Classification of Landslides

- **Falls**

  involve <u>immediate separation</u> of the falling material from parent rock or soil mass

- **Slide**

  moving material <u>remains in contact</u> and movement takes place along discrete shear surfaces

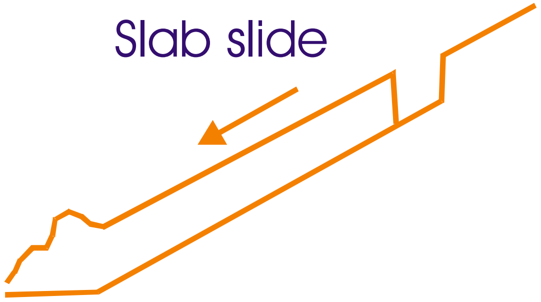
- **Flows**

  material becomes <u>disaggregated</u> and movement occurs without necessarily forming discrete shear surfaces

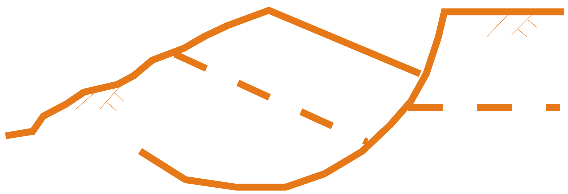
Large landslides often change from one type to another as they progress

Slides

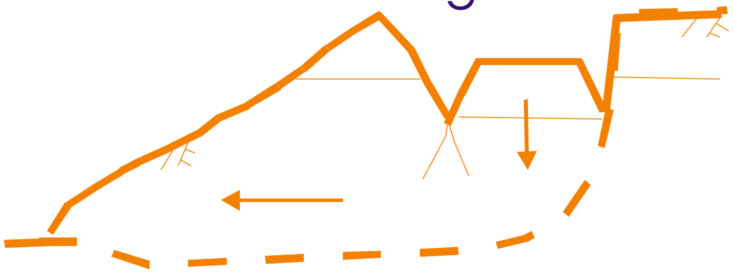
- Translational



- Rotational

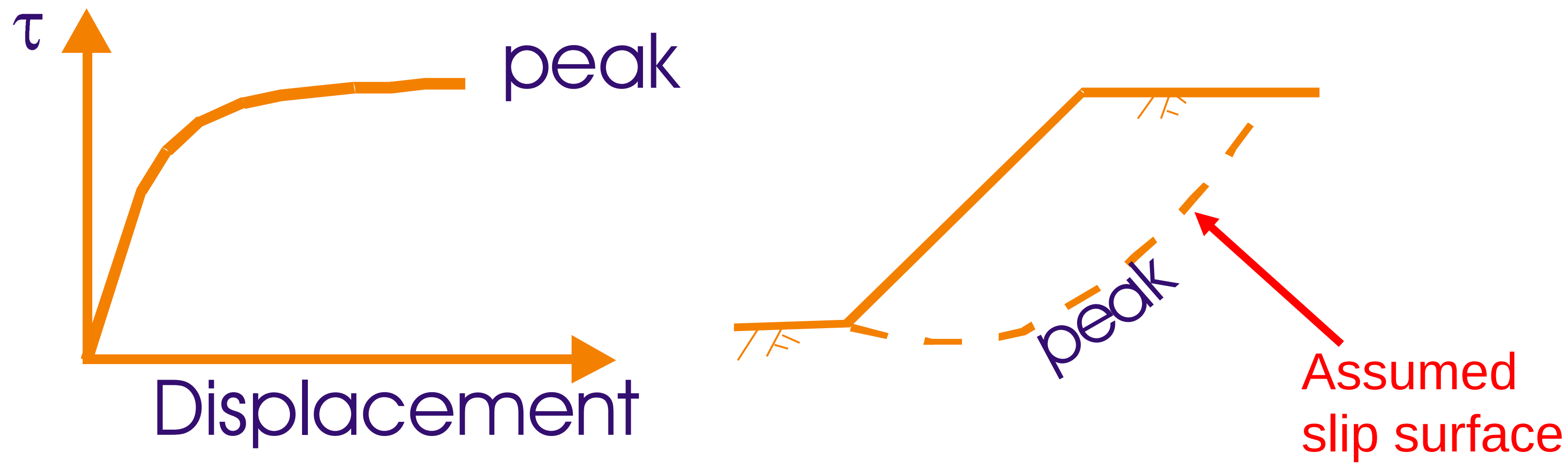


- Compound



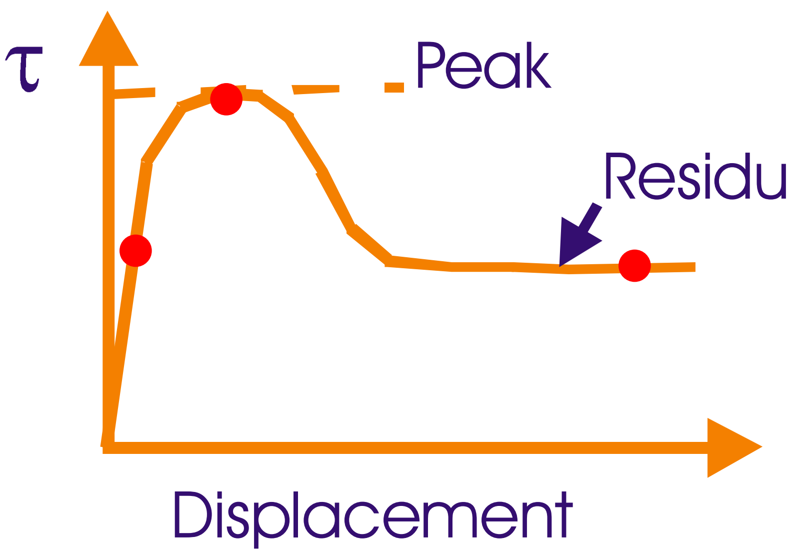
Geotechnical Classification of Landslides

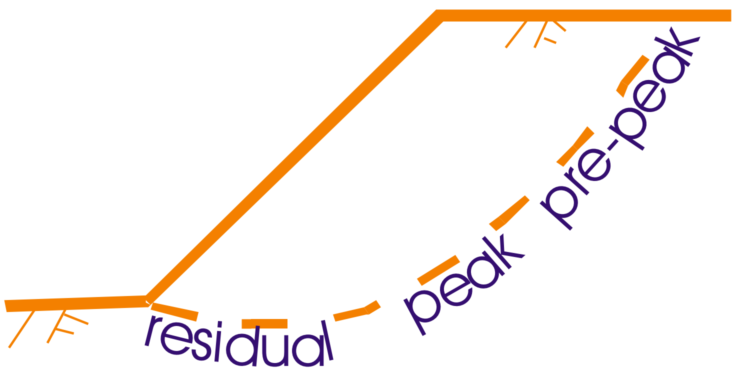
- “First time slides” in <u>low plasticity</u> soils



Geotechnical Classification of Landslides

- “First time slides” in <u>high plasticity</u> soils



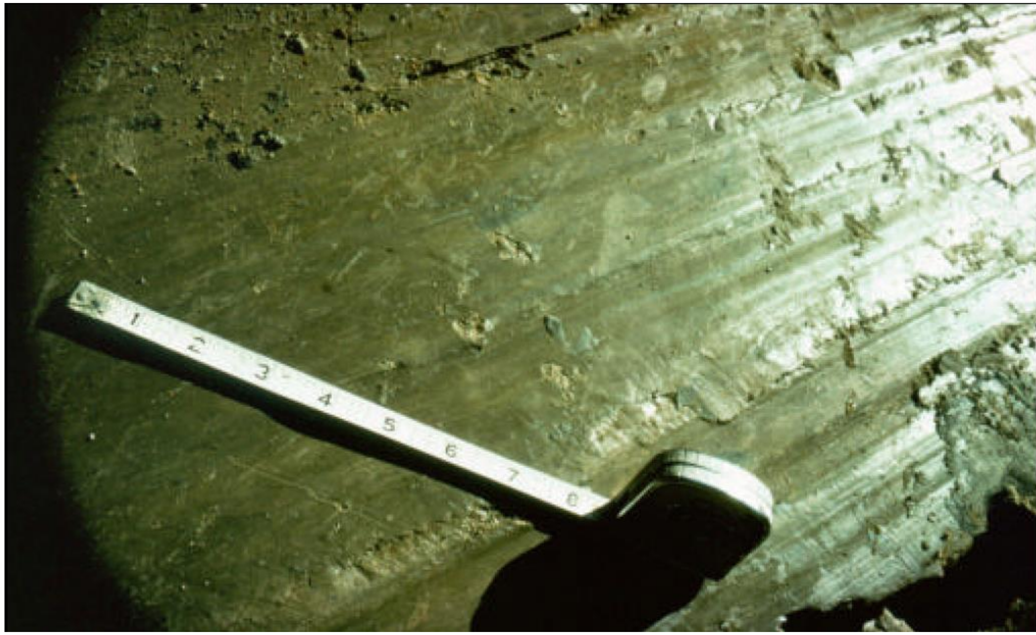


<u>Progressive Failure</u>: Failure does not occur on the whole surface at the same time

Geotechnical Classification of Landslides

- "First time slides"
- Slides on Pre-existing Shears

Usually involve
RESIDUAL strength



Shear surface exposed in a trial pit
From Bromhead (1992)

Pore Water Pressure Conditions

- Short-term failures
- Long-term failures
- Intermediate-term failures

How long does it take to reach pore pressure equilibrium????

It depends!!

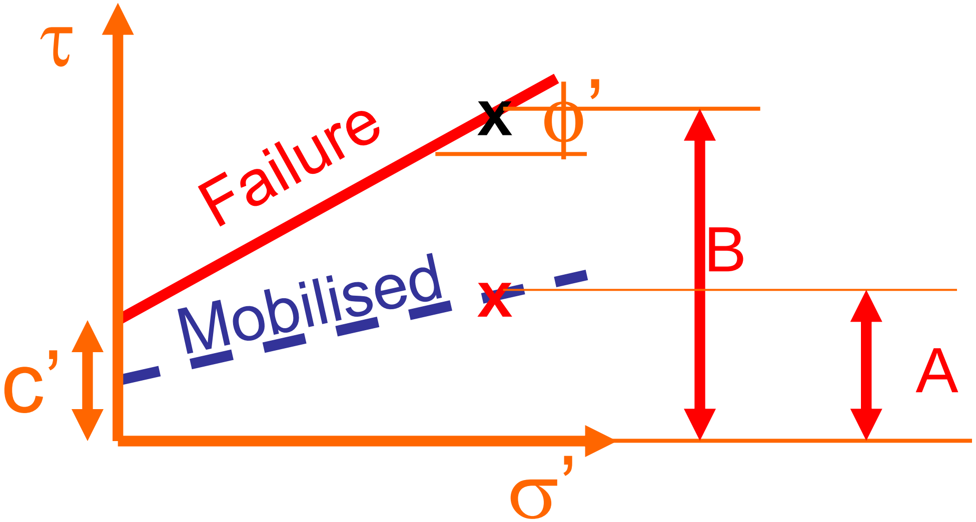
e.g.

