Lab 1

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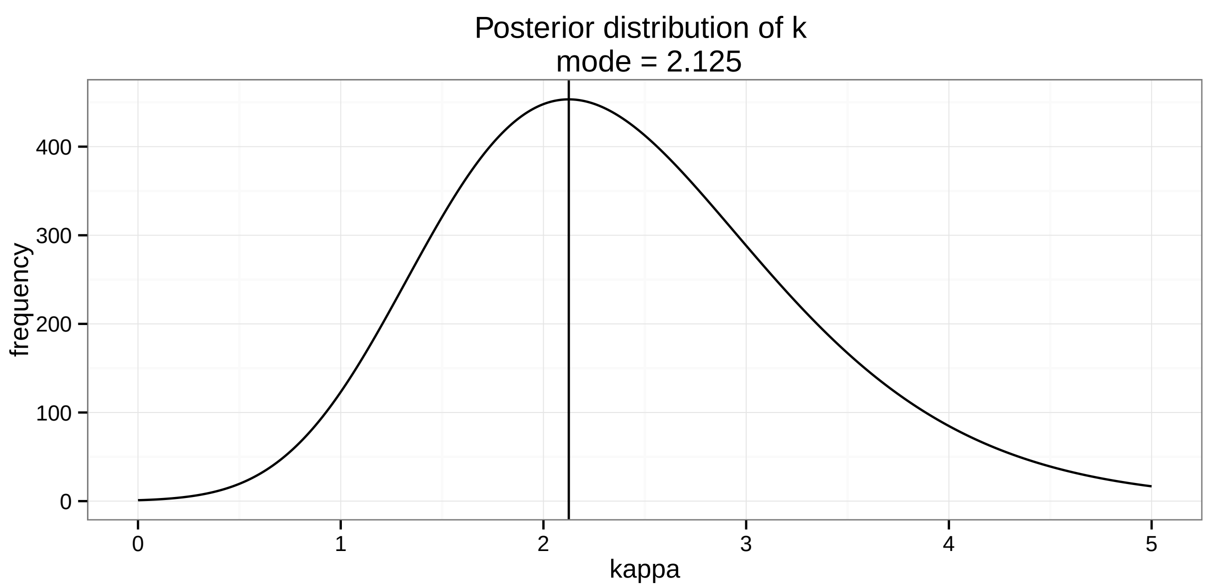
Assignment 3

a-b)

The posterior distribution for κ is proportional to

$$(\frac{1}{I_0(\kappa)})^n exp[\kappa * \sum cos(y_i - \mu) - \kappa]$$

This posterior distribution of κ is plotted over a grid of κ values going from 0 to 5 by steps of 0.001.



The mode value for the posterior distribution is 2.125. This is the value with the maximum posterior probability for the posterior distribution of κ for the wind direction data.

Assignment 4

a)

The prior distribution of λ is proportional to

$$\lambda^{\alpha_0 - 1} e^{-\beta_0 \lambda}$$

Y follows a Poisson($\frac{n_i \lambda}{100000}$) distribution and its likelihood is proportional to

$$\lambda^{\sum Y_i} e^{-\lambda n}$$

Where λ is equal to $\frac{n_i \lambda}{100000}$.

The posterior is equal to the prior times the likelihood

$$\lambda^{\alpha-1} e^{-\beta\lambda} * \lambda^{\sum Y_i} e^{-\lambda n}$$

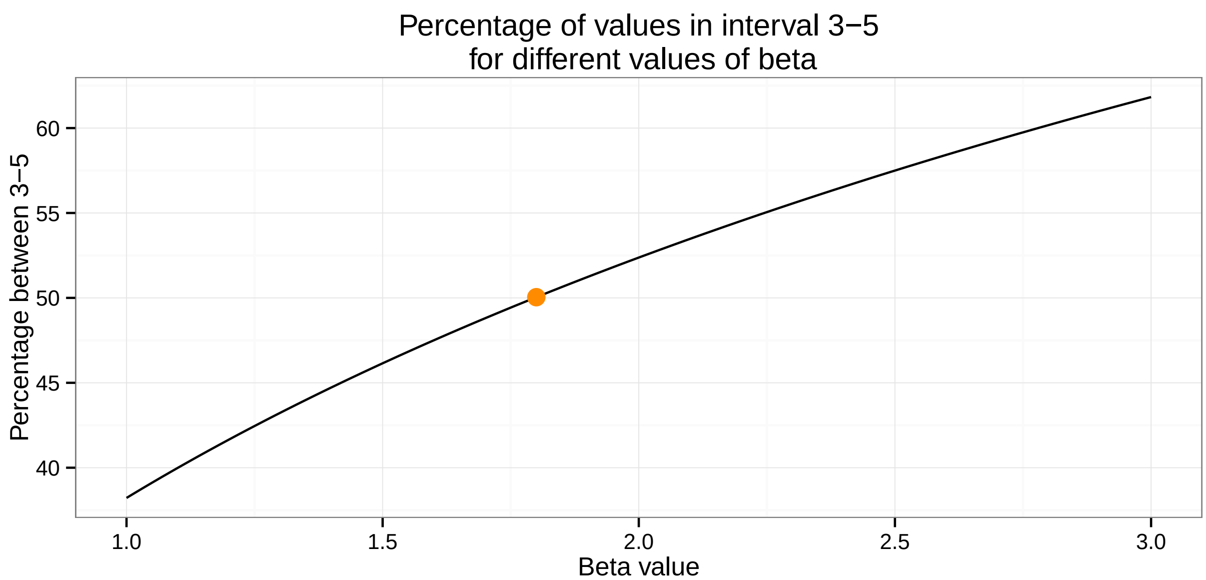
Which can be written as (remember that λ is equal to $\frac{n_i \lambda}{100000}$)

