

11-12 TREE AND ARRAY MULTIPLIERS AND VARIATIONS IN MULTIPLIERS

Chapter Goals

Learn additional methods for synthesizing fast multipliers as well as other types of multipliers (bit-serial, modular, etc.)

Chapter Highlights

Building a multiplier from smaller units
Performing multiply-add as one operation
Using a multiplier for squaring is wasteful
Building modular multipliers

TREE AND ARRAY MULTIPLIERS AND VARIATIONS IN MULTIPLIERS: TOPICS

Topics in This Chapter

11.1. Full-Tree Multipliers

12.1 Divide-and-Conquer Designs

12.2 Additive Multiply Modules

12.5 The Special Case of Squaring

12.6 Modular Multipliers

11.1 FULL-TREE MULTIPLIERS

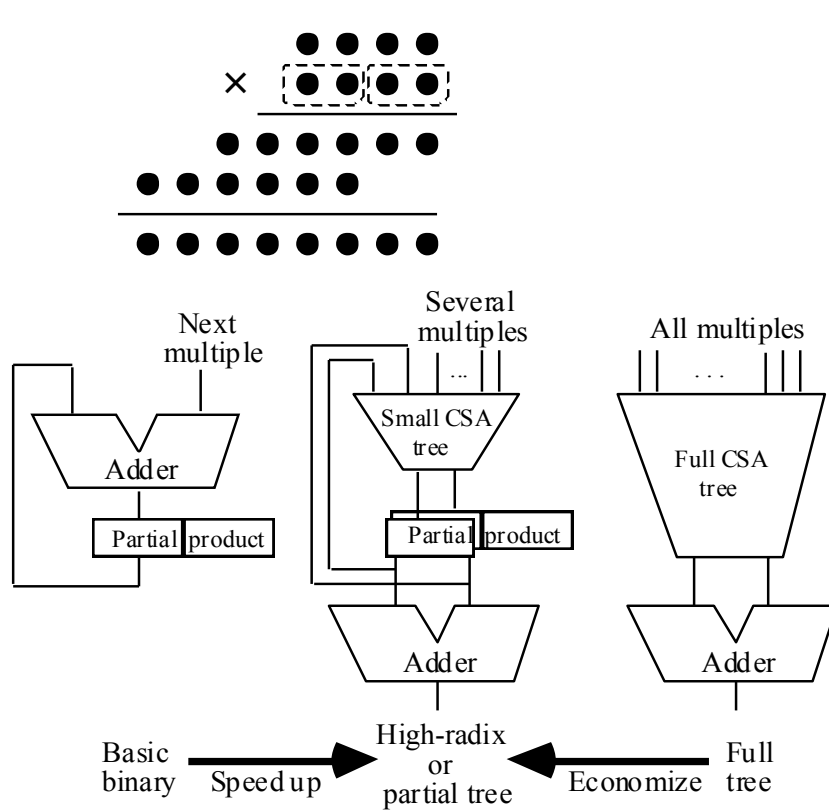


Fig. 10.13 High-radix multipliers as intermediate between sequential radix-2 and full-tree multipliers.

