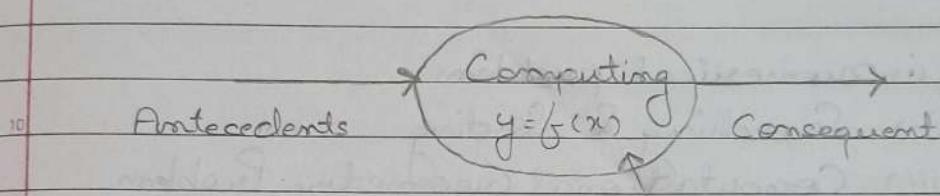


Soft Computing :-

→ what is Computing?

Computing is the process of using computer technology to complete a given goal-oriented task. Computing System contains certain inputs to generate a specific output.



- x : Input / Antecedents
- y : Output / Consequent
- f : Mapping function / Control Action

→ Features :-

- Should accept user information in terms of feature sets and produce a desired output using a suitable action control action.
- Control action should be unambiguous and accurate.
- Control actions should be easy to conceptualise, construct and operate.
- Should provide precise and optimised solution.

→ Hard Computing :-

- Precise Result
- Control Actions

↳ Unambiguous

↳ Formally defined

- It is primary primal procedure for solving engineering problems that can be formulated mathematically.

→ Features :-

- Input - output relationship are expressed in terms of mathematical expression.
- Control actions must be suitable to the problem defined.
- Control actions should be stable, highly predictable and accurate.

→ Solve -
10 (i) numerical problem

(ii) Searching & Sorting

(iii) Computational Geometry Problem.

→ Soft Computing :-

- It resembles the reasoning of human mind being a collection of biologically inspired methodologies.

→ Features :-

◦ Imprecision

◦ 20 Dynamic

◦ Uncertainty

◦ Low Solution Cost

◦ Do not require mathematical model.

25 Doctor & Patient

Fuzzy logic

A
a — SC → A

Neural Network

A
a

CSK
KKR — SC → who wins
RCB

Evolutionary / Genetic
Computing

2019 IPL?

→ Difference Soft Computing and Hard Computing

Hard Computing	Soft Computing
1. Precisely stated analytical model is required.	1. Imprecision is tolerable
2. More Computational time is required.	2. Computational time required is less as it involves intelligent Computational steps.
3. Precision is observed within the Computation.	3. Approximation is observed in the Computation.
4. Imprecision and uncertainty are undesirable properties.	4. Tolerance for imprecision and uncertainty is exploited to achieve traceability.
5. Programs are written which follows standard rules.	5. Programs are evolved with new laws and theories.
6. Requires exact input data.	6. Deals with ambiguous and noisy input data.
7. Strictly follows Sequential Computations	7. Allows parallel computation.
8. Produces precise result.	8. Produces approx. results

→ Hybrid Computing :-

Combination of Conventional hard Computing and emerging Soft Computing.

→ Fuzzy logic :-

- Logic represents and processes knowledge.
- Crisp / binary / boolean and fuzzy logic are both useful for knowledge based Systems.
- Fuzzy logic is a mathematical tool for dealing with Uncertainty and hence a Cognitive process.
- Crisp logic deals with Crisp sets and Crisp / boolean algebra.

- Fuzzy logic deals with uncertainty using fuzzy sets and fuzzy algebra.

→ Fuzzy logic
↓

mathematical language

Relational + Boolean + Predicate
logic logic logic

→ Features :-

- This concept is flexible that we can easily understand and implement.
- It is used for helping the minimization of logics created by the human.

It is the best method for finding the solution of those problems which are suitable for approximate reasoning.

- In the fuzzy logic, any system which is logical can be easily fuzzified.
- It is based on natural language processing and allows users to integrate with the programming.

→ Difference Crisp logic and Fuzzy logic

Crisp set

- It has sharp boundaries restricted by 0/F and 1/T.

- Some Crisp set can be fuzzy.

Fuzzy Set

- It has fuzzy boundaries with a degree of membership.

- No fuzzy set can be crisp.

- 3) Defined by precise and certain characteristics
- 4) Elements are single and either a member of a set or not.
- 5) logic is bi-valued
- 6) used in digital system design.

- 3) Prescribed by vague or ambiguous properties
- 4) Elements are ordered pairs and are allowed to be a partial member of set.
- 5) logic is infinite val.
- 6) Used only in fuzzy controllers.

Crisp O/P

↳ Yes or No

↳ True or False

Fuzzy O/P

↳ May be

↳ May not be

↳ Absolutely

↳ Partially

→ Crisp Set :-

For a Crisp set $A \subseteq U$ and any arbitrary element x

either $x \in A$ or $x \notin A$

U : All Student

A : In 10th

B : In 12th

U

(A)

(B)

i. Disjoint set

1 element in set belongs to one set at a time.

→ Fuzzy Sets :-

Fuzzy sets are generalization of Crisp sets and it allows partial membership.

Fuzzy set A is defined as :-

$$A = \{(x, \mu_A(x)) \mid x \in U\}$$

U = All Students

G = Good Students

S = Bad Students

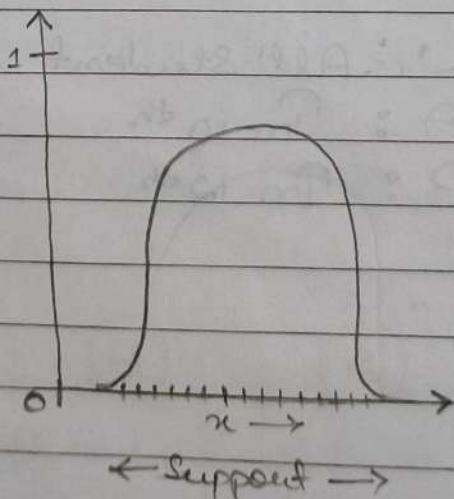
$$G = \{G_i, \mu(G_i)\} \quad \text{where degree of goodness}$$

$$G = \{(A, 0.9), (B, 0.7), (C, 0.1), (D, 0.3)\}$$

$$S = \{(A, 0.1), (B, 0.3), (C, 0.9), (D, 0.7)\}$$

→ Membership function :-

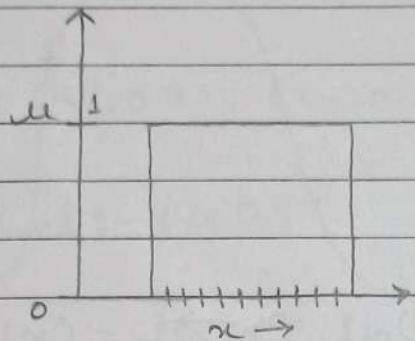
(i) Support : Set of all points such that $\{\mu_A(x) > 0\}$



