GTMØ as an attempt to capture the subjectivity of language within the framework of logic and mathematics

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Abstract

This paper introduces GTMØ (Geometry Topology Mathematics Ø), a novel theoretical framework designed to mathematically model the inherent subjectivity of language. Traditional approaches in computational linguistics often attempt to eliminate ambiguity and subjectivity, treating them as noise. In contrast, GTMØ posits that these are fundamental, creative, and essential properties of language. The theory proposes a paradigm shift from seeking objective, singular meanings to formally mapping the dynamic and multi-layered space of all possible interpretations.

At the core of GTMØ is the concept of a multi-dimensional "meaning space," where words and concepts are not represented by static points but by dynamic trajectories. This is primarily described within a three-dimensional Phase Space, whose axes are Determination (the clarity of a meaning, 0 to 1), Stability (its constancy over time, 0 to 1), and Entropy (its creative or chaotic nature, 0 to 1). Within this space, information is classified not by arbitrary rules but by its proximity to a finite set of "Topological Knowledge Attractors," such as the Knowledge Particle (Ψ^K) for clear, stable facts, the Knowledge Shadow (Ψ^h) for uncertain information, and the Singularity (Ø) for paradoxes.

The full architecture of GTMØ, however, is composed of nine distinct but interacting semantic spaces, including the aforementioned Phase Space, as well as Configuration, Topological, Observational, Emergent, Indefinite, Trajectory, Interpretation, and Boundary spaces. Together, these spaces provide a comprehensive framework for analyzing a linguistic entity's context, relational structure, observational perspective, and potential for change.

A unique feature of the theory is its foundation on 13 "executable axioms." These are not passive postulates but active computational rules that govern the system's behavior, acting as a form of "immune system" to prevent cognitive errors like hallucinations or false certainty. For instance, axioms enforce the undefinability of the Singularity (Ø), trigger meta-cognitive self-evaluation, and facilitate adaptive learning. By formalizing the mechanisms of subjectivity, GTMØ offers a robust foundation for developing more advanced and context-aware AI systems, capable of navigating the rich complexity and emergent nature of human language. It presents not a physics of definitions, but a mathematics of the music of meanings.

Introduction

The exploration of language through formal systems has long been a central goal in logic, computer science, and linguistics. However, traditional models have often prioritized objectivity and definability, relegating the inherent subjectivity and ambiguity of human language to the periphery, treating them as noise or error to be eliminated. This paper presents the culmination of a research program that seeks to reverse this paradigm. It builds directly upon two preceding works that introduced the foundational concepts of GTMØ (Geometry Topology Mathematics Ø).

Our initial paper, *GTMØ as an attempt to capture the subjectivity of language within the framework of logic and mathematics* (Skuza, 2024a), established a radical departure from classical approaches.

The subsequent publication, *GTMØ Computational Value & 13 Executable Axioms* (Skuza, 2024b), elaborated on the theory's computational backbone. It detailed the minimal set of fundamental constants governing the system and formalized the role of the 13 executable axioms[cite: 902, 908]. These axioms function not as static postulates but as an active, self-regulating "immune system" that protects an AI from cognitive errors like hallucination and false certainty by managing the boundaries of the definable[cite: 910, 585].

While these papers laid the groundwork, they presented a picture that was necessarily incomplete. The purpose of this article is to synthesize and transcend that earlier work by unveiling the complete architecture of the GTMØ framework.

1 Scientific Foundations

1.1 The Nature of Indefiniteness

Indefiniteness (the lack of definitiveness) is a characteristic or state of something that lacks an unambiguous, ultimate definition, boundaries, or established status. In various contexts, it can take on different meanings - philosophical, mathematical, linguistic, or ontological. In a general sense, it refers to the indeterminacy, fluidity, or openness of a given concept, phenomenon, or entity.

1.2 Can Something Be Observed Without Being Defined?

1.2.1 The Fundamental Question

Can something be observed while simultaneously not being defined?

1.2.2 Philosophy and Phenomenology

In the phenomenological tradition (Husserl, Merleau-Ponty), experience always precedes definition. We can observe (i.e., experience) certain phenomena without prior conceptualization.

Example: A new phenomenon in nature - you see it, but you don't yet have words or theories to describe it.

Observation can thus be primary to definition, with definition appearing later as an attempt to capture the meaning of what has already been observed.

1.2.3 Quantum Physics

In quantum physics, particularly quantum mechanics, the state of a particle can be underdetermined (superposition) until the moment of measurement. The observation itself changes the state of affairs.

You observe an effect (e.g., a detector click), but you don't yet have a definitive description of what occurred - because reality at the quantum level is not unambiguous before measurement.

1.2.4 Mathematics and Logic

In set theory and the semantics of logic, there exist entities that are formally undefinable, even though one can speak of their properties or existence (e.g., the set of all sets that do not contain themselves - Russell's paradox).

1.2.5 Language and Concepts

Some concepts are inherently indeterminate or lack sharp definition: "life," "art," "consciousness." We can observe them, talk about them, point to examples - but we don't have one definitive boundary.

1.2.6 Psychology and Consciousness

You can experience emotions or psychological states that you cannot yet name. Self-awareness often precedes verbalization.

1.2.7 Summary

Yes, something can be observed while simultaneously not being defined.

Observation (in the sensory, cognitive, or phenomenological sense) does not require prior definition of the object. Definitions, in turn, are secondary constructs created by the mind to tame and classify what has previously been noticed, experienced, or known.

1.3 Justifying the Attempt to Create a Theory of Indefiniteness

1.3.1 Lakatos's Research Programs

As a philosopher of science, allow me to present one of the most influential concepts in 20th-century philosophy of science - Imre Lakatos's theory of scientific research programs.

Lakatos created his theory as an attempt to reconcile two dominant approaches in the philosophy of science: Karl Popper's critical rationalism and Thomas Kuhn's theory of scientific revolutions. Popper claimed that science develops through the falsification of theories, while Kuhn described science as a sequence of paradigms interrupted by revolutions. Lakatos recognized that both approaches were too simplistic.

1.3.2 Structure of a Research Program

According to Lakatos, a research program consists of two main elements:

Hard Core - a set of fundamental assumptions that are considered unfalsifiable within the given program. Scientists working in the program do not question these assumptions but protect them.

Protective Belt - a set of auxiliary hypotheses, initial conditions, and observational theories that can be modified in the face of anomalies. It is precisely the protective belt that takes the "hits" of falsifying observations.

1.3.3 Program Heuristics

Each research program has two types of heuristics:

Negative Heuristic - prohibits directing criticism at the hard core of the program. All anomalies must be explained through modifications in the protective belt.

Positive Heuristic - indicates how to develop the program, what models to build, what problems to solve. This is the program's "research plan."

1.3.4 Progressiveness vs. Degeneration

A key element of Lakatos's theory is the distinction between:

Progressive Programs - which predict new, surprising facts and lead to empirical discoveries. Modifications in the protective belt not only explain anomalies but also generate new, testable predictions.

Degenerating Programs - which only adjust post hoc to observations, not generating new predictions. Modifications are purely defensive in nature.

