3DOF 기구학

로봇팔 세미나 - 김혜윤 -

Contents

3DOF Manipulator

3DOF Manipulaor

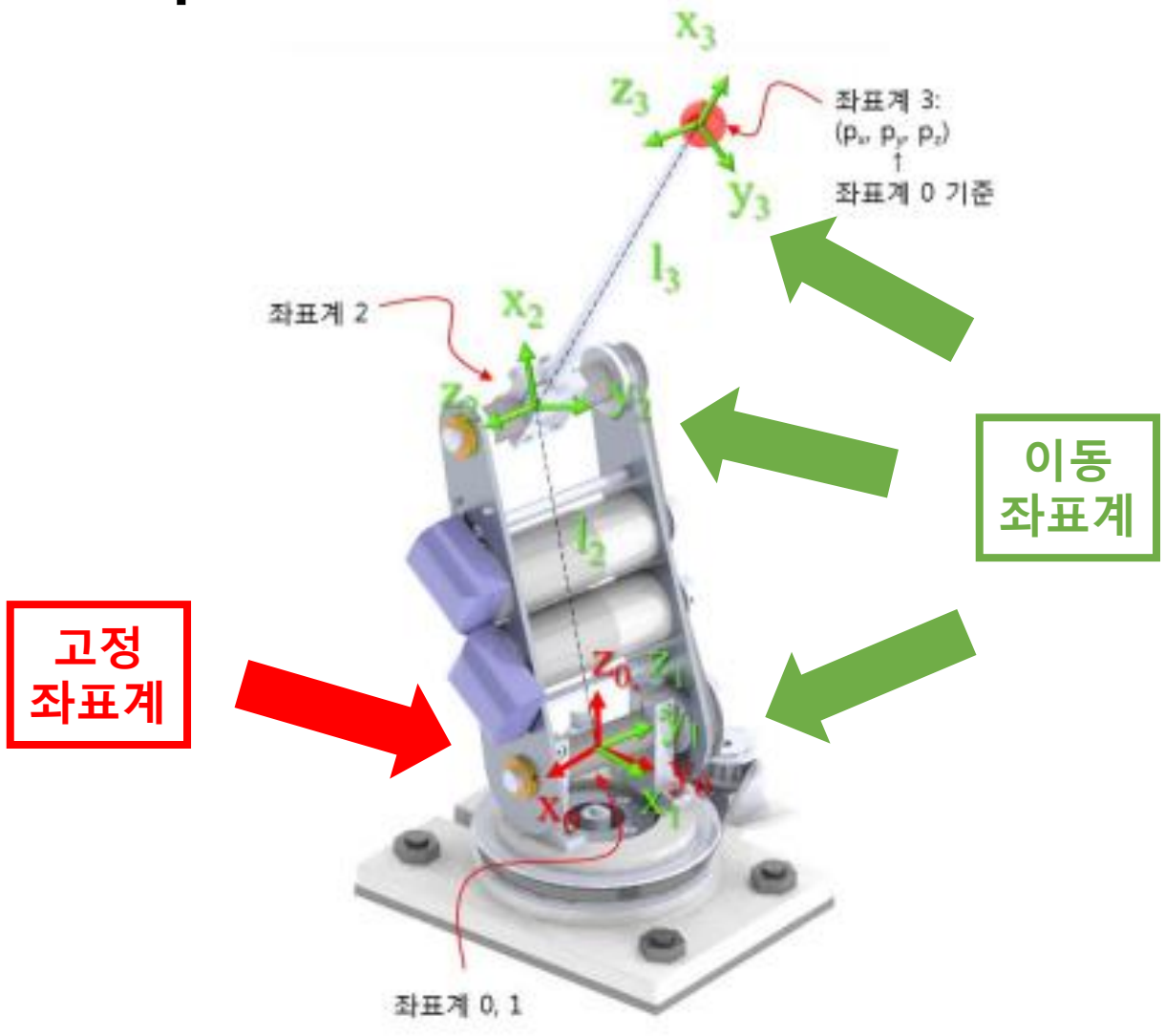