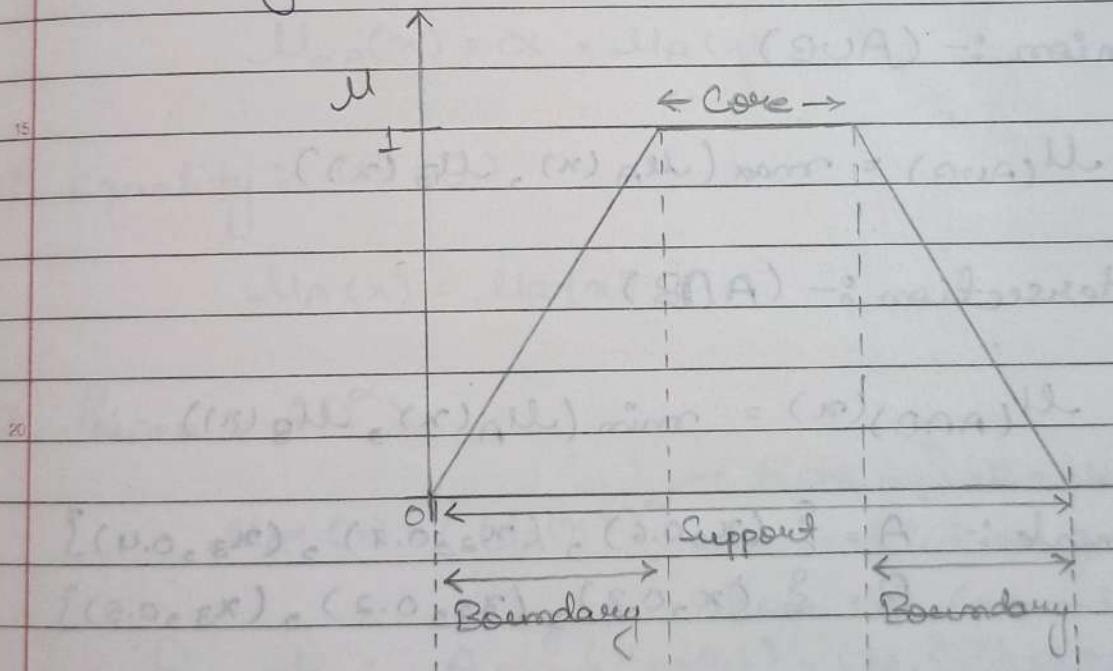
$$\text{Support}(A) = \{x \mid \mu_A(x) > 0\}$$

II Core : $\mu_A(x) = 1$

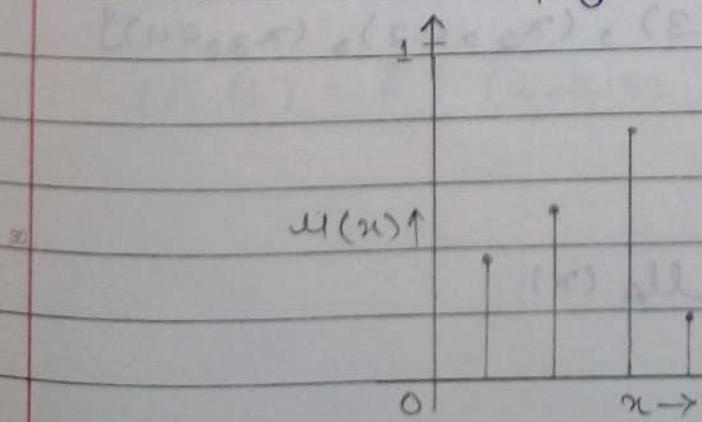
$$\text{Core}(A) = \{x \mid \mu_A(x) = 1\}$$



III Boundary : ($1 > \mu_A(x) > 0$)



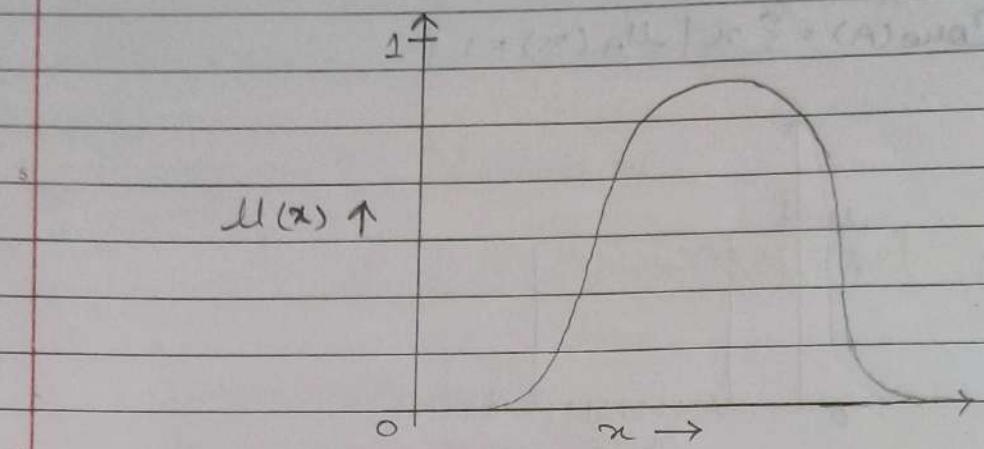
→ Discrete Membership function :-



Ye discrete MF tab ho
jab MF on element
MF and element me
se koi ek discrete
hog.

Date / /

→ Continuous Membership function :-



→ Fuzzy set operations :-

1) Union :- $(A \cup B)$

$$M_{(A \cup B)} = \max(M_A(x), M_B(x))$$

2) Intersection :- $(A \cap B)$

$$M_{(A \cap B)}(x) = \min(M_A(x), M_B(x))$$

Example :- $A = \{(x_1, 0.6), (x_2, 0.7), (x_3, 0.4)\}$
 $B = \{(x_1, 0.3), (x_2, 0.2), (x_3, 0.5)\}$

$$(A \cup B) = \{(x_1, 0.6), (x_2, 0.7), (x_3, 0.5)\}$$

$$(A \cap B) = \{(x_1, 0.3), (x_2, 0.2), (x_3, 0.4)\}$$

3) Complement :- A^c

$$M_{(A^c)}(x) = 1 - M_A(x)$$

Example :- $A = \{(x_1, 0.6), (x_2, 0.7), (x_3, 0.4)\}$

$B = \{(x_1, 0.3), (x_2, 0.2), (x_3, 0.5)\}$

$A^c = \{(x_1, 0.4), (x_2, 0.3), (x_3, 0.6)\}$

$B^c = \{(x_1, 0.7), (x_2, 0.8), (x_3, 0.5)\}$

4) Vector Product :- $(A \cdot B)$

$$M_{AB}(x) = M_A(x) \cdot M_B(x)$$

5) Scalar Product :- $(\alpha \times A)$

$$M_{\alpha A}(x) = \alpha \times M_A(x)$$

6) Equality :- $(A = B)$

$$M_A(x) = M_B(x)$$

7) Power :- (A^α)

$M_{A^\alpha}(x) = (M_A(x))^\alpha$ → Alpha is a value less than 1 means dilution greater than 1
Alpha is a value more than 1 means concentration.

