

The Relationship of Language to Mathematics and Mathematics to Language

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1 Introduction

The contemporary digital era, dominated by algorithms and driven by advances in artificial intelligence, gives rise to a natural temptation to reduce language, the most human of tools, to the level of a mathematical problem. The successes of statistical models and neural networks in natural language processing (NLP) seem to suggest that its complexity is ultimately "computable" and that its structures can be fully described by formal systems.

Underlying this tendency is a pursuit of engineering precision—a vision of a system where meanings are unambiguous, communication is free from misunderstanding, and every utterance is subject to binary verification of truth and falsehood. This is a perspective in which language is perceived primarily as a carrier of information, and its effectiveness is measured by its ability to losslessly encode and decode facts.

This article, however, posits that such a reduction is fundamentally impossible and is based on a foundational category error. Mathematics, in its essence, is a formal system whose purpose is to describe objective, axiomatic truths about structures and relations. Meanwhile, language, as a complex semiotic system, is a tool with an incomparably broader spectrum of applications. Its descriptive function (describing reality) is but one of many. Equally important, if not more so, are its abilities to create fiction (the poetic function), influence beliefs (the persuasive function), establish obligations and states of affairs (the performative function), and construct subjective yet binding "social truths" that do not yield to mathematical logic.

Understanding this fundamental difference is of particular importance in the face of two key contemporary challenges.

First, in the age of disinformation and "post-truth," it is precisely the non-logical, persuasive, and emotional aspects of language that are exploited to manipulate public opinion, and a purely mathematical approach to content analysis proves helpless against these phenomena.

Second, the emergence of AI systems capable of autonomous communication raises unprecedented questions about what "language-games" they will create among themselves. In response to these challenges, this article will analyze the relationship between language and mathematics, using the paradoxical statement $2 + 2 = 5$ as an example. Subsequently, using the tools of epistemic logic and the philosophy of Ludwig Wittgenstein, it will demonstrate the limits of mathematization and discuss its profound implications.

2 Foundational Assumptions and Definitions

To precisely analyze the relationship between language and mathematics, it is necessary to deepen their definitions beyond common understanding. They must be viewed as systems with different goals, structures, and, most importantly, limitations.

2.1 What is Mathematics (M)?

Mathematics (M) is often defined as a formal system based on axioms and rules of inference, aiming to precisely describe abstract structures and relations. In this view, close to the formalism of David Hilbert, mathematics is seen as a symbolic game. As Max Tegmark notes in a paper published on arXiv, where he argues for the mathematical universe hypothesis: “A mathematical structure is [...] a set of abstract entities with relations between them” (arXiv:0704.0646v1). From this perspective, objectivity and consistency are fundamental—a system cannot simultaneously prove a theorem and its negation.

However, this definition is incomplete. A more contemporary view sees mathematics as "the science of patterns." It deals not only with numbers and symbols but with the systematic study of all possible patterns and structures, both real and imagined. This approach emphasizes the creative and exploratory nature of mathematics.

Crucially, the fundamental limitation of any such formal system was demonstrated by Kurt Gödel in his incompleteness theorems. As a review paper on arXiv summarizes: “In any consistent formal system F, powerful enough to formalize arithmetic, there exist true but unprovable sentences” (arXiv:math/0408144v1). This means the set of mathematical truths is inherently larger than the set of theorems that can be proven within any single, consistent system of axioms. Mathematics itself thus points to its own limits, separating the concept of universal truth from systemic provability.

2.2 What is Language (L)?

Language (L) is a system of signs used for communication, expression, and the construction of meaning. Unlike mathematics, its purpose is not limited to describing objective reality. Attempts to formalize language, especially in the work of Noam Chomsky, defined it in a way close to mathematics. In his view, “a language is a set (finite or infinite) of sentences, each finite in length and constructed out of a finite set of elements” (cited in arXiv:1412.2442v1). This definition was revolutionary for computational linguistics but focused almost exclusively on syntax.

A more complete picture of language is provided by Charles W. Morris’s classic semiotic model, which divides the study of signs into three branches, a distinction reflected in many works in AI and linguistics (e.g., arXiv:1307.0087v1):

- **Syntax:** Studies the formal relations between signs, i.e., grammar and structure. At this level, language most resembles a mathematical system—we are interested in the rules for combining symbols, not their meaning. " $2 + 2 = 5$ " is a syntactically correct sentence, even though it is semantically false.
- **Semantics:** Studies the relations between signs and the objects they refer to, i.e., meaning and truth conditions. It is here that the sentence " $2 + 2 = 5$ " is evaluated as false because it does not correspond to the state of affairs in the axiomatic system of arithmetic. The descriptive function of language operates mainly at this level.