- 10 days in Bangkok clay for 4m excavation depth
- 1 month in Mexico city clay for 4.5-8m excavation depth
- London Clay: ~ 50 years for 6-12m deep cuts
  ~ 2000 years for 44m high cliffs at Warden Point

Limit Equilibrium

- A mechanism of collapse is assumed involving a failure surface that may be planar, curved or some combination of these
- A failure criterion (in terms of shear strength parameters, either total or effective) holds everywhere along the failure surface
- Only the global equilibrium of the rigid blocks of soil between the failure surfaces and the boundaries of the problem is considered
- The internal distribution within the blocks is not considered

Factor of Safety



Effective Stress Analysis

F=B/A

Shear strength at failure

B= c'+σ' tan φ'

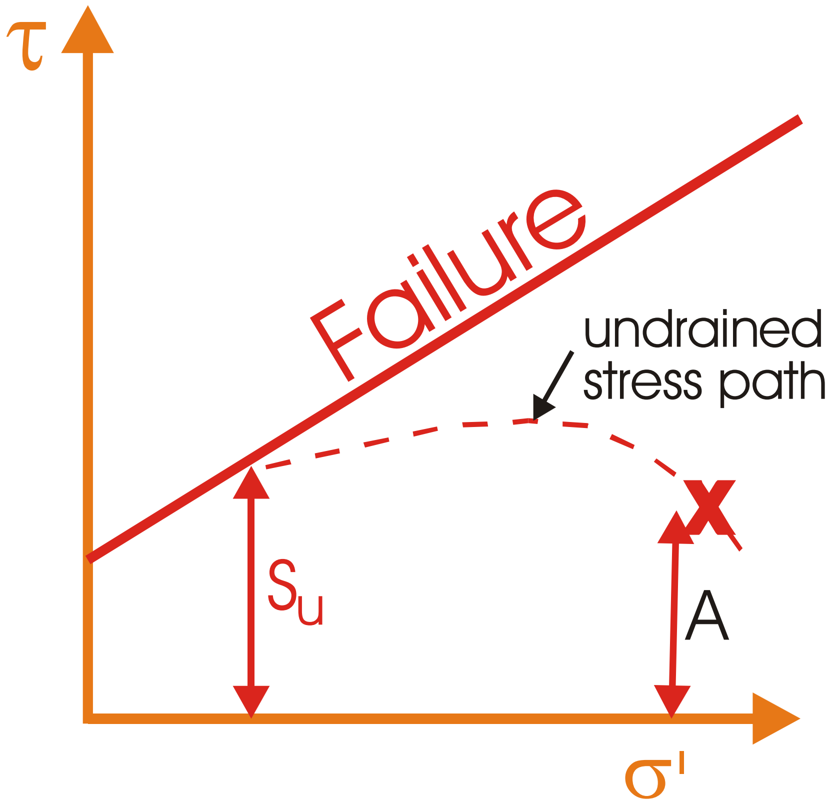
Mobilised Shear strength

A= c'_m+ σ' tan ϕ'_m

$c'_m = c'/F$

$\tan \phi'_m = \tan \phi'/F$

Factor of Safety



Total Stress Analysis

$$F = S_u / A$$

$$\boxed{\tau_{mob} = S_u / F}$$

<u>Important Assumption</u>:
FoS assumed to be constant along the slip surface

Factor of Safety

Other definitions

$$F = \frac{\sum \text{Resisting forces}}{\sum \text{Disturbing forces}}$$

Planar failure surfaces

$$F = \frac{\sum \text{Resisting moments}}{\sum \text{Overturning moments}}$$

Rotational failure surfaces

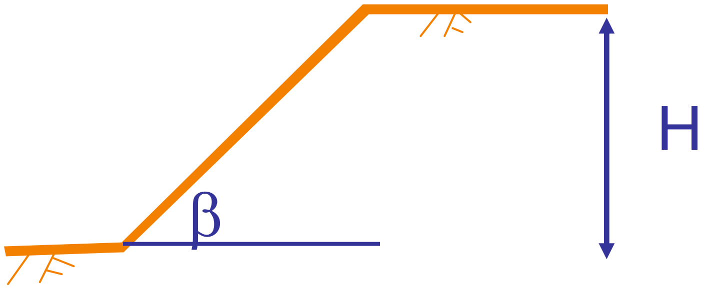
In most cases the FoS on <u>shear strength</u> is adopted

Objectives of Limit Equilibrium Analysis

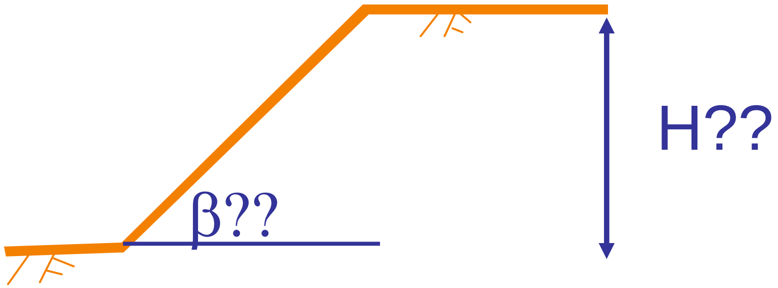
- Knowing the geometry of the slope i.e. H, β

  ↓

  FoS????

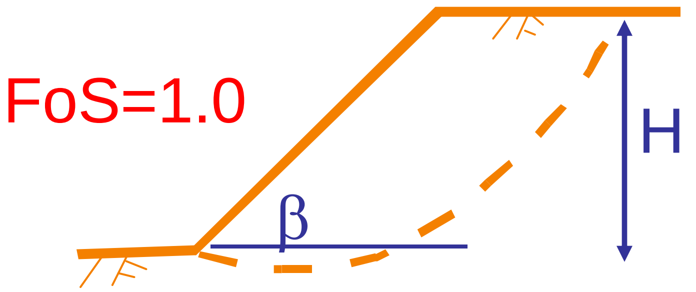


- For a given FoS to design a slope



- Back analysis of existing slopes

  S_u?? c', φ'??



Limit Equilibrium Procedures

- Methods for Planar Movements
- Methods for Rotational Movements
- Methods for Non-rotational Movements

↕

Slides

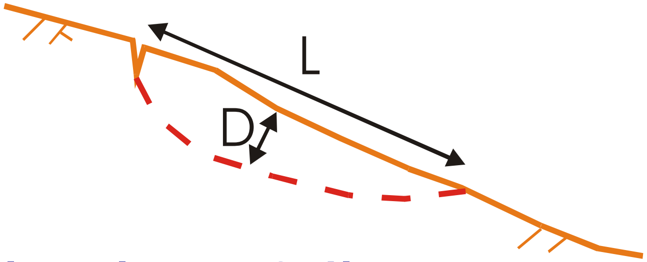
- Translational
- Rotational
- Compound

Planar Movements

Infinite Slope Analysis

<u>Applications</u>

- Elongated landslides, i.e. The ratio of depth to failure surface to length of failure zone is relatively small (D/L<10%)

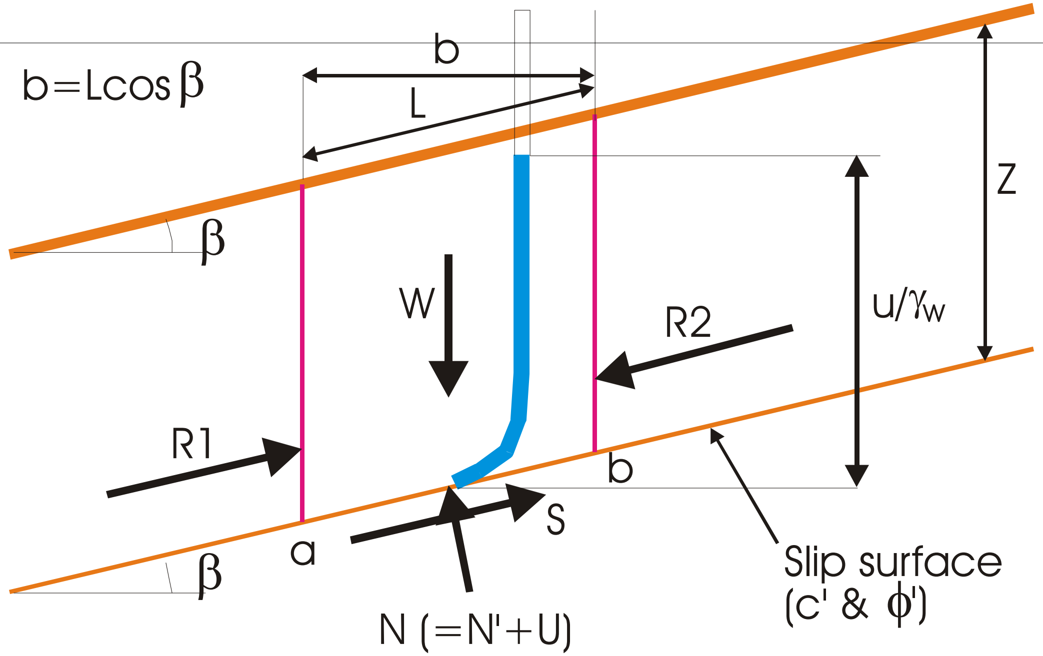


- Translational failures along a single plane failure surface parallel to slope surface

<u>Additional Assumptions</u>

- The slope is infinitely long
- The failure surface is parallel to the ground surface
- Uniform pwp conditions exist

Infinite Slope Analysis



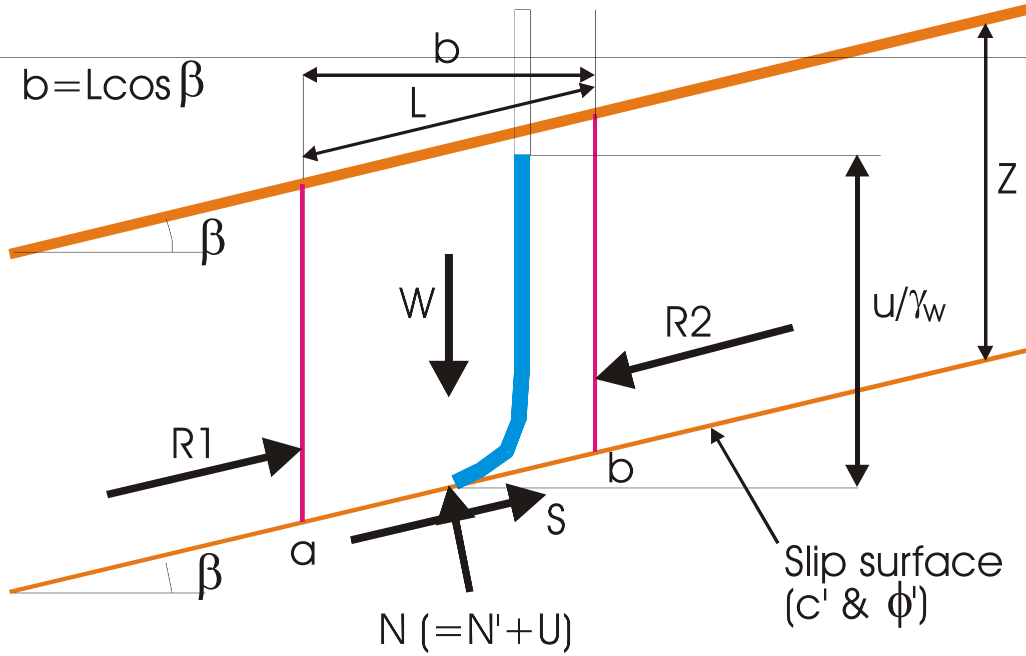
Available shear strength along the failure plane:

$$\tau_{max} = c' + \left(\frac{N'}{L}\right)\tan\phi'$$

Mobilised shear strength along the failure plane:

$$\tau_{mob} = \frac{S}{L}$$

Infinite Slope Analysis



Resolving forces perpendicular to the slip surface

$$N' = W\cos\beta - U$$

Resolving forces parallel to the slip surface

$$S = W\sin\beta$$

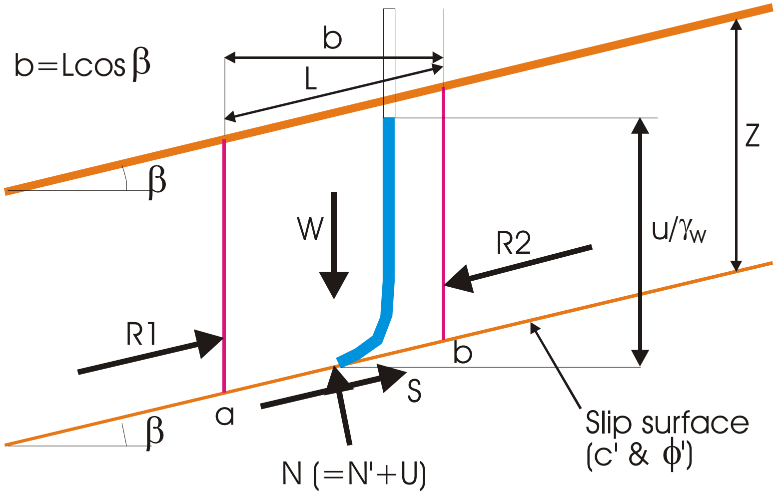
Infinite Slope Analysis

$$\tau_{max} = c' + \left(\frac{N'}{L}\right)\tan\phi'$$

$$\tau_{mob} = \frac{S}{L}$$

$$N' = W\cos\beta - U$$

$$S = W\sin\beta$$



$$\boxed{F = \frac{\tau_{max}}{\tau_{mob}} = \frac{c' + \left(\gamma Z\cos^2\beta - u\right)\tan\phi'}{\gamma Z\sin\beta\cos\beta}}$$

Infinite Slope Analysis

$$F = \frac{\tau_{max}}{\tau_{mob}} = \frac{c' + \left(\gamma Z \cos^2 \beta - u\right) \tan \phi'}{\gamma Z \sin \beta \cos \beta}$$

Special cases:

- Dry cohesionless soil, c'=u=0

$$F = \frac{\tan\varphi'}{\tan\beta}$$

FoS is independent of the depth of the slip surface

- Saturated, cohesionless soil, c'=0

$$F = \frac{\gamma'}{\gamma} \frac{\tan\phi'}{\tan\beta} \approx \frac{1}{2} \frac{\tan\phi'}{\tan\beta}$$

