Circuito RF

Setembro 2020

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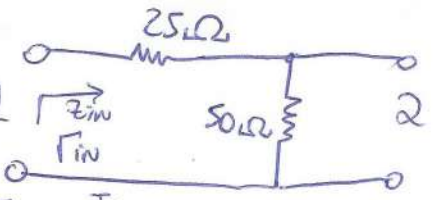
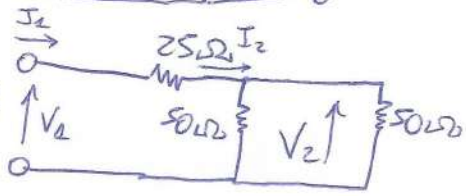
$Z_0 = 50\,\Omega$

S parameters Matrix?

3)



S_{11}



$\Rightarrow z_{in} = 25 + 50 // 50 = 50\,\Omega$

$\Rightarrow S_{11} = \Gamma_{in} = \frac{z_{in} - z_0}{z_{in} + z_0}$

$= \frac{50-50}{50+50} = 0$

S_{21}

$\Rightarrow V_1 = V_1^+ + V_1^- = V_1^+ \cdot (1 + S_{11})$

$\Rightarrow V_1^+ = \frac{V_1}{1+S_{11}} = V_1 \Rightarrow V_1^- = 0$

$\Rightarrow V_1 = I_1 \cdot z_{in} = 50 \cdot I_1 = 50 \cdot I_2 \Rightarrow I_1 = I_2$

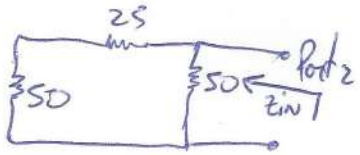
$\Rightarrow V_2^- = V_2 = I_2 \cdot (50 // 50) = 25 \cdot I_2$

$\Rightarrow S_{21} = \frac{V_2^-}{V_1^+} = \frac{25}{50} = 0{,}5$

S_{12}

$\Rightarrow S_{12} = S_{21}$ (reciprocal network)

S_{22}



$\Rightarrow z_{in} = 50 // (25 + 50)$

$= 30\,\Omega$

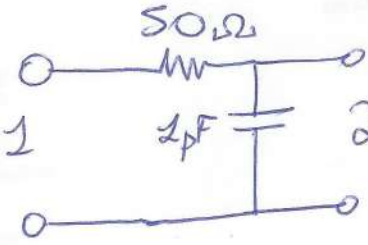
$\Rightarrow S_{22} = \Gamma_{in} = \frac{30-50}{30+50} = -0{,}25$

S

$$\Rightarrow S = \begin{bmatrix} 0 & 0{,}5 \\ 0{,}5 & -0{,}25 \end{bmatrix}$$

Two-port S-parameters 1 GHz

4)



$50\,\Omega = Z_0$

$Z_C = \frac{1}{j\omega C} = \frac{1}{j2\pi fC}$

$\Rightarrow Z_C = \frac{-j}{2\pi \cdot 10^9 \cdot 10^{-12}} \approx -j159{,}15\,\Omega$

$\Rightarrow Z_{IN} = 50 + (Z_C \,//\, 50)$

$= 95{,}51 - j14{,}3\,\Omega$

$\Rightarrow \boxed{S_{11}} = \Gamma_{IN} = \frac{Z_{IN} - Z_0}{Z_{IN} + Z_0} = \boxed{0{,}3193 - j0{,}0669}$

$\boxed{S_{21}} \Rightarrow V_1^+ = \frac{V_1}{1 + S_{11}} = (0{,}756 + j0{,}038) \cdot V_1$

$\boxed{S_{12}} \Rightarrow V_1 = I_1 \cdot Z_{IN} = I_1 \cdot (95{,}51 - j14{,}3) \qquad I_1 = I_2$

$\Rightarrow V_2^- = V_2 = I_2 \cdot (50 \,//\, Z_C) = I_2 \cdot (45{,}51 - j14{,}3)$