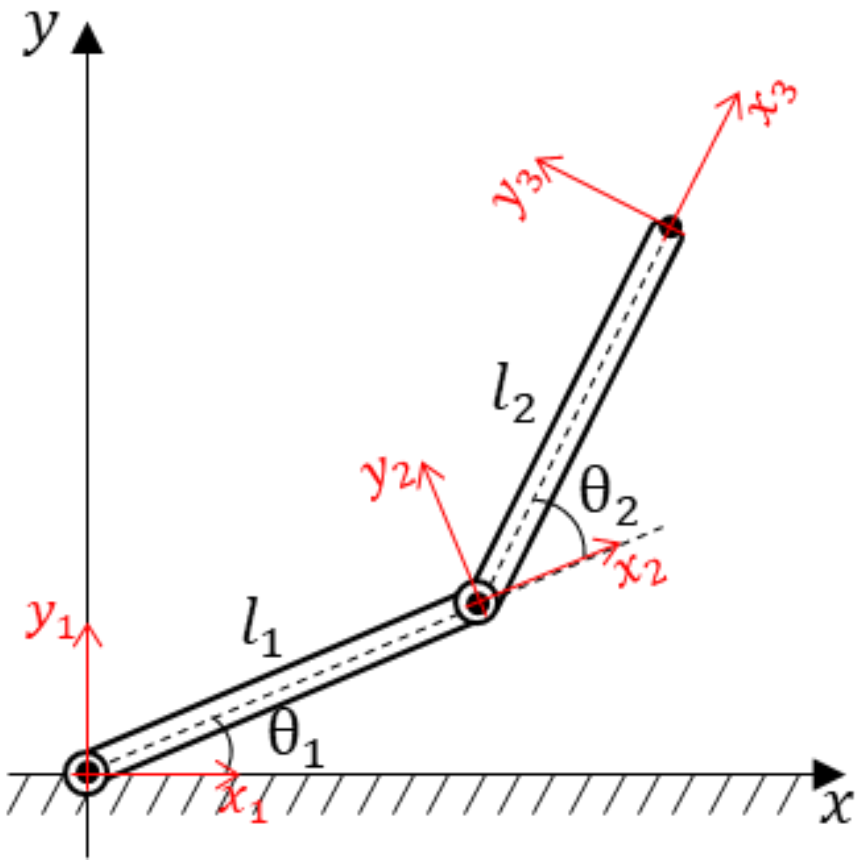
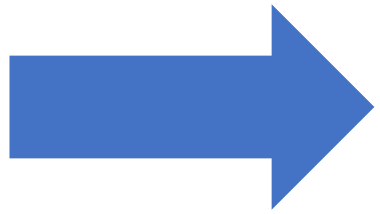