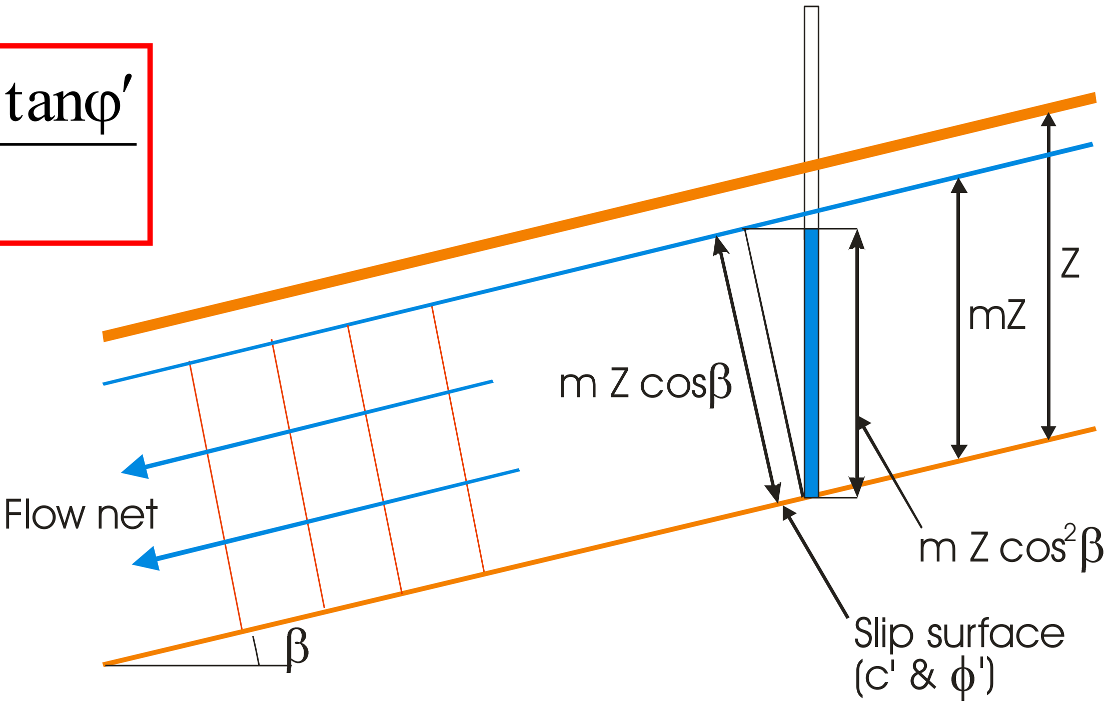
FoS is still independent of the depth of the slip surface, but it is approximately halved

Infinite Slope Analysis

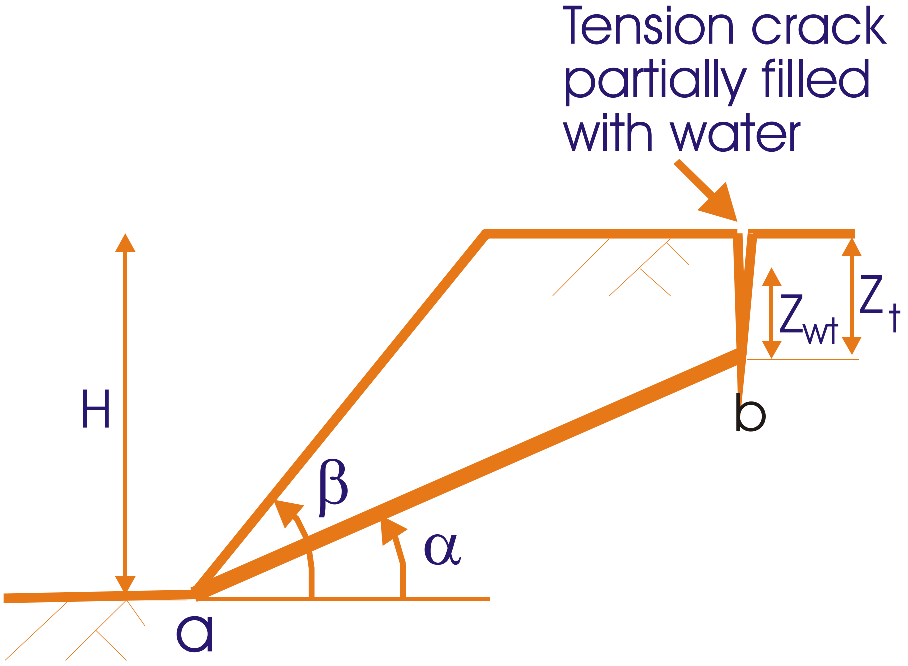
Steady seepage parallel to the ground surface with phreatic surface at vertical height m×Z above the slip surface

$$u = \gamma_w \, m Z \cos^2\beta$$

$$\boxed{F = \frac{c' + (\gamma - m\gamma_w) Z \cos^2\beta \tan\varphi'}{\gamma Z \sin\beta \cos\beta}}$$

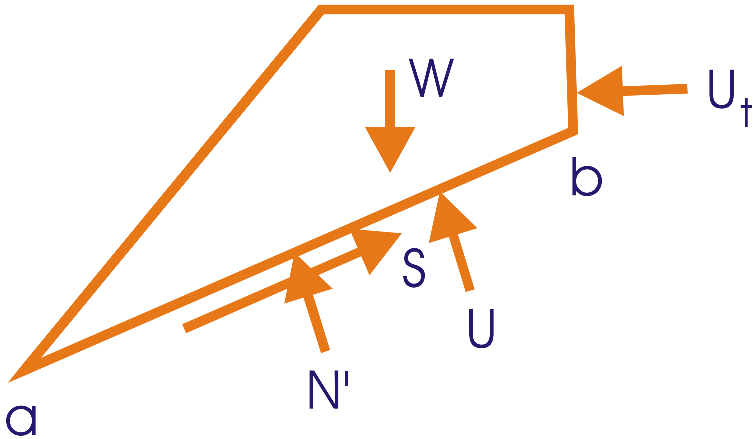


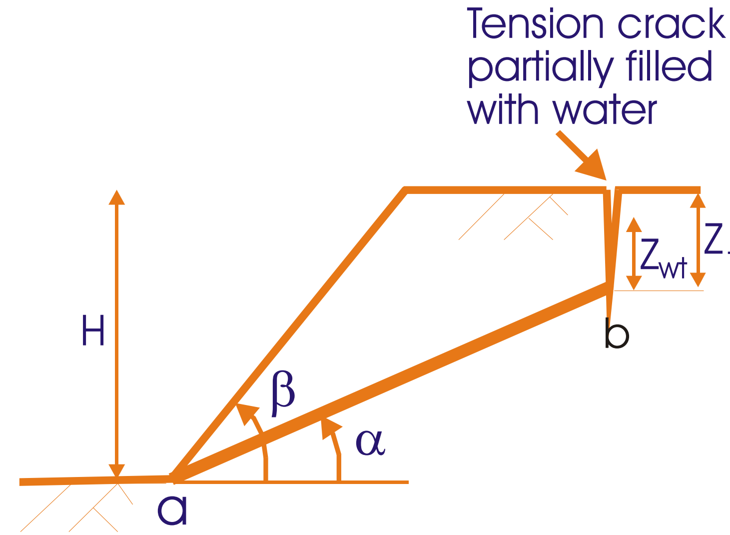
Planar surface analysis



Good approximation for slopes with a thin layer of soil that has low strength in comparison to overlying materials.

Planar surface analysis





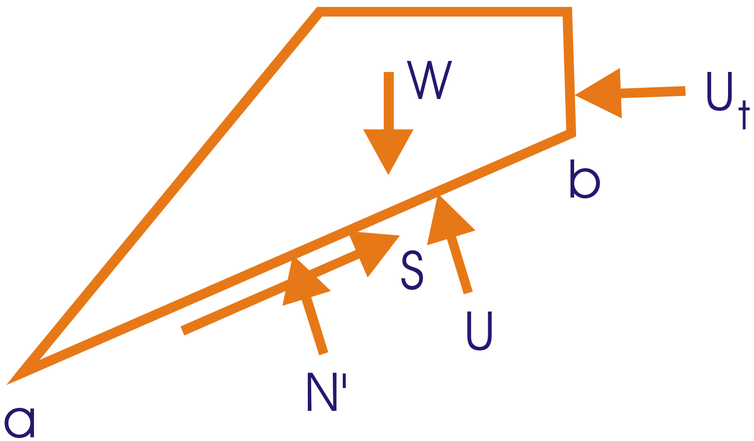
Resolving forces perpendicular to the slip surface

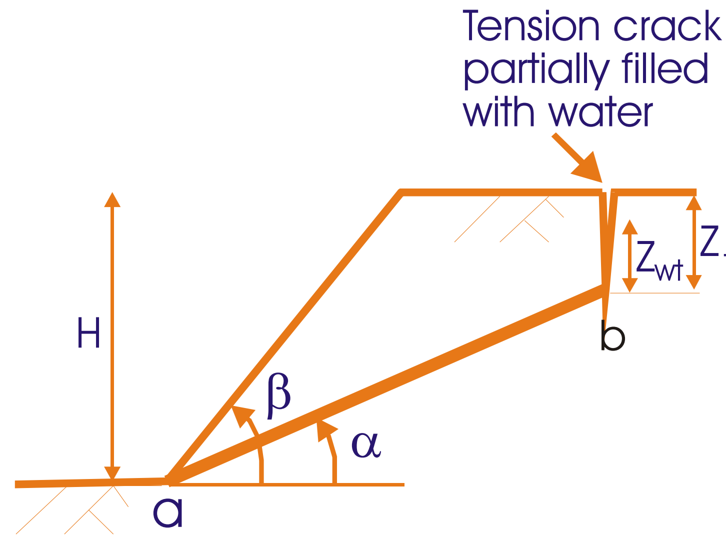
$$N' = W\cos\alpha - U - U_t \sin\alpha$$

Resolving forces parallel to the slip surface

$$S = W\sin\alpha + U_t \cos\alpha$$

Planar surface analysis





$$N' = W\cos\alpha - U - U_t \sin\alpha \longrightarrow \tau_{max} = c' + \left(\frac{N'}{L}\right)\tan\phi'$$

$$S = W\sin\alpha + U_t \cos\alpha \longrightarrow \tau_{mob} = \frac{S}{L}$$

$$\boxed{F = \frac{L\,c' + (W\cos\alpha - U - U_t \sin\alpha)\tan\phi'}{W\sin\alpha + U_t \cos\alpha}}$$

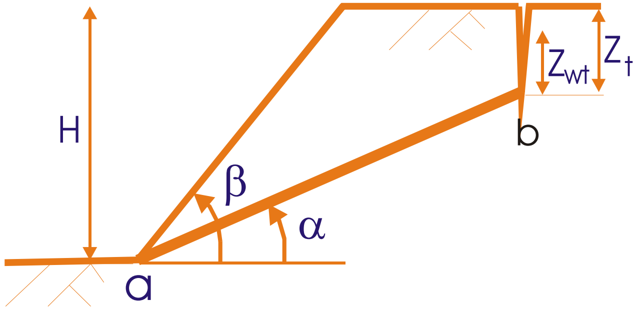
Planar surface analysis

$$\boxed{F = \frac{L\,c' + \left(W\cos\alpha - U - U_t\sin\alpha\right)\tan\phi'}{W\sin\alpha + U_t\cos\alpha}}$$

Special cases:

➢ Dry cohesionless soil, $c'=U=U_t=0$

$$F = \frac{\tan\phi'}{\tan\alpha}$$



Slope reaches limiting equilibrium when $\alpha=\varphi'$. Stability is independent of height, H, depending only on geometry and not scale.

