac{\Sigma(6)}{\Sigma(1)}$$

In these cases the use of the modified analysis outlined above is to be recommended, which, if carried only as far as the $(X_n - X_{n+1}) = 0$ stage, can readily be used for routine work.

Little field evidence is yet available for checking the overall accuracy of the stability calculation except in those cases where the " $\phi = 0$ " analysis has been applicable, and this emphasizes the need for failure records complete with pore-pressure data for both natural slopes and trial embankments.

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