$\Rightarrow S_{21} \Rightarrow V_2^- / V_1^+ = \boxed{2 + j0{,}314}$

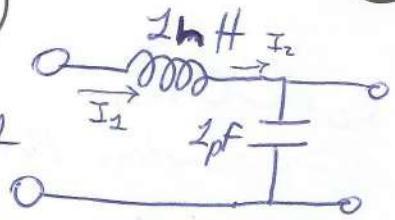
$S_{21} = S_{12} = V_2^- / V_1^+ = \boxed{0{,}638 - j0{,}13}$

$\boxed{S_{22}} \Rightarrow Z_{IN} = (50 + 50) \,//\, Z_C = 71{,}69 - j45{,}05\,\Omega$

$\Rightarrow S_{22} = \Gamma_{IN} = \frac{Z_{IN} - Z_0}{Z_{IN} + Z_0} = \boxed{0{,}277 - j0{,}267}$

$$S = \begin{bmatrix} 0{,}3193 - j0{,}0669 & 0{,}638 - j0{,}13 \\ 0{,}638 - j0{,}13 & 0{,}277 - j0{,}267 \end{bmatrix}$$

5)



Two port 50Ω
S-parameters f = 1GHz

$\Rightarrow Z_L = j\omega L = j \cdot 2\pi \cdot 10^9 \cdot 10^{-9} = j6{,}28\,\Omega$

$\Rightarrow Z_C = \frac{-j}{\omega C} = -j \cdot \frac{1}{2\pi \cdot 10^9 \cdot 10^{-12}} = -j\,159{,}15\,\Omega$

S_{11} $\Rightarrow Z_{IN} = Z_L + (Z_C // Z_0) = j6{,}28 + (50 // -j159{,}25)$
$= 45{,}51 + j\,8{,}02\,\Omega$

$\Rightarrow S_{11} = \Gamma_{IN} = \frac{Z_{IN} - Z_0}{Z_{IN} + Z_0} = \boxed{-0{,}0397 + j\,0{,}0872}$

S_{21}
S_{12}
$\Rightarrow V_1^+ = \frac{V_1}{1 + S_{11}} = (1{,}03 + j\,0{,}094)\,V_2$

$\Rightarrow V_1 = I_1 \cdot Z_{IN} = I_{IN} \cdot (45{,}51 - j\,8{,}02)$

$\Rightarrow V_1^+ = (45{,}51 - j\,8{,}02) \cdot (1{,}03 + j\,0{,}094) \cdot I_1$ $\quad I_1 = I_2$
$= (47{,}75 - j\,4) \cdot I_1$

$\Rightarrow V_2^- = I_2 \cdot (50 // Z_C) = I_1 \cdot (45{,}51 - j\,14{,}3)$

$\Rightarrow S_{21} = S_{12} = \frac{V_2^-}{V_1^+} = \boxed{0{,}971 - j\,0{,}218}$

S_{22} $\Rightarrow Z_{IN} = (50 + Z_L) // Z_C = 48{,}96 - j\,9{,}47\,\Omega$

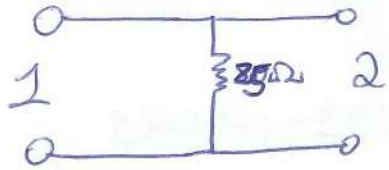
$\Rightarrow S_{22} = \Gamma_{IN} = \boxed{-0{,}00136 - j\,0{,}0958}$

$$S = \begin{bmatrix} 9{,}59 \angle -115° & 0{,}995 \angle -12{,}64° \\ 0{,}995 \angle -12{,}64° & 0{,}0958 \angle -90{,}81 \end{bmatrix}$$

6) Two port S parameters

Short 25Ω resistor

$Z_0 = 50\Omega$



$Y = \frac{1}{Z} = \frac{1}{25} = 0{,}04\ A/V$

$Y_0 = \frac{1}{Z_0} = \frac{1}{50} = 0{,}02\ S$

$\boxed{S_{11}} \Rightarrow S_{11} = \frac{Y_0 - Y_{IN}}{Y_0 + Y_{IN}}$

$\Rightarrow S_{11} = \frac{-Y}{2Y_0 + Y} = \frac{-0{,}04}{2 \cdot 0{,}02 + 0{,}04} = -0{,}5$