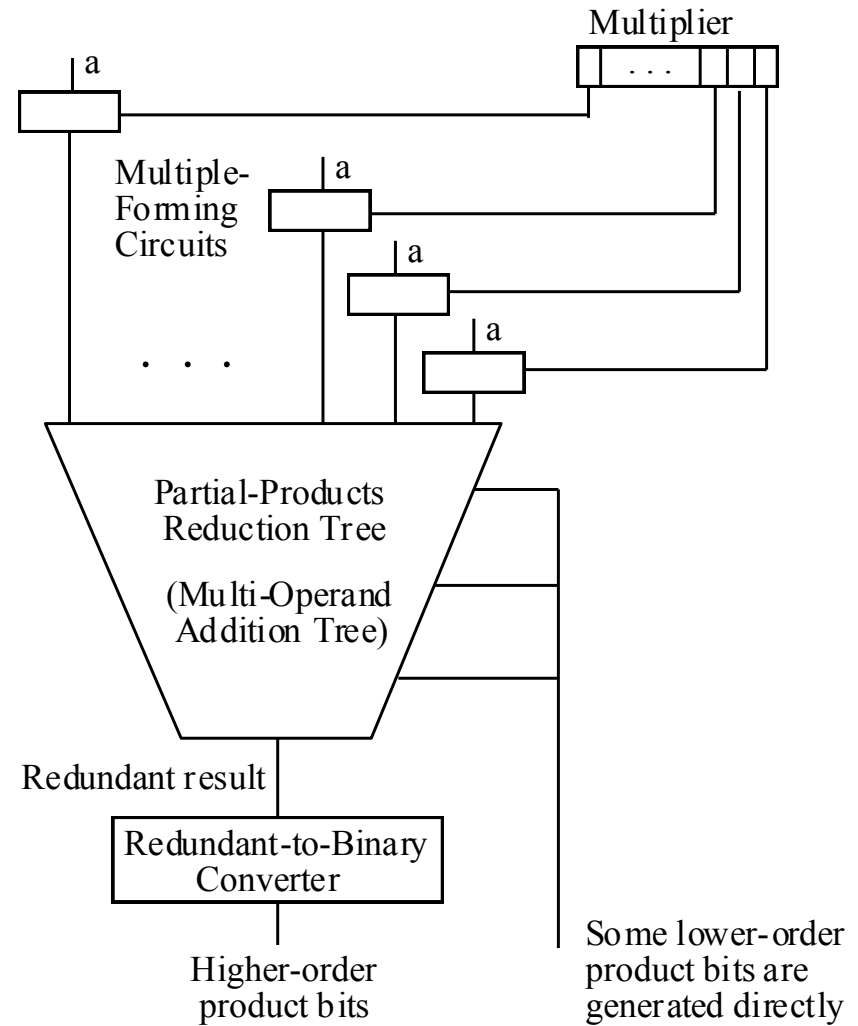
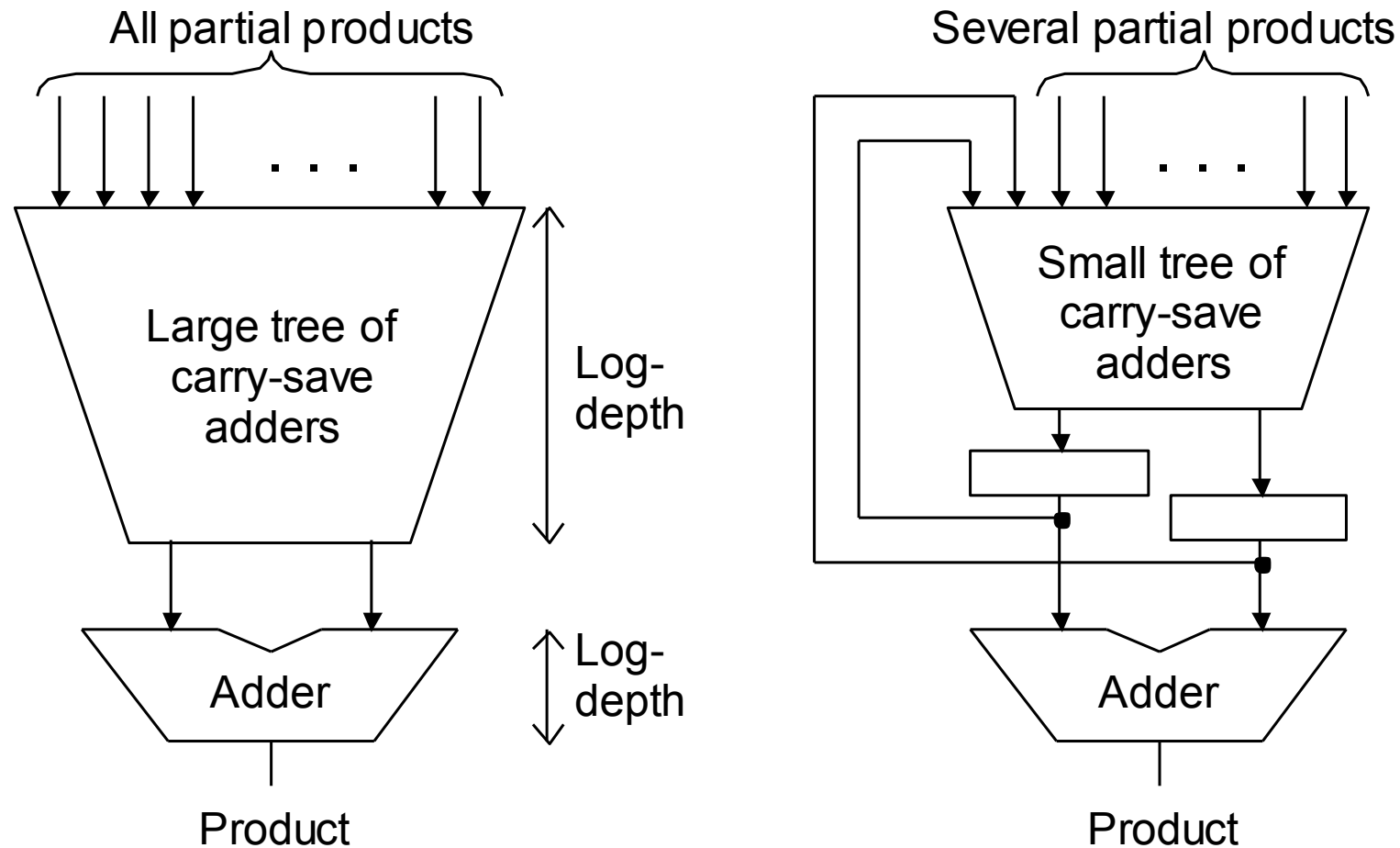