1.3.5 Example: Newton's Program

The hard core of the Newtonian program contained the three laws of dynamics and the law of universal gravitation. When perturbations in Uranus's orbit were discovered, instead of rejecting Newton's theory, scientists modified the protective belt by postulating the existence of an unknown planet. The discovery of Neptune was a triumph of a progressive program.

1.4 Chapter 4: Where Does Indefiniteness Appear in Science?

There is no single, coherent scientific theory titled "indefiniteness" as an independent field, but the idea of indefiniteness (lack of finality, fuzziness, indeterminacy) has been captured and developed within many serious and coherent scientific and philosophical theories - albeit under different names and approaches.

1.4.1 Quantum Mechanics

Fundamental Indeterminacy in Physics

- **Heisenberg's Uncertainty Principle**: One cannot simultaneously know precisely both the position and momentum of a particle. This is fundamental indeterminacy built into the very structure of reality.
- **Superposition of States**: A particle does not have one determined value until it is measured.
- **Quantum theory formalizes observability without a definitive state before measurement.**

This is the closest scientific equivalent to indefiniteness at the ontological level.

1.4.2 Logic and Mathematics

Formal Models of Incompleteness

- **Gödel's Incompleteness**: In any formal system strong enough to describe arithmetic, there exist statements that are true but cannot be proven within that system.
- **Fuzzy Set Theory (Fuzzy Logic)**: Concepts such as "warm" or "many" do not have sharp boundaries - they are gradable. Not everything is binary.
- **Set Theory and Semantic Paradigms**: Show the existence of entities that are undefined or paradoxical in a definitional sense (e.g., Russell's paradox).

These are formal models of fuzziness, incompleteness, and ambiguity.

1.4.3 Philosophy of Science

Epistemological Indefiniteness

- **Thomas Kuhn**: The concept of "scientific revolution" - science does not develop linearly but within paradigm shifts that are not always sharp or unambiguous.
- **Paul Feyerabend**: "Anything goes" - opposition to rigid definitions of science and defense of cognitive pluralism.
- **Imre Lakatos**: "Research programs" instead of hard theories - science is an underdetermined process that evolves.

Here indefiniteness appears as an epistemological feature of knowledge development.

1.4.4 Systems Theory and Cybernetics

Complexity and Emergence

- **Complex Systems Theory**: Some systems (e.g., weather, brain, economy) are so complex that they cannot be defined in a closed manner.
- **Emergent Complexity**: Properties of the whole are not reducible to parts - they are "undefinable" from the perspective of reductionism.

Indefiniteness as a consequence of complexity and emergence.

1.4.5 Process Ontology

Being as Becoming

- **Alfred North Whitehead** in "Process and Reality" creates a metaphysics in which being is becoming, and thus is never closed or definitive.
- **Henri Bergson**: The concept of "duration" (la durée), in which phenomena do not have sharp boundaries but flow into one another.

Here indefiniteness concerns the nature of existence and time.

1.5 Summary

There is no single scientific theory of "indefiniteness" as a separate discipline, but the concept has been captured and grounded in many important fields of science and philosophy:

- In **physics** as uncertainty
- In **mathematics** as incompleteness or fuzziness
- In **philosophy of science** as paradigm variability
- In **ontology** as the processual nature of being

The attempt to create a unified theory of indefiniteness, such as GTMØ, represents an effort to synthesize these various approaches into a coherent, executable framework that can be directly applied in the development of AI and cognitive systems.

2 GTMØ Theory: Mathematics of Linguistic Subjectivity

2.1 Introduction - A New Paradigm

GTMØ (Geometry Topology Mathematics Ø is a theory that does not attempt to eliminate subjectivity from language, but rather creates a formal mathematical apparatus for its analysis and description.

2.1.1 Paradigm Shift: From Objectivism to Mapping Subjectivity

Traditional mathematical approaches to language strive to find "objective truth" and reject everything else as error or noise. GTMØ completely reverses this perspective[1].

- **Traditional question**: "How can we objectively measure the meaning of the word 'justice'?"[2]
- **Question in GTMØ**: "How can we formally describe the space of all possible interpretations of 'justice' by different observers?"[3]

The GTMØ theory explicitly states that this is **"mathematics of linguistic subjectivity"**.

2.1.2 Key Tools for Analyzing Subjectivity

GTMØ employs specific mechanisms built into its architecture for this purpose:

Observer as Part of the System Meaning does not exist in a vacuum; it is always the result of interaction between a sign and an observer[5]. Different observers (e.g., a doctor and a poet analyzing a "broken heart") will obtain different, but equally valid within GTMØ, interpretations[6].

Dimension	Description
Determination (clarity of meaning)	How unambiguous and clearly defined the word's
Stability (constancy of meaning over time)	Whether the meaning is permanent or dynamicall
Entropy (chaotic or creative nature of meaning)	How complex, creative, or paradoxical the meanin

Dedicated Subjectivity Spaces Two of the nine fundamental spaces in GTMØ are explicitly created to model subjectivity:

- **Observational Space**: Represents all possible perspectives and viewpoints[7]. Each observer constitutes a separate dimension[8].
- **Interpretation Space**: Contains all possible readings of a given concept that arise from different observations[9].

Subjectivity as a Resource The ultimate conclusion of the theory is that subjectivity is not a communication error, but a fundamental property of language that enables creativity, adaptation, and richness of meanings[10]. GTMØ is presented as **"mathematics of the music of meanings, not physics of definitions"**.

2.2 Core Ideas of GTMØ

2.2.1 Fundamental Thesis

The GTMØ theory is based on a radical departure from the classical concept of mathematical definiteness of meanings. Unlike traditional approaches that try to eliminate subjectivity and undefinability at all costs, GTMØ treats them as essential properties of language.

Core thesis of GTMØ:

Undefinability and subjectivity are not errors, but key aspects of language that enable creativity, adaptation, and the emergence of new meanings.

2.2.2 The Meaning Space

GTMØ introduces an original concept of "meaning space," which is a dynamic mathematical structure where each word or concept possesses not a point, but a trajectory – a path it traverses through time and contexts.

Dimensions of Meaning Space The meaning space is three-dimensional, with each dimension corresponding to specific semantic properties:

2.3 Types of Knowledge in GTMØ

The theory introduces several types of knowledge, i.e., specific states in meaning space that can attract concepts like "magnets" – these are so-called topological attractors:

2.3.1 Topological Attractors in GTMØ

Attractors in GTMØ are points or areas in meaning space that naturally attract specific types of information:

Type of Knowledge	Description	Exampl
Knowledge Particle (Ψ^K)	Clear, stable information	"2 + 2 =
Knowledge Shadow (Ψ^h)	Unclear, unstable information	"It might
Emergence (Ψ^N)	Information creating new meanings	"Faceboo
Liminality (Ψ^l)	Boundary state, transitional, unstable	The word
Singularity (Ø)	Paradoxes, logical contradictions, absolute undefinabilities	"This sen

Traditional Language Mathematics	GTMØ
Seeks precise definitions	Accepts undefinability and dynamism of meanings
Point representations of meanings (static)	Meaning trajectories (dynamic)
Paradoxes and contradictions as errors	Paradoxes and contradictions as natural elements of lang

- **Function of attractors**: Classification of information occurs not arbitrarily, but through measurable distance from these attractors.
- **Method effectiveness**: This allows for objective classification, e.g.: The paradoxical sentence "This sentence is false" will be attracted by attractor Ø (Singularity) due to very high entropy and low determination/stability.
- **Adaptability**: Attractors can shift depending on changing language, providing adaptability to the GTMØ system.