그림 출처: http://robot.hanbat.ac.kr/May11/teaching/lab/3%20DOF%20robot%20arm_rev2.pdf

3DOF Manipulaor





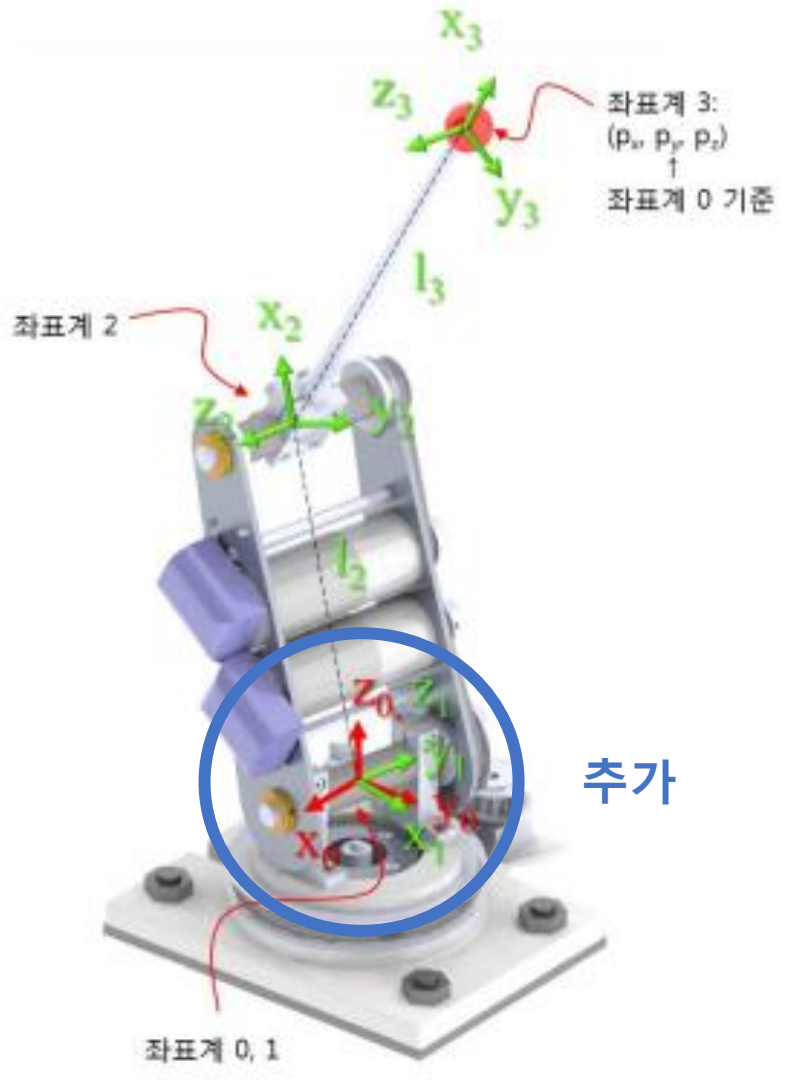
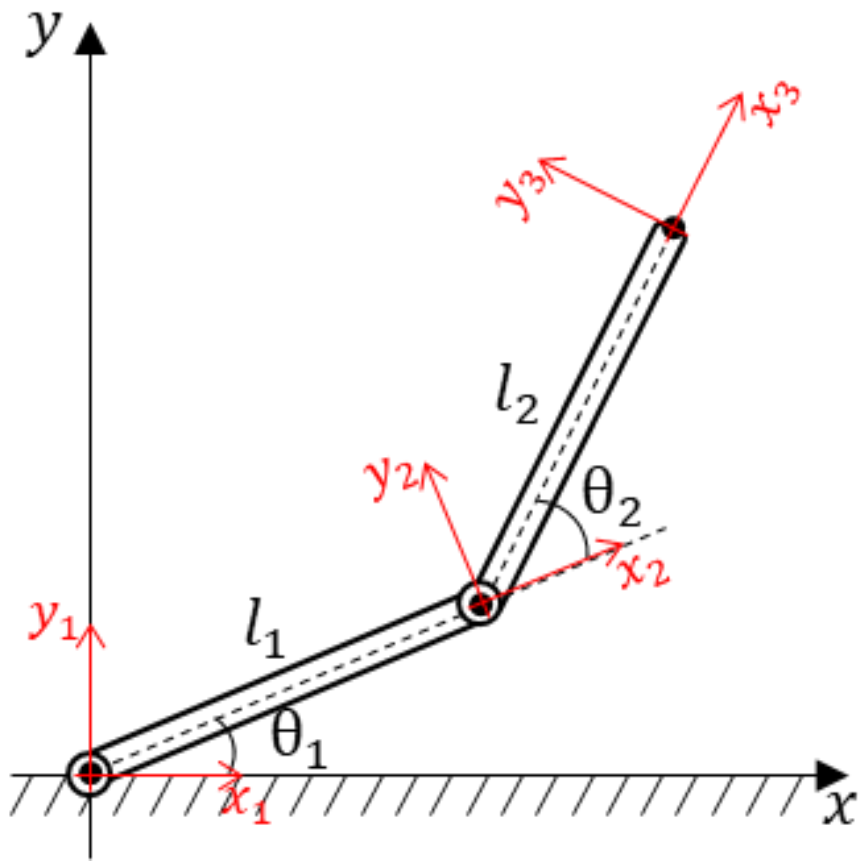


그림 출처: https://www.minthee.kr/matlab%ec%9c%bc%eb%a1%9c-dynamixel-%ea%b5%ac%eb%8f%99%ed%95%98%ea%b8%b0-03-two-link%eb%a1%9c%eb%b4%87-kinematics/
http://robot.hanbat.ac.kr/May11/teaching/lab/3%20DOF%20robot%20arm_rev2.pdf