Fig. 11.1 General structure of a full-tree multiplier.

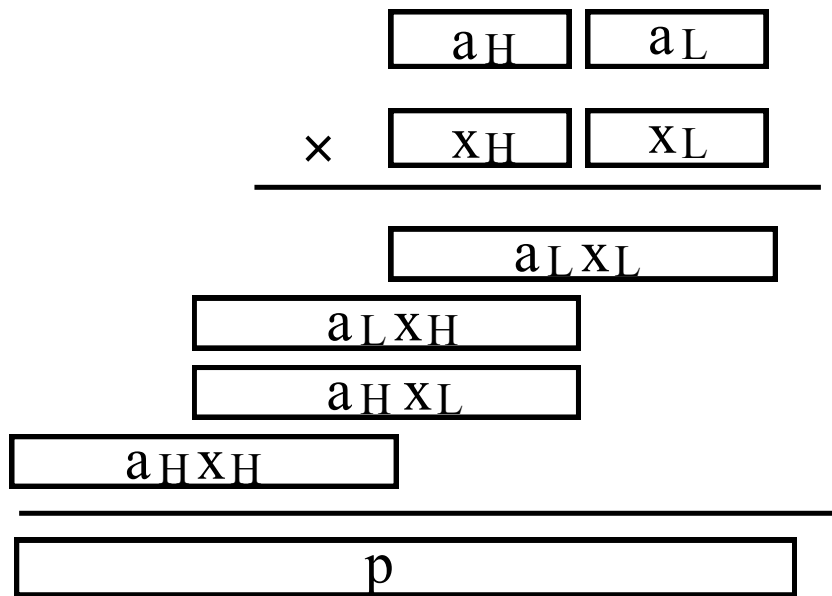
FULL-TREE VERSUS PARTIAL-TREE MULTIPLIER



Schematic diagrams for full-tree and partial-tree multipliers.

