Computational Linear Algebra

Grant Smith

Spring 2022

1 Matrices

A table of numbers

1.1 Matrix Multiplication Properties

$$(AB)C = A(BC) = ABC$$

$$A(B + C) = AB + AC$$

$$(AB)^T = B^T A^T$$

1.2 Special Matrices

- Zero Matrix
- Identity Matrix
- Symmetric Matrices $A^T = A$
- Triagnle matrices or trapezoidal, and diagonal is a subset

1.2.1 Upper triangular matrix

- easy to solve
- eigenvalues are right there
- determinants are right there too

1.2.2 Lower Triangular

if they are both upper triangular and lower triangular, then they are diagonal I don't know much about lower triangular matrices

1.2.3 Diagonal Matrices

Very easy to solve a linear system

2 Essential Problems of Linear Algebra

2.1 Solve Linear Systems

$Ax = b$

Use Gauss Jordan Elimination to solve

Computing determinants is $n!$, so we essentially never do it

2.2 Eigenvalue Problems

Eigenvalues are vectors such that

$$Av = \lambda v$$

bridges are essentially eigenvalue problems

2.3 Singular Value Problem

This is about finding vectors u and v such that

$$Au = \sigma v$$

Or

$$A^* u = \sigma v$$

2.4 Matrix Decomposition

Diagonalization is an example, and SVD is another example. The goal here is to write a matrix as a product of two other matrices. Here are a couple examples

2.4.1 LU

$$A = LU$$

Where L is lower triangular and U is upper triangular

Why is this particular composiiton useful? All the work is done in computing the two matrices L and U, and then once you have that, you can solve everything else cheaply. You're essentially done.

2.4.2 QR

This example is

$$A = QR$$

Where Q is unitary and R is upper triangular

2.4.3 SVD

This one is:

$$A = U\Sigma V^*$$

where U and V^* are unitary and Σ is diagonal