$\Rightarrow S_{11} = S_{22}$

$\boxed{S_{21}} \Rightarrow S_{21} = \frac{2Y_0}{Y + 2Y_0} = \frac{2 \cdot 0{,}02}{0{,}04 + 2 \cdot 0{,}02} = 0{,}5$

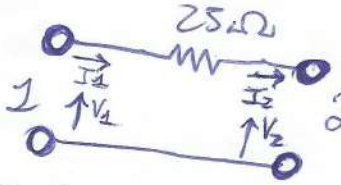
$S_{21} = S_{12}$

$$S = \begin{bmatrix} -0{,}5 & 0{,}5 \\ 0{,}5 & -0{,}5 \end{bmatrix}$$

7) Two port S parameters

Series 25Ω resistor

$Z_0 = 100\Omega$



$I_2 = I_1$

S_{11} $\Rightarrow Z_{IN} = 25 + 100 = 125\Omega$

$\Rightarrow S_{11} = \Gamma_{IN} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$

$= \frac{125 - 100}{125 + 100} = 0,11$

S_{21} $\Rightarrow V_1 = V_1^+ + V_1^- = V_1^+ (1 + S_{11})$

$\Rightarrow V_1^+ = \frac{V_1}{(1 + S_{11})} = 0,9 V_1$

$\Rightarrow V_1 = I_1 \cdot Z_{IN} = 125 I_1$

$\Rightarrow V_1^+ = 112,5 \cdot I_1$

$\Rightarrow V_2^- = V_2 = I_2 \cdot 100 = 100 I_1$

$\Rightarrow S_{21} = \frac{V_2^-}{V_1^+} = \frac{100}{112,5} = 0,89$

$\Rightarrow S_{21} = S_{12}$

rede recíproca

S_{22} $\Rightarrow Z_{IN} = 25 + 100 = 125$

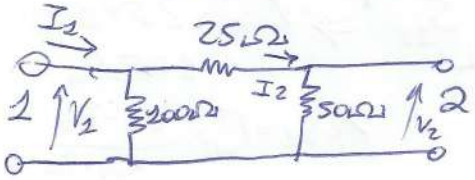
$\Rightarrow S_{22} = \Gamma_{IN} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = 0,11$

rede simétrica

Logo $S = \begin{bmatrix} 0,11 & 0,89 \\ 0,89 & 0,11 \end{bmatrix}$

8) Two port Π - network

$Z_0 = 50\,\Omega$



S_{11}

$\Rightarrow Z_{in} = 100 // (25 + (50 // 150)) = 33{,}33\,\Omega$

$\Rightarrow S_{11} = \Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = -0{,}2$

$S_{21} \Rightarrow V_1^+ = \frac{V_2}{1 + S_{11}} = \frac{1}{1 - 0{,}2} \cdot V_1 = 1{,}25 \cdot V_1$

$\Rightarrow V_1 = I_1 \cdot Z_{in} = 33{,}33 \cdot I_1$

$\Rightarrow V_1^+ = 1{,}25 \cdot 33{,}33 \cdot I_1 = 41{,}66 \cdot I_1$

$\Rightarrow V_1^+ = I_2 \cdot 1{,}25 \cdot (25 + 50 // 150) = 62{,}5 \cdot I_2$

$\Rightarrow V_2^- = V_2 = I_2 \cdot (50 // 150) = 25 \cdot I_2$

$\Rightarrow S_{21} = \frac{V_2^-}{V_1^+} = \frac{25 \cdot I_2}{62{,}5 \cdot I_2} = 0{,}4$

$S_{21} = S_{12}$ rede recíproca

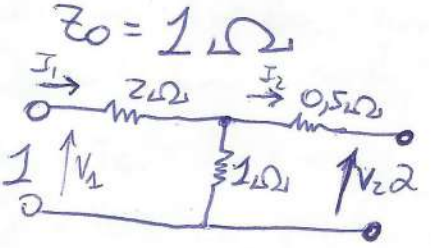
$S_{22} \Rightarrow Z_{in} = 50 // (25 + (50 // 100)) = 26{,}92\,\Omega$