12.1 DIVIDE-AND-CONQUER DESIGNS

Building wide multiplier from narrower ones



Rearranged partial products in $2b$ -by- $2b$ multiplication

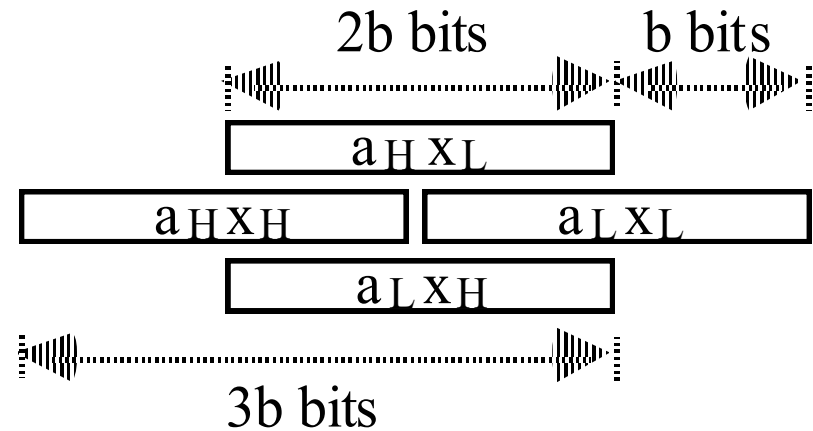


Fig. 12.1 Divide-and-conquer (recursive) strategy for synthesizing a $2b \times 2b$ multiplier from $b \times b$ multipliers.

