Stochastic Processes

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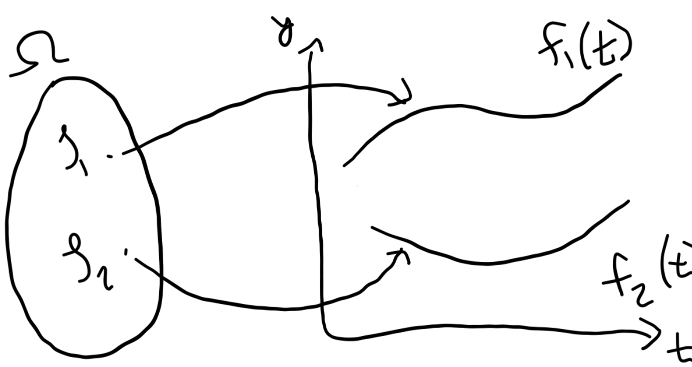
1 Introduction

We are just going to show a few things about stochastic integration and differentiation.

$$X_t = x : \Omega \to \mathbb{R}^+ \to \mathbb{R} = x : \left(\Omega, \mathbb{R}^+\right) \to \mathbb{R}$$

And we have that

$$x(\varsigma_1) = f_1(t)$$
$$x(\varsigma_2) = f_2(t)$$
$$x(\varsigma) = f_\varsigma(t)$$



1.1 Differentiation

We have that:

$$X_t' = \frac{\partial x}{\partial t}(\varsigma, t) = g(\varsigma, t) = \frac{x(\varsigma, t+h) - x(\varsigma, t)}{h} = \frac{f_\varsigma(t+h) - f_\varsigma(t)}{h} = \frac{df_\varsigma}{dt}$$

Which means that X_t' is just the probability weighted derivatives of the trajectories.

Side note that this does not work in Brownian Motion because the individual trajectories in Brownian Motion are not differentiable (anywhere). I think we might be able to do it with some form of a weak derivative, though, just not the regular one.

1.2 Integration

We have that:

$$\int X_t = h(\varsigma, t) = \int_{s=0}^{s=t} X_s ds = \int_{s=0}^{s=t} x(\varsigma, s)\, ds = \int_{s=0}^{s=t} f_\varsigma(s)\, ds$$

Which means that $\int X_t$ is just the probability weighted integrals of the trajectories.

I think we can do this with Brownian Motion because even though Brownian Motion trajectories are not (in the normal sense) differentiable, they are integrable.