

2DOF 기구학

로봇팔 세미나 - 김혜윤 -

Contents

1. 2DOF Manipulator

2. 2DOF 정기구학

- 2DOF 동차 변환 행렬

3. 2DOF 역기구학

2DOF Manipulator

2DOF Manipulaor

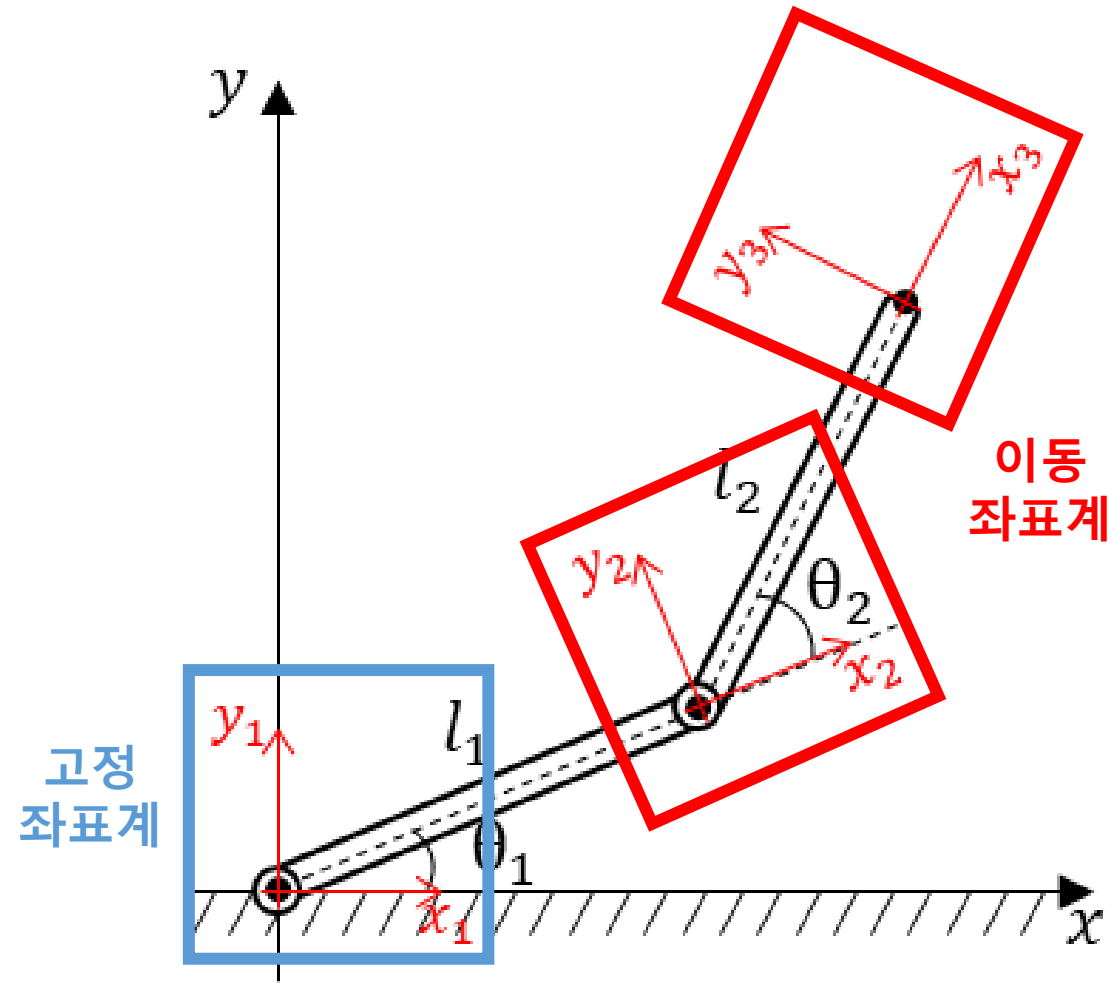


그림 출처: <https://www.minthee.kr/matlab%ec%9c%bc%eb%a1%9c-dynamixel-%ea%b5%ac%eb%8f%99%ed%95%98%ea%b8%b0-03-two-link%eb%a1%9c%eb%b4%87-kinematics/>

2DOF 정기구학

동차 변환 행렬 (회전 / 이동)

$$A(x, \theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A(y, \theta) = \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