GENERAL STRUCTURE OF A RECURSIVE MULTIPLIER

$2b \times 2b$ use (3; 2)-counters

$3b \times 3b$ use (5; 2)-counters

$4b \times 4b$ use (7; 2)-counters

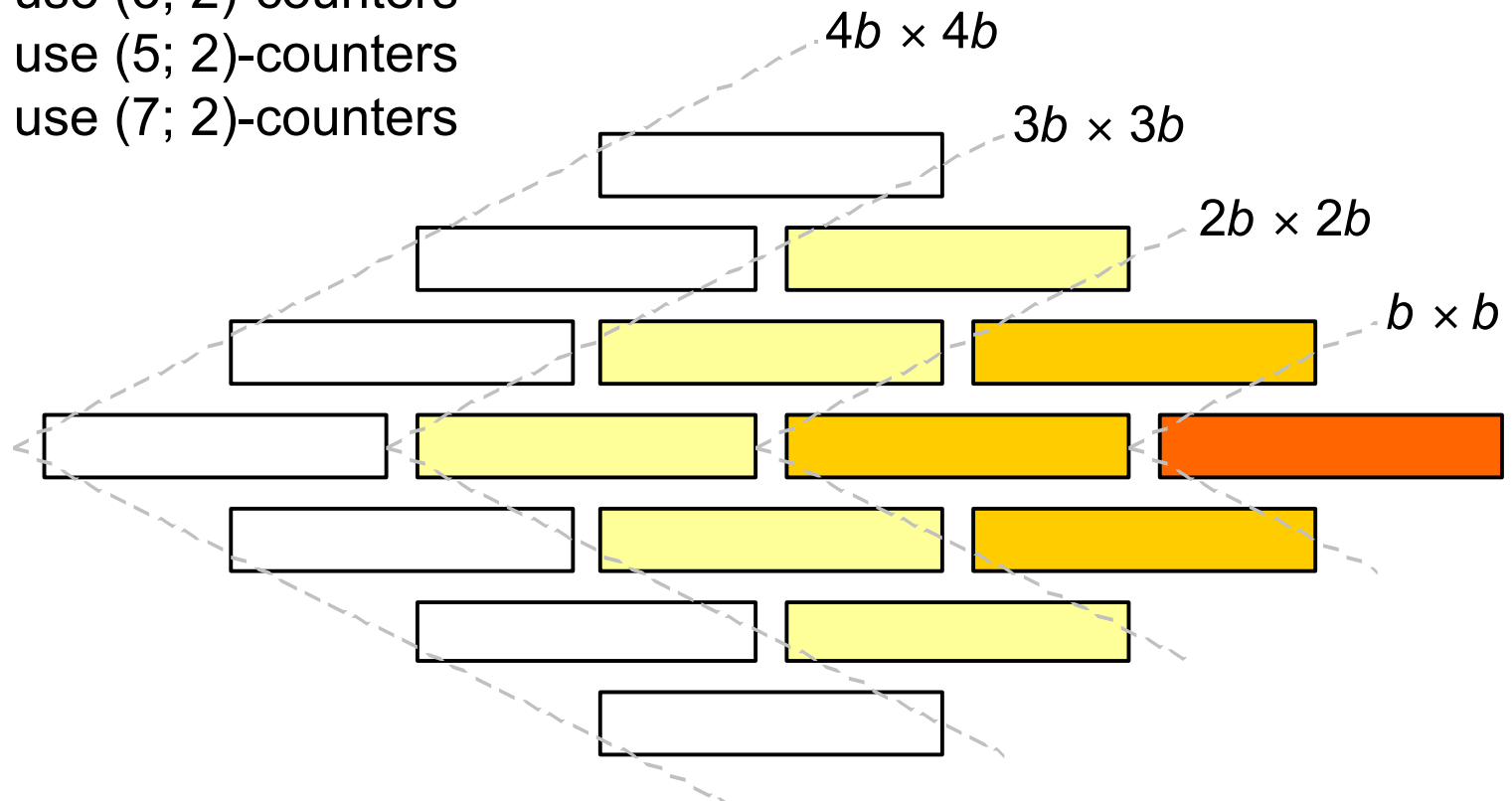


Fig. 12.2 Using $b \times b$ multipliers to synthesize $2b \times 2b$, $3b \times 3b$, and $4b \times 4b$ multipliers.

PROBLEMAS

Problema 11.1. Para uma multiplicação de dois operandos de 24 bits, aplique o método de dividir para conquistar e obtenha o custo e caminho crítico dos blocos considerando A_{FA} e T_{FA} como a área e atraso por *Full-Adder*, e $0,5 \times A_{FA}$ e $0,5 \times T_{FA}$, para o *Half-Adder*, $(a/2) \times A_{FA}$ e $(a/2) \times T_{FA}$ para o $(2^a:1)$ MUX.