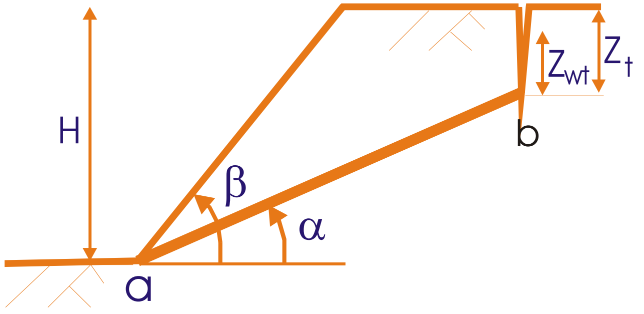
Planar surface analysis

$$\boxed{F = \frac{L\,c' + \left(W\cos\alpha - U - U_t\sin\alpha\right)\tan\phi'}{W\sin\alpha + U_t\cos\alpha}}$$

Special cases:

- Total Stress Analysis

$$\boxed{F = \frac{S_u\,L}{W\sin\alpha + U_t\cos\alpha}}$$



Planar surface analysis

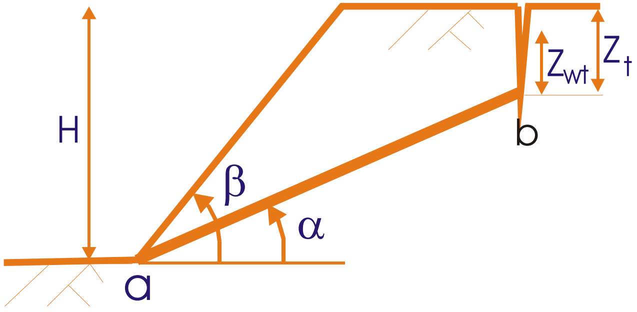
$$F = \frac{L\,c' + \left(W\cos\alpha - U - U_t \sin\alpha\right)\tan\phi'}{W\sin\alpha + U_t \cos\alpha}$$

Special cases:

- Water level in tension crack is Z_{wt} & pwp reduces linearly along the slip surface from $\gamma_w Z_{wt}$ at **b** to zero at **a**. U_t and U are given by:

$$U_t = \frac{1}{2}\gamma_w Z_{wt}^2$$

$$U = \frac{1}{2}\gamma_w Z_{wt} L$$



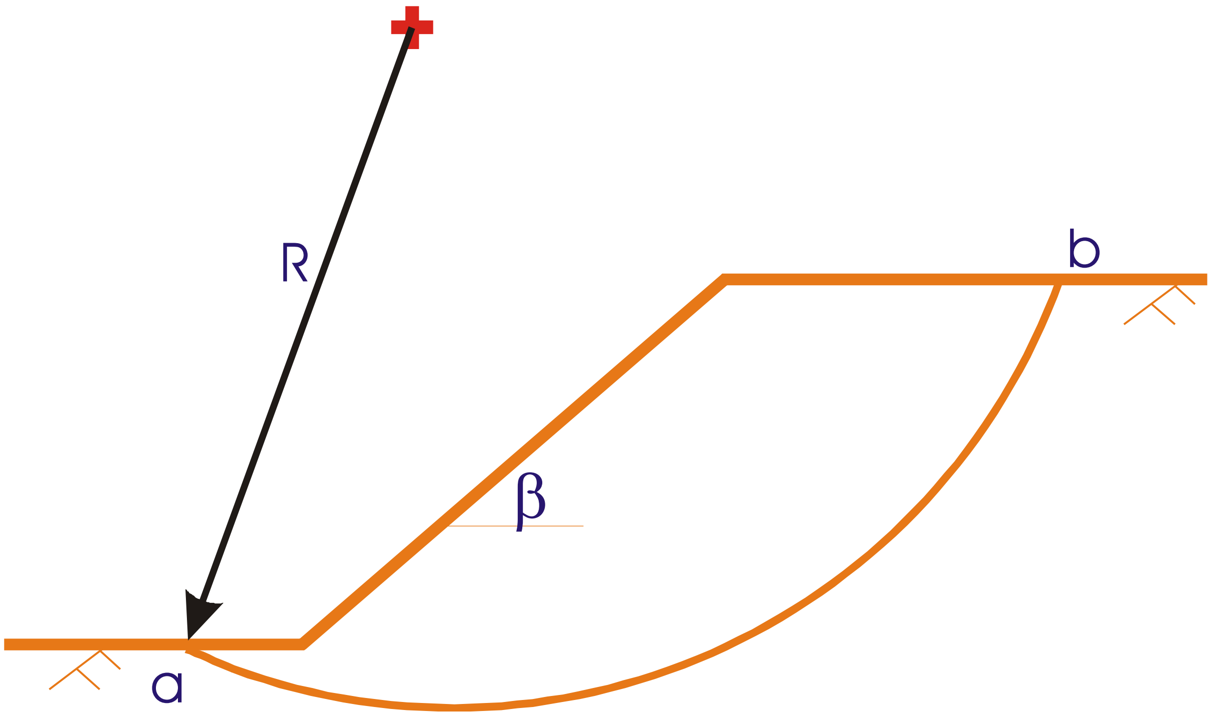
Planar Movements

- Only force equilibrium has been used to obtain the above solutions.
- For moment equilibrium to be satisfied the forces acting on the failing block must give no net moment.
- As the distribution of normal stress σ_n' along the slip surface **ab** has not been considered the point of action of N' is unknown.
- Application of moment equilibrium would enable this to be determined.
- However, it would not affect the solutions obtained above.

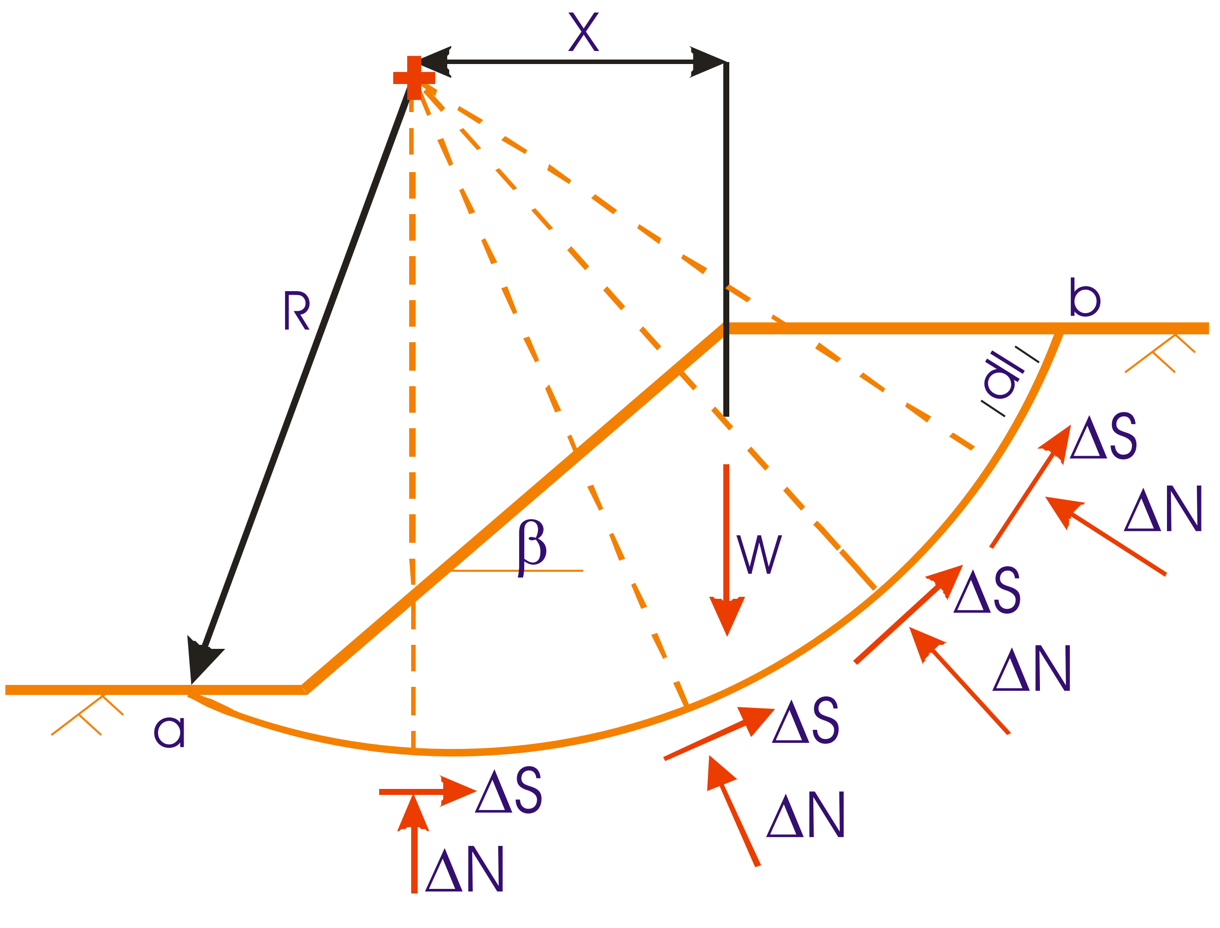
Rotational Movements

Circular Arc (or $\varphi_u = 0$) Method

- The failure surface is assumed to be an arc of a circle
- The shear strength is defined by the undrained strength



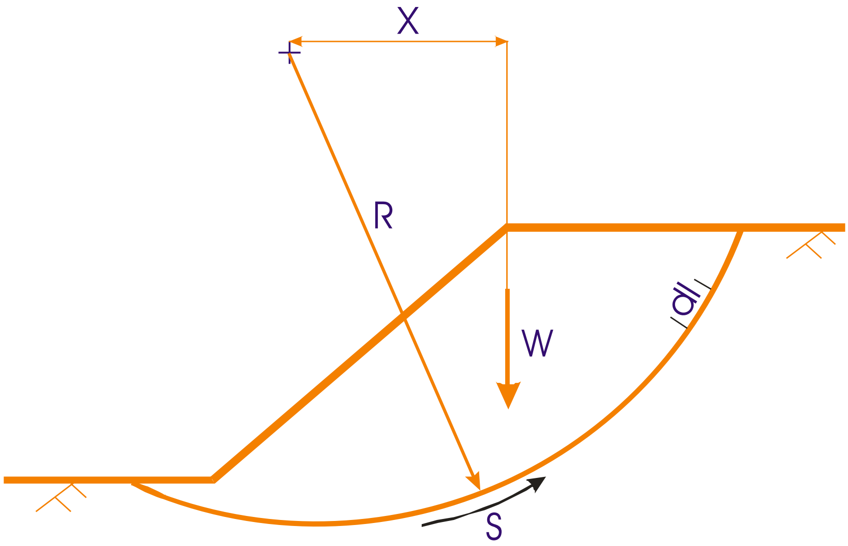
Circular Arc (or φ_u=0) Method



Circular Arc (or φ_u=0) Method

Resisting moment

$$M_R = S R = R \frac{S_u}{F} \sum dl = R \frac{S_u L}{F}$$



Disturbing moment

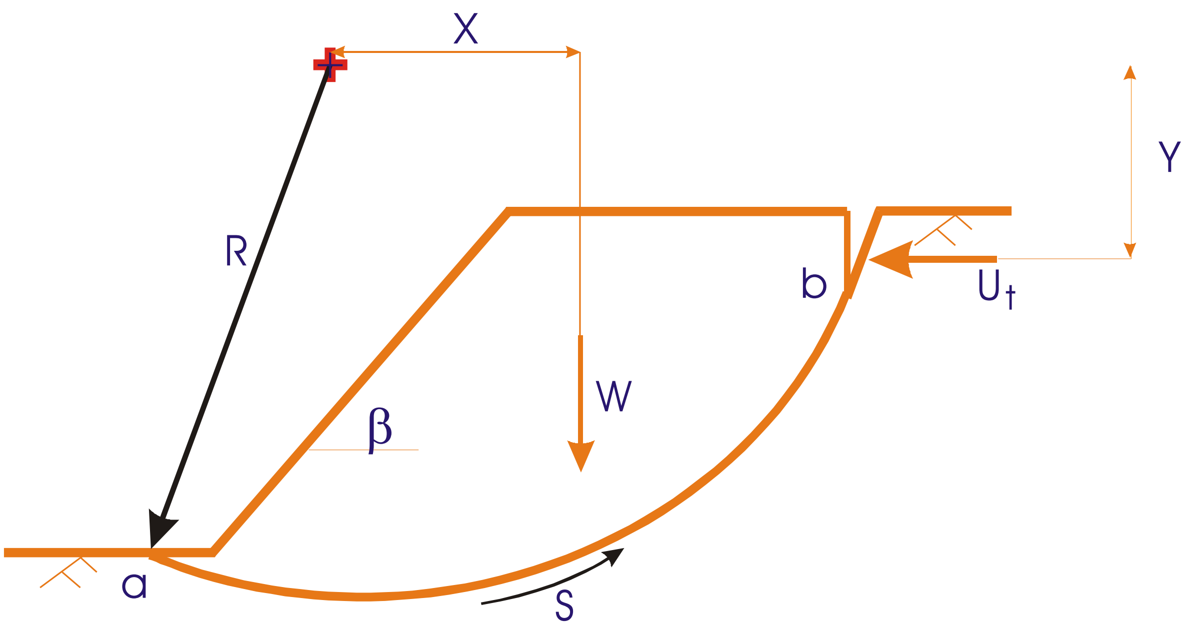
$$M_D = W X$$

Equating the above two equations and rearranging an expression for the FoS is obtained:

$$F = \frac{R S_u L}{W X}$$

Circular Arc (or φ_u=0) Method

- The above result can also be obtained by applying the Upper Bound (unsafe) theorem of plasticity.
- Can include tension cracks at the crest of the slope