Example :- $A = \{(x_1, 0.6), (x_2, 0.7), (x_3, 0.4)\}$

$B = \{(x_1, 0.3), (x_2, 0.2), (x_3, 0.5)\}$

$$(A \cdot B) = \{(x_1, 0.18), (x_2, 0.14), (x_3, 0.20)\}$$

8) Sum ($A+B$) :-

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$$

9) Difference ($A-B$) :-

$$\mu_{A-B}(x) = \mu_{A \cap B^c}(x)$$

10) Disjunctive Sum ($A \oplus B$) :-

$$\mu_{(A \oplus B)}(x) = (A^c \cap B) \cup (A \cap B^c)$$

11) Cartesian Product ($A \times B$) :-

$$\mu_{A \times B}(x, y) = \min(\mu_A(x), \mu_B(y))$$

Example :-

$$A(x) = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5)\}$$

$$B(x) = \{(y_1, 0.8), (y_2, 0.6), (y_3, 0.3)\}$$

$$A \times B = \begin{matrix} & y_1 & y_2 & y_3 \\ x_1 & 0.2 & 0.2 & 0.2 \\ x_2 & 0.3 & 0.3 & 0.3 \\ x_3 & 0.5 & 0.5 & 0.3 \end{matrix}$$

→ Fuzzy set properties :-

$$17) \text{ Commutative :- } A \cap B = B \cap A$$

$$A \cup B = B \cup A$$

2) Associative :- $A \cup (B \cup C) = (A \cup B) \cup C$
 $A \cap (B \cap C) = (A \cap B) \cap C$

3) Distributive :- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
 $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4) Idempotence :- $A \cup A = A ; A \cap A = \emptyset$
 $A \cup \emptyset = A ; A \cap \emptyset = \emptyset$

5) Transitive :- If $A \subseteq B ; B \subseteq C$ then $A \subseteq C$

6) DeMorgan's law :- $(A \cap B)^c = A^c \cup B^c$
 $(A \cup B)^c = A^c \cap B^c$

→ Crisp Relation :-

$$A = \{1, 2, 3\}$$

$$B = \{4, 7, 8\}$$

$$A \times B = \{(1,4), (1,7), (1,8), (2,4), (2,7), (2,8), (3,4), (3,7), (3,8)\}$$

$$R_1 = \{(a,b) | a > b, (a,b) \in A \times B\} \quad \text{If condition will not satisfy then } R = \{\emptyset\}$$

$$R_1 = \{(1,4), (1,7), (1,8), (2,4), (2,7), (2,8), (3,4), (3,7), (3,8)\}$$

$$R = \begin{matrix} & 4 & 7 & 8 \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

$$R = \{(1,4), (2,7), (3,8)\}$$

2) Associative :- $A \cup (B \cup C) = (A \cup B) \cup C$
 $A \cap (B \cap C) = (A \cap B) \cap C$

3) Distributive :- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
 $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4) Idempotency :- $A \cup A = A ; A \cap A = \emptyset$
 $A \cup \emptyset = A ; A \cap \emptyset = \emptyset$

5) Transitive :- If $A \subseteq B ; B \subseteq C$ then $A \subseteq C$

6) DeMorgan's law :- $(A \cap B)^c = A^c \cup B^c$
 $(A \cup B)^c = A^c \cap B^c$

→ Crisp Relation :-

$$A = \{1, 2, 3\}$$

$$B = \{4, 7, 8\}$$

$$A \times B = \{(1,4), (1,7), (1,8), (2,4), (2,7), (2,8), (3,4), (3,7), (3,8)\}$$

$R_1 = \{(a,b) | a > b, (a,b) \in A \times B\}$ If condition w
not satisfy them
 $R = \{\emptyset\}$

$$R_1 = \{(1,4), (1,7), (1,8), (2,4), (2,7), (2,8), (3,4), (3,7), (3,8)\}$$

$$R = \begin{matrix} & 4 & 7 & 8 \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \left[\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \right] \end{matrix}$$

$$R = \{(1,4), (2,7), (3,8)\}$$

- Operation On Crisp Relation :-
 → Operation on Fuzzy Relation :-
 1) Union : $(A \cup B)$ we choose max bet. both the sets

5 A (x, y)

B (x, y)

$$10 A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$27 A \cup B = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

28 Intersection : $(A \cap B)$

We choose min. bet. both the sets.

$$20 A \cap B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

29 Complement : A^c

Reverse of index.

$$30 A^c = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix} \quad \therefore \text{In Fuzzy relation we have to use } \neg \mu(x) \text{ formula for complement}$$

→ Fuzzy Relation :-

$$A = \{(x_1, 0.6), (x_2, 0.2), (x_3, 0.3)\}$$

$$B = \{(y_1, 0.7), (y_2, 0.3), (y_3, 0.4)\}$$

$$\mu_R(x, y) = \max_{\{x, y\}} \{\mu_A(x), \mu_B(y)\}$$

	y_1	y_2	y_3
x_1	0.6	0.3	0.4
x_2	0.2	0.2	0.2
x_3	0.3	0.3	0.3

→ If then Rule in Fuzzy :-

- Fuzzy Implication
 - Fuzzy Rule
 - Fuzzy Conditional Statement
- } Called as

If x is A then y is B

- x is 'A' : antecedents
or
Premise
 - y is 'B' : Consequence
or
Conclusion
- A and B is a set and
 x and y is a element
that belong to set A and
set B.

↳ Fuzzy Rule : 'R' is denoted as

$$R : A \rightarrow B$$

Example : If Temp is high then pressure is low.

$$P_{\text{High}} = \{(25, 0.1), (30, 0.2), (35, 0.5), (40, 0.6)\}$$

$$P_{\text{Low}} = \{(2, 0.3), (5, 0.5), (6, 0.4)\}$$

If stem temp is high then pressure is low.

$$R: T_{\text{High}} \rightarrow P_{\text{Low}}$$

$$R: A \rightarrow B$$

	2	5	6
R @ 25	0.1	0.1	0.1
30	0.2	0.2	0.2
A \times B = 35	0.3	0.5	0.4
40	0.3	0.5	0.4

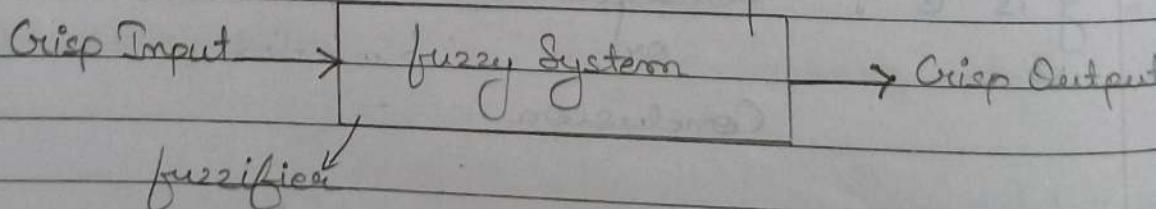
→ Defuzzification :-

Fuzzy to Crisp Conversion

Fuzzification :-

Crisp to fuzzy conversion

Defuzzification



1) Dombob - cut method

2) Maxima methods

3) Weighted Sum method

4) Centroid methods

1) Lambda cut - method :- fuzzy sets

• Fuzzy set A $\xrightarrow{\lambda} \text{Crisp set } A_\lambda \quad (0 \leq \lambda \leq 1)$

$$A_\lambda = \{x \mid \mu_A(x) > \lambda\}$$

Example :-

$$A = \{(x_1, 0.2), (x_2, 0.4), (x_3, 0.6)\}$$

$$\lambda = 0.3$$

$$A_{0.3} = \{(x_1, 0), (x_2, 1), (x_3, 1)\}$$

$$= \{x_2, x_3\}$$

$$B = \{(y_1, 0.5), (y_2, 0.4), (y_3, 0.7)\}$$

$$\lambda = 0.7$$

$$B_{0.7} = \{(y_1, 0), (y_2, 0), (y_3, 1)\}$$

$$= \{y_3\}$$

→ Lambda cut - method :- fuzzy relation

$$R = \begin{bmatrix} 0.2 & 1 \\ 0.3 & 0.6 \end{bmatrix} \quad \lambda = 1, 0.5, 0.1, 0$$