$\Rightarrow S_{22} = \Gamma_{in} = \frac{26{,}92 - 50}{26{,}92 + 50} = -0{,}3$

Logo

$$S = \begin{bmatrix} -0{,}2 & 0{,}4 \\ 0{,}4 & -0{,}3 \end{bmatrix}$$

g) Two port T-network



$S_{11} \Rightarrow z_{iN} = 2 + (1 \,//\, (0{,}5+1)) = 2{,}6\,\Omega$

$\Rightarrow S_{11} = \Gamma_{iN} = \frac{z_{iN} - z_0}{z_{iN} + z_0} = \frac{2{,}6-1}{2{,}6+1} = 4/9 \approx 0{,}44$

$S_{21} \Rightarrow V_1^+ = \frac{V_1}{1+S_{11}} = 0{,}69 \cdot V_1$

$\Rightarrow V_1 = I_1 \cdot z_{iN} = 2{,}6 \cdot I_1$

$\Rightarrow V_1^+ = 2{,}6 \cdot 0{,}69 \cdot I_1 \approx 1{,}8 \cdot I_1$

$\Rightarrow I_2 = I_1 \cdot \frac{1}{1+0{,}5+1} = 0{,}4 \cdot I_1$

$\Rightarrow V_2^- = V_2 = z_0 \cdot I_2 = I_2$

$\Rightarrow S_{21} = \frac{V_2^-}{V_1^+} = \frac{0{,}4 \cdot I_1}{1{,}8 \cdot I_1} = 2/9 \approx 0{,}22$

$S_{21} = S_{12}$ rede recíproca

$S_{22} \Rightarrow z_{iN} = 0{,}5 + (1 \,//\, (2+1)) = 1{,}25\,\Omega$

$\Rightarrow S_{22} = \Gamma_{iN} = \frac{1{,}25-1}{1{,}25+1} = 1/9 \approx 0{,}11$

Logo

$$S = \begin{bmatrix} 0{,}44 & 0{,}22 \\ 0{,}22 & 0{,}11 \end{bmatrix}$$

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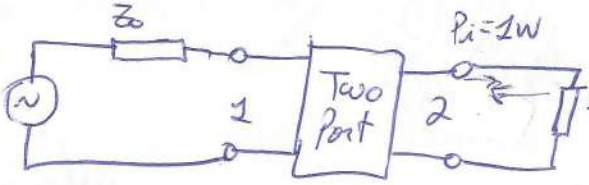
B) $S = \begin{bmatrix} 0,5+j0,5 & 0,95+j0,25 \\ 0,15-j0,05 & 0,5-j0,5 \end{bmatrix}$

$Z_0 = 50\Omega$

a) Não é recíproco, pois $S_{12} \neq S_{21}$

b) 1W



$Z_L = Z_0 = 50\Omega$ Casamento

$RL_{dB} = -20 \log |S_{11}| \Rightarrow |S_{11}| = \sqrt{Re(S_{11})^2 + Im(S_{11})^2}$

RL ↓ Return Loss

$= \sqrt{0,5^2 + 0,5^2}$

$= 0,7071$

$\Rightarrow RL_{dB} = -20 \log(0,7071) \approx 3,01\ dB$

$\Rightarrow RL_{dB} = 10 \log \frac{P_i}{P_r} = 3,01 \Rightarrow P_i/P_r = 10^{3,01/10} = 2$