3DOF 정기구학

2DOF Manipulaor 동차 변환 행렬



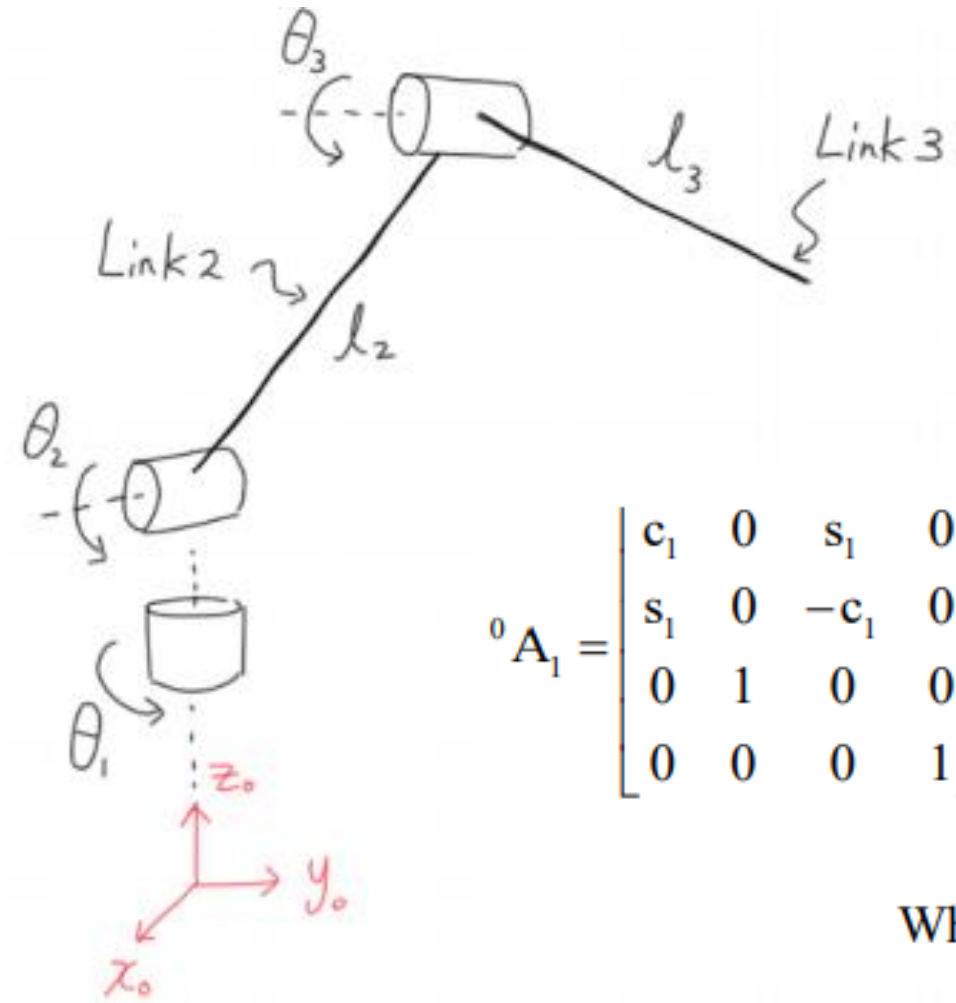
동차 변환 행렬

$$
{}^{0}\mathrm{T}_{1} = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & l_1\cos(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) & 0 & l_1\sin(\theta_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}\mathrm{T}_{2} = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & l_2\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & l_2\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

그림 출처: https://www.minthee.kr/matlab%ec%9c%bc%eb%a1%9c-dynamixel-%ea%b5%ac%eb%8f%99%ed%95%98%ea%b8%b0-03-two-link%eb%a1%9c%eb%b4%87-kinematics/

3DOF Manipulaor 동차 변환 행렬



$$
{}^{0}A_{1} = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^{1}A_{2} = \begin{bmatrix} c_2 & -s_2 & 0 & c_2 l_2 \\ s_2 & c_2 & 0 & s_2 l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^{2}A_{3} = \begin{bmatrix} c_3 & -s_3 & 0 & c_3 l_3 \\ s_3 & c_3 & 0 & s_3 l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Where $c_i = \cos(\theta_i)$, $s_i = \sin(\theta_i)$ $(i = 1,2,3)$