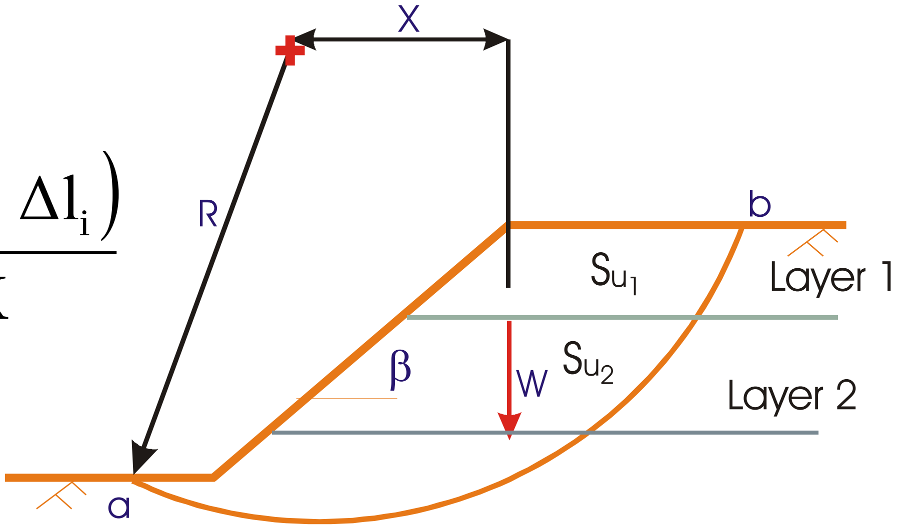
Circular Arc (or $\varphi_u = 0$) Method

- The above result can also be obtained by applying the Upper Bound (unsafe) theorem of plasticity.
- Can include tension cracks at crest of slope
- Can deal with contained water at toe (i.e. if a canal or reservoir embankment)

Circular Arc (or φ_u=0) Method

- The above result can also be obtained by applying the Upper Bound (unsafe) theorem of plasticity.
- Can include tension cracks at crest of slope
- Can deal with contained water at toe (i.e. if a canal or reservoir embankment)
- Can include layered soils or soils in which S_u varies spatially.

$$F = \frac{R\,S_u\,L}{W\,X} \longrightarrow F = \frac{R\sum\left(S_{u_i}\,\Delta l_i\right)}{W\,X}$$

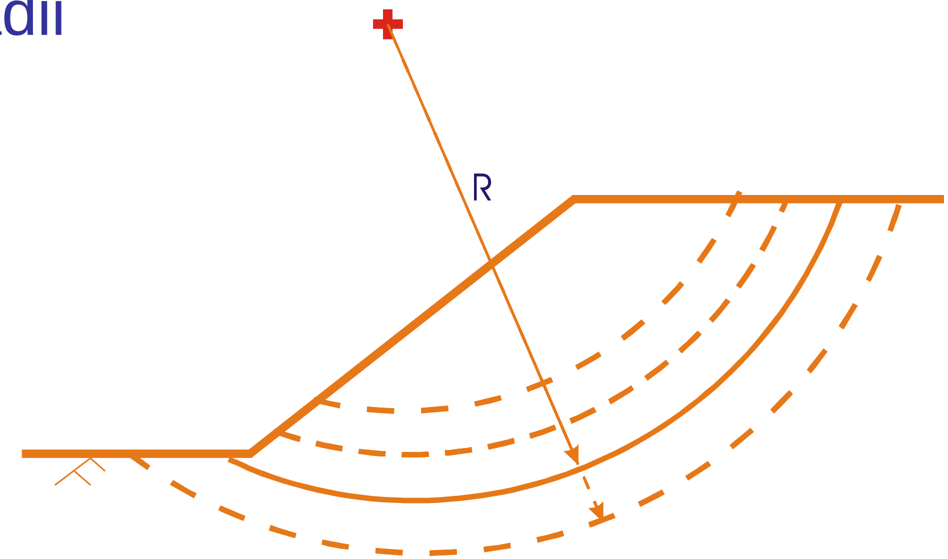


Circular Arc (or φ_u=0) Method

- The above result can also be obtained by applying the Upper Bound (unsafe) theorem of plasticity.
- Can include tension cracks at crest of slope
- Can deal with contained water at toe (i.e. if a canal or reservoir embankment)
- Can include layered soils or soils in which S_u varies spatially.
- A search must be made for the most critical slip surface (i.e. one with lowest factor of safety)

Critical Slip Surface

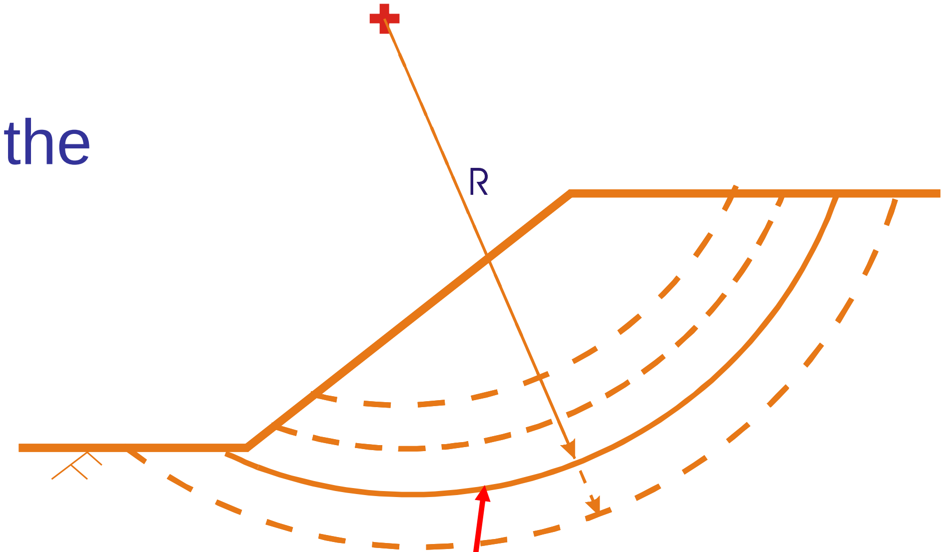
Step 1: Choose a centre of rotation and consider a series of slip surfaces of different radii

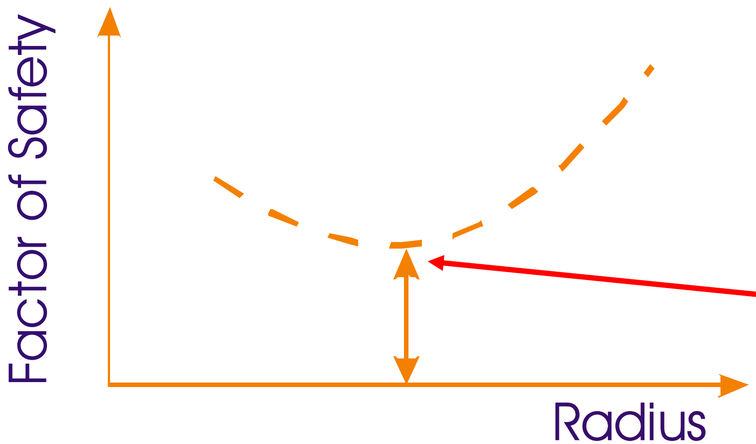


Critical Slip Surface

Step 1: Choose a centre of rotation and consider a series of slip surfaces of different radii

Step 2: Plot the FoS against the radius

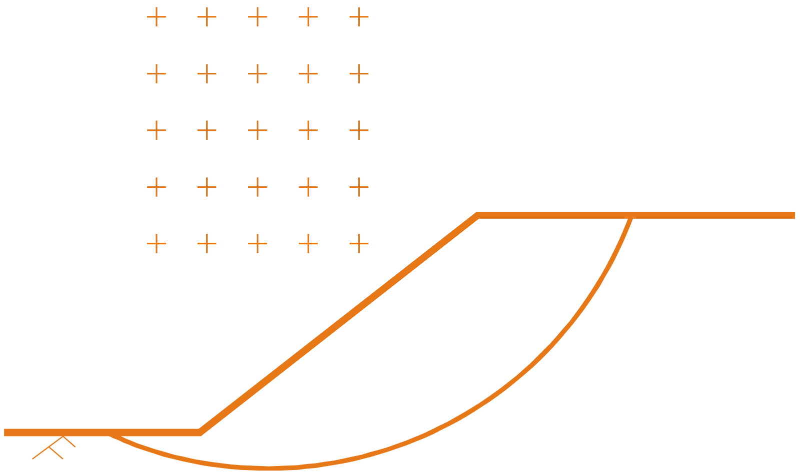




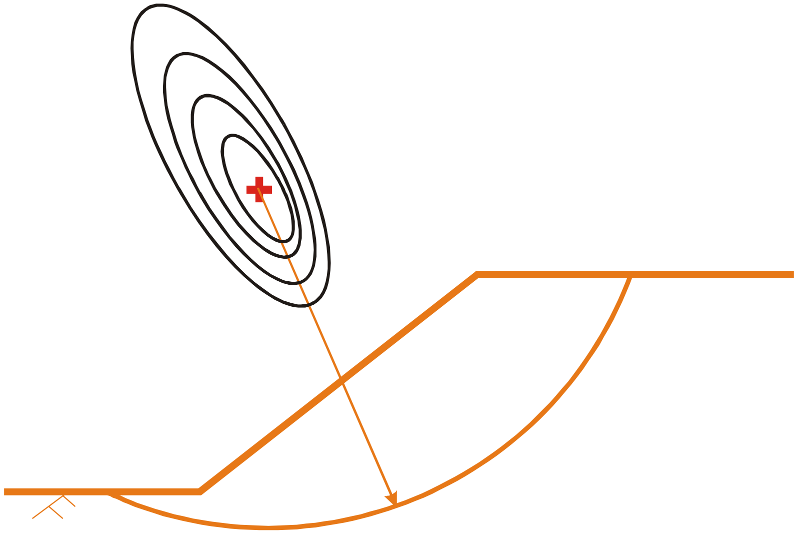
Critical circle
for a specific centre

Critical Slip Surface

<u>Step 3</u>: Repeat steps 1 and 2 for an array of centres



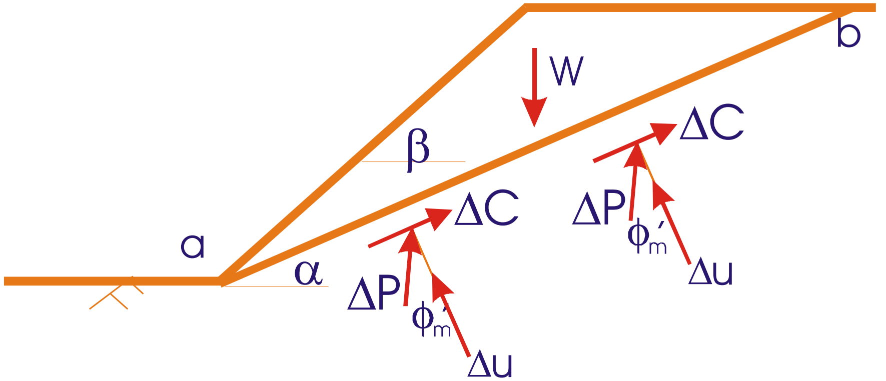
<u>Step 4</u>: Draw contours of FoS to locate the overall worst critical circle



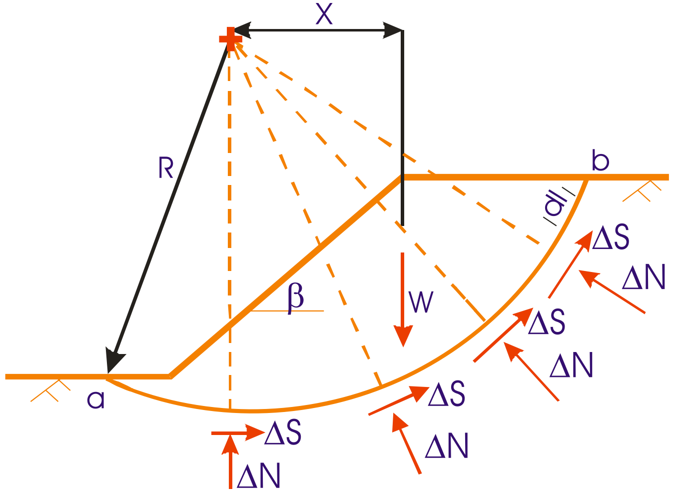
Statically determinate problems

- For planar slip surfaces, the resultant direction of **$\sum\Delta P$** is known

$$\Delta S = \Delta C + \Delta P \sin\phi'_m$$



- For total stress analysis using circular slip surfaces the resultant direction of **$\sum\Delta P$**, passes through the centre of the slip surface