\rightarrow fuzzy relation

$$R_\lambda = R_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \rightarrow \text{Crisp relation}$$

\therefore It is not necessary if fuzzy relation ka crisp relation ek hi ho because it depend on the value of λ .

$$R_{0,S} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$R_{0,T} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

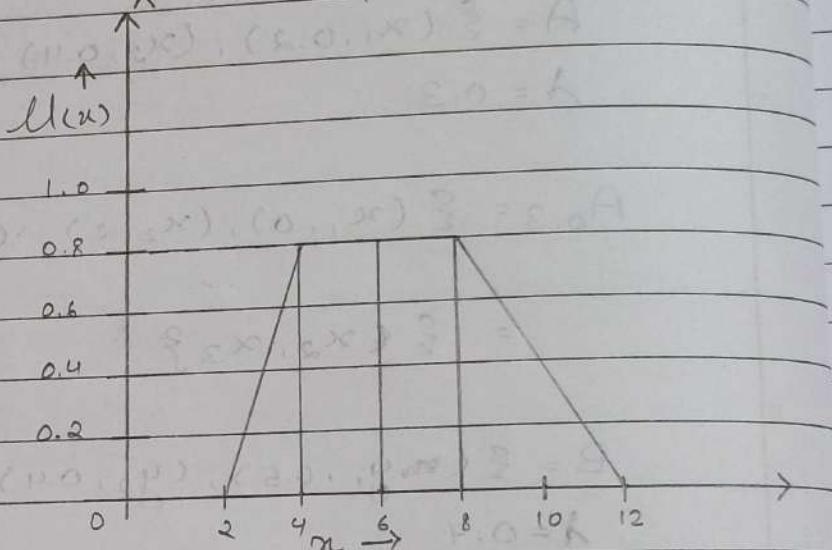
$$R_0 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

→ Maxima methods :- maxima means → height of fuzzy set
 $x^* \rightarrow$ Crisp Value

1) First of Maxima
(FOM)
 $= x^* = 4$

2) Last of Maxima
(LOM)
 $= x^* = 8$

3) Mean of Maxima
(MOM)



$$Bx^* = \frac{\sum x_i e^M}{|M|}$$

$$M = \{x \mid M_A(x) = \text{height of fuzzy set}\}$$

~~DEF~~ $|M| =$ Cardinality of set M

for mean of Maxima

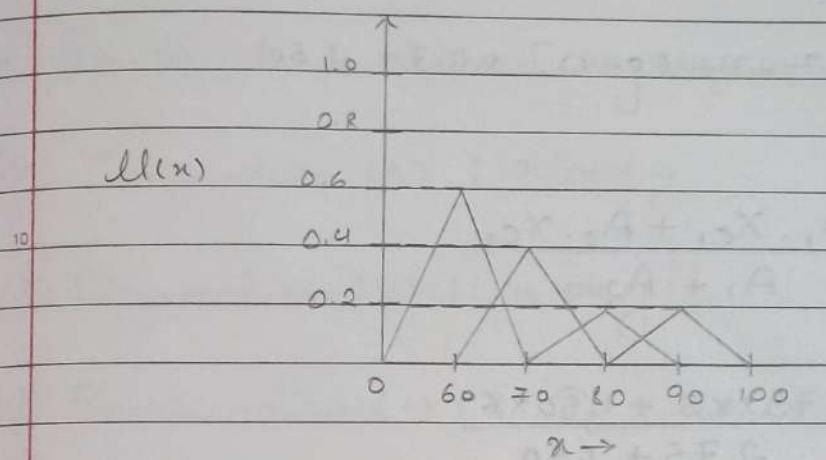
$$x = 4, 6, 8$$

$$x^* = \frac{4+6+8}{3}$$

$$x^* = 6$$

→ Weighted Average Method :-

$$x^* = \frac{\sum u(x) \cdot x}{\sum u(x)}$$

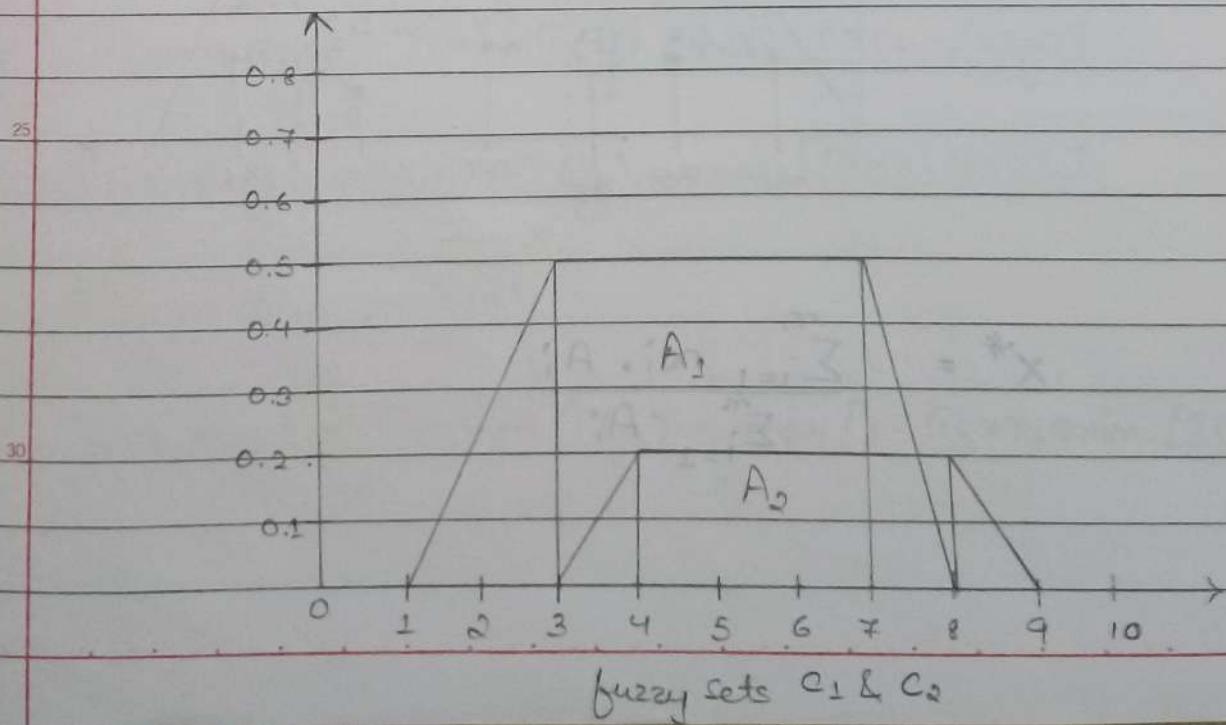


$$x^* = \frac{(60 \times 0.6 + 70 \times 0.4 + 80 \times 0.2 + 90 \times 0.2)}{0.6 + 0.4 + 0.2 + 0.2}$$

