Lab2

Gustav Sternelöv

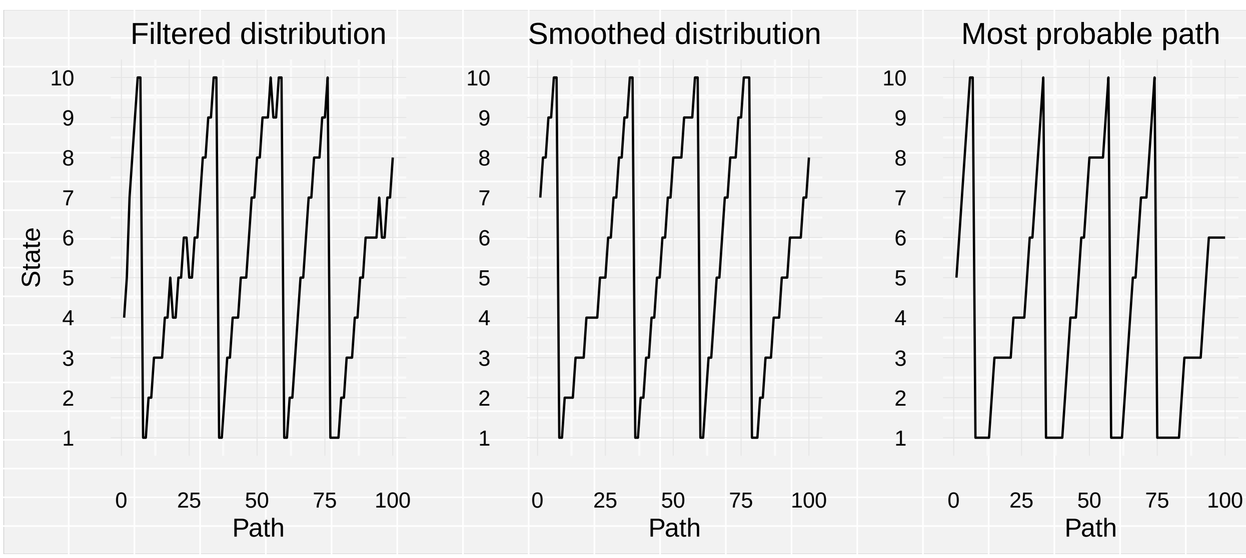
15 september 2016

(1-2) Build a HMM for the scenario described above. Simulate the HMM for 100 time steps.

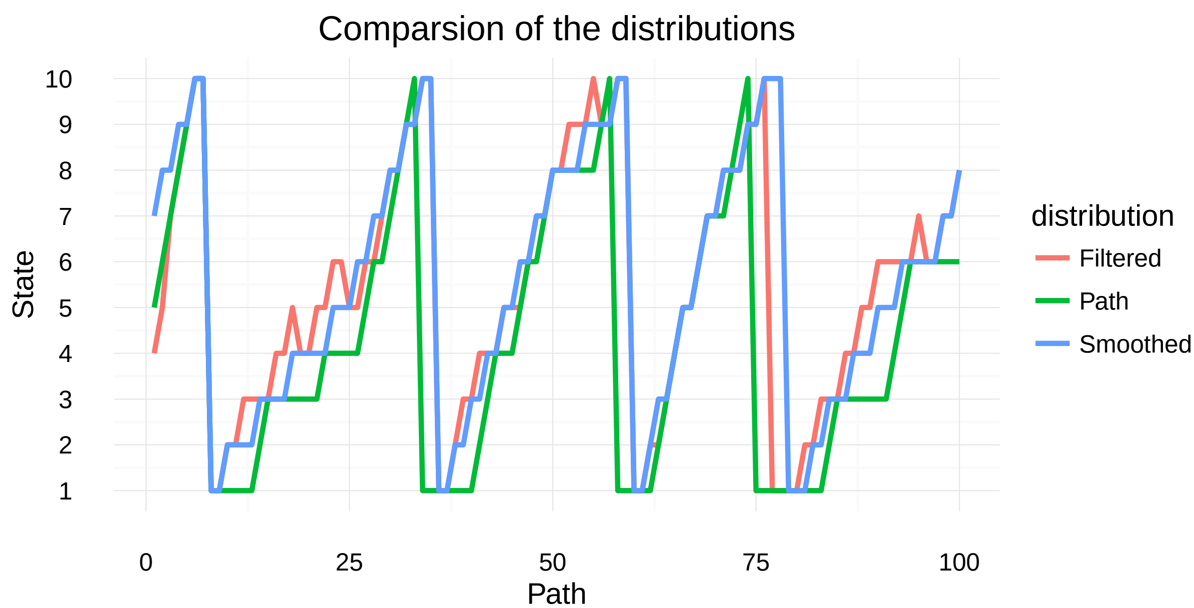
A HMM model with the described scenario is implemented with the *initHMM* function and in the model is the starting probability equal for all different states. For this model is then 100 time steps simulated.