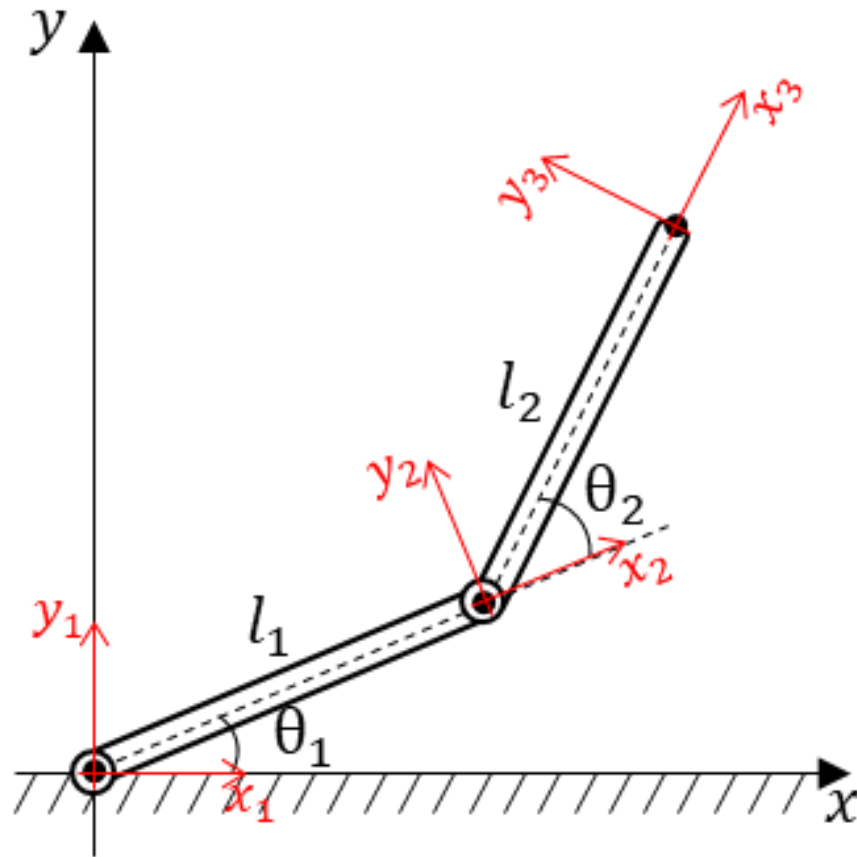
$$A(z, \theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

z축 회전

이동 좌표

$$A(p_{0x'}, p_{0y'}, p_{0z'}) = \begin{bmatrix} 1 & 0 & 0 & p_{0x} \\ 0 & 1 & 0 & p_{0y} \\ 0 & 0 & 1 & p_{0z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2DOF Manipulaor 동차 변환 행렬



동차 변환 행렬

$${}^0T_1 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & l_1\cos(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) & 0 & l_1\sin(\theta_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1T_2 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & l_2\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & l_2\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2DOF Manipulaor 동차 변환 행렬

동차 변환 행렬

$${}^0T_1 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & l_1\cos(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) & 0 & l_1\sin(\theta_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1T_2 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & l_2\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & l_2\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

전체 동차 변환 행렬

$${}^0T_2 = \begin{bmatrix} \cos(\theta_{12}) & -\sin(\theta_{12}) & 0 & l_1\cos(\theta_1) + l_2\cos(\theta_{12}) \\ \sin(\theta_{12}) & \cos(\theta_{12}) & 0 & l_1\sin(\theta_1) + l_2\sin(\theta_{12}) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\theta_{12} = \theta_1 + \theta_2$$

2DOF 역기구학

2DOF Manipulaor 동차 변환 행렬

동차 변환 행렬

$${}^0T_1 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & l_1\cos(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) & 0 & l_1\sin(\theta_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1T_2 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & l_2\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & l_2\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

전체 동차 변환 행렬

끝점 x y 좌표

$${}^0T_2 = \begin{bmatrix} \cos(\theta_{12}) & -\sin(\theta_{12}) & 0 & l_1\cos(\theta_1) + l_2\cos(\theta_{12}) \\ \sin(\theta_{12}) & \cos(\theta_{12}) & 0 & l_1\sin(\theta_1) + l_2\sin(\theta_{12}) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\theta_{12} = \theta_1 + \theta_2$$