- **Pragmatics:** Studies the relations between signs and their users (interpreters). As a recent review on arXiv puts it: “Pragmatics studies how context affects the meaning of language [...] and how people use language in real-world situations to convey hidden meanings, emotions, and intentions” (arXiv:2502.12378v2). It is in pragmatics that the key difference between M and L lies. Language functions such as persuasion, performativity (e.g., making a promise, issuing a command), or expression are purely pragmatic phenomena that cannot be evaluated in terms of mathematical truth and falsehood, but only in terms of effectiveness, appropriateness, or sincerity.

In summary, while mathematics focuses on syntactic correctness and semantic truth within a closed system, natural language derives its power from pragmatic flexibility, which allows it not only to describe the world but to actively shape it.

2.3 Systemic Divergence: Mathematical Truth versus the Communicative Act

The fundamental reason mathematics cannot encompass the entirety of language lies in their different goals and natures. Mathematics is a closed and truth-functional system, meaning the value (truth/falsehood) of complex expressions is fully determined by the value of their components and established rules. Its domain is provability within a consistent axiomatic system. In contrast, natural language is an open and intentional system, whose primary goal is not to prove truths but to achieve communicative aims in an unlimited number of contexts.

This divergence is best understood by referring to the philosophy of language, particularly J.L. Austin’s theory of speech acts. Austin noted that utterances do not merely describe the world (stating facts), which he called the locutionary act. The key is the illocutionary act—the action we perform by speaking. When we say, “I promise I will come,” we are not describing a fact but creating an obligation. When we say, “Please close the door,” we are not stating a truth but attempting to influence someone’s behavior.

Mathematics is almost entirely the domain of locutionary acts—the statement “ $2+2=4$ ” simply asserts a certain relation. It has no built-in apparatus to handle illocutionary acts like requests, promises, threats, warnings, or questions. It is this “illocutionary force,” the ability to act in the social world, that constitutes the essence of language’s pragmatics, which completely eludes mathematical formalization. This is why a false statement like “ $2+2=5$,” while worthless in system M, can perform an effective illocutionary function in system L—it can be a provocation, a joke, a test of alertness, or a tool of disinformation. It is not its logical value that matters, but its potential impact on the receiver.

2.4 Contemporary Research at the Intersection of Language and Formal Systems

Theoretical considerations about the differences between language and mathematics now find empirical confirmation in artificial intelligence research. Attempts to teach machines to use natural language highlight the limits of formalization.

2.4.1 Computational Pragmatics and the "Linguistic Blind Spots" of AI Models

One of the most active research fields is computational pragmatics, which attempts to model and implement the ability to understand language in context. The latest works, such as the review "Pragmatics in the Era of Large Language Models" (arXiv:2502.12378v1), show that although Large Language Models (LLMs) excel at syntax and semantics on a statistical level, they have enormous difficulty understanding a speaker's intent.

- **Problem:** Models often fail to understand irony, sarcasm, metaphors, or implicatures (what is suggested but not explicitly stated). This is because these phenomena require not only text analysis but also a "theory of mind"—the ability to model the knowledge state, goals, and beliefs of the interlocutor.
- **Findings:** A study described in "Linguistic Blind Spots of Large Language Models" (arXiv:2503.19260v1) indicates that even the most powerful models make blatant errors in identifying complex linguistic structures that are obvious to humans.
- **Conclusion for our thesis:** These difficulties are empirical proof that pragmatics is a layer of meaning that cannot be reduced to statistical patterns in text data. This confirms that language is more than a formal system of signs.

2.4.2 Disinformation Analysis: From Facts to Narratives

Research into automatic disinformation detection shows a similar limit. Initially, the focus was on fact-checking, which is a semantic task. This quickly proved insufficient.

- **Problem:** The most effective disinformation is not based on overt lies but on pragmatic manipulation: the use of emotionally charged language, the creation of deceptive narratives, the application of persuasive logical fallacies, and the evocation of a sense of group belonging.
- **Research approach:** Modern systems (described, among others, in a CORDIS report, 2024) attempt to analyze not only what is said but how and why. They analyze style, emotional payload, and rhetorical techniques to assess the author's intent.
- **Conclusion for our thesis:** An effective fight against disinformation requires tools that go beyond logic and fact-checking, entering the realm of pragmatics and the psychology of language. This proves that linguistic influence is a non-mathematical phenomenon.