(3) Discard the hidden states from the sample obtained above. Use the remaining observations to compute the filtered and smoothed probability distributions for each of the 100 time points. Compute also the most probable path.

The filtered and the smoothed probability distribution and the most probable path for the 100 time points are visualised with the plots below.



All the distributions seem to be rather similar. The general pattern is the same in the plots for each distribution. To compare the respective distributions may be easier when all the distributions are in the same plot.



It is notable that the line for the most probable path seem to get stuck at state 1 for relatively large periods. The filtered and the smoothed distribution appears to be rather similar for this set of 100 simulated time steps.

(4) Compute the accuracy of the filtered and smoothed probability distributions, and of the most probable path. That is, compute the percentage of the true hidden states that are guessed by each method.

The accuracy for the respective distributions is presented in the table below. For the actual sample of 100 time steps is the smoothed probability the most accuarate one with a margin of 9 percentages down to the filtered distribution. The least accurate of the methods for this set of simulated time steps was the most probable path which was right in 44 % of the time steps.

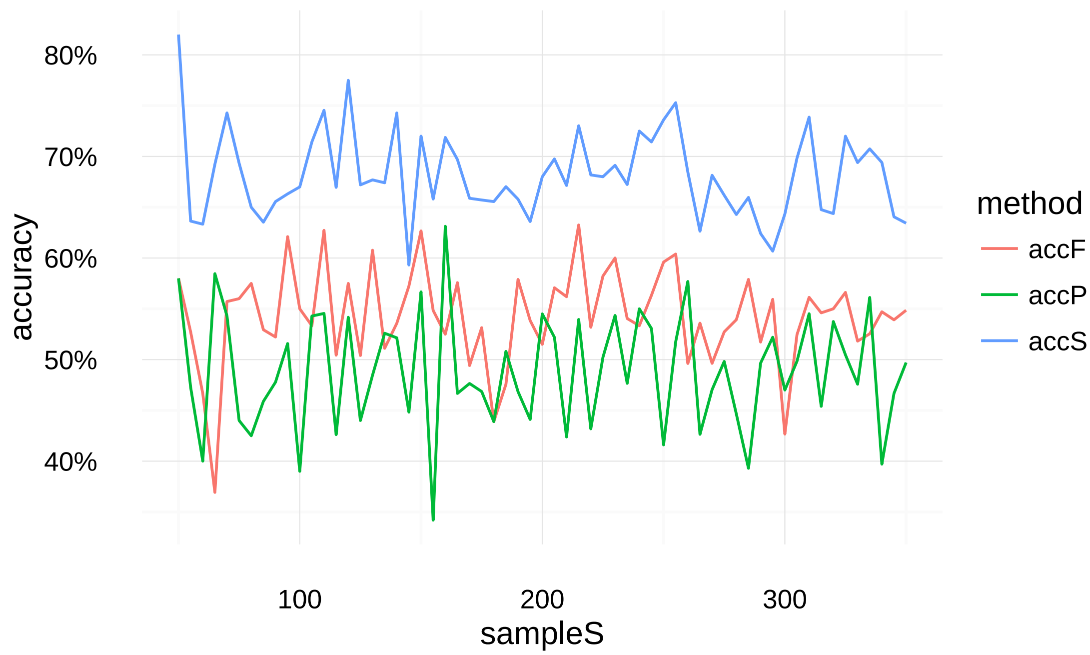
```
##   AccuracyFiltered AccuracySmoothed AccuracyPath
## 1             0.57             0.66         0.44
```

The probability for each state for the respective distributions is shown in the following table. The smoothed distribution has the most values that are 0.1 or close to 0.1. For the filtered distribution is the values also relatively close to 0.1 but for the most probable path there are som values that are quite far from 0.1. Especially for state 1 is the probability, 0.27, very high.