그림 출처: http://robot.hanbat.ac.kr/May11/teaching/lab/3%20DOF%20robot%20arm_rev2.pdf

3DOF Manipulaor 동차 변환 행렬

$$ {}^{0}A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}T_2 = \begin{bmatrix} \cos(\theta_{12}) & -\sin(\theta_{12}) & 0 & l_1\cos(\theta_1) + l_2\cos(\theta_{12}) \\ \sin(\theta_{12}) & \cos(\theta_{12}) & 0 & l_1\sin(\theta_1) + l_2\sin(\theta_{12}) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \theta_{12} = \theta_1 + \theta_2 $$

Where $c_i = \cos(\theta_i)$, $s_i = \sin(\theta_i)$ $(i = 1,2,3)$

$$ {}^{0}T_3 = \begin{bmatrix} c_1\cos(\theta_{23}) & -c_1\sin(\theta_{23}) & s_1 & c_1(l_2c_2 + l_3\cos(\theta_{23})) \\ s_1\cos(\theta_{23}) & -s_1\sin(\theta_{23}) & -c_1 & s_1(l_2c_2 + l_3\cos(\theta_{23})) \\ sin(\theta_{23}) & \cos(\theta_{23}) & 0 & (l_2s_2 + l_3\sin(\theta_{23}) \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

그림 출처: https://www.minthee.kr/matlab%ec%9c%bc%eb%a1%9c-dynamixel-%ea%b5%ac%eb%8f%99%ed%95%98%ea%b8%b0-03-two-link%eb%a1%9c%eb%b4%87-kinematics/
http://robot.hanbat.ac.kr/May11/teaching/lab/3%20DOF%20robot%20arm_rev2.pdf

3DOF 역기구학

역기구학 풀이

$$
{}^{0}T_{3} = \begin{bmatrix} c_1\cos(\theta_{23}) & -c_1\sin(\theta_{23}) & s_1 & c_1(l_2c_2 + l_3\cos(\theta_{23})) \\ s_1\cos(\theta_{23}) & -s_1\sin(\theta_{23}) & -c_1 & s_1(l_2c_2 + l_3\cos(\theta_{23})) \\ sin(\theta_{23}) & \cos(\theta_{23}) & 0 & (l_2s_2 + l_3\sin(\theta_{23}) \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} r_1 & r_2 & r_3 & p_x \\ r_4 & r_5 & r_6 & p_y \\ r_7 & r_8 & r_9 & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
p_x = c_1(l_2c_2 + l_3\cos(\theta_{23}))
$$

$$
p_y = s_1(l_2c_2 + l_3\cos(\theta_{23}))
$$

$$
\theta_1 = \mathrm{Atan2}(p_y, p_x)
$$

그림 출처: http://robot.hanbat.ac.kr/May11/teaching/lab/3%20DOF%20robot%20arm_rev2.pdf

역기구학 풀이

$$
{}^{0}T_{3} = \begin{bmatrix} c_1\cos(\theta_{23}) & -c_1\sin(\theta_{23}) & s_1 & c_1(l_2c_2 + l_3\cos(\theta_{23})) \\ s_1\cos(\theta_{23}) & -s_1\sin(\theta_{23}) & -c_1 & s_1(l_2c_2 + l_3\cos(\theta_{23})) \\ sin(\theta_{23}) & \cos(\theta_{23}) & 0 & (l_2s_2 + l_3\sin(\theta_{23}) \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} r_1 & r_2 & r_3 & p_x \\ r_4 & r_5 & r_6 & p_y \\ r_7 & r_8 & r_9 & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
(p_x\ c_1 + p_y\ s_1)^2 = (c_1^2 + s_1^2)^2\ (l_2c_2 + l_3\cos(\theta_{23}))^2
$$