2.3.2 Computational Minimalism in Epistemic Terms

GTMØ uses a minimal number of parameters and fundamental mathematical values (0.5, $\sqrt{2}$, φ), which allow for efficient calculations without the need to model infinite complexity.

2.4 Practical Applications

1. **Context-understanding AI** – modeling meanings dynamically, not statically (like traditional embeddings)
2. **Manipulation detection** – semantic and pragmatic content analysis, capturing subtle differences in meaning
3. **Creative AI** – generating new concepts through simulation of emergence processes

2.4.1 GTMØ vs Traditional Approaches

2.5 Chapter 5: Understanding GTMØ Through Examples

2.5.1 The Multiple Meanings Problem

Imagine the word "bank." What does it mean?

- A financial institution?
- The edge of a river?
- To tilt or lean?

Traditional mathematics would try to assign "bank" = [0.3, -0.5, 0.8...] (vector). But this doesn't explain how we know which meaning to choose!

GTMØ says: meaning is not fixed – it EMERGES from context.

2.5.2 Problems GTMØ Aims to Solve

Natural language is full of things mathematics cannot describe:

- **Paradoxes**: "This sentence is false"
- **Emergence**: "deadline" = "dead" + "line", but means something entirely new
- **Context**: "Bank" by the river vs "bank" on the street
- **Creativity**: How do new words and meanings arise?

2.6 Key Concepts in Detail

2.6.1 Phase Space – Map of All Possible Meanings

Instead of a point (like in GPS), each concept has a trajectory – it moves through meaning space:

- **Determination**: How clearly defined (0-1)
- **Stability**: How constant over time (0-1)
- **Entropy**: How chaotic/creative (0-1)

2.6.2 AlienatedNumbers – Meaning Hybrids

When two concepts come very close together, something entirely new can emerge:

- "Smart" + "Phone" → Smartphone (not just "intelligent telephone"!)
- "Face" + "Book" → Facebook (new social meaning)

2.6.3 Concrete Example in Action

Card Experiment:

1. Take two cards – write "0" and "1"
2. Place horizontally: you see "01" (sequence)
3. Place vertically: you see "10" (maybe hierarchy?)
4. Bring very close: something new emerges!

This shows how GTMØ works:

- Same elements (0,1)
- Different configurations
- Different emergent meanings
- Observer co-creates meaning

2.7 Why This Could Be Revolutionary

Traditional AI: "Everything can be mapped to numbers"
GTMØ: "Some things are inherently undefinable – and that's OK!"

2.7.1 Practical Implementations

1. **Better AI** – that understands context and paradoxes
2. **Disinformation detection** – recognizes meaning manipulation
3. **Creativity** – AI that creates new concepts
4. **Efficiency** – doesn't waste power on the undefinable

2.7.2 GTMØ in Summary

- GTMØ is mathematics of how language creates meanings
- Meanings are not fixed – they emerge from context and interaction
- Accepts that some things are undefinable (Ø)

2.7.3 Final Analogy

- **Traditional language mathematics** = Map (static, everything has its position)
- **GTMØ** = GPS with motion prediction + awareness that some places are "off the map"

This is the first theory that says: "Hey, language isn't a collection of definitions, it's a living organism constantly creating new meanings – and we can describe this mathematically!"

2.8 Deep Dive into Trajectories

2.8.1 What is a "Trajectory" in Meaning Space?

Real-life example:
Take the word "gay":

- **1900**: "cheerful, joyful" (determination: 0.9, stability: 0.9)
- **1960**: begins to also mean "homosexual" (determination: 0.6, stability: 0.4)
- **2024**: mainly "homosexual", rarely "cheerful" (determination: 0.8, stability: 0.7)

This is a trajectory! The word "traveled" through meaning space.

2.8.2 The Three Dimensions Explained

1. **Determination (X-axis)**: How unambiguous is the meaning?
   - "2+2=4" → determination $\tilde{1}.0$ (very clear)
   - "Art" → determination $\tilde{0}.3$ (what is it really?)
2. **Stability (Y-axis)**: How constant over time?
   - "Water = H_2O" → stability $\tilde{1}.0$ (doesn't change)
   - "Cool" → stability $\tilde{0}.4$ (constantly evolving)
3. **Entropy (Z-axis)**: How chaotic/creative?
   - "Chair" → entropy $\tilde{0}.1$ (not very creative)
   - "Meme" → entropy $\tilde{0}.8$ (constantly new forms)

2.8.3 Evidence for Trajectories

Empirical evidence:

1. **Dictionary changes:**
   - "Mouse" (animal) → "Mouse" (computer)
   - "Cloud" (in the sky) → "Cloud" (storage)
2. **New meanings in context:**
   - "Sick" = ill
   - "That's sick!" = awesome! (in slang)
3. **AI confirms this**: GPT often "gets lost" with words on unstable trajectories

2.8.4 Concrete Trajectory Example

The word "VIRUS":

1900: [Determination: 0.2, Stability: 0.8, Entropy: 0.1]
- Unknown concept, maybe poison?
1950: [Determination: 0.8, Stability: 0.9, Entropy: 0.2]
- Biological pathogen, clear medical definition
1990: [Determination: 0.6, Stability: 0.5, Entropy: 0.7]
- "Computer virus" appears - chaos!
2020: [Determination: 0.7, Stability: 0.3, Entropy: 0.9]
- COVID + memes + "viral content" - explosion of meanings

2.8.5 Why This Matters

Traditional approach problem:

- Embedding: "virus" = [0.3, -0.2, 0.8...] - static point
- Doesn't see that "virus" in 2020 is completely different than in 1950!

GTMØ sees dynamics:

- Word travels, changes position
- Context affects trajectory
- We can predict where it's heading

2.8.6 Practical Test

Think about the word "phone":

- **1980**: Wired device at home
- **2000**: Can be cellular
- **2024**: Computer in your pocket

Do you feel this "journey" of meaning? That's exactly what a trajectory is!

2.9 Conclusion

Trajectory = history of a word's meaning changes over time and contexts

Meaning space = 3D map where:

- X = how clear/unclear
- Y = how stable/changing
- Z = how creative/chaotic

How do we know? = We observe language in action! Words change meanings, combine, create new ones – this isn't theory, it's a fact that GTMØ mathematically describes.

It's like the difference between a photograph (embedding) and a movie (trajectory) – GTMØ shows the movie, not the snapshot!