2.4.3 Emergent Communication in Multi-Agent Systems

A fascinating field is the study of emergent communication, where autonomous AI agents (e.g., robots) must develop a communication protocol on their own to collectively solve a problem.

- **Problem:** How to design systems that learn to communicate from scratch, without an imposed human grammar?

- **Findings:** As research shows (e.g., arXiv:2502.06148v1), the "languages" developed by agents are often extremely dissimilar to human ones. They can be holistic (one signal means an entire complex concept) and strongly tied to a specific task, rather than compositional (where the meaning of a sentence derives from the meaning of words and syntax rules), which is a feature of both natural and formal languages.
- **Conclusion for our thesis:** This research suggests there is no single, universally "optimal" way to communicate that must resemble mathematical logic. The nature of an emerging language depends on the goals, environment, and cognitive capabilities of the agents. This shows that communication is an adaptive and pragmatic phenomenon at its very core.

3 Formalization of the Problem

3.1 The Paradoxical Example: A Multi-level Analysis of " $2 + 2 = 5$ "

Let us consider the statement: "two plus two equals five and that is true, and two plus two equals four is false." To understand why this poses such a profound challenge to mathematical reductionism, we must analyze it on three distinct levels:

- **Syntactic Level (Structural):** From the perspective of English grammar, this sentence is flawless. It has a correctly constructed subject, predicate, and conjunctions. Similarly, if treated as a logical formula $p \wedge \neg q$, it is perfectly valid in terms of propositional logic syntax. System M, like L, accepts this structure as formally permissible. At this level, there is no conflict.
- **Semantic Level (Meaning):** Here, the first discrepancy appears. The sentence posits a state of affairs that is in direct contradiction to the axioms of Peano arithmetic. Within the "language-game" known as mathematics, it is unequivocally false. However, in system L, its semantic falsehood does not invalidate it. The sentence remains fully understandable—every receiver knows what it means and to what referential truth it relates, even while negating it. In mathematics, a semantic contradiction ends the analysis; in language, it is only the beginning.
- **Pragmatic Level (Use):** This is the key level where language completely out-classes mathematics. We must ask: what does this utterance do, not just what does it mean? Its purpose is not to describe reality but to perform an illocutionary act—an action intended to produce a specific effect on the receiver. It is an act of provocation, a tool for testing loyalty, or, most importantly, an instrument of power. The most famous example is George Orwell's dystopia *Nineteen Eighty-Four*, where the ability to accept that $2 + 2 = 5$ was the ultimate proof of the mind's subordination to the Party's will. The goal was not to teach new mathematics but to destroy the idea of an objective truth independent of power.

3.1.1 Contrast: The Role of Contradiction in Both Systems

In system M (mathematics), encountering a contradiction ($p \wedge \neg p$) signals a critical error. A contradiction invalidates the entire proof and is a logical endpoint that forces a revision

of assumptions. Its "influence" is purely negative and systemic—it stops the process.

In system L (language), the same contradiction can be an incredibly productive starting point. It can initiate a political narrative, become the foundation of an ideology, serve as a tool for building group cohesion (by rejecting an external "truth"), or be a deliberate disinformation strategy. Its influence is positive and creative—it starts a social process.

This is precisely why the statement, useless to a mathematician, becomes a powerful tool in the hands of a politician, a poet, or a propagandist. Its "value" is not measured in terms of truth/falsehood but in terms of its potential impact on people's beliefs and behaviors. This prepares the ground for an attempt to formalize this phenomenon.

3.2 Logical Formalization: From Truth to Belief and Influence

To precisely capture the nature of the problem, we will translate our intuitions into the language of logic. The goal of the model below is not to create a fully functional social simulator but to use formalisms to rigorously define concepts and map the relationships between them. The key is to distinguish between objective, axiomatic truth (T), which is the domain of mathematics, and subjective belief (B), which is the currency in the world of language.

3.2.1 Choice of Conceptual Apparatus

To build the model, we will use a combination of three logical tools, each addressing a different aspect of the problem:

- **Epistemic Logic (Logic of Knowledge and Belief):** Essential for formally distinguishing truth from belief. The operator $B_o(x)$ ("observer o believes that x") allows us to talk about an agent's internal mental state, which can be independent of the external truth state $T(x)$. It is this divergence between $T(x)$ and $B_o(x)$ that is the source of the phenomenon under analysis.
- **Modal Logic:** Introducing the operator \Diamond ("it is possible that...") allows us to model potentiality. In a natural language system, belief in a falsehood is neither a necessity nor an impossibility—it is precisely a possibility. Modal logic provides a tool to express this key feature of open systems, in contrast to the determinism of purely mathematical systems.
- **Probabilistic Approach:** Social processes like persuasion are rarely binary. The use of probability $P(\dots)$ in the final formula reflects this reality. Repeating a lie does not make someone believe it with certainty; it merely increases the probability of such an event. This adds realism to the model and shows that influence is a stochastic phenomenon.