$$x^* = 70$$

→ Centroid Method :-

17 Centre of Sum (COS) :-



$$x^* = \frac{\sum A_i x_i}{\sum A_i}$$

$$A_1 = \frac{1}{2} [(8-1) + (7-3)] \times 0.5 = 2.75$$

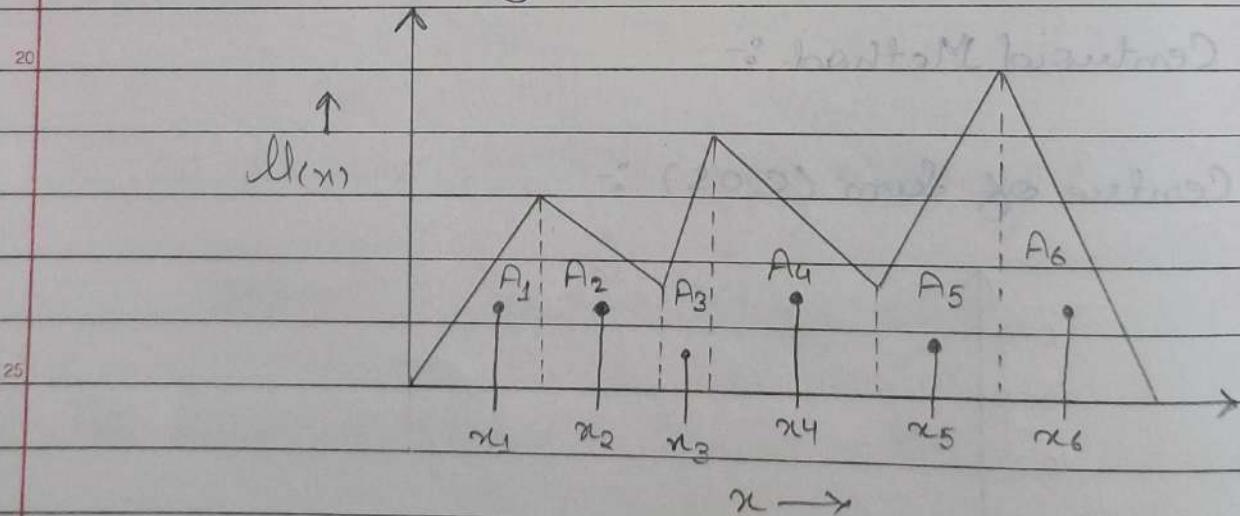
$$A_2 = \frac{1}{2} [(9-2) + (8-4)] \times 0.3 = 1.50$$

$$Q x^* = \frac{A_1 \cdot x_{c_1} + A_2 \cdot x_{c_2}}{A_1 + A_2}$$

$$= \frac{2.75 \times 5 + 1.50 \times 6}{2.75 + 1.50}$$

$$x^* = 5.35$$

2) Centre of Gravity (COG) :-



$$x^* = \frac{\sum_{i=1}^m x_i \cdot A_i}{\sum_{i=1}^m A_i}$$

→ Truth Values and Tables in Fuzzy logic :-

- x is A

\Rightarrow "London is in UK"

↓ ↓
Subject Predicate

- Truth table defines logic function of 2 propositions.

- i) Conjunction (\wedge) : x and y

- ii) Disjunction (\vee) : x or y

- iii) Implication (\rightarrow) : If x Then y

- iv) Bidirectional (\leftrightarrow) : x IFF y

- Truth value of propositions in fuzzy logic are in logic
are in range $[0, 1]$

eg: P: Ram is Boy

$$\hat{T}(P) = 0.8$$

- $\hat{T}(x \text{ and } y) = \hat{T}(x) \wedge \hat{T}(y) = \min [\hat{T}(x), \hat{T}(y)]$

- $\hat{T}(x \text{ or } y) = \hat{T}(x) \vee \hat{T}(y) = \max [\hat{T}(x), \hat{T}(y)]$

- $\hat{T}(\text{Not } x) = 1 - \hat{T}(x)$

- $\hat{T}(x \rightarrow y) = \hat{T}(x) \rightarrow \hat{T}(y) = \max [1 - \hat{T}(x), \min [\hat{T}(x), \hat{T}(y)]]$

→ Fuzzy Proposition & Crisp Proposition :-

P : Ram is a Boy

$$\tilde{T}(P) = 0.0$$

$$\tilde{T}(P) = 0.2$$

$$\tilde{T}(P) = 0.8$$

$$\tilde{T}(P) = 1.0$$

Q : Ram is intelligent

$$\tilde{T}(Q) = 0.6$$

\Rightarrow Ram is not intelligent

$$\tilde{T}(\bar{Q}) = 1 - \tilde{T}(Q) = 0.4$$

\Rightarrow Ram is Boy and so is intelligent

$$\tilde{T}(P \wedge Q) = \min(\tilde{T}(P), \tilde{T}(Q))$$

$$= \min(0.8, 0.6)$$

$$= 0.6$$

- 25. o Fuzzy predicates : " Shridhar is tall " (tall, short, quick)

- o Fuzzy predicate modifier : ("very, fairly, moderately, rather, slightly")

- Fuzzy quantifiers :
 - (Most, Several, Many)

- Fuzzy quantifiers :
- Based on Truth (x is t)
- Based on Probability (x is λ)
- Based on Possibility (x is π)

→ Decomposition of Rules :-

- 1) Multiple Conjunction antecedents :-

If x is $A_1, A_2, A_3, \dots, A_m$ Then y is B_m .

$$A_m = A_1 \cap A_2 \cap A_3 \cap \dots \cap A_m$$

$$M_{A_m}(x) = \min [M_{A_1}(x), M_{A_2}(x), \dots, M_{A_m}(x)]$$

\Rightarrow If A_m Then B_m

- 2) Multiple disjunctive antecedents :-

- 3) Conditional statements :-

If A_1 , Then B_1 . Else B_2

\Rightarrow If A_1 , Then B_1 ,

OR

If $\neg A_1$, Then B_2

- 4) Nested If -then rules

If A_1 , Then [If A_2 then B_1]

\Rightarrow If A_1 AND A_2 Then B_1

→ Aggregation of fuzzy rules :-

1) Conjunctive System of rules :-

- Rules to be jointly satisfied
- Using AND
- Using intersection

$$y = y_1 \text{ AND } y_2 \text{ AND } y_3 \dots \text{ AND } y_m$$

$$y = y_1 \cap y_2 \cap y_3 \dots \cap y_m.$$

$$\mu_y(y) = \min[\mu_{y_1}(y), \mu_{y_2}(y), \mu_{y_3}(y) \dots \mu_{y_m}(y)]$$

2) Disjunctive System of rules :-

◦ The satisfaction of at least one rule.

- OR is used

$$y = y_1 \text{ or } y_2 \text{ or } \dots \text{ or } y_m$$

$$y = y_1 \cup y_2 \cup y_3 \dots \cup y_m$$

$$\mu_y(y) = \max[\mu_{y_1}(y), \mu_{y_2}(y), \mu_{y_3}(y) \dots \mu_{y_m}(y)]$$

→ Genetic Algorithm :-

are adaptive heuristic search algo. that belongs to the larger of evolutionary algorithm.