Effective Stress analysis

ΔC forces act circumferentially at a constant radius R

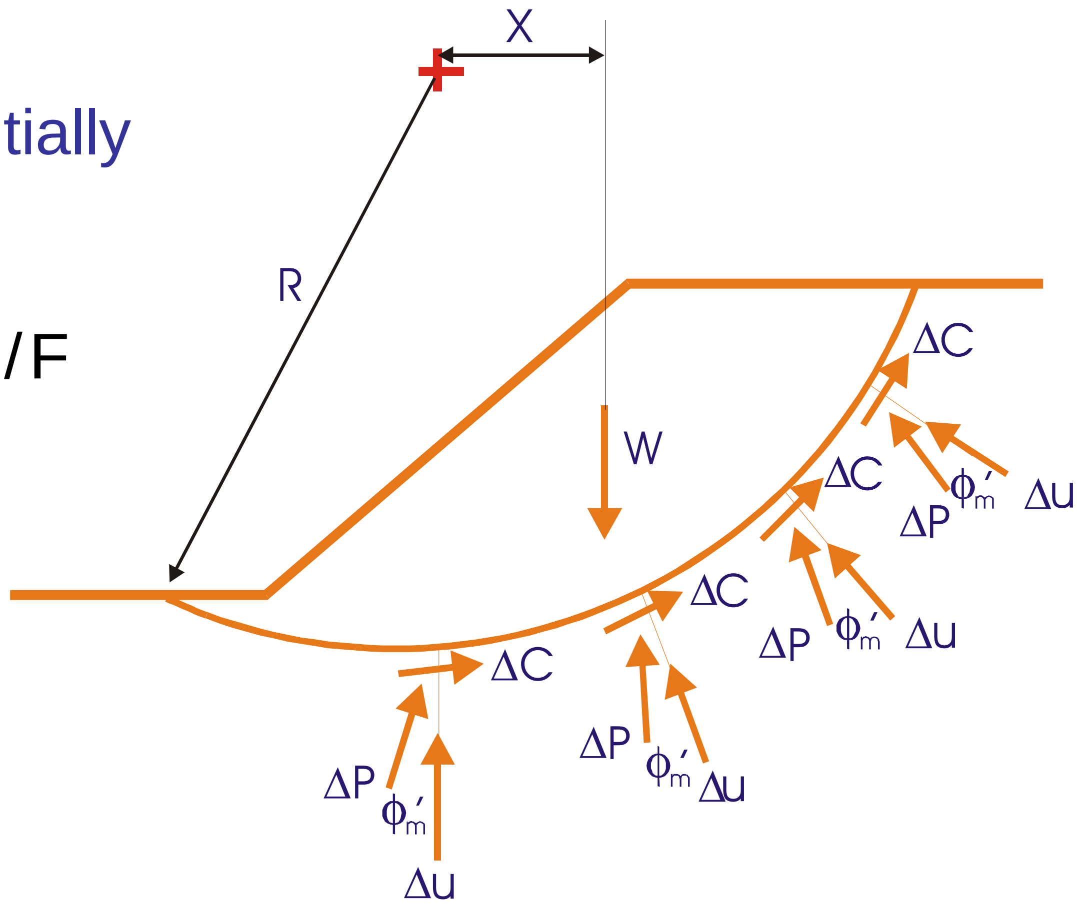
$$M_{\Delta C} = R\sum \Delta C = R\sum c' dl / F$$

$$\Delta P = dl\left(\frac{\sigma_n - u}{\cos\phi'_m}\right)$$

$$\phi'_m = \arctan\left(\frac{\tan\phi'}{F}\right)$$

$$\Delta S = \Delta C + \Delta P \sin\phi'_m$$

Magnitude, direction and line of action of the resultant force **∑ΔP** is unknown



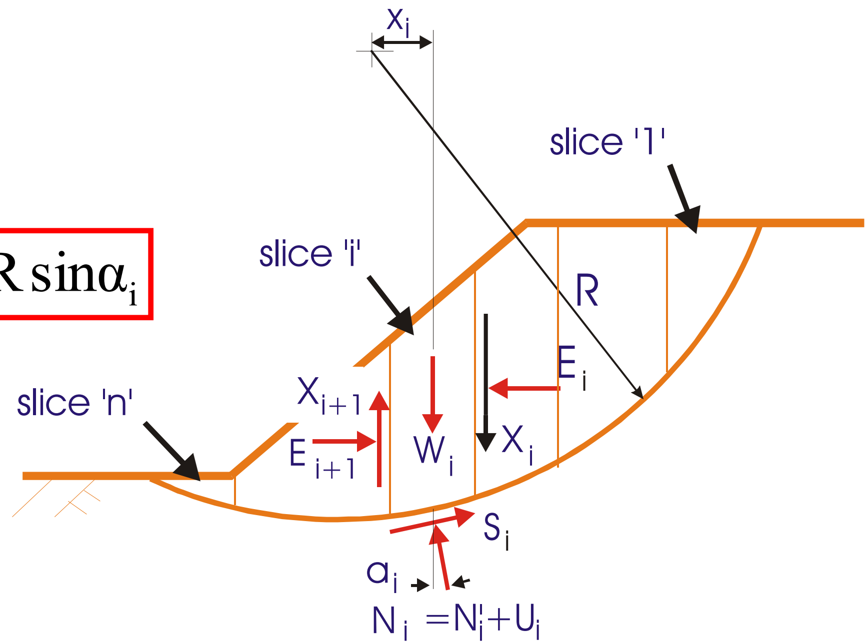
The problem is statically <u>indeterminate</u>

Effective Stress analysis

- For effective stress analysis using circular slip surfaces, an <u>additional</u> assumption is needed to render the problem *statically determinate*
- Several different alternatives available
  - Log-spiral method
  - Friction circle method
  - Method of Slices

Method of Slices

$$\boxed{x_i = R\sin\alpha_i}$$

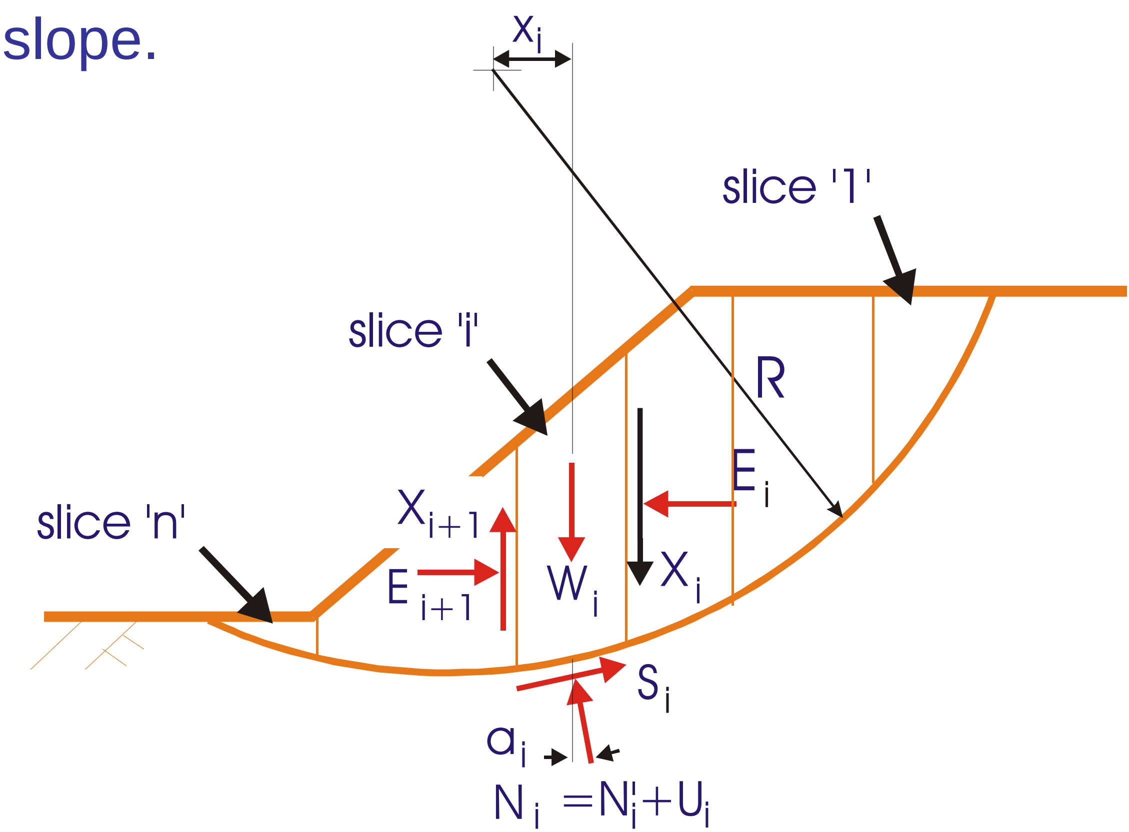


- This is essentially a numerical approach in which the sliding soil mass is divided into 'n' slices
- Equilibrium is then applied to each slice in turn
- The method is formulated so that the factor of safety is calculated for a known slope geometry

Method of Slices

Known:

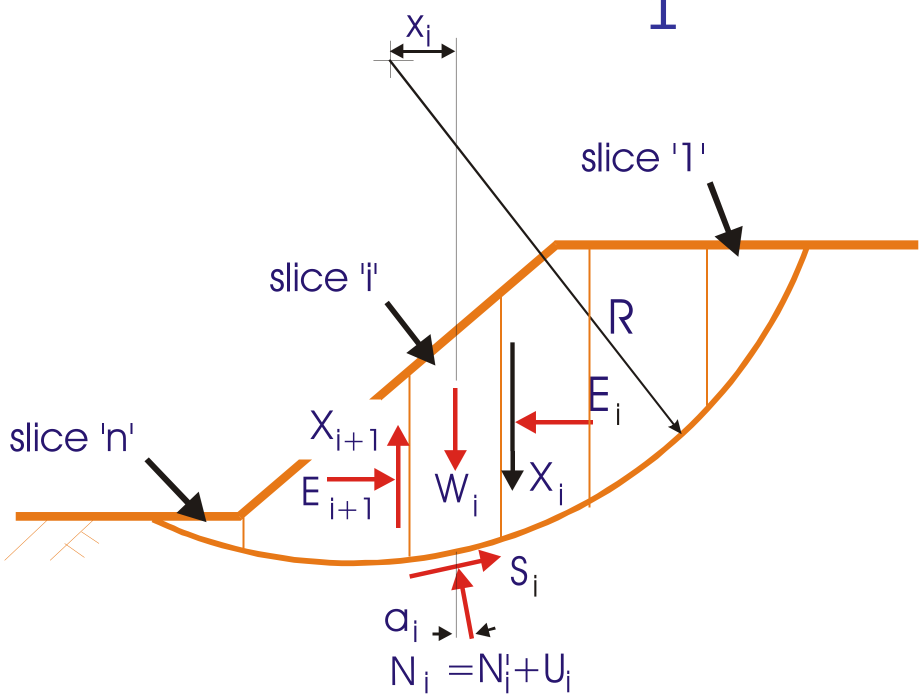
- Geometry of the slope
- Bulk unit weight, γ, and strength parameters, c' and φ' (or S_u) for the soil
- Distribution of pwp in the slope.



Method of Slices

Unknown:	Number
•Magnitude N (or N')	n
•Magnitude of shear force, S	n
•Magnitude of inter-slice force, E (or E')	n-1
•Magnitude of interslice force, X	n-1
•Point of application (i.e. line of action) of N (or N')	n
•Point of application (i.e. line of action) of E (or E')	n-1
•Factor of safety, F	1

$\boxed{\text{Total unknown } \sum = \mathbf{6n\text{-}2}}$



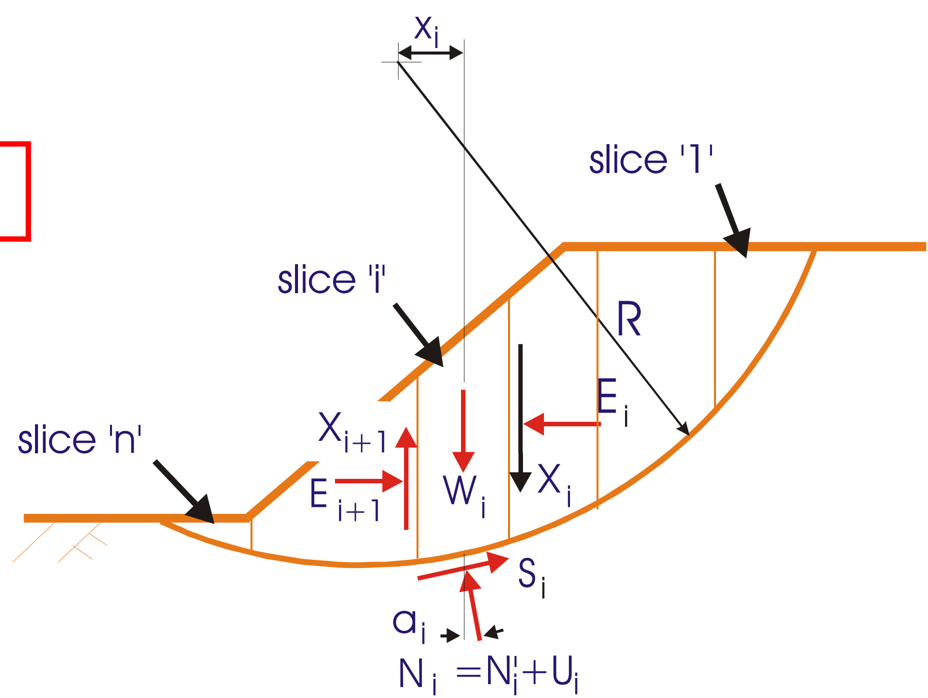
Method of Slices

Total unknown: $\sum$ **= 6n-2**

Equations:	Number
•Force equilibrium (2 per slice)	2n
•Moment equilibrium (1 per slice)	n
•Failure criterion, (1 per slice)	n

Total number of equations: $\sum = 4n$

STATICALLY INDETERMINATE



Method of Slices

Total unknown: $\sum$ **= 6n-2**

Total number of equations: $\sum = 4n$

2n-2 additional independent assumptions are needed

- There are many (infinite ?) different combinations of assumptions that can be made
- Consequently numerous methods of slices have been developed
- Some methods actually make too many assumptions (i.e. more than 2n-2) and, consequently- either do not satisfy all 3 equations of equilibrium or have to iterate to obtain a consistent solution.