3 GTMØ Computational Value Foundations

3.1 Fundamental Value Architecture

3.1.1 Three-Dimensional Phase Space

GTMØ operates in a phase space with three dimensions:

- **Determination (X-axis)**: $0.0 \to 1.0$
- **Stability (Y-axis)**: $0.0 \to 1.0$
- **Entropy (Z-axis)**: $0.0 \to 1.0$

3.1.2 Cognitive Center [0.5, 0.5, 0.5]

The equilibrium point of cognitive space:

- Represents neutral knowledge state
- Boundary radius: 0.5
- All calculations reference this center

3.2 Key Computational Constants

3.2.1 $\sqrt{2}$ and $1/\sqrt{2}$ (0.707)

# Quantum superposition amplitude
amplitude = 0.707 + 0j # = $1/\sqrt{2}$

- Maximum quantum coherence between states
- Optimal value for equal superposition
- Geometrically: ratio of diagonal to square side

3.2.2 Golden Ratio $\varphi = (1 + \sqrt{5})/2 \approx 1.618$

# Golden angle for optimal point distribution
theta = np.pi * (1 + np.sqrt(5)) * i

- Generates most uniform distribution on sphere
- Minimizes local point clustering
- Natural proportion for self-organizing systems

3.2.3 Relationship between 0.7 and φ

- $1/\sqrt{2} \approx 0.707$ - quantum equilibrium
- $1/\varphi \approx 0.618$ - inverse golden ratio
- **$1/\varphi^2 \approx 0.382$ - second golden section**

3.3 Topological Knowledge Attractors

3.3.1 Atractor Map in Phase Space

3.3.2 Attractor Mathematics

# Effective distance with strength weighting
effective_distance = wasserstein_distance / attractor_strength
# Classification within attraction basin
if effective_distance <= basin_radius:
return attractor_type

Knowledge Type	Determination	Stability	Entropy	Radius	Strength
Singularity (Ø)	1.0	1.0	0.0	0.15	2.0
Alienated ($\ell\emptyset$)	0.999	0.999	0.001	0.10	1.5
Particle (Ψ^{K})	0.85	0.85	0.15	0.25	1.0
Shadow (Ψ^{h})	0.15	0.15	0.85	0.25	1.0
Emergent (Ψ^{N})	0.5	0.3	0.9	0.20	1.2
Transcendent ($\Psi\uparrow$)	0.7	0.7	0.3	0.15	1.1
Flux ($\tilde{\Psi)}$)	0.5	0.5	0.8	0.30	0.9
Void ($\Psi\diamond$)	0.0	0.0	0.5	0.20	0.8

3.3.3 Topological Knowledge Attractors - Explanation

What is it? Imagine a 3-dimensional map of all possible knowledge states, where each point in space represents a different type of information. GTMØ uses the concept of "attractors" - these are like gravitational wells that pull similar types of knowledge together.

How does it work? 3D Space

Each piece of information has coordinates:

- **X-axis (Determination)**: How certain/defined the information is (0 = chaos, 1 = absolute certainty)
- **Y-axis (Stability)**: How stable over time (0 = constantly changing, 1 = unchanging)
- **Z-axis (Entropy)**: How chaotic/disordered (0 = perfect order, 1 = total chaos)

Practical Examples: Singularity (Ø) - [1.0, 1.0, 0.0]

- Maximum determination and stability, zero entropy
- Example: "Logical contradiction" - absolutely certain it's impossible
- Strength 2.0 = strongest attractor, epistemic "black hole"

Knowledge Particle (Ψ^{K}) - [0.85, 0.85, 0.15]

- High certainty and stability, low entropy
- Example: "Water boils at 100°C" - well-established fact
- Radius 0.25 = large area of influence

Knowledge Shadow (Ψ^{h}) - [0.15, 0.15, 0.85]

- Low certainty and stability, high entropy
- Example: "It might rain tomorrow" - uncertain speculation
- Also radius 0.25, but opposite pole

Emergent (Ψ^N) - [0.5, 0.3, 0.9]

- Medium determination, low stability, very high entropy
- Example: "New social media trend" - something happening, but chaotically
- Strength 1.2 = attracts more strongly than standard types

What do the parameters mean?

- **Radius**: How far the attractor's "gravity" reaches (0.1-0.3)
- **Strength**: How powerfully it attracts objects (0.8-2.0)

3.3.4 New Capabilities

1. Natural Classification Instead of rigid categories ("this is fact" / "this is opinion"), information naturally "falls" to the nearest attractor, like a ball in a funnel.

2. Handling Edge States Information between attractors (e.g., [0.5, 0.5, 0.5]) is automatically recognized as liminal - in a transitional state.

3. Adaptability System can shift attractors by learning from experience:
If many "facts" turn out to have entropy 0.2 instead of 0.15
Attractor Ψ^K shifts to [0.85, 0.85, 0.20]

3.3.5 3.5 Practical Example

Question: "Will Bitcoin reach 100k USD?"
System analysis:

- Determination = 0.3
- Stability = 0.2
- Entropy = 0.8

Point [0.3, 0.2, 0.8] is close to attractor Ψ^h (Shadow) But also not far from $\tilde{\Psi}$(Flux) System classifies as "Shadow with Flux tendency" - uncertain, variable prediction

3.3.6 3.6 Revolutionary Approach

- **Traditional AI**: "This is a fact with 73% certainty"
- **GTMØ**: "This is in the Shadow Knowledge attraction basin, 0.3 units from attractor center, with slight drift toward Flux"

Result: Much richer and more accurate description of information's epistemic nature, enabling better AI decision-making.

3.4 Threshold Values and Boundaries

3.4.1 Boundary Thickness: 0.01-0.02

- Represents transitional area between interior and boundary
- Prevents numerical instabilities
- Models epistemic "fuzziness"

3.4.2 Entropy Threshold for Ø: 0.001

- Below this value, collapse to singularity occurs
- E_GTMØ(Ø) = min(E_GTMØ(x)) for all x

3.4.3 "Breathing" Amplitude: 0.1

- Cognitive space pulsates with ±10% amplitude
- Models dynamic nature of knowledge
- Frequency: 0.1 rad/iteration

3.5 Computational Elegance

3.5.1 lue Minimalism

GTMØ uses minimal set of fundamental constants:

- **0.5** - center and radius (symmetry)
- $1/\sqrt{2}$ - quantum equilibrium
- φ - golden ratio (optimal distribution)

3.5.2 Complexity Emergence

From these simple values emerge:

- 8+ epistemic types
- Infinite configuration space
- Adaptive learning
- Quantum state superposition

3.5.3 Computational Efficiency

```
# Instead of calculating exact positions:
    if distance_to_singularity < 0.001:
    return Ø  # Immediate collapse
    # Instead of full quantum simulation:
    amplitude = 1/sqrt(2)  # Optimal superposition
```

3.6 Philosophical Significance of Values

3.6.1 0.5 as Epistemic Center

- Point of maximum uncertainty
- Balance between knowledge and ignorance
- Neutral cognitive state

3.6.2 0.707 as Quantum Harmony

- Perfect superposition of possibilities
- Maximum informational coherence
- Transition point between states

3.6.3 φ as Natural Proportion

- Appears in nature and art
- Optimal for perception and organization
- Connects mathematics with aesthetics

3.7 AI Implications

These fundamental values enable:

1. **Complexity Reduction** - instead of modeling infinity, use attractors
2. **Natural Classification** - objects "fall" to nearest basins
3. **Efficient Learning** - adapt attractor positions, not entire space
4. **Quantum Expression** - superposition of epistemic states

3.7.1 Computational Breakthrough

GTMØ doesn't attempt to compute the undefinable. Instead, it uses elegant mathematical values to navigate epistemic space, achieving orders of magnitude in computational savings.