3.2.2 Definitions of Symbols

Constants and Variables:

- p := the proposition " $2 + 2 = 5$ " (paradigmatic falsehood)
- q := the proposition " $2 + 2 = 4$ " (paradigmatic truth)
- o := any observer (an agent in the system)

- $G :=$ the set of all observers

Predicates and Operators:

- $T(x) :=$ " x is true in an axiomatic sense." Thus, $T(q)$ is true, and $\neg T(p)$ is false.
- $B_o(x) :=$ "Observer o believes that x is true."
- $R(x) :=$ "The proposition x is repeatedly stated" (an indicator of exposure).
- $A(o) :=$ "Observer o is susceptible to new information" (an indicator of receptiveness).
- $\text{Influence}(x) := \sum_{o \in G} B_o(x) \times S(o)$, where $S(o)$ is the "social influence score" of observer o (e.g., authority, number of followers)

3.2.3 Analysis of Logical Formulas

The Reductionist Hypothesis: The World as Mathematics

$$(L \subseteq M) \implies (\neg T(p) \implies \neg(\text{Influence}(p) > 0))$$

Description: This formula describes an ideal world where language is entirely subordinate to the rules of mathematics. It states that if a proposition p is axiomatically false ($\neg T(p)$), then its social influence must be zero.

Implications: In such a world, influence is inextricably linked to truth. Since no one could believe a falsehood ($B_o(p)$ would always be 0 for a false p), the entire sum in the formula for $\text{Influence}(p)$ would be zero. False statements would be informational "dead ends," devoid of any power to spread.

The Reality of Natural Language: Openness to Error

$$\neg(L \subseteq M) \implies \Diamond \exists o : B_o(p)$$

Description: This formula is the formal negation of the previous one. It states that precisely because language is not a subset of mathematics, it becomes possible (\Diamond) that there will be at least one (\exists) observer who believes the false proposition p .

Implications: This is the crucial moment where the linguistic system "opens up" to phenomena impossible in strict mathematics. It is the logical entry point for fiction, myth, cognitive bias, and intentional disinformation. The formula does not say that someone will definitely believe a lie, but that the system allows for such a possibility.

The Mechanism of Social "Truth": The Propagation Engine

$$\forall o : [R(p) \wedge A(o)] \implies [P(B_o(p)) > P_0]$$

Description: This formula describes the mechanism by which a potential belief in falsehood can become a widespread phenomenon. It states that for any observer, if a false proposition is intensely repeated ($R(p)$) and the observer is receptive ($A(o)$), then the probability (P) that they will believe that proposition exceeds some baseline threshold P_0 .

Implications: This formula models the process of "infecting" minds. $R(p)$ and $A(o)$ are, of course, abstractions for very complex phenomena— $R(p)$ could represent the actions of media, bots, or propaganda, while $A(o)$ could depend on education, group affiliation, or trust in the source. However, the model shows that influence is not magic but the result of the interaction of measurable (at least in theory) factors: the intensity of the message and the receptiveness of the audience.

4 Language and Mathematics in the Process of Scientific Discovery

4.1 Intuition and Discovery: The Engine of Scientific Revolutions

The greatest scientific breakthroughs are not born from purely formal deductions but from intuition and imagination, which operate in the domain of language.

- **Einstein's "Holy Curiosity":** He claimed to think in images and feelings, with words and mathematics being only a "secondary stage" of formalization. His thought experiments (e.g., with the elevator) were linguistic narratives that preceded the formulation of the general theory of relativity.
- **Poincaré's Distinction:** He believed that logic only verifies proofs, but it is intuition that creates them, providing the "fertile" leap toward new knowledge.
- **Gödel's Mathematical Intuition:** He believed in the existence of a "mathematical intuition" that allows us to grasp the truth of axioms. His incompleteness theorems suggest that mathematical truth extends beyond any single formal system.

These examples support the thesis that scientific theories (like the theories of relativity, which we test today under increasingly extreme conditions) would not have arisen if their creators had not used both language (for "out-of-the-box" thinking) and mathematics (for formal description).