Conventional Method of Slices

<u>Assumptions</u>: Number

Neglects the inter-slice forces (i.e. X=E=0):

- Magnitude of inter-slice force, E (E=0) n-1
- Magnitude of inter-slice force, X (X=0) n-1

The normal force, N acts through the centre of the base of the slice:

- Point of application (i.e. line of action) of N n

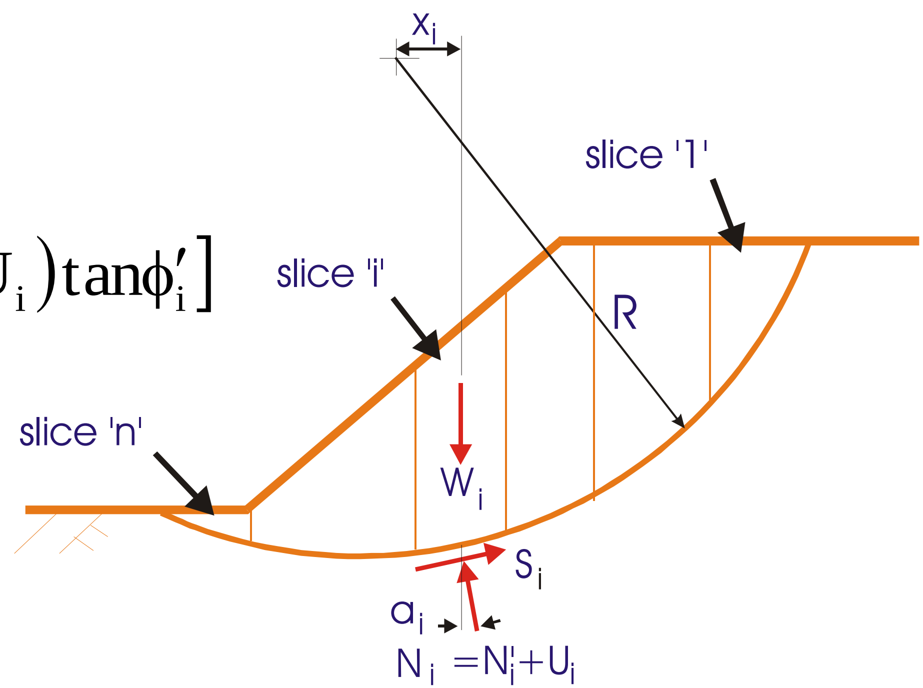
Total number of assumptions: $\sum = 3n-2$

- Consequently there are n more assumptions than required
- Only the moment and one force equilibrium equation considered
- Therefore force equilibrium is not fully satisfied

Conventional Method of Slices

Resisting moment

$$M_R = R\sum S_i = \frac{R}{F}\sum\left[c_i' l_i + \left(N_i - U_i\right)\tan\phi_i'\right]$$



Disturbing moment

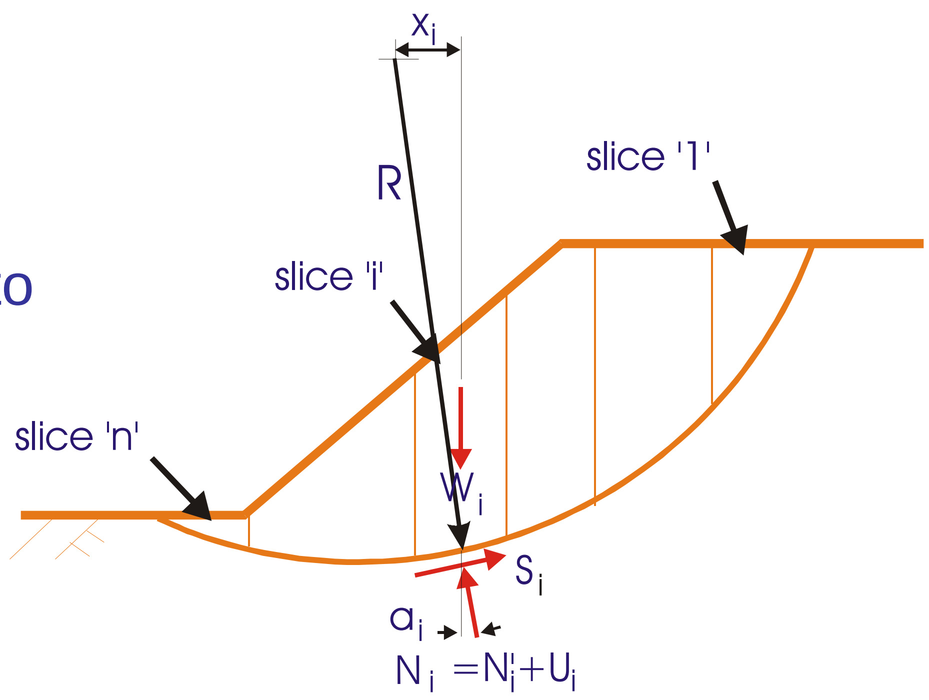
$$M_D = \sum W_i x_i$$

Equating the above two equations and rearranging an expression for the FoS is obtained:

$$F = R\frac{\sum\left[c_i' l_i + \left(N_i - U_i\right)\tan\phi_i'\right]}{\sum W_i x_i}$$

Conventional Method of Slices

$$F = R\frac{\sum\left[c_i' l_i + \left(N_i - U_i\right)\tan\phi_i'\right]}{\sum W_i x_i}$$



Resolving forces perpendicular to base of slice

$$N_i = W_i \cos\alpha_i$$

and noting that $x_i = R\sin\alpha_i$

$$\boxed{F = \frac{\sum\left[c_i' l_i + \left(W\cos\alpha_i - U_i\right)\tan\phi_i'\right]}{\sum W\sin\alpha_i}}$$

Conventional Method of Slices

- Conventional method is the only one that allows the FoS to be calculated directly (i.e. non-iterative solution)
- Force equilibrium is not fully satisfied
- Yields conservative results due to inconsistent calculation of effective stresses at the base of the slices
- Accurate only when the central angle of the arc forming the slip circle is relatively small

Bishop Simplified Method of Slices

Assumptions:	Number
Neglects inter-slice shear forces (i.e. X=0):	
•Magnitude of inter-slice force, X (X=0)	n-1
The normal force, N acts through the centre of the base of the slice:	
•Point of application (i.e. line of action) of N	n

$\boxed{\text{Total number of assumptions: } \sum = 2n\text{-}1}$

- Consequently there is 1 more assumption than required -one constraint cannot be satisfied
- This usually manifests itself as failure to satisfy horizontal equilibrium in one slice. The error is usually small- acceptable

Bishop Simplified Method of Slices

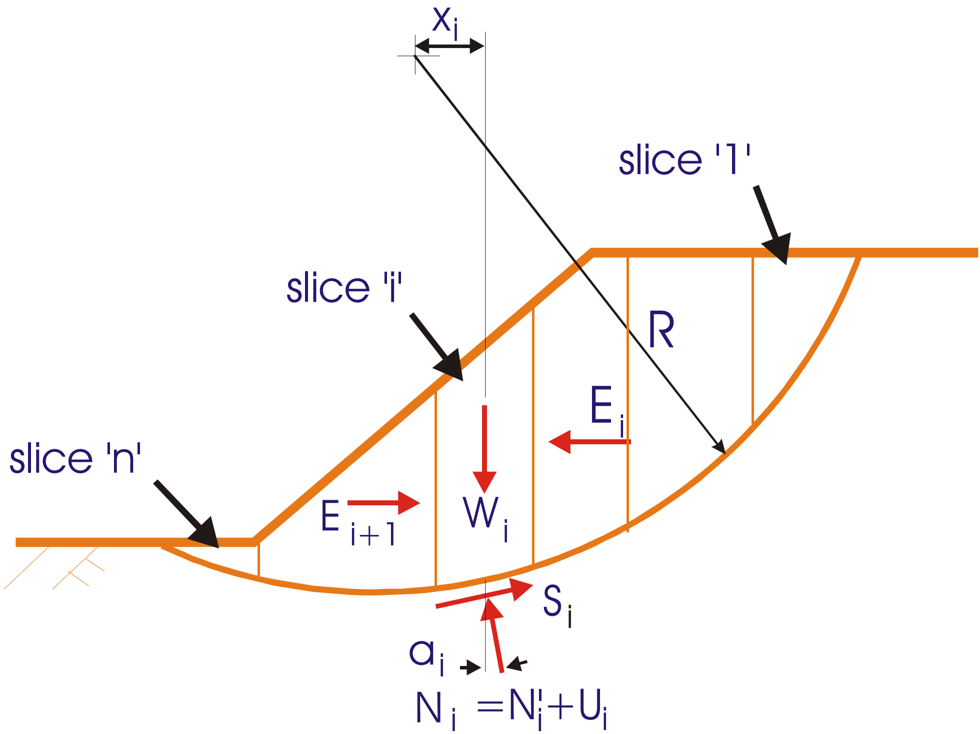
The equation for the factor of safety is derived in a similar way to that for the conventional method

$$F = R\frac{\sum\left[c'_i\, l_i + \left(N_i - U_i\right)\tan\phi'_i\right]}{\sum W_i\, x_i}$$

Resolving forces vertically:

$$N_i \cos\alpha + S_i \sin\alpha_i - W_i = 0$$

$$N_i = \frac{W_i - \dfrac{\sin\alpha_i}{F}\left(c'_i\, l_i - U_i \tan\phi'_i\right)}{\cos\alpha_i + \dfrac{\sin\alpha_i \tan\phi'_{ii}}{F}}$$



Bishop Simplified Method of Slices

$$\boxed{F = \frac{1}{\sum W_i \sin\alpha_i} \sum \frac{c_i' l_i \cos\alpha_i + \left(W_i - U_i \cos\alpha_i\right)\tan\phi_i'}{\cos\alpha_i + \dfrac{\sin\alpha_i \tan\phi_i'}{F}}}$$

- An initial value of F is assumed and used to evaluate the right hand side of the above equation and thus determine a new value of F
- The procedure is then repeated with this new value of F
- This continues until successive changes in the value of F are small.

Methods of Slices for Rotational Slides

	Conventional Method	Bishop Simplified Method
Assumptions	1. inter-slice shear forces =0 2. inter-slice normal forces=0 3. The normal force, N acts through the centre of the base of the slice	1. inter-slice shear forces =0 2. The normal force, N acts through the centre of the base of the slice
Equilibrium Equations used	1. Moment equilibrium 2. Force equilibrium perpendicular to the slip surface.	1. Moment equilibrium 2. Force equilibrium in the vertical direction

Slope Stability analysis

- Planar movements
- Rotational movements
  - Circular Arc method (total stress analysis)
  - Method of slices

Stability Charts

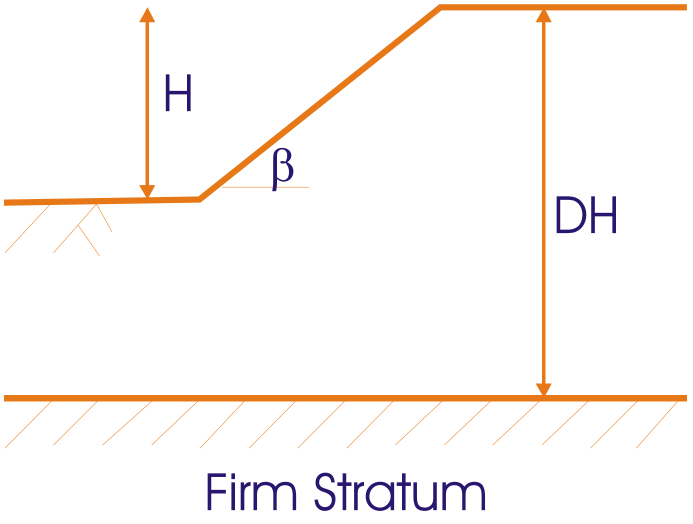
- Limit equilibrium solutions involve time consuming and repetitive calculations
- For simple problems one can use a corpus of established solutions in the form of stability charts
- These charts rely on dimensionless relationships that exist between the FoS and other parameters that describe the slope geometry, shear strength and pore water pressures
- They provide the ***minimum* FoS** and thus eliminate the need to search for a critical slip surface.