$\Rightarrow P_r = 1/2 = 0,5W$

Logo $P_L = 0,5W$

c) Máxima Transferência de Potência

$\Rightarrow RL_{dB} = 0 = 10 \log \left| \frac{1}{\rho^2} \right|$

$\Rightarrow \rho^2 = 1$

$\Rightarrow \rho = 1$

13) c) CONTINUAÇÃO

$$\Rightarrow S_{11} + \frac{\Gamma_L \cdot S_{12} \cdot S_{21}}{(1 - \Gamma_L \cdot S_{22})} = \rho = 1$$

$$\Rightarrow \Gamma_L \cdot S_{12} \cdot S_{21} = (1 - S_{11}) \cdot (1 - \Gamma_L \cdot S_{22})$$

$$\Rightarrow \Gamma_L \cdot S_{12} \cdot S_{21} = 1 - \Gamma_L \cdot S_{22} - S_{11} + \Gamma_L \cdot S_{11} \cdot S_{22}$$

$$\Rightarrow \Gamma_L \cdot (S_{12} \cdot S_{21} + S_{22} - S_{11} \cdot S_{22}) = 1 - S_{11}$$

$$\Rightarrow \Gamma_L = (1 - S_{11}) / (S_{12} \cdot S_{21} + S_{22} - S_{11} \cdot S_{22})$$

$$= (1 - 0{,}5 - j\,0{,}5) / [(0{,}95 + j\,0{,}25) \cdot (0{,}15 - j\,0{,}05) + 0{,}5 - j\,0{,}5 - (0{,}5 + j\,0{,}5) \cdot (0{,}5 - j\,0{,}5)]$$

$$\Rightarrow \Gamma_L = 1{,}17 + j\,0{,}62$$

$$= 1{,}33 \angle 28{,}09^\circ$$

14) Two port

$Z_0 = 50\Omega$

$S_{11} = 0,3 - j0,4$

$S_{21} = 0,5$

a) Recíproca

Se

então

$S_{12} = S_{21} = 0,5$

b) Sem perdas (Lossless) se $|S_{11}|^2 + |S_{21}|^2 = 1$ (I)

e

$|S_{12}|^2 + |S_{22}|^2 = 1$ (II)

Condição (I) confere pois $\sqrt{0,3^2+0,4^2} + \sqrt{0,5^2} = 1$

Se $S_{12} = 0,5$, então S_{22} deve ser 0,5 para que seja sem perdas.

c) $Z_L = Z_0 = 50\Omega$ (Casamento)

$P_i = 0 dBm = 1 mW$

$\Rightarrow RL_{dB} = -20 \log |S_{11}| = -20 \log(0,5) = 6,02 dB$

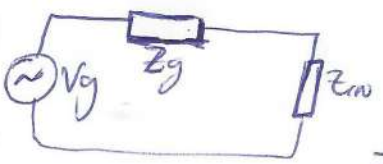
$\Rightarrow RL_{dB} = 10 \log\left(\frac{P_i}{P_r}\right) \Rightarrow \frac{P_i}{P_r} = 10^{\frac{6,02}{10}} = 4$

$\Rightarrow P_r = \frac{P_i}{4} = 0,25 mW$

$\Rightarrow P_L = 0,75 mW$

15) $S = \begin{bmatrix} 0,25 & 0 \\ 1,2 & 0,5 \end{bmatrix}$ $Z_0 = 50\,\Omega$, $Z_G = 50\,\Omega$, $P_s = 1\,mW$, $Z_L = 25\,\Omega$

a) Não recíproca, pois $S_{12} \neq S_{21}$

b) 

$\Rightarrow \Gamma_{in} = S_{11} + \frac{\Gamma_L \cdot S_{12} \cdot S_{21}}{(1 - \Gamma_L \cdot S_{22})}$ ($S_{12} \to 0$)

$= S_{11} = 0,25$

$\Rightarrow \Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \Rightarrow 0,25 \cdot Z_{in} + 0,25 Z_0 = Z_{in} - Z_0$

$\Rightarrow Z_{in} = \frac{1,25 \cdot 50}{0,75}$

$= 83,33\,\Omega$

$\Rightarrow P_s = \frac{V_g^2}{Z} \Rightarrow V_g = \sqrt{P_s \cdot Z}$

$= \sqrt{0,001 \cdot (50 + 83,33)}$

$\approx 0,365\,V$

c) $RL_{dB} = -20 \log |\Gamma_{in}| = -20 \log (0,25) = 12,04\,dB$

$= 10 \log \left(\frac{P_i}{P_r} \right) \Rightarrow P_i = P_s \cdot \frac{Z_{in}}{Z_{in} + Z_G} = 0,001 \cdot \frac{83,33}{83,33 + 50}$