4.2 The Wittgensteinian Perspective: Mathematics as a Language-Game

Ludwig Wittgenstein fundamentally reframes the entire debate. He argues that mathematics is not a description of a Platonic reality but a specific human activity—a collection of "language-games."

The sentence " $2 + 2 = 4$ " is simply a grammatical rule in the game called arithmetic. The meaning of this sentence lies not in its reference to abstract objects but in its use and its embeddedness in our "form of life"—in building bridges or programming computers.

In this perspective, mathematics (M) is just one of many applications of language (L). The statement " $2 + 2 = 5$ " is not an error in system L but a move in a different language-game (e.g., the game of political control or surrealist poetry).

5 Conclusions and Implications: Navigating the World of Language

The analysis of the fundamental difference between mathematics (M) and language (L) is not merely an abstract academic exercise. It has profound and practical implications for understanding the modern world, designing intelligent technologies, and educating future generations. The following conclusions outline the most significant of these consequences.

5.1 For the Contemporary World: The Anatomy of the Post-Truth Era

The phenomenon of "post-truth" becomes fully understandable in light of our model. The modern information ecosystem functions as an automated machine of persuasion that systematically promotes the mechanisms described in our formalization.

- **Maximization of $R(p)$:** Social media, with its viral nature and the operation of bots, acts as a powerful amplifier for the $R(x)$ function (repetition of the message). Every "share" or "like" is a micro-act of repetition that increases exposure to a given piece of content, regardless of its truthfulness.
- **Maximization of $A(o)$:** Recommendation algorithms create personalized echo chambers and filter bubbles that minimize contact with information contradicting pre-existing beliefs. This artificially increases $A(o)$ (observer susceptibility) by systematically weakening their critical evaluation skills.

As a result, the architecture of modern media is almost perfectly designed to make the situation where $\text{Influence}(p)$ (the influence of falsehood) surpasses $\text{Influence}(q)$ (the influence of truth) not an anomaly, but a frequent and predictable outcome.

5.2 For Artificial Intelligence: The Challenge of True Understanding

The emergence of Large Language Models (LLMs) is empirical proof of this article's theses. These machines have achieved mastery in operating at the syntactic and statistical-semantic level of language L, while having no access to axiomatic truth $T(x)$. They are, in a sense, the first non-trivial entities for which $\text{Influence}(p)$ is the sole measure of success.

- **The "Stochastic Parrot" Problem:** LLMs do not "understand" what they say. They generate text based on probability, which allows them to create incredibly persuasive, grammatically coherent, and stylistically consistent messages that are entirely false. They thus constitute a potentially infinite source for the $R(p)$ function.
- **Ethical Design Dilemmas:** The key question is not, "How do we stop AI from lying?" but rather, "Is it possible to design an AI that has a model of the world and 'cares' about the truth?" This would require creating systems that can not only process language but also verify it against grounded knowledge ($T(x)$) and understand the intentions of others ($B_o(x)$), which remains one of the most difficult challenges in the entire field of AI.

5.3 For Education: Towards Cognitive Bilingualism

Since our students operate in a world where L and M are constantly confused and instrumentalized, the education system must equip them with tools to navigate this complexity. This suggests the need to move away from siloed teaching towards developing "cognitive bilingualism"—fluency in both the formal language of mathematics and the pragmatic, contextual language of communication.

- **Interdisciplinarity in Practice:** This means that a mathematics lesson should include a module on how statistics can be used for manipulation (the pragmatics of numbers). An English lesson should teach not only about metaphors but also about analyzing the logical structure of arguments in a text.
- **The New Key Question:** Education must teach students to ask not only the question, "Is this statement true?" but, above all, the questions: "What is this statement doing? What is its author's goal? Who benefits if I believe it?"

5.4 Research Perspectives: Towards a Scientific "Logic of Fiction"

The most promising research direction emerging from this analysis is the need to formalize the principles governing the worlds that language constantly creates. We need a scientific "Logic of Fiction"—a formal system capable of analyzing the internal consistency and rules of narratives, literature, mythology, and even conspiracy theories.