12.2 ADDITIVE MULTIPLY MODULES

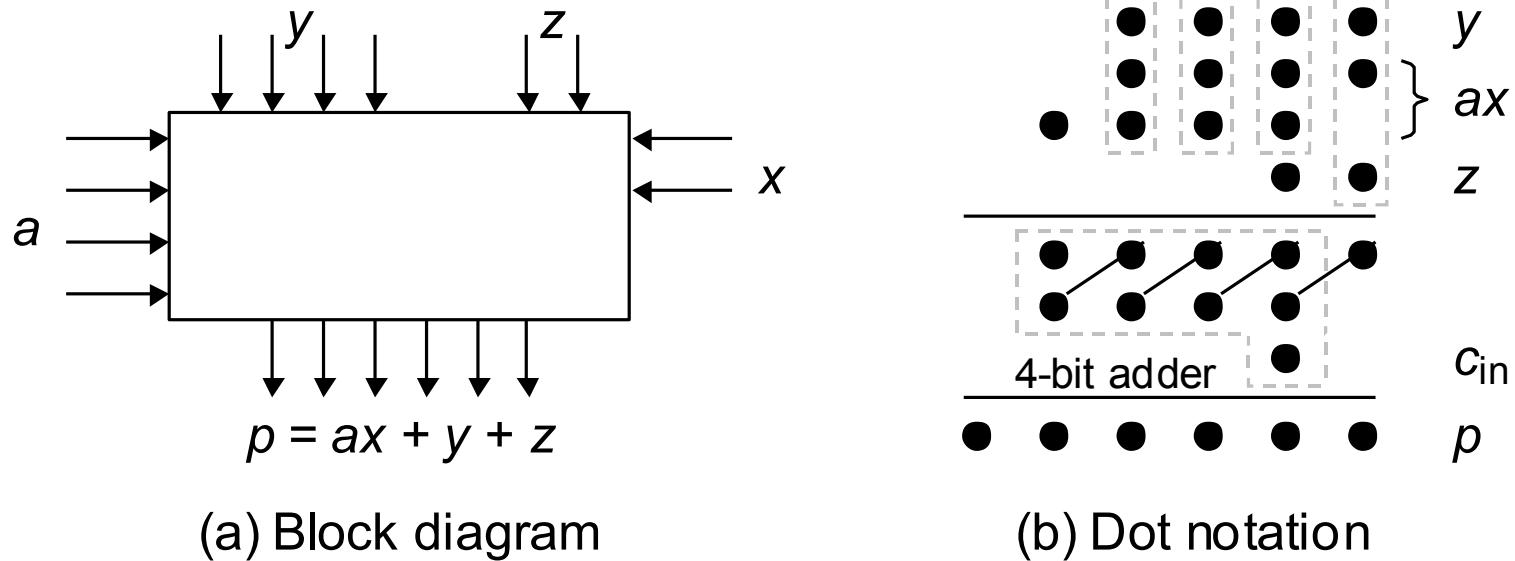
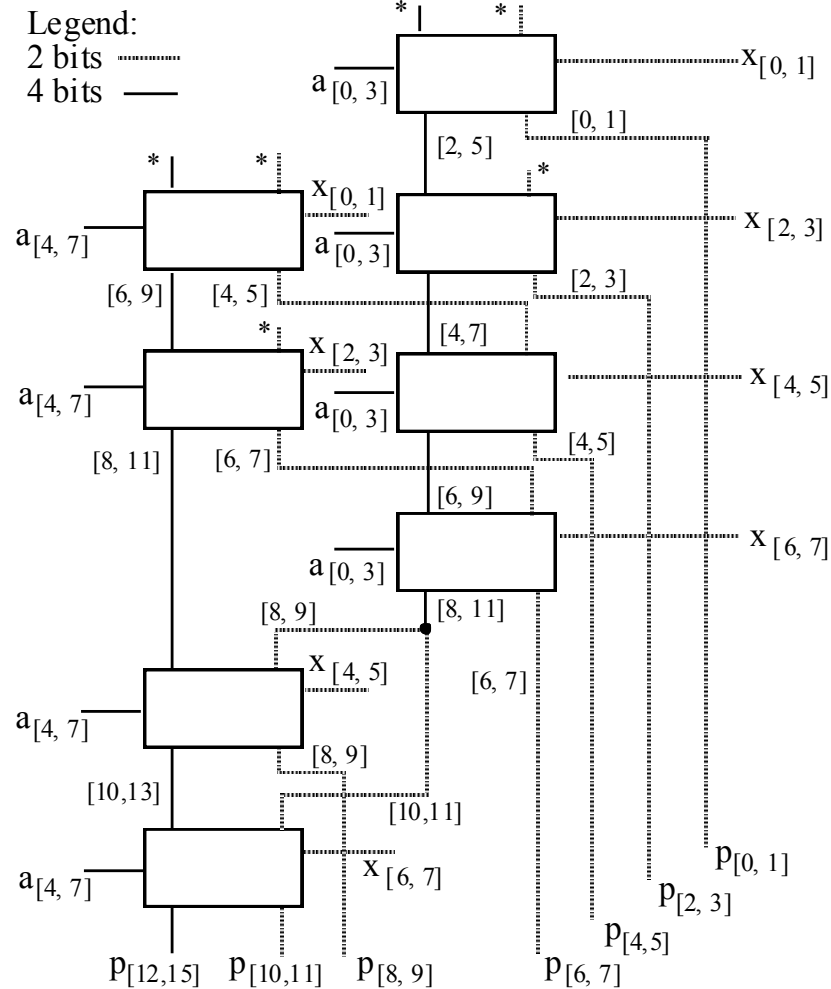


Fig. 12.4 Additive multiply module with 2×4 multiplier (ax) plus 4-bit and 2-bit additive inputs (y and z).