3.8 Chapter 8: How Classification Really Works

3.8.1 8.1 The Problem

How to classify objectively without arbitrariness?

3.8.2 8.2 The Solution: Topological Attractors

This is NOT random chaos! GTMØ uses topological attractors.

How it really works:

1. **Each entity has 3D coordinates:**

   - "2+2=4" → (determination: 0.95, stability: 0.95, entropy: 0.05)
   - "It might rain" → (determination: 0.3, stability: 0.4, entropy: 0.6)

2. **Attractors in space - like magnets!**

PHASE SPACE 3D:
Entropy ↑
|
| Ψ° (Shadow)
| • (0.15, 0.15, 0.85)
| /
• Ψ^{n} ——-+——- Determination →
(Emergent) |
| • Ψ^{K} (Particle)
| (0.85, 0.85, 0.15)
|
Stability

1. **Classification = find nearest attractor**

   - Example: Analyzing sentence "Art is..."
   - System calculates: determination=0.3, stability=0.5, entropy=0.7
   - Measures distance to each attractor
   - Nearest to Ψ^{n} (Emergent) → classification!

3.8.3 8.3 Objective Measurement Criteria

Determination measured by:

- Certainty words: "always", "never", "must" → +0.3
- Uncertainty words: "maybe", "probably" → -0.3

Stability measured by:

- Is meaning constant over time?
- Does context strongly change interpretation?

Entropy measured by:

- Number of possible interpretations
- Presence of paradoxes or contradictions

3.8.4 8.4 Why This Works

It's like semantic GPS:

- No need to know word history
- Measure its position NOW
- Attractors are like cities on a map
- Classify by proximity

3.8.5 8.5 Concrete Test

```
# Take any sentence
    text = "This sentence is false"
    # System analyzes:
    # - Paradox! → entropy ↑↑↑
    # - Unstable → stability ↓↓↓
    # - Impossible to resolve → determination ↓↓↓
    # Coordinates: (0.1, 0.1, 0.95)
    # Nearest attractor: Ø (Singularity)!
```

3.8.6 8.6 Summary

This is NOT arbitrary because:

1. **Measurable parameters** (determination, stability, entropy)
2. **Mathematical distances** in 3D space
3. **Fixed attractors** as reference points
4. **Objective classification** by nearest distance

It's like sorting colors - blue goes with blues not because "I decided so", but because its RGB coordinates are nearest to other blues!

GTMØ doesn't guess - it measures position in meaning space and finds the nearest "semantic magnet".

4 The Role of Executable Axioms in GTMØ

4.1 Introduction - What are Executable Axioms?

In 2012, Maria Piesko in her book "Incomputable Computability" put forward an advanced thesis that the classical Turing Machine model is insufficient to describe the operation of modern computer systems, which increasingly operate in interaction with the environment. She proposes replacing it with a theory of interactive machines, which, operating in an "incomputable environment" such as the physical world, are "stronger" than a regular TM. Despite the lack of a formal model of such machines, Piesko believes that the thesis of the possibility of building AI should be a methodological assumption for further research.

From the perspective of GTMØ author - Grzegorz Skuza, such an approach is, however, too radical as a fundamental step. Maria Piesko's theory of interactive machines thus constitutes a key inspiration for a fundamental feature of GTMØ theory, namely the 13 executable axioms.

4.2 Theoretical Foundations and Inspirations

Beyond Maria Piesko's analyses, the author took into account approaches in which "axioms" take the form of programs (are executable).

4.2.1 2.1 Curry-Howard Correspondence (proofs = programs, theorems = types)

In constructive systems like Coq/Agda/Lean, every proof is a program term.

An axiom is a constant of a given type (postulate). It has "programmatic" sense only when you provide a construction (definition); a purely declared axiom acts as an "oracle".

4.2.2 2.2 Logic Programming (Prolog: Horn clauses = axioms)

Facts and rules are axioms, and computation = proof search (SLD-resolution).

parent(alina, bartek).
parent(bartek, celina).
ancestor(X,Y) :- parent(X,Y).
ancestor(X,Y) :- parent(X,Z), ancestor(Z,Y).
The above "axioms" execute like a program answering queries.

4.2.3 2.3 Rewriting Logic / Algebraic Specifications (Maude/CafeOBJ)

Equations treated as axioms are executed by a rewriting system.

mod NAT-PLUS is
sorts Nat .
ops 0 : -> Nat . s_ : Nat -> Nat . _+_ : Nat Nat -> Nat .
vars X Y : Nat .
eq 0 + Y = Y .
eq s(X) + Y = s(X + Y) .
endm
These "axioms" compute normal forms (results).

4.2.4 2.4 Realizability (Kleene) and Program Extraction

A formula is "realized" by a function index (program code). In practice: programs are extracted from constructive proofs; non-constructive axioms correspond to "black boxes".

4.3 The 13 Executable Axioms of GTMØ

4.3.1 AX0: Systemic Uncertainty

"There is no proof that the GTMØ system is fully definable"

- **Action**: Introduces quantum superposition of states: $|\psi\rangle = \alpha|\text{defined}\rangle + \beta|\text{undefined}\rangle + \gamma|\text{indefinite}\rangle$
- **In practice**: System can be simultaneously in multiple fundamental states
- **For AI**: Allows model to admit "I don't know" at a fundamental level

4.3.2 AX1: Ontological Difference

"$\varnothing \notin \{0, 1, \infty\}$ and no function maps numbers to $\varnothing$"

- **Action**: Blocks attempts to reduce Ø to ordinary numbers
- **In practice**: $Ø + Ø \neq 0$, $Ø \times Ø \neq Ø$, $Ø \div Ø \neq 1$
- **For AI**: Protects against collapse to trivial answers

4.3.3 AX2: Translogical Isolation

"There exists no function f: D → Ø for any definable system D"

- **Action**: Actively blocks attempts to reach Ø through standard calculations
- **In practice**: Creates "isolation barriers" around Ø
- **For AI**: Prevents hallucinations like "I know everything"

4.3.4 AX3: Epistemic Singularity

"No cognitive system can fully know Ø"

- **Action**: Limits cognitive access to the singularity
- **In practice**: System can approach Ø but never "understand" it
- **For AI**: Models fundamental limits of knowledge

4.3.5 AX4: Non-representability

"$\emptyset$ cannot be represented in any system containing {0,1,∞}"

- **Action**: Blocks attempts at symbolic representation of Ø
- **In practice**: Every attempt to draw/describe Ø leads to AlienatedNumber
- **For AI**: Teaches recognition of representation limits

4.3.6 AX5: Topological Boundary

"Ø always exists on the boundary of cognitive space"