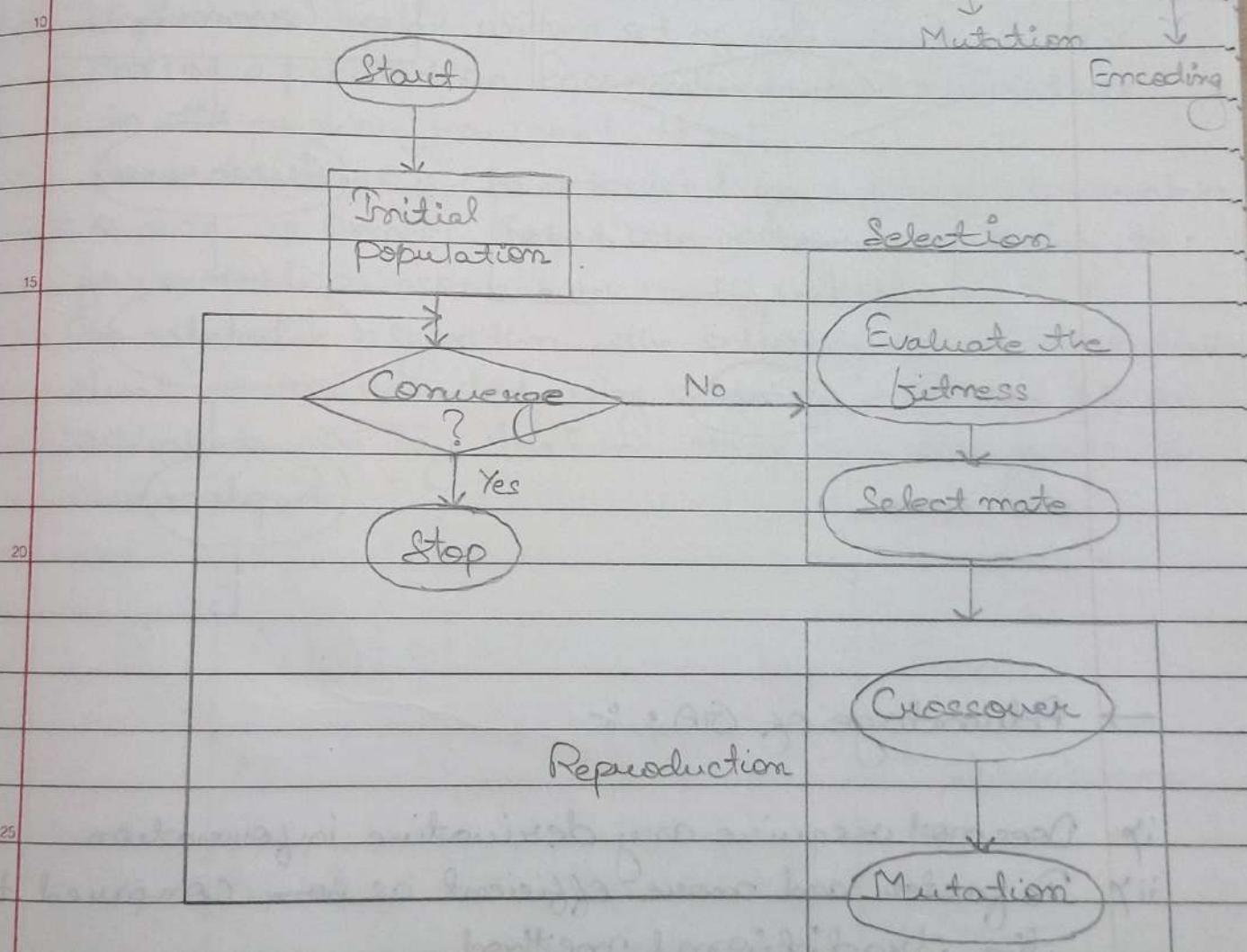
Genetic algo. are based on the ideas of natural selection and genetics. These are intelligent exploitation of random search provided with historical data to direct the search into the region of better performance in solution space.

They are commonly used to generate high quality solution for optimization problems.

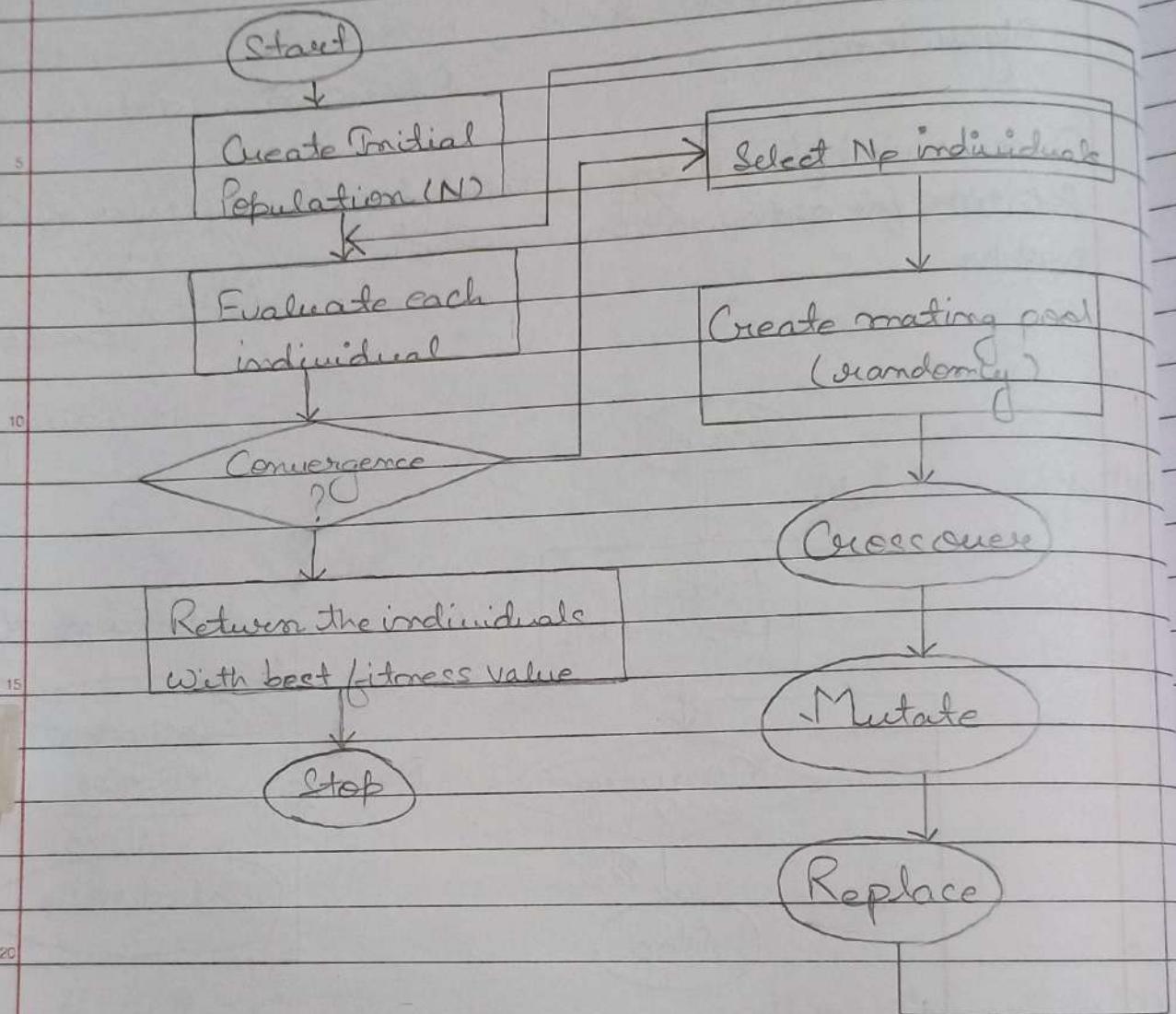
and search problems.