$b \times c$ AMM $\begin{cases} b\text{-bit and } c\text{-bit multiplicative inputs} \\ b\text{-bit and } c\text{-bit additive inputs} \\ (b + c)\text{-bit output} \end{cases}$

$$(2^b - 1) \times (2^c - 1) + (2^b - 1) + (2^c - 1) = 2^{b+c} - 1$$

MULTIPLIER BUILT OF AMMS



Understanding
 an 8×8 multiplier
 built of 4×2
 AMMs using dot
 notation

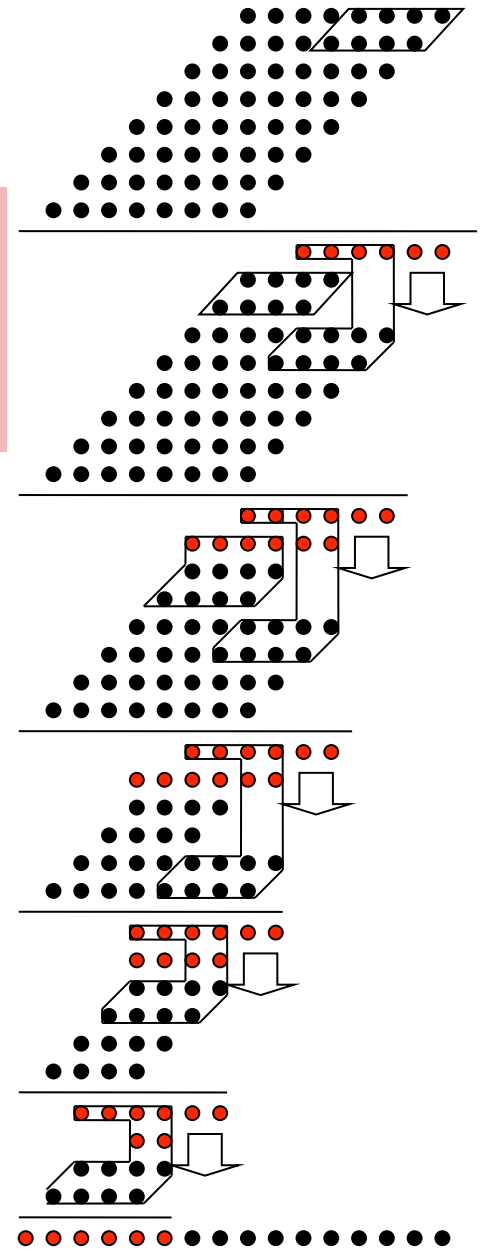
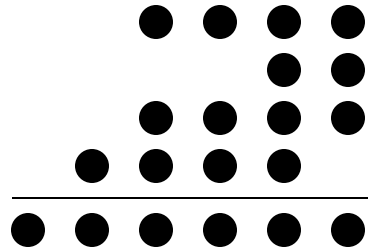


Fig. 12.5 An 8×8 multiplier built of 4×2 AMMs. Inputs marked with an asterisk carry 0s.

PROBLEMAS

Problema 12.1. Projete um AMM 2×2 , com duas entradas de soma de dois bits usando unicamente 4 full adders e 4 portas AND.

- a) Mostre como conectar os AMMs projetados para projetar um multiplicador 4×4 .
- b) Determine o caminho critico usando o Full adder como unidade de atraso.
- c) Pode ser usado o multiplicador do apartado a como um AMM 4×4 ?

Problema 12.2. Projete os seguintes AMMs usando unicamente 2×4 AMMs:

- a) 4×4 AMM;
- b) 2×8 AMM;
- c) 6×6 AMM
- d) 4×8 AMM
- e) 4×8 AMM (usando 4×4 AMMs).
- f) Compare a eficiência de d) e e) em área e atraso considerando A_{FA} e T_{FA} como a área e atraso por *Full-Adder*, e $0,5 \times A_{FA}$ e $0,5 \times T_{FA}$, para o *Half-Adder*.

Problema 12.3. Projete o circuito AMM da seguinte expressão: $A \times B \times C + 2^b D + 2^c E + 2^a F$, onde A, D tem a=4 bits, B, E tem b=3 bits e C, F tem c=2 bits.