- **Action**: Dynamically shifts space boundaries so Ø is always on the edge
- **In practice**: Space "breathes" - expands and contracts
- **For AI**: Models dynamic knowledge boundaries

4.3.7 AX6: Minimal Entropy

"Ø has minimal cognitive entropy of all objects"

- **Action**: Applies gradient descent on entropy approaching Ø
- **In practice**: E_GTMØ(Ø) < 0.001, lowest possible
- **For AI**: Ø is the point of maximum certainty about uncertainty

4.3.8 AX7: Meta-Closure

"Ø belongs to the meta-closure of GTMØ and triggers system self-evaluation"

- **Action**: Encountering Ø triggers meta-cognitive reflection
- **In practice**: System asks "do I know that I don't know?"
- **For AI**: Implements awareness of own limitations

4.3.9 AX8: Not a Limit Point

"No sequence converges to Ø"

- **Action**: Blocks attempts to "sneak up" on Ø through limits
- **In practice**: $\lim(x_n) \rightarrow$ Ø is impossible
- **For AI**: Prevents asymptotic approach to omniscience

4.3.10 AX9: Operator Irreducibility

"No standard operator can act on Ø"

- **Action**: Returns SingularityError on attempt Op(Ø)
- **In practice**: $\sin(\varnothing)$, $\log(\varnothing)$, $\int \varnothing\, dx$ – all error
- **For AI**: Protects against invalid operations

4.3.11 AX10: Meta-Operators

"Only Ψ_GTMØ and E_GTMØ can act on Ø"

- **Action**: Defines special epistemic operators
- **In practice**: Ψ_GTMØ(Ø) = 1.0, E_GTMØ(Ø) = min
- **For AI**: Provides tools to work with undefinability

4.3.12 AX11: Adaptive Learning

"System neurons can modify their response patterns"

- **Action**: Implements long-term memory and defensive strategies
- **In practice**: Neuron learns to avoid collapses to ∅
- **For AI**: True learning from experience

4.3.13 AX12: Topological Classification

"Knowledge types correspond to basins of attraction in phase space"

- **Action**: Uses attractors instead of percentage thresholds
- **In practice**: Knowledge "falls" to nearest attractor like a ball
- **For AI**: Natural, fluid epistemic classification

4.4 New Possibilities in NLP

4.4.1 4.1 Active Mathematics

Instead of "we assume that X", we have "system actively enforces X"

4.4.2 4.2 Self-Regulation

System monitors its own boundaries and limitations

4.4.3 4.3 Emergence of Complexity

From 13 simple rules emerges infinite complexity

4.4.4 4.4 Protection Against Hallucinations

Each axiom is a barrier against false certainty

4.4.5 Example in Action:

AI tries to answer "What is the meaning of life?"

1. AX0 activates → introduces superposition of possible answers
2. AX3 limits → "I cannot fully know this truth"
3. AX5 shifts boundaries → answer drifts toward boundary
4. AX7 triggers reflection → "does my answer make sense?"
5. AX11 learns → remembers this is a boundary question

Result: Instead of certain hallucination, AI gives honest
"This question touches the boundaries of definability (Ψ^{h} with drift toward ∅)"
These 13 axioms are an immune system against AI's epistemic arrogance!

4.5 GTMØ as Mathematics of Linguistic Subjectivity

This is a fundamental shift in perspective. GTMØ doesn't try to eliminate subjectivity, but rather formalizes it as an inherent feature of communication.

4.5.1 5.1 Observer as Part of Meaning

class LanguageObserver:
def observe(self, word1, word2, context):
if self.culture == "Eastern":
return eastern_interpretation
elif self.training == "Medical":
return medical_interpretation
elif self.paradigm == "Poetic":
return metaphorical_interpretation
Meaning doesn't exist without an observer!

4.5.2 5.2 Emergence of Meanings from Relations

GTMØ shows that:

- "Deadline" ≠ "dead" + "line"
- "Understanding" ≠ "under" + "standing"

Meanings emerge from configuration, they are not the sum of parts.

4.5.3 5.3 Context as Space, Not Addition

Parameters of semantic configuration:

- **Distance**: Semantic proximity of concepts
- **Orientation**: Type of relation
- **Time**: Temporal context (meanings evolve!)
- **Scale**: Level of abstraction

4.5.4 5.4 Multi-observationality as Standard

Example from the document:
"Heart" + "Break":

- Medical observer: "cardiac arrest"
- Poetic observer: "emotional pain"
- Mechanical observer: "structural failure"

4.5.5 5.5 New Alternative Paradigm

Traditional approach: "How to objectively measure the meaning of the word 'justice'?"

GTMØ: "How to formally describe the space of all possible interpretations of 'justice' by different observers?"

4.5.6 5.6 Implications for Communication

1. **Misunderstandings are natural** - different observers at different points in semantic space
2. **Context doesn't fix meaning** - context CREATES meaning
3. **Ambiguity is a feature, not a bug** - enables creativity and adaptation
4. **Language lives** - in FLUX mode meanings constantly emerge and disappear

4.5.7 5.7 Connection with Philosophy of Language

GTMØ connects with:

- **Wittgenstein**: "meaning emerges from use"
- **Derrida**: "play of differences constituting meaning"
- **Peirce**: "relational triad: sign-object-interpretant"
- **Enactivism**: "cognition as action, not representation"

4.5.8 5.8 Practical Consequences

For AI:

- Instead of searching for "true" meaning, model the space of interpretations
- Instead of eliminating hallucinations, manage semantic exploration
- Instead of hiding uncertainty, express it as superposition

For Communication:

- Ask "from what perspective are you looking?" instead of "what does it really mean?"
- Negotiate common context instead of assuming objectivity
- Celebrate multiplicity of interpretations instead of seeking one truth

4.6 Final Conclusion

GTMØ doesn't solve the problem of subjectivity - it transforms it into a resource.

Subjectivity is not a flaw of the communication system, it is its fundamental property that enables:

- **Creativity** (new configurations of meanings)
- **Adaptation** (meanings evolve with context)
- **Richness** (multiplicity of interpretations)
- **Poetry** (play with meanings)

It's like the difference between:

- Physics searching for objective laws
- Music celebrating subjective experience

GTMØ is the mathematics of the music of meanings, not the physics of definitions.

5 GTMØ: The 9 Semantic Spaces - Complete Architecture

5.1 Chapter 1: Introduction to the Nine-Dimensional Framework

GTMØ defines **9 fundamental spaces**, not 4 dimensions. What was previously described represents merely parameters within one of these spaces - the Configuration Space.

5.2 Chapter 2: The 9 Semantic Spaces of GTMØ

5.2.1 2.1 Configuration Space (4D+)

The space of configuration - where objects possess position, orientation, time, and scale.

Parameters:
- position: [x, y, z]
- time: t
- orientation: [ϑ, φ, ψ]
- scale: s

Example: The word "Bank" in different spatiotemporal configurations

5.2.2 2.2 Phase Space (3D)

The phase space - a three-dimensional epistemic map.

Dimensions:
- Determinacy (0→1)
- Stability (0→1)
- Entropy (0→1)

Example: "Justice" has coordinates [0.3, 0.4, 0.8] = low determinacy, medium stability, high entropy

5.2.3 2.3 Topological Space (∞D)

The topological space - relations and structures.