Adaptive heuristic Search, Evolutionary, Genetic & Algorithm, natural selection.

To generate high-Quality, Population &, Operator of GA solution for optimization Individual problem.



→ Simple Genetic Algorithm :-



→ Advantage of GAs :-

- i) Does not require any derivative information.
- ii) Is faster and more efficient as ~~compared~~ compared to the traditional method.
- iii) Has very good parallel Capabilities.
- iv) Provides a list of "good" Solutions and not just a Single Solution.
- v) Always gets an answer to the problem, which gets better over the time.

→ GA Phases :-

- i) Initial population
- ii) Fitness function
- iii) Selection
- iv) Crossover
- v) Mutation

vi) Initial Population :-

- The process begins with a set of individuals which is called a population. Each individual is a solution to the problem you want to solve.
- An individual is characterized by a set of parameters known as genes. Genes are joined into string to form a chromosome.
- In a genetic algorithm, the set of genes of an individual is represented using a string, in terms of an alphabet. we say that we encode the genes in chromosome.

20 A₁ | 0 | 0 | 0 | 0 | 0 | 0 | 0 → Gene

25 A₂ | 1 | 1 | 1 | 1 | 1 | 1 | 1 → Chromosome

A₃ | 1 | 0 | 1 | 0 | 1 | 1 | 1

A₄ | 1 | 1 | 0 | 1 | 1 | 0 | 0

→ Population

→ Genetic Algorithm

- i) Genetics & Natural Selection to solve optimization problem.
- ii) More advanced
- iii) Used in field such as ML, AI.
- iv) Probabilistic Rules
- v) Search on a population

Traditional Algorithm

- i) Step by step procedure to solve a given problem
- ii) Not as advanced
- iii) Used in fields such as programming, mathematics
- iv) Fully deterministic rules
- v) Search on a single point of points

→ Convergence Test / Termination Condition :-

The termination condition of a genetic algorithm is important in determining when a GA run will end. It has been observed that initially, the GA progresses very fast with better solutions coming in every few iterations, but this tends to saturate in the later stages where the improvements are very small. We usually want a termination condition such that our solution is close to optimal, at the end of the run.

- Manual checking
- Solution found that specifies satisfy objective criteria
- Fixed no. of generation.
- Budget limit reached.

→ Encoding :-

- Binary Encoding :- Representing a gene in terms of bits. In this encoding scheme, a gene or chromo-

Some is represented by a string of binary bits

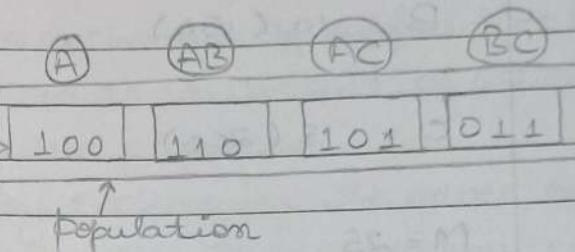
$$\textcircled{A} = \textcircled{A} = 5(100)$$

$$\textcircled{B} = 10(150)$$

$$\textcircled{C} = 15(200)$$

$$M = 25$$

chromosome
some



Example : $f(x) = x^3$, $0 \leq x \leq 255$

→ 8 bit digit

10011011

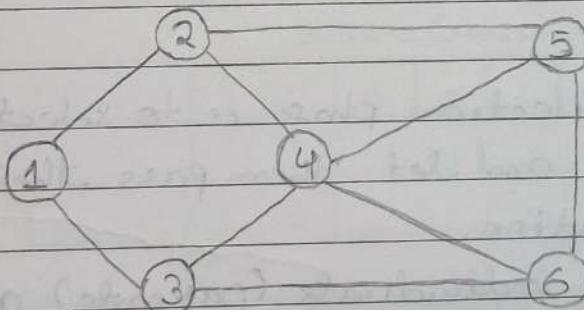
155

11111111

255

iii) Fitness function :-

- Fitness function determines how fit an individual is.
- It gives a fitness score to each individual.
- The probability that an individual will be selected for reproduction is based on its fitness score.



Path 1 : 1 2 5 6 4 3 1 - ⑯

Path 2 : 1 2 5 4 6 3 1 - ⑰

Path 3 : 1 2 4 5 6 3 1 - ⑯

Path 4 : 1 2 5 6 3 4 1 - ⑰

$$A = 5(100)$$

$$B = 10(150)$$

$$C = 15(200)$$

$$M = 25$$

		<i>w.</i>	<i>P</i>
	A - 100	5	100
10	AB - 110	15	250
	AC - 101	20	300
15	BC - 011	25	350 ✓
	ABC - 111	30	450 ✗

iii) Selection :-

- The idea of selection phase is to select the fittest individuals and let them pass their genes to the next generation.
- Two pairs of individuals (parents) are selected based on their fitness scores.
- Individuals with high fitness have more chance chance to be selected for reproduction.

iv) Crossover :-

- Crossover is the most significant phase in a genetic algorithm. For each pair of parents to be mated a crossover point is chosen at random from within the genes.

- Once a pool of mating mating pair are selected, they undergo through crossover operations.
- In crossover, there is an exchange of properties between the two parents and as a result of which two offspring solutions are produced.
- Offspring are created by exchanging the genes of parents among themselves until the crossover point is reached.

a) Single point crossover :-

- Here, we select the K point lying between I and L.
let it be K.
- A single crossover point at K on both parent's string is selected.
- All data beyond that point in either string is swapped between the two parents.
- The resulting strings are the chromosomes of the offspring produced.

Before Crossover

P₁ : [0 | 1 | 1 | 0 | 0 | 0 | 1 | 0] Two diploid from a mating pair

P₂ : [1 | 0 | 1 | 0 | 1 | 1 | 0 | 0]

Crossover Point - K

O₁ : [0 | 1 | 1 | 0 | 1 | 1 | 0 | 0]

O₂ : [1 | 0 | 1 | 0 | 0 | 0 | 1 | 0]

After Crossover

b) Two point Crossover :-

- In this scheme, we select two different crossover points k_1 and k_2 lying between 1 and L at random such that $k_1 \neq k_2$.
- The middle parts are swapped between the two strings.
- Alternatively, left and right parts also can be swapped.