- **Problem and Goal:** Classical logic, based on correspondence with a single, objective world, is helpless in the face of fiction. The question "Is it true that Frodo Baggins destroyed the Ring?" is meaningless in the logic of $T(x)$. A Logic of Fiction would not ask about absolute truth, but about truth within a given narrative universe. Its goal would be to create an apparatus to assess whether a given statement is true in the world of *The Lord of the Rings* ($T_W(p)$).
- **Foundations and Tools:** Such a logic could draw from existing theories of possible worlds (Kripke, Lewis). Each fictional world W would be defined by a set of its narrative axioms A_W (e.g., "The Ring is evil," "Elves are immortal") and a set of inference rules R_W , which may or may not coincide with the logic of our world. We would need new modal operators capable of operating on these worlds.
- **Example of Formalization:** Let W_{HP} denote the world presented in the *Harry Potter* novels. Let $p :=$ "Harry Potter used the Avada Kedavra curse." Let $q :=$ "Voldemort used the Avada Kedavra curse."

Within a Logic of Fiction, we could formally demonstrate that $\neg T_{W_{HP}}(p)$ (the statement p is false in the HP world, as it clashes with the hero's moral axioms and actions), while $T_{W_{HP}}(q)$ is true, as it is consistent with the axioms describing that character.

- **Practical Applications:**
 - **Analysis of Ideology and Disinformation:** Conspiracy theories can be treated as closed fictional worlds. Instead of fighting their claims on the basis

of facts (which is often ineffective), one could formally analyze their internal logic, exposing their contradictions and unjustified inferential leaps.

- **Advanced AI:** An AI equipped with a Logic of Fiction could generate much more coherent and believable narratives, as well as identify "plot holes" in existing texts.
- **The Humanities:** It would provide literary and cultural scholars with a powerful, formal tool for analyzing the structure of myths and narratives.

Creating such a logic would be the ultimate recognition that language's ability to create worlds is not its flaw or error, but its most powerful and fundamental feature.

6 Summary

This article has attempted to demonstrate that the effort to completely reduce language (L) to mathematical structures (M) is fundamentally impossible and is based on a misunderstanding of the nature of both systems. We began with the analytical paradox of " $2 + 2 = 5$," which served as a tool to illustrate the fundamental difference between axiomatic truth ($T(x)$)—the domain of mathematics—and subjective belief ($B_o(x)$), which is the key currency in the sphere of language. We have shown that while in mathematics a contradiction is a signal of error and the end of analysis, in language it can be an incredibly productive beginning of a narrative, an act of persuasion, or a manifestation of power.

The analysis presented, supported by an attempt at formalization using epistemic and modal logic, allowed for a precise modeling of this divergence. The analysis proved that it is precisely language's departure from the strict framework of mathematics ($\neg(L \subseteq M)$) that creates the possibility (\Diamond) for belief in falsehood to arise, and that social mechanisms, such as intense repetition ($R(x)$) and audience susceptibility ($A(o)$), can transform this possibility into a powerful force with real influence.

The ultimate explanation for this phenomenon finds its reference point in the philosophy of Ludwig Wittgenstein. His concept of language-games allowed us to abandon a false dichotomy and understand that mathematics is not a system superior to language but one of many possible, highly formalized games we can play using language. " $2 + 2 = 4$ " is a rule in the game called arithmetic; " $2 + 2 = 5$ " is a move in a completely different game, for example, the game of political power.

Understanding this relationship is not merely an academic curiosity—it is crucial in an era where the boundaries between objective and social truth are blurring at an unprecedented rate. The architecture of social media and the operation of AI models, which are masters of language ungrounded in truth, create an environment where the pragmatic power of language can be used on an industrial scale to shape reality. This requires a new approach to education, promoting "cognitive bilingualism" and resilience to manipulation.

Therefore, the dispute over the primacy of language or mathematics is illusory. Instead of trying to force language into the framework of mathematics, we should move in the opposite direction: using the precise tools of M to better understand and appreciate the power of L.

The proposed vision of a scientific "Logic of Fiction" is an expression of this new synthesis—an endeavor to create formal methods for analyzing language's ability to create entire worlds. Ultimately, mathematics and language are not rival systems but two

different, though inextricably linked, tools in the same workshop of human cognition and communication.

A natural consequence of this analysis is the fundamental hypothesis that language, especially in the age of AI, requires a different mathematics for its analysis than the one that describes logical reality. This statement is not a call to abandon mathematical rigor but a summons for a fundamental change in tools and perspective.

"Classical" mathematics (M_{logic}), based on set theory, binary logic, and algebra, is perfectly suited for describing the physical world and abstract structures—a world of objective, unchanging, and intention-free truths. Attempting to apply it uncritically to the analysis of natural language (L) is like trying to measure temperature with a ruler; the tool is precise, but unsuited to the nature of the phenomenon being studied.

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IV. Research on Large Language Models

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