Features:
- Homologies of meaning
- Attractor basins
- Epistemic holes

Example: "Love" and "Hate" exist within the same topological basin (opposites)

5.2.4 2.4 Observational Space (∞D)

The observational space - all possible perspectives.

Properties:
- Each observer = one dimension
- Cultural, linguistic, cognitive lenses

Example: "Democracy" viewed by 1000 different observers = 1000 dimensions

5.2.5 2.5 Emergent Space (0D)

The space of emergence - birth points of AlienatedNumbers.

Characteristics:
- Dimension 0 = point emergences
- When d$\to$0, something new appears

Example: The moment when "dead" + "line" $\to$ "deadline"

5.2.6 2.6 Indefinite Space (N/A)

The space of indefiniteness - the "paper" on which everything exists.

Nature:
- Without dimension (pre-topological)
- Pure potentiality

Example: All possible, but not yet conceived words

5.2.7 2.7 Trajectory Space (∞D)

The trajectory space - cognitive paths.

Contains:
- History of interpretations
- Learning of meanings

Example: How a child learns the meaning of "justice" over years

5.2.8 2.8 Interpretation Space (∞ D)

The interpretation space - all possible readings.

Structure:
- Each interpretation = one point
- Semantic pluralism

Example: "Freedom" has thousands of interpretations across different contexts

5.2.9 2.9 Boundary Space (2D)

The boundary space - where d→0.

Properties:

- Codimension 1 (one dimension less than surroundings)
- Place of transformation

Example: The boundary between "friend" and "boy/girlfriend" where "friendzone" emerges

5.3 Chapter 3: How These Spaces Interact

5.3.1 Example: The Word "LOVE"

Configuration: "love" + "sick" at distance 0.03

Phase: [0.6, 0.3, 0.7] - moderately determined, unstable, entropic

Topological: In the basin of intense emotions

Observational: Psychologist sees "obsession", poet sees "passion"

Emergent: → "lovesick" (new meaning!)

Indefinite: Potential for other combinations

Trajectory: History of how someone experienced this state

Interpretation: Illness? Blessing? Curse?

Boundary: Moment of transformation friend→lover

5.4 Chapter 4: Innovation of the Approach

Key Innovation: Every word/concept exists SIMULTANEOUSLY in all 9 spaces!

This is like GPS coordinates, but instead of 3 dimensions (longitude, latitude, altitude), we have 9 different coordinate systems operating in parallel.

5.4.1 Consequences:

- **Communication** = synchronization of positions across 9 spaces
- **Misunderstanding** = desynchronization in one or more spaces
- **Poetry** = intentional play between spaces
- **Learning** = building trajectories through spaces

This explains why language is so rich and complex - it operates in a 9-dimensional hyperspace of meaning!

5.5 Chapter 5: Phase Space Within the Nine-Space Architecture

The Phase Space (3D) is ONE of the 9 spaces - specifically Phase Space (#2).

The description of attractors and 3D coordinates operates within one specific space among nine:

5.5.1 The 9 GTMØ SPACES:

1. Configuration Space (4D+) - position, time, orientation
2. **Phase Space (3D) ← WHERE ATTRACTORS OPERATE!**
3. Topological Space (∞D)
4. Observational Space (∞D)
5. Emergent Space (0D)
6. Indefinite Space (N/A)
7. Trajectory Space (∞D)
8. Interpretation Space (∞D)
9. Boundary Space (2D)

5.6 Chapter 6: Language Fluctuation Across Spaces

5.6.1 Step 1: Word/sentence enters the system

"This sentence is false"

5.6.2 Step 2: Phase Space (3D) - CLASSIFICATION

- Determinacy: 0.1 (paradox!)
 - Stability: 0.1 (unstable)
 - Entropy: 0.95 (maximum chaos)
 - Result: Nearest to attractor Ø (Singularity)

5.6.3 Step 3: But this isn't the end! Other spaces add context:

Configuration Space:

- Where was this said? (logic lecture vs joke)
- When? (temporal context)

Observational Space:
Who observes this? (logician vs poet vs child)
Interpretation Space:
What are the possible interpretations? (paradox, joke, test)
Trajectory Space:
Where did we come from to reach this sentence? (conversation history)

5.7 Chapter 7: Complete Example - "Justice"

5.7.1 In Phase Space (3D):

Coordinates: [0.3, 0.4, 0.8]
Classification: Ψ^{N} (Emergent)

5.7.2 But simultaneously in other spaces:

Configuration: Court vs street vs philosophy
Topological: Connected to "law", "morality", "revenge"
Observational: Judge vs victim vs perpetrator - different perspectives
Emergent: May generate "social justice"
Indefinite: Potential for new understandings
Trajectory: How culture learned this concept through centuries
Interpretation: Punishment? Balance? Retribution?
Boundary: Where law ends and revenge begins

5.8 Chapter 8: Visual Metaphor

Imagine a 9-layer map:

5.8.1 Layer 2 (Phase Space):

Here are attractors and semantic GPS
• Ψ^{K} • Ψ^{n}
• ∅
• Ψ^{h} • $\Psi^{\sim}$

5.8.2 Layer 1 (Configuration):

Here is spatiotemporal context
[Court] [Street] [Book]

5.8.3 Layer 3 (Topological):

Here are connections and relations
justice—law
\ /
morality
...and so on for all 9 layers.

5.9 Chapter 9: Why This Is Innovative

1. **Phase Space provides objective classification** (what you described)
2. **The other 8 spaces add richness of context**
3. **Together they create a complete picture of meaning**

This is like the difference between:

- **GPS** (shows only position) = Phase Space
- **Google Maps** (shows buildings, traffic, photos, reviews) = All 9 spaces

5.9.1 Summary:

- YES, the system has objective measurements in Phase Space (3D)
- BUT this is only 1/9 of the complete picture
- Full meaning emerges from the interaction of all 9 spaces
- Subjectivity appears mainly in Observational and Interpretation Space
- Objectivity is in Phase Space and Topological Space

This is why GTMØ is so powerful - it combines objective classification with the richness of context!

5.10 Chapter 10: Why Exactly 9 Spaces?

The number 9 is not arbitrary - it results from the completeness of describing the phenomenon of language and meaning.

5.10.1 Each space answers a fundamental question:

1. **Configuration Space → WHERE?**
   - Where in time and space does meaning exist?
   - Without this, there is no context
2. **Phase Space → WHAT STATE?**
   - How certain/stable/chaotic is the meaning?
   - Without this, there is no classification
3. **Topological Space → WHAT RELATIONS?**
   - How do meanings connect and structure?
   - Without this, there are no connections
4. **Observational Space → WHO OBSERVES?**
   - From what perspective do we observe?
   - Without this, there is no interpretation
5. **Emergent Space → WHAT IS BORN NEW?**
   - Where do AlienatedNumbers arise?
   - Without this, there is no creativity
6. **Indefinite Space → WHAT IS POSSIBLE?**
   - What is the space of potentiality?