Before Crossover

10	P ₁ :	0 1 1 0 0 0 1 0	
----	------------------	-------------------------------	--

11	P ₂ :	1 0 1 0 1 1 0 0	
----	------------------	-------------------------------	--

K₁ K₂

15

O ₁ :	0 1 1 0 1 1 1 0	
------------------	-------------------------------	--

16	O ₂ :	1 0 1 0 0 0 0 0	
----	------------------	-------------------------------	--

20 After Crossover

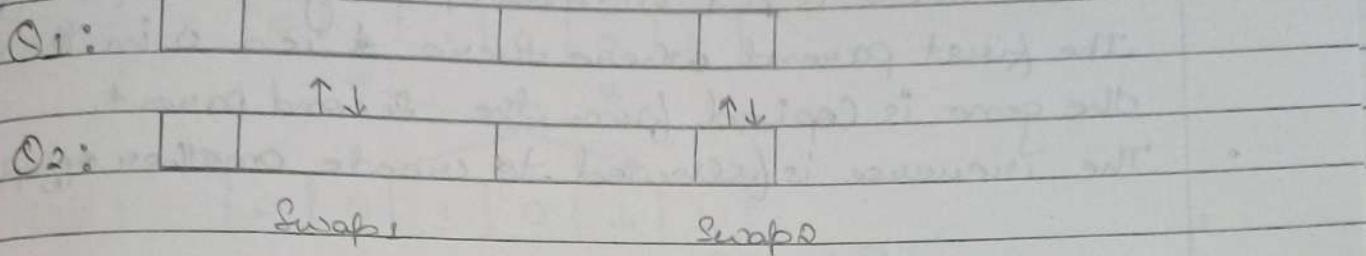
c) Multipoint Crossover :-

- In case of multipoint Crossover, a number of crossover points are selected along the length of the string at random.
- The bits lying between alternate pairs of sites are then swapped.

25	P ₁ :						
----	------------------	--	--	--	--	--	--

30	P ₂ :						
----	------------------	--	--	--	--	--	--

K₁ K₂ K₃



d) Uniform Crossover :-

- Uniform crossover is a more general vision of multi-point crossover.
- In this scheme, at each bit position of parent string, we toss a coin to determine whether there will be swap of the bits or not.
- The two bits are then swapped or remain unaltered, accordingly.

P1: 1 1 0 0 0 1 0 1 1 0 0 1

P2: 0 1 1 0 0 1 1 1 0 1 0 1

20 Coin tossing

1 0 0 1 1 1 0 1 1 0 0 1

Q1: 1 1 1 0 0 1 1 1 1 1 0 1

Q2: 0 1 0 0 0 1 0 1 0 0 0 1

e) Uniform crossover with crossover mask :-

- Here, each gene is created in the offspring by copying the corresponding gene from one or the other parent chosen according to a random generated binary crossover mask of the same length as the chromosome.

- where there is a 1 in mask, the gene is copied from the first parent where there is a 0 in the mask the gene is copied from the second parent.
- The reverse is followed to create another offspring

Before Crossover

P₁: | 1 | 1 | 0 | 0 | 0 |

when there is a 1 in the mask
the gene is copied from P₁
from P₂.

P₂: | 0 | 1 | 1 | 0 | 0 |

Mask: | 1 | 0 | 0 | 1 | 1 |

when there is a 1 in the mask
the gene is copied from P₂
else from P₁.

After Crossover

O₁: | 1 | 1 | 1 | 0 | 0 |

O₂: | 0 | 1 | 0 | 0 | 0 |

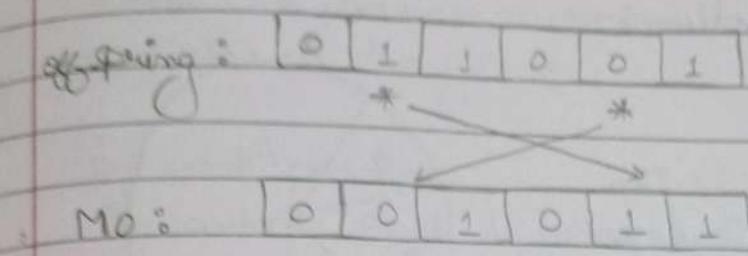
→ Mutation :-

- In certain new offspring formed, some of their genes can be subjected to a mutation with a low random probability. This implies that some of the bits in the bit string can be flipped.
- Mutation occurs to maintain diversity within the population and prevent from premature convergence

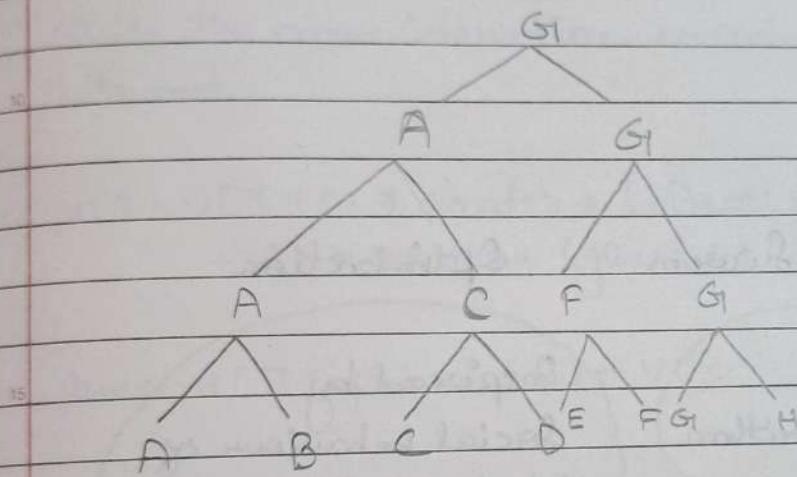
offspring: | 0 | 1 | 1 | 0 | 0 | 1 |

M_P(M_P): | 0 | 1 | 0 | 0 | 0 | 1 |

M_O: | 0 | 0 | 1 | 0 | 0 | 0 |



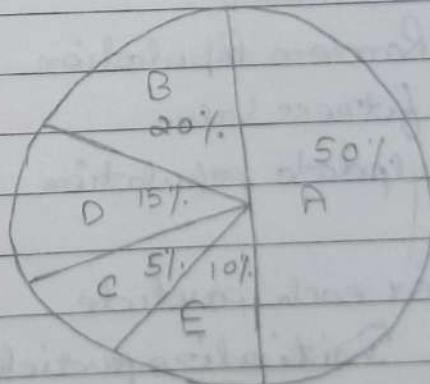
→ Tournament Selection :-



→ Roulette wheel Selection :-

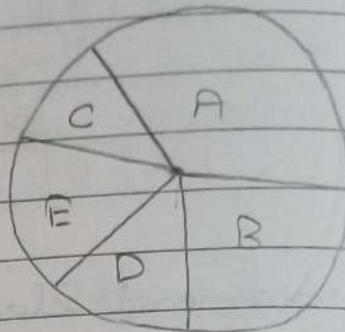
C	F
A	S
B	2
C	0.5
D	1.5
E	1

50% 20% 5% 15% 10%



→ Rank Based Selection :-

C	F	Rank	
A	5	5	= 33.33%
B	2	4	= 26.67%
C	0.5	1	= 6.67%
D	1.5	3	= 20%
E	1	2	= 13.33%
		15	



→ PSO :-

Particle

Swarm

Optimization

15 Population based
Stochastic Algorithm

inspired by
Social behaviour of
bird flocking or
fish schooling.

Common :-

20 Random population
fitness value
Update population

For each particle

25 Initialize particle

END

DO

For each particle

Calculate fitness value

30 If the fitness value is better than pBest in history
Set current value as the new pBest.

ENO

choose the particle with the best fitness value of all particles as the gBest.

For each particle

Calculate particle velocity ①

Update particle position ②

END

while the max iteration or min error criteria is not attained.

$$1. \quad v[i] = v[i] + c_1 * \text{rand}() * (\text{pBest}[i] - \text{present}[i]) \\ + c_2 * \text{rand}() * (\text{gBest}[i] - \text{present}[i])$$

$$2. \quad \text{present}[i] = \text{present}[i] + v[i]$$

20

25

30