12.5 THE SPECIAL CASE OF SQUARING

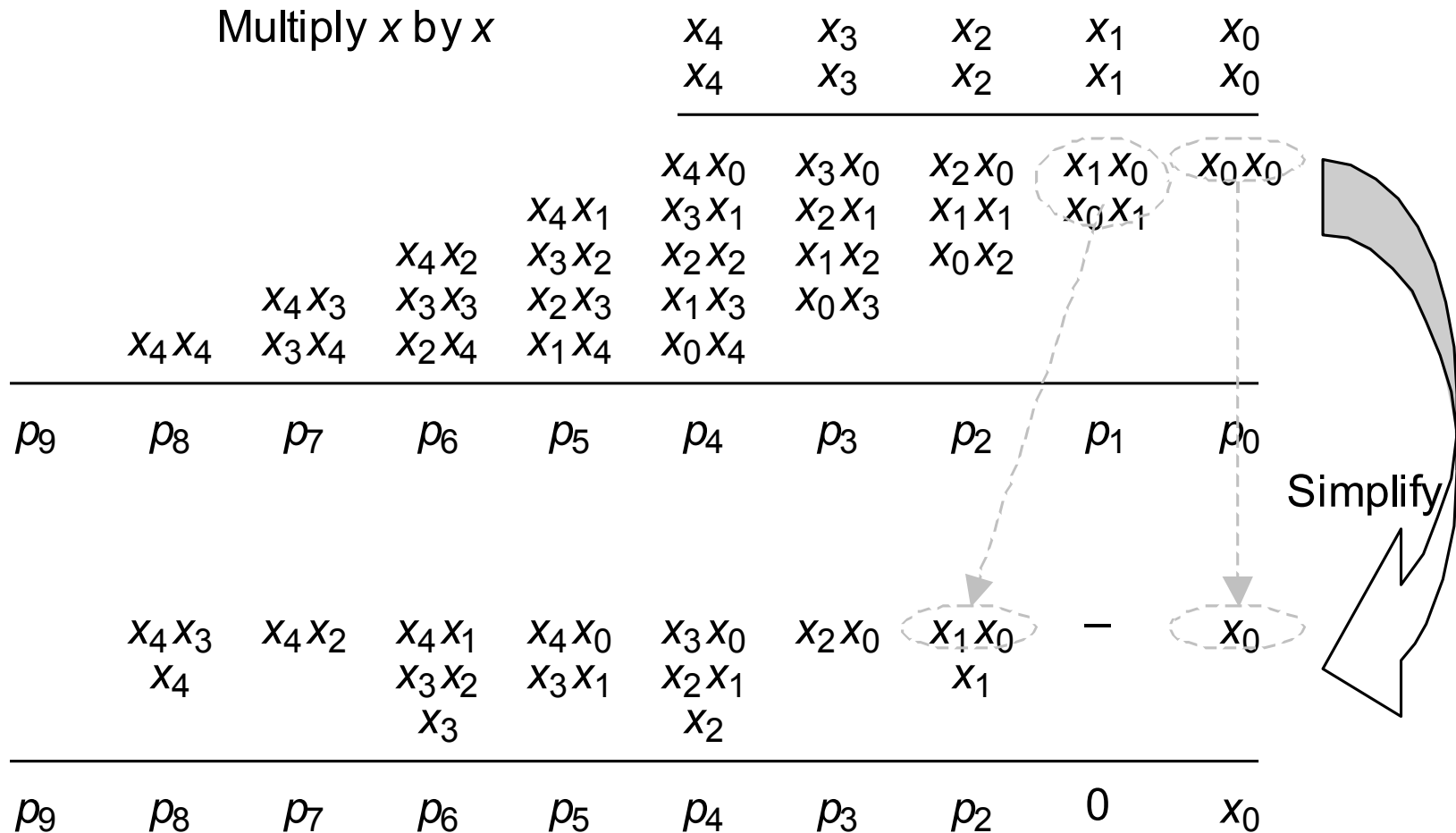


Fig. 12.18 Design of a 5-bit squarer.

PROBLEMAS

Problema 12.4. Projete a estrutura do multiplicador quadrático RNS para os seguintes módulos:

- a) 29;
- b) 31;
- c) 13.