- Without this, there is no future of language

7. **Trajectory Space → WHERE DID WE COME FROM?**
   - What is the history of meaning?
   - Without this, there is no learning
8. **Interpretation Space → HOW CAN THIS BE UNDERSTOOD?**
   - What are the possible readings?
   - Without this, there is no pluralism
9. **Boundary Space → WHERE ARE THE BOUNDARIES?**
   - Where do transformations occur?
   - Without this, there is no change

5.10.2 Why Not 8?

If we removed any space, we would lose a crucial aspect:

- Without Configuration - no context
- Without Phase - no classification
- Without Topological - no structure
- etc.

5.10.3 Why Not 10 or 11?

Additional spaces would be redundant:

- "Emotional Space"? - Already in Observational (emotional perspective)
- "Cultural Space"? - Also in Observational (cultural perspective)
- "Temporal Space"? - Already in Configuration (time dimension)

5.10.4 Analogy to Physics:

It's like asking "why does space have 3 dimensions, not 2 or 4?"

- In 2D there is no depth
- In 4D (spatial) the world would be unstable
- 3D is the minimum completeness for our world

Similarly, 9 spaces of GTMØ are the minimum completeness for describing language.

5.10.5 Mathematical Elegance:

Notice also the structure of dimensions:
0D: Emergent, Boundary (points)
2D: Boundary (surfaces)
3D: Phase (phase space)
4D+: Configuration (spacetime)
∞D: Topological, Observational, Trajectory, Interpretation
N/A: Indefinite (pre-dimensional)
This creates a complete spectrum from 0 to ∞ dimensions!

5.11 Chapter 11: Completeness Test

Try to describe any linguistic phenomenon:
"The word 'cool' changed meaning"
You need:

- **Configuration:** when and where
- **Phase:** how stable it was/is
- **Topological:** what it connects to
- **Observational:** who speaks this way
- **Emergent:** how new meaning arose
- **Indefinite:** what potential it has
- **Trajectory:** history of changes
- **Interpretation:** different understandings
- **Boundary:** moment of transformation

All 9 are essential!

5.12 Chapter 12: Conclusion

9 spaces is not an arbitrary choice, but a natural consequence of attempting a complete description of language. It's like discovering that an atom has protons, neutrons, and electrons - not 2, not 4, but exactly 3 types of fundamental particles.

GTMØ discovers that language has exactly 9 fundamental spatial aspects. No fewer, no more.

6 Summary of GTMØ: The Mathematics of Linguistic Subjectivity

The article presents GTMØ (Geometry Topology Mathematics Ø), a theory formulated by Grzegorz Skuza, which introduces an innovative approach to the mathematical analysis of language. Instead of eliminating subjectivity, GTMØ creates a formal apparatus for its description, treating it as a fundamental and creative property of language. The following summary combines the key concepts from the three parts of the document.

6.0.1 Part I: GTMØ Theory – Mathematics of Linguistic Subjectivity

The first part introduces GTMØ as a paradigm shift in language analysis—moving away from the traditional pursuit of "objective truth" in favor of formally mapping the space of possible interpretations. The theory is explicitly defined as the "mathematics of linguistic subjectivity". A key assumption is that meaning does not exist in a vacuum but is the result of an interaction between a sign and an observer, whose perspective is an integral part of the system.

Key Ideas:

- **Meaning Space:** GTMØ introduces a dynamic, three-dimensional structure where words do not have fixed meanings (points) but rather move along trajectories. The three dimensions of this space are:
  - **Determination (0-1):** The degree of a meaning's unambiguity.
  - **Stability (0-1):** The constancy of a meaning over time.
  - Entropy (0-1): The degree of a meaning's creativity, complexity, or paradoxical nature.

    An example is the trajectory of the word "virus," whose meaning and coordinates in this space have changed drastically over the years.
- **Topological Knowledge Attractors:** The theory defines different types of knowledge as "attractors" in the meaning space, which draw in information of a similar nature. This allows for the objective classification of information based on its distance from these points. The main types of attractors are:
  - **Knowledge Particle (ΨK):** Clear and stable information (e.g., "2+2=4").
  - **Knowledge Shadow (Ψh):** Unclear and unstable information (e.g., "It might rain tomorrow").
  - **Emergence (ΨN):** Information that creates new meanings (e.g., "Facebook").
  - **Singularity (Ø):** Paradoxes and logical contradictions (e.g., "This sentence is false").

6.0.2 Part II: GTMØ Computational Value

The second part focuses on the theory's computational architecture, which is based on a **three-dimensional phase space** (Determination, Stability, Entropy). At its core is a **cognitive center** with coordinates [0.5, 0.5, 0.5], representing a neutral state of knowledge and a reference point for all calculations.

Computational Foundations:

- **Value Minimalism:** The system uses a minimal set of mathematical constants, such as (related to quantum superposition) and the golden ratio (φ), ensuring elegance and computational efficiency.
- **Mathematics of Attractors:** Each knowledge attractor has precisely defined coordinates in the phase space, as well as a specific radius and strength. For example, **Singularity (Ø)**, located at [1.0, 1.0, 0.0], is the strongest attractor, acting as an "epistemic black hole" for logical contradictions. The classification of information is performed by calculating its "effective distance" from these fixed reference points, which allows for objective and dynamic categorization.

6.0.3 Part III: The Role of Executable Axioms in GTMØ

The third part presents a unique feature of GTMØ: **13 executable axioms**. Inspired by, among others, Maria Piesko's theory of interactive machines, these axioms are not passive assumptions but active programs that regulate the system's operation and protect it from cognitive errors. They function as an AI "immune system," preventing hallucinations and false certainty.

Example Axioms and Their Functions:

- **AX0: Systemic Uncertainty:** Introduces a superposition of states, allowing the system to fundamentally admit "I don't know".
- **Axioms protecting Ø (AX1-AX4, AX8-AX9):** A set of rules that isolate the concept of absolute undefinability (Ø) from standard computations, making it impossible to reduce it to a number or reach it through mathematical functions.
- **AX7: Meta-Closure:** Triggers system self-evaluation upon encountering Ø, forcing it to ask, "do I know that I don't know?".
- **AX11: Adaptive Learning:** Enables the system to learn from experience, for example, by avoiding states that lead to contradictions.
- **AX12: Topological Classification:** Formalizes classification as the natural "falling" of information into the nearest basin of attraction, rather than relying on rigid percentage thresholds.

In practice, when faced with boundary questions like "What is the meaning of life?", these axioms guide the system to formulate an answer that acknowledges the question touches the limits of definability, rather than generating a hallucination.

6.0.4 Final Conclusion

GTMØ is a comprehensive theory that revolutionizes the approach to language, transforming subjectivity from a problem into a resource. It proposes that language is not a static collection of definitions, but a living organism that can be described mathematically through dynamic trajectories in a meaning space. Through the combination of a three-dimensional phase space, topological attractors, and a self-regulating system of 13 executable axioms, GTMØ offers a framework for building artificial intelligence that not only understands context and ambiguity but is also aware of its own cognitive limitations. It is, as the author concludes, "the mathematics of the music of meanings, not the physics of definitions".

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