$$\lambda^{\alpha_0 + \sum Y_i - 1} e^{-\lambda(b_0 + \frac{\sum n_i}{100000})}$$

It is then easy to see that the posterior distribution also is a gamma distribution but with the parameters $\alpha_0 + \sum Y_i$ and $b_0 + \frac{\sum n_i}{100000}$

b)

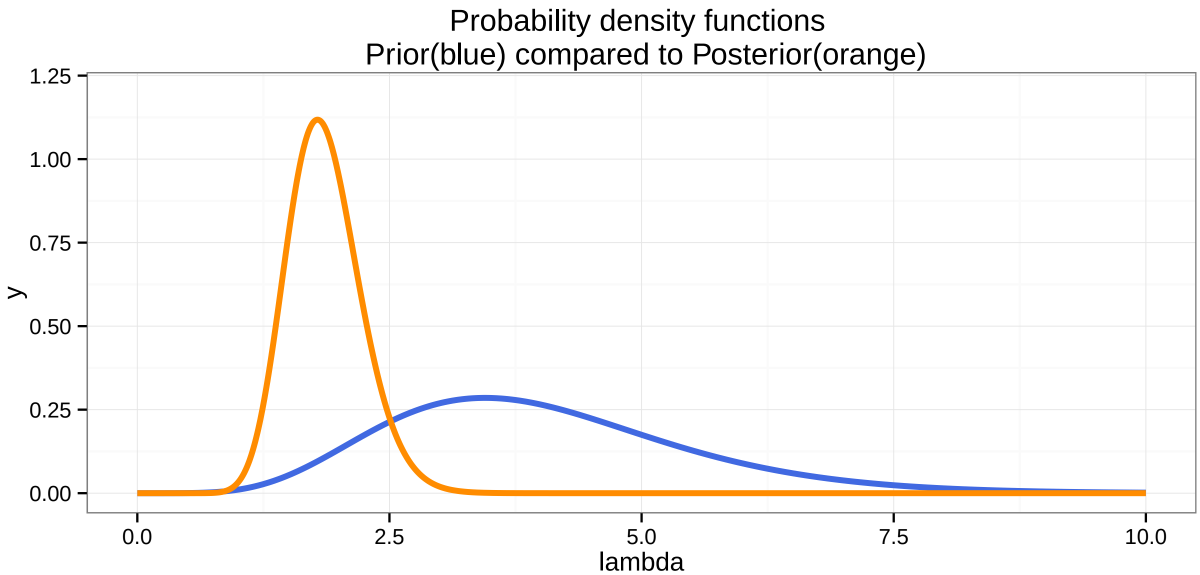
A trial and error approach is used for finding the parameter values for the prior. λ follows a Gamma(4β, β) distribution and the aim is to find a value of β such that $\Pr(3 \leq \lambda \leq 5) \approx 0.5$. Values in the range 1 to 3 by steps of 0.01 is tested and the results are presented in the plot below.



The value closest to 0.5 is given when β is equal to 1.80. Hence, the chosen prior distribution for λ is a Gamma(4*1.8, 1.8).

c)

The prior for λ is updated in line with the results obtained in *b)*. With the updated information about the prior is it a gamma distribution with the following parameter settings: $\lambda \sim Gamma(7.2, 1.8)$. The posterior distribution is a also a gamma distribution but with updated parameters, $\lambda \sim Gamma(7.2+19, 1.8+12.31663)$. The probability density function of λ for both the prior and the posterior are compared with the following plot.



The observed data has had a rather heavy impact on the distribution specified by the prior. As shown by the plot has the curve for the probability density function of the posterior shifted to the left by quite some margin. For example is the posterior probability that λ is between 3 and 5 just 0.313 %.