

Constant Acceleration

$$d = d_0 + v_0 t + \frac{1}{2} a t^2$$

$$v = v_0 + a t$$

$$v^2 = v_0^2 + 2a(\Delta x)$$

Projectile Motion

$$v_x(t) = v_{x0} = v_0 \cos \theta_0$$

$$x(t) = x_0 + v_0 \cos(\theta_i) t$$

$$v_y(t) = v_0 \sin \theta_i - g t$$

$$y(t) = y_0 + v_0 \sin(\theta_i) t + \frac{1}{2} a t^2$$

$$v = v_{0y} - g t$$

$$v^2 = v_y^2 - 2 a d$$

Vectors with angles

$$A_x = A \cos \theta$$

$$A_y = A \sin \theta$$

$$\tan \theta = \frac{A_y}{A_x}$$

$$\theta = \arctan \frac{A_y}{A_x}$$

Angular Velocity

$$s = r \theta$$

$$v = r \omega$$

$$v = \frac{2\pi r}{T}$$

$$\omega = \frac{v}{r}$$

$$\omega = \frac{d\theta}{dt} = \frac{2\pi}{T}$$

$$T = \frac{2\pi r}{v}$$

Angular Acc.

$$r = \text{radius}$$

$$\alpha = \text{acceleration } \left(\frac{\text{rad}}{\text{s}^2}\right)$$

$$\omega = \text{velocity } \left(\frac{\text{rad}}{\text{s}}\right)$$

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

$$s = \text{arc length}$$

$$s = s_0 + v_0 t + \frac{1}{2} a t^2$$

$$s = r \Delta \theta$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\Delta \theta = \frac{w_f^2 - w_0^2}{2\alpha}$$

$$v_f = r \omega$$

$$v_f^2 = v_0^2 + 2a(\Delta s)$$

$$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta \theta)$$

$$\omega_f = \omega_0 + \alpha \Delta t$$

$$a_c = \frac{v_f^2}{r} \text{ (centripetal)}$$

$$a_r = \omega^2 r$$

$$a_t = r \alpha \text{ (tangential)}$$

$$v_{ang} = \frac{r}{T}$$

$$a_{ang} = \frac{v_{ang}}{T}$$

$$a_{total} = \sqrt{a_t^2 + a_c^2}$$

$$\sum F_r = m a_c = \frac{1}{2} m v^2$$

Work/Energy

$$K = \text{Kinetic Energy}$$

$$U = \text{Potential Energy}$$

$$\text{Master equation}$$

$$\Delta E^m = -f_k d$$

$$K(U) = \frac{1}{2} m v^2$$

$$PE = m g d$$

$$W = PE + KE$$

$$\text{Conservation of Energy}$$

$$K_1 + U_1 = K_2 + U_2$$

$$U_{sp} = \frac{1}{2} k x^2$$

Momentum

$$p = \text{Momentum}$$

$$J = \text{Joules}$$

$$p = m v$$

$$F_{net} = \frac{dp}{dt}, m \frac{v}{dt}$$

$$F_{avg} = \frac{\Delta p}{\Delta t}$$

$$\text{Impulse} = \int F dt = J$$

Collisions/Explosions

General Eq. (for inelastic, share common final velocity.):

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

- During inelastic collision, some KE goes to thermal.
- If KE is conserved, called perfectly elastic.
- Momentum is always conserved

KE in elastic collision (general, 2D):

$$m_1 \frac{v_{1i}^2}{2} + m_2 \frac{v_{2i}^2}{2} = m_1 \frac{v_{1f}^2}{2} + m_2 \frac{v_{2f}^2}{2}$$

KE in 1D: $v_{1i} - v_{2i} = v_{2f} - v_{1f}$

If needed to split to x/y components

$$\theta = \tan^{-1} \left(\frac{v_y}{v_x} \right)$$

$$v_f = \sqrt{v_y^2 + v_x^2}$$

Friction

$$f_k = \mu_k m g$$

$$\sum F_x = m a$$

$$\sum F_y = n - m g = 0$$

$$\vec{F}_{net} = \sum \vec{F}_x + \sum \vec{F}_y$$

$$a = \frac{f_{net}}{m}$$

$$\mu m g = m a$$

Newton's Laws

$$F = m a$$

$$\sum F = T - m g = m a$$

$$\sum F = m a + m g$$

$$\sum F = \frac{T - f_k}{m}$$

$$\sum F_{m_a + m_b} = (m_a + m_b) a$$

Center of Mass

$$x_{cm} = \frac{m_1x_1...(+m_2x_2)}{m_1...(+m_2)}$$
$$y_{cm} = \frac{m_1y_1...(+m_2y_2)}{m_1...(+m_2)}$$
$$r_{cm} = \frac{1}{M} \sum m_ix_i$$
$$K_{rot} = \frac{\omega^2}{2} \sum m_ir_i^2$$

Rotation of a Rigid Body

$$K_{sys} = \frac{1}{2}L\omega^2$$
$$I = \sum m_ir_i^2, I = \int r^2dm$$
$$\frac{1}{2}I\omega_i^2 + Mg \cdot y_{cm,i} = \frac{1}{2}I\omega_f^2 + Mg \cdot y_{cm,f}$$

Parallel-Axis Theorem

$$I_{cm} = \frac{1}{12}ML^2 \text{ (if at center.)}$$
$$I_A = I_{cm} + Md^2 \equiv \frac{1}{3}ML^2$$
$$I_{cyl} = \frac{1}{2} \cdot MR^2$$

Torque

$$t = rF \sin \varphi$$
$$t = mgx$$

(-,cw : +,ccw)

$$d = r \sin \varphi$$
$$\theta = \varphi - 90$$
$$t_g = Mgd \equiv t_g = Mgx_{cm}$$
$$t_{net} = I\alpha \equiv (\sum m_ir_i^2)\alpha$$
$$\alpha = \frac{t}{I}$$

Misc. Equations

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$rpm \rightarrow rad = \frac{rev}{min} \times \frac{2\pi rad \cdot min}{rev \cdot 60s}$$
$$rad \rightarrow rev = \frac{rad}{2\pi}$$

Rolling Motion

$$x_{cm} = R\theta, v_{cm} = R\omega, a_{cm} = R\alpha$$
$$K_{total} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$$

Angular Momentum

$$t = r \times F$$
$$[a_2 \cdot b_3 - a_3 \cdot b_2]i$$
$$[a_3 \cdot b_1 - a_1 \cdot b_3]j$$
$$[a_1 \cdot b_2 - a_2 \cdot b_1]k$$

Simple Harmonic Motion

w = angular freq., A = amplitude, equilibrium = x(0).

$$x(t) = A \cos \omega t$$
$$\omega = 2\pi f$$
$$\phi = \omega t$$
$$T = \frac{2\pi}{\omega}$$