$= 0,625\,mW$

$\Rightarrow P_r = \frac{0,625}{10^{(12,04/10)}} = 39,06\,\mu W$

15) d) $\delta_S = (1-S_{11})\cdot(1-S_{22}) - S_{12}\cdot S_{21}$

$= (1-0,25)\cdot(1-0,5) = 0,375$

$z_{11} = [(1+S_{11})(1-S_{22}) + S_{12}\cdot S_{21}] / \delta_S$

$= (1+0,25)(1-0,5)/0,375 \approx 1,67$

$z_{12} = 2\cdot S_{12}/\delta_S = 2\cdot 0/0,375 = 0$

$z_{21} = 2\cdot S_{21}/\delta_S = 2\cdot 1,2/0,375 = 6,4$

$z_{22} = [(1-S_{11})(1+S_{22}) + S_{12}\cdot S_{21}] / \delta_S$

$= [(1-0,25)(1+0,5)]/0,375 = 3$

↓

Parâmetros normalizados por $z_0 = 50\Omega$

Parâmetros atuais = $z'_{11} = z_{11}\cdot z_0 = 83,5$

$z'_{12} = z_{12}\cdot z_0 = 0$

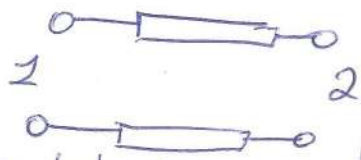
$z'_{21} = z_{21}\cdot z_0 = 320$

$z'_{22} = z_{22}\cdot z_0 = 150$

$$Z = \begin{bmatrix} 83,5 & 0 \\ 320 & 150 \end{bmatrix}$$

17) Transmission Line

comprimento $l = \frac{\lambda}{4}$



β: constante de propagação

$\beta = \frac{2\pi}{\lambda}$

$\Rightarrow A = \cos(\beta l)$

$\Rightarrow B = j \cdot z_0 \cdot \text{sen}(\beta l)$

$\Rightarrow C = j \cdot Y_0 \cdot \text{sen}(\beta l)$

$\Rightarrow D = \cos(\beta l)$

$\Rightarrow A = \cos(\beta l) = \cos\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

$\Rightarrow B = j \cdot z_0 \cdot \text{sen}(\beta l) = j \cdot z_0 \cdot \text{sen}\left(\frac{\pi}{2}\right) = j z_0$

$\Rightarrow C = j \cdot Y_0 \cdot \text{sen}(\beta l) = j Y_0$

$\Rightarrow D = \cos(\beta l) = 0$

$\Rightarrow \Delta = A + B/z_0 + C \cdot z_0 + D = j2$

$\Rightarrow S_{11} = \frac{A + B/z_0 - C \cdot z_0 - D}{\Delta} = 0$

$\Rightarrow S_{22} = \frac{-A + B/z_0 - C \cdot z_0 + D}{\Delta} = 0$

$\Rightarrow S_{21} = \frac{2}{\Delta} = -j$

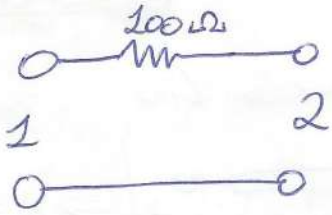
$\Rightarrow S_{12} = \frac{2(AD - BC)}{\Delta} = -j$