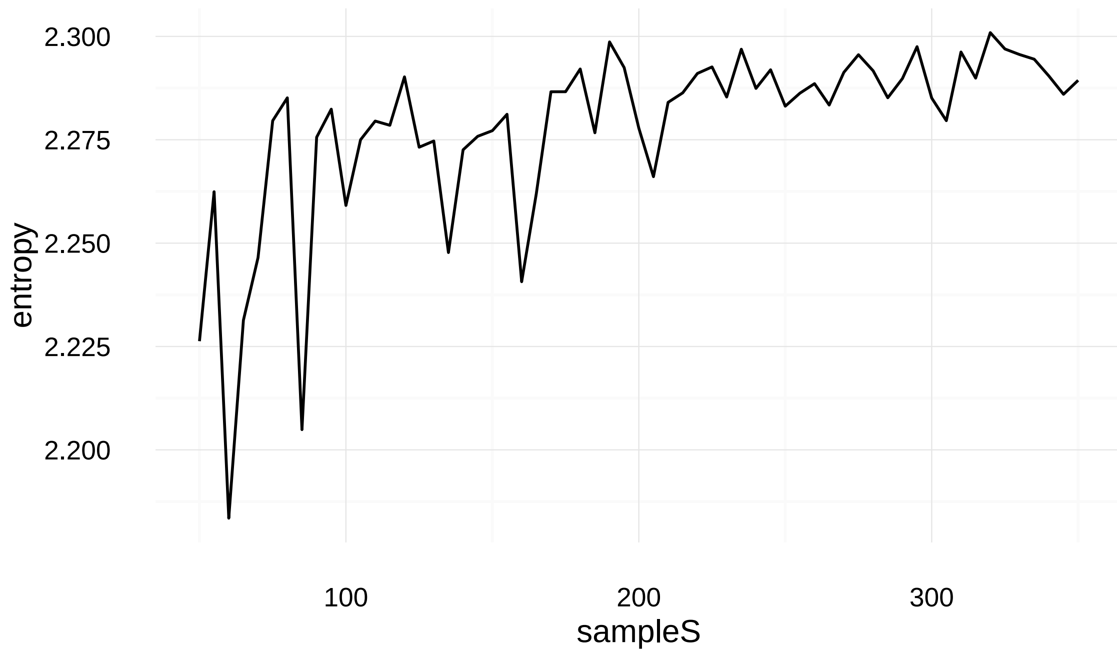
```
##    Filtered Smoothed Path
## 1      0.10     0.09 0.27
## 2      0.07     0.09 0.04
## 3      0.10     0.11 0.16
## 4      0.11     0.11 0.10
## 5      0.13     0.10 0.06
## 6      0.13     0.10 0.13
## 7      0.09     0.09 0.06
## 8      0.09     0.12 0.09
```

```
## 9       0.10      0.10 0.04
## 10      0.08      0.09 0.05
```

(5) Repeat the previous exercise with different simulated samples. In general, the smoothed distributions should be more accurate than the filtered distributions. Why ? In general, the smoothed distributions should be more accurate than the most probable path, too. Why ?



(6) Is it true that the more observations you have the better you know where the robot is?



(7) Consider any of the samples above of length 100. Compute the probabilities of the hidden states for the time step 101.

Z^{100} is simulated to be 7, so in the transition matrix is the probability for state 7 and 8 set to 0.5 and for the rest of the states is the probability 0. This tranisition matrix is multiplicated with the 100th row of the filtering matrix and normalized so that the sum of the probabilties is equal to 1. The result of these computations are presented in the table below and it can be noted that the probability for Z^{101} to be in state 7 is 0.4 and the probability to be in state 8 is 0.6.

```
##       [,1]
##  [1,]  0.0
##  [2,]  0.0
##  [3,]  0.0
##  [4,]  0.0
##  [5,]  0.0
##  [6,]  0.0
##  [7,]  0.4
##  [8,]  0.6
##  [9,]  0.0
## [10,]  0.0
```