1 Week 1

- A set is a collection of unique/district objects. ie, $\{3, 8, 6\}$ It must not contain any duplicates.
- X is an element of a $x \in A$, or X is not an element $x \notin A$ Empty set is $0 = \{\}$
- A subset of another set (B) is if every element in the first is (A) in the other, $A \subseteq B$
- An event is a subset of a sample space, ie $\{2, 4, 6\} \subseteq \{1, 2, 3, 4, 5, 6\}$
- A union is the set of all elements in A or B, or both, $A \cup B$
- An intersection of two sets is where A and B happen, $A \cap B$, or if its empty, $A \cap B = 0$ is disjoint.
- A compliment is all the elements in A where A did not occur, $A^c = 1 - A$

2 Week 2

- Demorgans law: $(A \cup B)^c = A^c \cap B^c$, $(A \cap B)^c = A^c \cup B^c$
- A sample space is a set of all possible outcomes.
- $P(A \cup B) = P(A) + P(B)$ only if $P(A \cap B) = 0$, if not, $= P(A) + (B) - A \cap B$
- If we have two experiements with sample space ω then new sample space is $\omega \times \omega$ or ω^n
- Example: pick a number between 1 and 100... What is the possibility of:
  The number has one digit? $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $P(A) = \frac{|A|}{|\omega|} = \frac{9}{100}$
  The number has two digits? $B = \{10, 11, ..., 99\}$, $P(B) = \frac{9}{10}$
  The probability of %4? $C = \{4, 8, 12, 16, ..., 100\}$, $P(C) = \frac{25}{100} = \frac{1}{4}$
  What is C^c? $P(C^c) = 1 - P(C) = 1 - \frac{1}{4} = \frac{1}{4}$
- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- Example: We toss three fair coins. What is the prob. that the first coin turns up as T given there was one T?
  $\omega = 2$, $\omega \times \omega \times \omega = \omega^3 = 8$
  $P(A) =$ First coin toss is T, $P(B) =$ One coin is T.
  $P(B) = \{(H, H, T), (T, H, H), (H, T, H)\} = 3$
  $P(A \cap B) = \{(T, H, H)\} = 1$, $\frac{P(A \cap B)}{P(B)} = \frac{1}{3}$

3 Week 3

- $A^c = \omega \backslash A$, $A \backslash B = A \cap B^c c$
- Example (mad cow),T=Test is positive, B=has disease.: Given $P(T|B) = 0.7$, $P(T|B^c) = 0.1$, $P(T) = 0.2$, what is $P(B)$? ($P(B) = x$)
  $P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$, $P(T) = P(T|B)P(B) + P(T|B^c)P(B^c) == 0.2 = 0.7x + 0.1(1-x) = \frac{1}{6}$
- Conditional probability: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

4 Week 4

- To check if A and B are independent, if $P(A \cap B) = P(A)P(B)$
- A, B, C are events, A, B ,C are jointly independent if all the following hold:
  $P(A|B \cap C) = P(A)$
  $P(A|B \cap C^c) = P(A)$
  $P(A|B^c \cap C) = P(A)$

$P(A|B^c \cap C^c) = P(A)$

- Bayes rule: $P(A|B) = P(B|A)\frac{P(A)}{P(B)}$
- Example: Supposed that a test for a certain rare disease is 95% accurate. Ie, suppose that for a person with the disease, the test is positive with probability 0.95 and for a person without the disease, 0.95. Suppose that a random person drawn from the population has the disease with probability 0.0001 ($\frac{1}{10^4}$). Suppose that person A gets the test and it is positive. What is the probibiility of them actually having the disease?

  $P(D) =$ Person has disease $= \frac{1}{10^4}$, $P(D|T) =$ Test is positive - What we are looking for.
  $P(T|D) = 0.95$, $P(T^c|D) = 0.05$, $P(T|D^c) = 0.05$, $P(T^c|D^c) = 0.95$
  Use Bayes rule: $P(D|T) = P(T|D)\frac{P(D)}{P(T)}$ but we don't know $P(T)$, so use law of total probability.
  $P(T) = P(T|D)P(D) + P(T|D^c)P(D^c) == P(T) = 0.95(0.0001) + 0.05(1 - \frac{1}{10^4})$, $P(T) = 0.0019$

5 Week 5

- A factory makes coins of which 10% are defetive. They have a $\frac{3}{4}$ probability of falling heads. If you toss a coin three times and you see three heads, what is the probability that it is defective? Hint: Draw tree or use Bayes Rule.
  - D - The event that a coin is defective.
  - X - The event that we see three heads.
  - Find $P(D|X), = P(X|D)\frac{P(D)}{P(X)} = \frac{P(X \cap D)}{P(D)}\frac{P(D)}{P(X)} = \frac{P(X \cap D)}{P(X)}$
  - What is $P(X|D)$? »$(\frac{3}{4})^3 = \frac{27}{64}$
  - $P(X|D^c) = (\frac{1}{2})^2$
  - $P(D|X) = \frac{\frac{27}{63} \times 0.1}{\frac{27}{63} \times 0.1 + 0.9 \times \frac{1}{8}}$
- A random variable is a function that maps every outcome in a sample space to a real number. Formally, given ω, a random variable X is a mapping from ω to $\mathbb{R}$
- Probability mass function (pmf)
  - The "Mass function" of a random variable X is a function f such that $f(z)$ gives the probability that the variable X takes the value z.
  - Ex 1: X - outcome of throwing a dice $(1, 2, 3, 4, 5, 6)$
  - Ex 2: X - outcome of tossing a coin 10 times and being heads. What is $P(X) = 0$? $\frac{1}{2^{10}}$
    $= \frac{C_4^{10}}{2^{10}}$, $C = \frac{m!}{k!(n-k)!}$