$$S = \begin{bmatrix} 0 & -j \\ -j & 0 \end{bmatrix}$$

rede recíproca e simétrica

18) Two Port

$Z_0 = 50\,\Omega$



a) $\Rightarrow A = 1 \qquad B = z = 100\,\Omega$

$C = 0 \qquad D = 1$

$\Rightarrow \Delta = A + B/z_0 + C \cdot z_0 + D = 4$

$\Rightarrow S_{11} = \dfrac{A + B/z_0 - C \cdot z_0 - D}{\Delta} = 0{,}5$

$\Rightarrow S_{12} = \dfrac{2 \cdot (AD - BC)}{\Delta} = 0{,}5$

$\Rightarrow S_{21} = \dfrac{2}{\Delta} = 0{,}5$

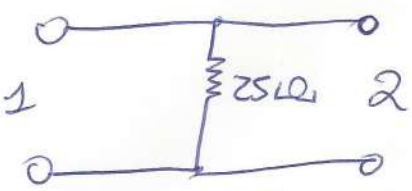
$\Rightarrow S_{22} = \dfrac{-A + B/z_0 - (C \cdot z_0 + D)}{\Delta} = 0{,}5$

$$S = \begin{bmatrix} 0{,}5 & 0{,}5 \\ 0{,}5 & 0{,}5 \end{bmatrix}$$

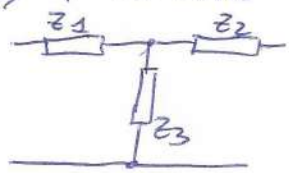
b)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 100 \\ 0 & 1 \end{bmatrix}$$

19) Two port $\quad Z_0 = 50\,\Omega$



$\Rightarrow$ T-network



Pela tabela de parâmetros ABCD temos: Com $Z_1 = Z_2 = 0$

$\Rightarrow A = 1 \qquad \Rightarrow B = 0$

$\Rightarrow C = y = 1/z = \frac{1}{25} = 0{,}04 \qquad \Rightarrow D = 1$

$\Rightarrow \Delta = A + B/z_0 + C \cdot z_0 + D = 4$

$\Rightarrow S_{11} = \frac{A + \frac{B}{z_0} - C z_0 - D}{\Delta} = -0{,}5$

$\Rightarrow S_{21} = \frac{2}{\Delta} = 0{,}5$

$\Rightarrow S_{12} = \frac{2 \cdot (AD - BC)}{\Delta} = 0{,}5$

$\Rightarrow S_{22} = \frac{-A + \frac{B}{z_0} - C z_0 + D}{\Delta} = -0{,}5$

Logo $S = \begin{bmatrix} -0{,}5 & 0{,}5 \\ 0{,}5 & -0{,}5 \end{bmatrix}$

b) $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0{,}04 & 1 \end{bmatrix}$

22) Atenuador 50Ω, 10dB $\quad S_{11} = S_{22} = 0$

$Z_0 = 75\Omega$

I_L

a) Referência 50Ω: $\quad -10dB = 20\log(S_{21})$

$\Rightarrow S_{21} = S_{12} = 10^{-10/20} = 0,316$

$$S = \begin{bmatrix} 0 & 0,316 \\ 0,316 & 0 \end{bmatrix}$$

Referência 75Ω

$$\Rightarrow \Gamma_{75} = \frac{75-50}{75+50} = 0,2$$

$$\Rightarrow {}^{75}S = \begin{bmatrix} {}^{50}S_{11} - \Gamma_{75} & {}^{50}S_{12} \\ {}^{50}S_{21} & {}^{50}S_{22} - \Gamma_{75} \end{bmatrix} \cdot \begin{bmatrix} 1 - \Gamma_{75} \cdot {}^{50}S_{11} & \Gamma_{75} \cdot {}^{50}S_{12} \\ -\Gamma_{75} \cdot {}^{50}S_{21} & 1 - \Gamma_{75} \cdot {}^{50}S_{22} \end{bmatrix}$$

$$= \begin{bmatrix} -0,2 & 0,316 \\ 0,316 & -0,2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -0,0632 \\ -0,0632 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0,22 & 0,32884 \\ 0,3288 & -0,22 \end{bmatrix}$$

Logo, coeficiente de transmissão: $S_{21} = 0,328$

22) b) $^{75}IL_{dB} = 10 \log\left(\frac{1}{|^{75}S_{21}|^2}\right)$

$= 10 \log\left(\frac{1}{0,328}\right) = 4,84\,dB$

c) Carga casada $Z_L = Z_0^* = 75\,\Omega$

$\Rightarrow \Gamma_{IN} = S_{11} = -0,22$