Stability Charts for undrained analyses

Taylor (1948)- based on Circular Arc method

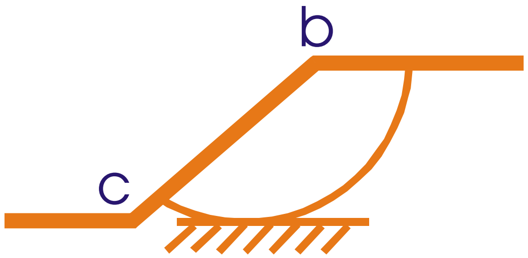
- Su is constant with depth
- There are no tension cracks
- No water pressures act on the slope

D: ratio of the depth of the homogenous layer to the height of the slope, H



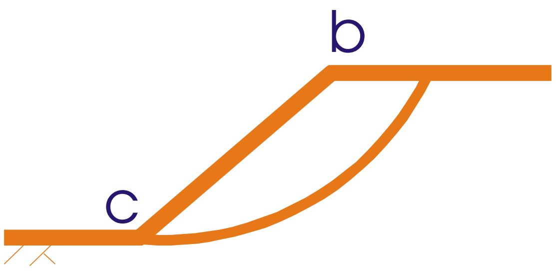
Stability Charts for undrained analyses

- Firm base is located at a short distance below the level **c**
- Critical circle tangential to firm base



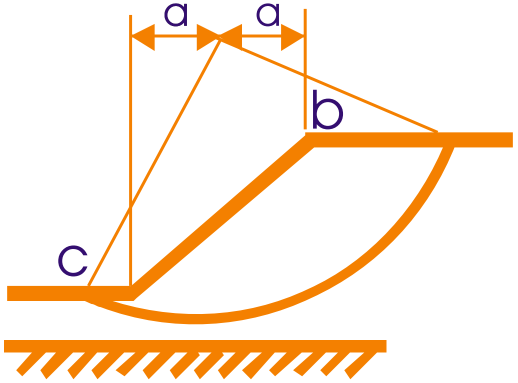
Slope circle

- Critical circle passes through the toe **c** of the slope



Toe circle

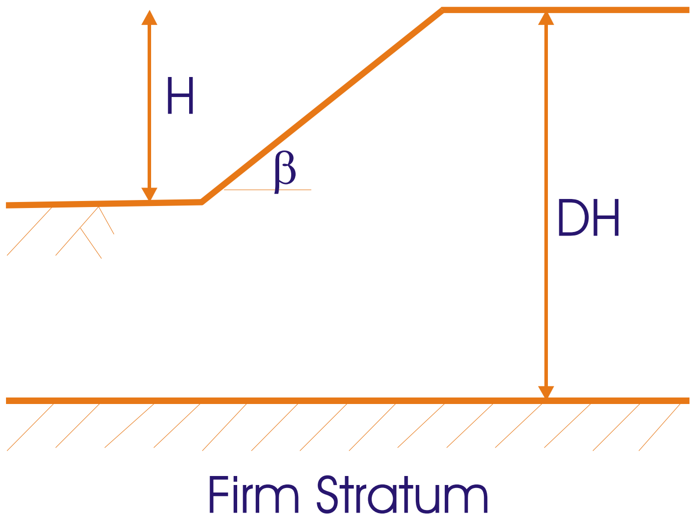
Stability Charts for undrained analyses



Foundation circle

- Base failure
- The centre of the foundation circle is located on a vertical line that passes through the midpoint of the slope.

Stability Charts for undrained analyses



Independent variables:

- Height H
- Slope angle β
- Mobilised strength Su/F
- Unit weight γ
- Depth factor D

Dimensionless Stability number

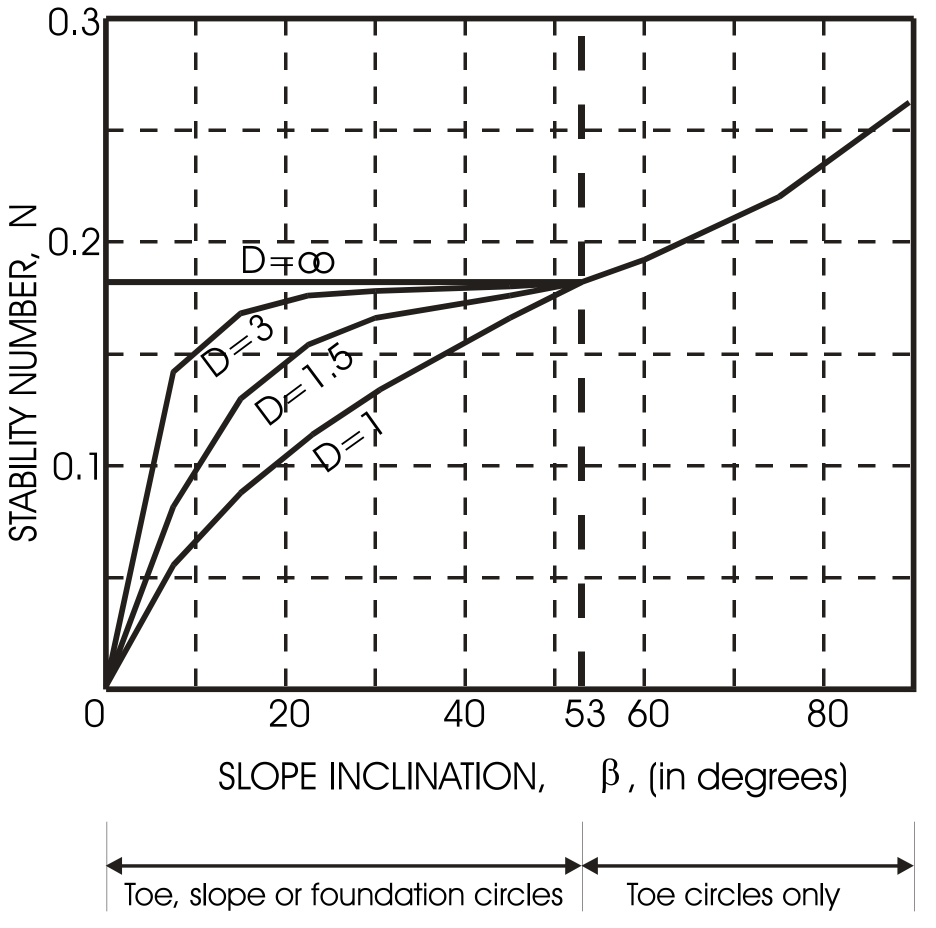
$$N = \frac{S_u}{F \gamma H} = f(\beta, D)$$

Stability Charts for undrained analyses

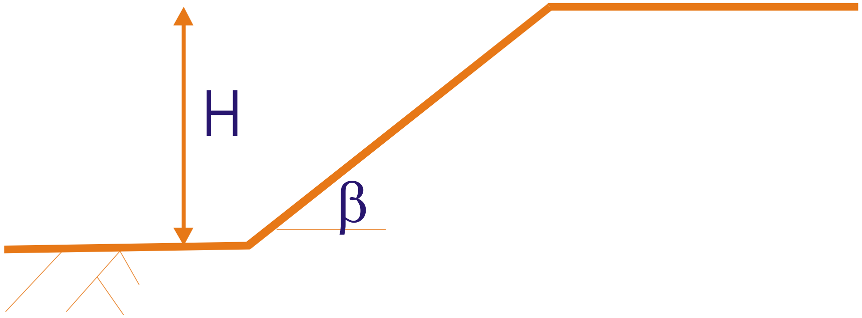
$\beta > 53^o$, failure occurs along a toe circle

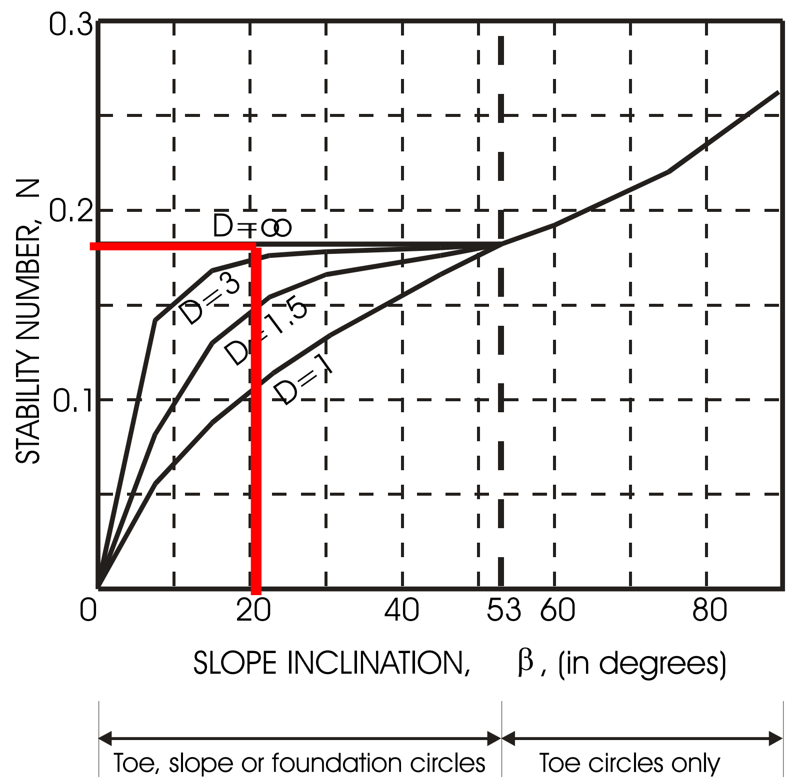
$\beta < 53^o$, the type of failure depends on the value of the depth factor D

$$N = \frac{S_u}{F \gamma H}$$



Stability Charts for undrained analyses





β=20°, H=8m, γ=19kN/m³,
Su=40kPa,

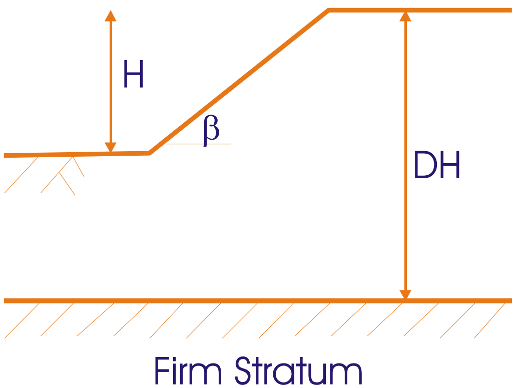
F=?

$$F = \frac{S_u}{N \gamma H}$$

$$N = \frac{S_u}{F \gamma H}$$

N=0.18, so F=1.46

Stability Charts for undrained analyses

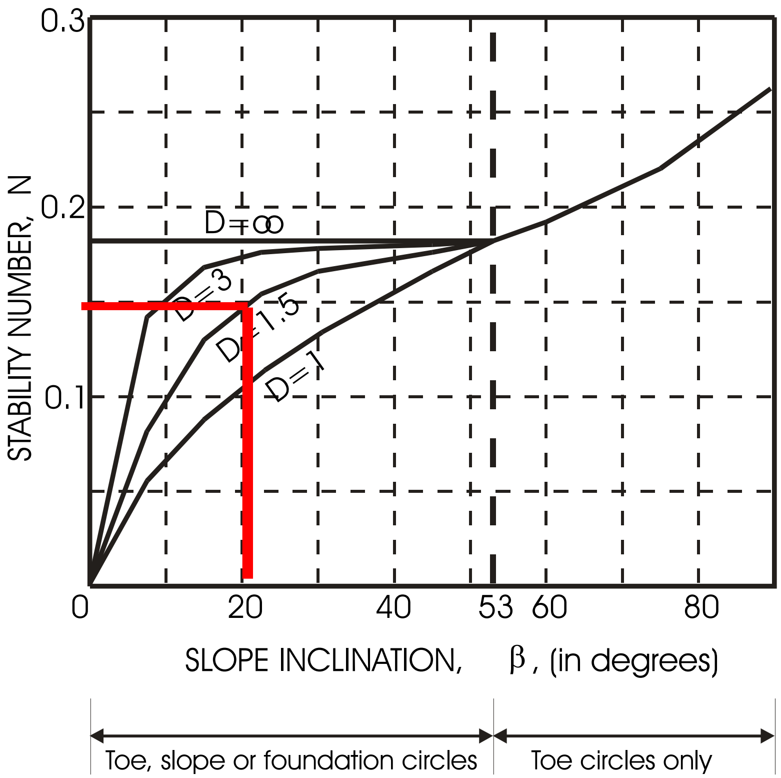


β=20º, H=8m, γ=19kN/m^3,
Su=40kPa, D=1.5

F=?

$$F = \frac{S_u}{N \gamma H}$$

N=0.15, so F=1.75



$$N = \frac{S_u}{F \gamma H}$$