$$
= (l_2c_2 + l_3\cos(\theta_{23}))^2
$$

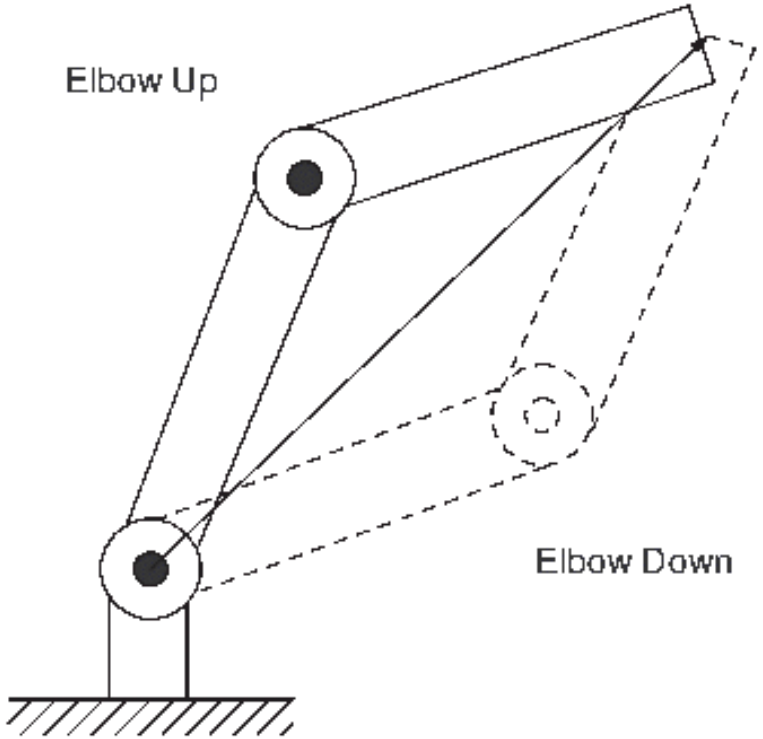
$$
(p_x\ c_1 + p_y\ s_1)^2 + p_z^2 - l_3^2 - l_2^2
$$

$$
= (l_2c_2 + l_3\cos(\theta_{23}))^2 + (l_2s_2 + l_3\sin(\theta_{23})^2 - l_2^2 - l_3^2
$$

$$
= 2l_2\ l_3(c_2\cos(\theta_{23}) + s_2\sin(\theta_{23})) = 2l_2\ l_3\cos\theta_3
$$

그림 출처: http://robot.hanbat.ac.kr/May11/teaching/lab/3%20DOF%20robot%20arm_rev2.pdf

역기구학 풀이



$$(\mathrm{p_x}\ c_1 + \mathrm{p_y}\ \mathrm{s_1})^2 + \mathrm{p_z}^2 - {l_3}^2 - {l_2}^2 = 2l_2\ l_3 \cos\theta_3$$

$$\mathrm{A_0} = \frac{(\mathrm{p_x}\ c_1 + \mathrm{p_y}\ \mathrm{s_1})^2 + \mathrm{p_z}^2 - {l_3}^2 - {l_2}^2}{2l_2 l_3} = \cos\theta_3$$

$$\mathrm{B_0} = \pm\sqrt{1 - \mathrm{A_0}^2} = \sin\theta_3$$

$$\theta_3 = \mathrm{Atan2}(\mathrm{B_0}, \mathrm{A_0})$$

그림 출처: https://ddangeun.tistory.com/27
https://www.minthee.kr/matlab%ec%9c%bc%eb%a1%9c-dynamixel-%ea%b5%ac%eb%8f%99%ed%95%98%ea%b8%b0-03-two-link%eb%a1%9c%eb%b4%87-kinematics/

역기구학 풀이

$$\mathrm{A}_0 = \cos\theta_3,\ \mathrm{B}_0 = \sin\theta_3$$

$$\mathrm{p_x}\ c_1 + \mathrm{p_y}\ \mathrm{s}_1 = ({c_1}^2 + {\mathrm{s}_1}^2)(l_2c_2 + l_3\cos(\theta_{23})) = l_2c_2 + l_3\cos(\theta_{23})$$

$$(\mathrm{A}_0 l_3 + l_2)(\mathrm{p_x}\ c_1 + \mathrm{p_y}\ \mathrm{s}_1) = (l_3\mathrm{c}_3 + l_2)(\ l_2c_2 + l_3\cos(\theta_{23}))$$