역기구학 풀이

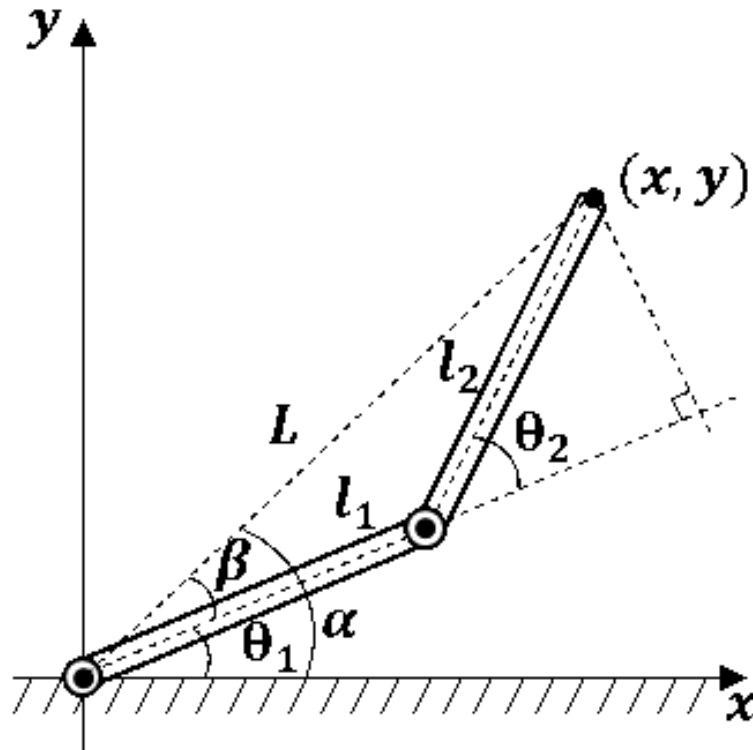
$$\begin{aligned}x &= l_1 \cos \theta_1 + l_2 \cos (\theta_1 + \theta_2) \\y &= l_1 \sin \theta_1 + l_2 \sin (\theta_1 + \theta_2)\end{aligned}$$

x, y 제곱해서 더하면

$$x^2 + y^2 = l_1^2 + l_2^2 + 2l_1l_2\cos\theta_2$$

$$\cos\theta_2 = \frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1l_2}, \sin\theta_2 = \pm \sqrt{1 - \cos^2\theta_2}$$

역기구학 풀이

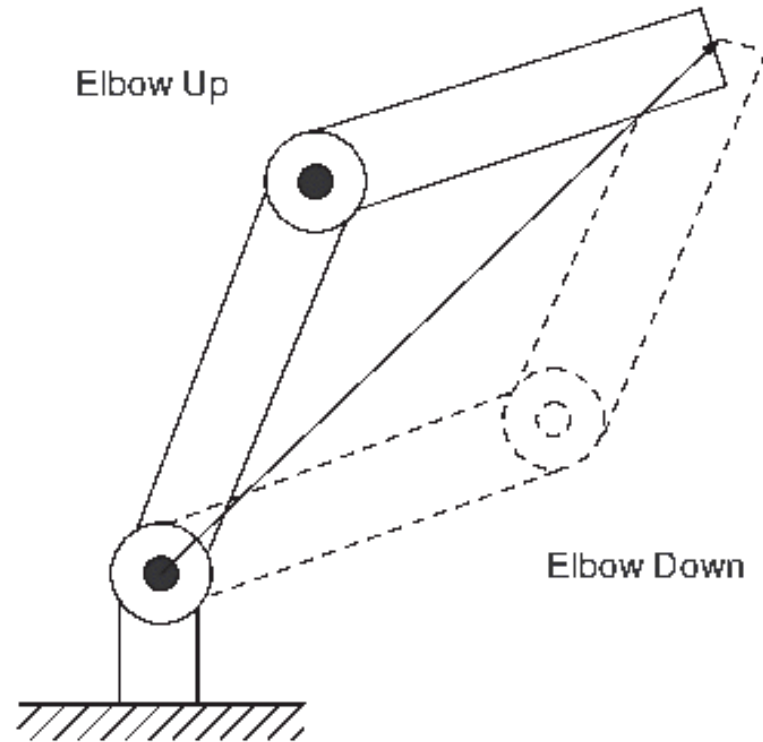


$$\theta_1 = \underbrace{\text{atan2}(y, x)}_{\alpha} - \underbrace{\text{atan2}(l_1 + l_2 \cos \theta_2, l_2 \sin \theta_2)}_{\beta}$$

α

β

역기구학 풀이



$$\cos\theta_2 = \frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1l_2}, \sin\theta_2 = \pm \sqrt{1 - \cos^2\theta_2}$$

$$\theta_1 = \underbrace{\text{atan2}(y, x)}_{\alpha} \pm \underbrace{\text{atan2}(l_1 + l_2\cos\theta_2, l_2\sin\theta_2)}_{\beta}$$

α

β

그림 출처: <https://ddangeun.tistory.com/27>

<https://www.minthee.kr/matlab%ec%9c%bc%eb%a1%9c-dynamixel-%ea%b5%ac%eb%8f%99%ed%95%98%ea%b8%b0-03-two-link%eb%a1%9c%eb%b4%87-kinematics/>