$$(\mathrm{A}_0 l_3 + l_2)(\mathrm{p_x}\ c_1 + \mathrm{p_y}\ \mathrm{s}_1) + \mathrm{B}_0 l_3 \mathrm{p_z}$$

$$= (l_3\mathrm{c}_3 + l_2)(\ l_2c_2 + l_3\cos(\theta_{23})) + l_3\mathrm{s}_3(l_2s_2 + l_3\sin(\theta_{23})$$

$$= l_2l_3(\mathrm{c}_2\mathrm{c}_3 + \cos(\theta_{23}) + \mathrm{s}_2\mathrm{s}_3) + {l_2}^2c_2 + {l_3}^2(c_3\cos(\theta_{23}) + \mathrm{s}_3\sin(\theta_{23}))$$

$$= \mathrm{c}_2(2l_2l_3\cos\theta_3 + {l_2}^2 + {l_3}^2)$$

$$(\mathrm{p_x}\ c_1 + \mathrm{p_y}\ \mathrm{s}_1)^2 + {\mathrm{p_z}}^2 = 2l_2\ l_3\cos\theta_3 + {l_2}^2 + {l_3}^2$$

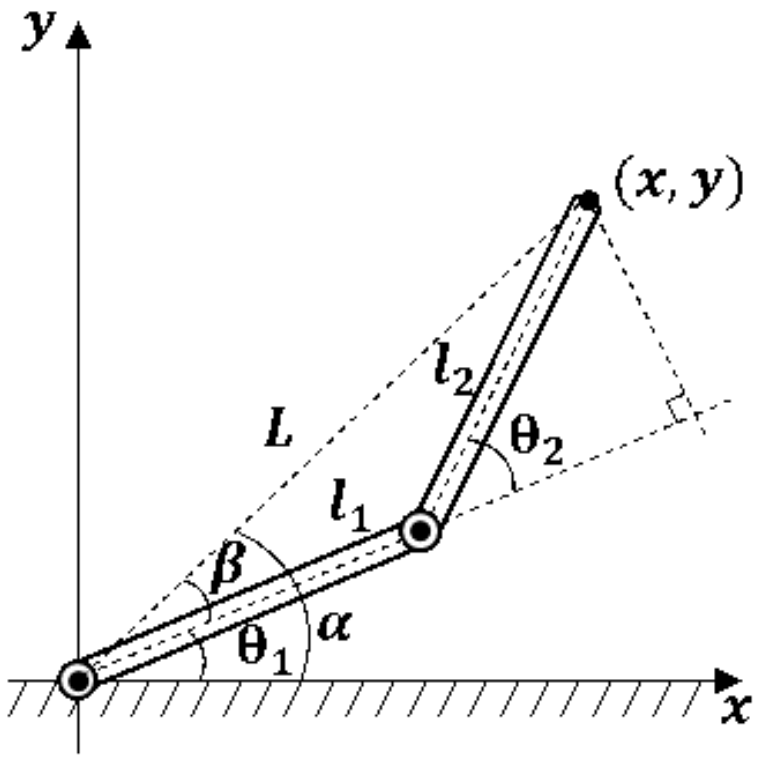
역기구학 풀이

$$A_1 = \frac{(A_0 l_3 + l_2)(p_x\ c_1 + p_y\ s_1) + B_0 l_3 p_z}{(p_x\ c_1 + p_y\ s_1)^2 + {p_z}^2} = \cos\theta_2$$

$$B_1 = \pm\sqrt{1 - {A_1}^2} = \sin\theta_2$$

$$\theta_2 = \mathrm{Atan2}(B_1, A_1)$$

2DOF 역기구학



$$\theta_1 = \boxed{atan2(y,x)} - \boxed{atan2(l_1 + l_2\cos\theta_2, l_2\sin\theta_2)}$$

α β

그림 출처: https://www.minthee.kr/matlab%ec%9c%bc%eb%a1%9c-dynamixel-%ea%b5%ac%eb%8f%99%ed%95%98%ea%b8%b0-03-two-link%eb%a1%9c%eb%b4%87-kinematics/