Constant Acceleration

$d = d_0 + v_0 t + \frac{1}{2}at^2$

$v = v_0 + at$

$v^2 = v_0^2 + 2a(\Delta x)$

Projectile Motion

$v_x(t) = v_{x0} = v_0 \cos\theta_0$

$x(t) = x_0 + v_0 \cos(\theta_i)t$

$v_y(t) = v_0 \sin\theta_i - gt$

$y(t) = y_0 + v_0 \sin(\theta_i)t + \frac{1}{2}at^2$

$v = v_{0y} - gt$

$v^2 = v_y^2 - 2ad$

Vectors with angles

$A_x = A\cos\theta$

$A_y = A\sin\theta$

$\tan\theta = \frac{A_y}{A_x}$

$\theta = \arctan\frac{A_y}{A_x}$

Angular Velocity

$s = r\theta$

$v = r\omega$

$v = \frac{2\pi r}{T}$

$\omega = \frac{v}{r}$

$\omega = \frac{d\theta}{dt} = \frac{2\pi}{T}$

$T = \frac{2\pi r}{v}$

Angular Acc.

$r =$ radius

$\alpha =$ acceleration $(\frac{rad}{s^2})$

$\omega =$ velocity $(\frac{rad}{s})$

$\alpha = \frac{\Delta\omega}{\Delta t}$

$s =$ arc length

$s = s_0 + v_0 t + \frac{1}{2}at^2$

$s = r\Delta\theta$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\Delta\theta = \frac{w_f^2 - w_0^2}{2\alpha}$

$v_f = r\omega$

$v_f^2 = v_0^2 + 2a(\Delta s)$

$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$

$\omega_f = \omega_0 + \alpha\Delta t$

$a_c = \frac{v_f^2}{r}$ (centripetal)

$a_r = \omega^2 r$

$a_t = r\alpha$ (tangential)

$v_{ang} = \frac{r}{T}$

$a_{ang} = \frac{v_{ang}}{T}$

$a_{total} = \sqrt{a_t^2 + a_c^2}$

$\sum F_r = ma_c = \frac{1}{2}mv^2$

Work/Energy

$K =$ Kinetic Energy

$U =$ Potential Energy

Master equation

$\Delta E^m = -f_k d$

$K(U) = \frac{1}{2}mv^2$

$PE = mgd$

$W = PE + KE$

Conservation of Energy

$K_1 + U_1 = K_2 + U_2$

$U_{sp} = \frac{1}{2}kx^2$

Momentum

$p =$ Momentum

$J =$ Joules

$p = mv$

$F_{net} = \frac{dp}{dt}, m\frac{v}{dt}$

$F_{avg} = \frac{\Delta p}{\Delta t}$

Impulse $= \int F dt = J$

Collisions/Explosions

General Eq. (for inelastic, share common final velocity.):

$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

- During inelastic collision, some KE goes to thermal.
- If KE is conserved, called perfectly elastic.
- Momentum is always conserved

KE in elastic collision (general, 2D):

$m_1 \frac{v_{1i}^2}{2} + m_2 \frac{v_{2i}^2}{2} = m_1 \frac{v_{1f}^2}{2} + m_2 \frac{v_{2f}^2}{2}$

KE in 1D: $v_{1i} - v_{2i} = v_{2f} - v_{1f}$

If needed to split to x/y components

$\theta = \tan^{-1}(\frac{v_y}{v_x})$

$v_f = \sqrt{v_y^2 + v_x^2}$

Friction

$f_k = \mu_k mg$

$\sum F_x = ma$

$\sum F_y = n - mg = 0$

$\overrightarrow{F_{net}} = \sum \overrightarrow{F_x} + \sum \overrightarrow{F_y}$

$a = \frac{f_{net}}{m}$

$\mu mg = ma$

Newton's Laws

$F = ma$

$\sum F = T - mg = ma$

$\sum F = ma + mg$

$\sum F = \frac{T - f_k}{m}$

$\sum F_{m_a + m_b} = (m_a + m_b)a$

Center of Mass

$x_{cm} = \frac{m_1 x_1 ...(+m_2 x_2)}{m_1 ...(+m_2)}$

$y_{cm} = \frac{m_1 y_1 ...(+m_2 y_2)}{m_1 ...(+m_2)}$

$r_{cm} = \frac{1}{M} \sum m_i x_i$

$K_{rot} = \frac{\omega^2}{2} \sum m_i r_i^2$

Rotation of a Rigid Body

$K_{sys} = \frac{1}{2} L\omega^2$

$I = \sum m_i r_i^2, I = \int r^2 dm$

$\frac{1}{2} I\omega_i^2 + Mg \cdot y_{cm,i} = \frac{1}{2} I\omega_f^2 + Mg \cdot y_{cm,f}$

Parallel-Axis Theorem

$I_{cm} = \frac{1}{12} ML^2$ (if at center.)

$I_A = I_{cm} + Md^2 \equiv \frac{1}{3} ML^2$

$I_{cyl} = \frac{1}{2} \cdot MR^2$

Torque

$\tau = rF \sin\varphi$

$\tau = mgx$

(-,cw : +,ccw)

$d = r \sin\varphi$

$\theta = \varphi - 90$

$\tau_g = Mgd \equiv \tau_g = Mgx_{cm}$

$\tau_{net} = I\alpha \equiv (\sum m_i r_i^2)\alpha$

$\alpha = \frac{\tau}{I}$

Misc. Equations

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$rpm -> rad = \frac{rev}{min} \times \frac{2\pi rad \cdot min}{rev \cdot 60s}$

$rad -> rev = \frac{rad}{2\pi}$

Rolling Motion

$x_{cm} = R\theta, v_{cm} = R\omega, a_{cm} = R\alpha$

$K_{total} = \frac{1}{2} mv_{cm}^2 + \frac{1}{2} I\omega^2$

Simple Harmonic Motion

ω = angular freq.

A = amplitude,

equilibrium = x(0)

f = Frequency

T = period

ϕ_0 = Phase shift

$x(t) = A\cos(\omega t + \phi_0)$

$\omega = 2\pi f$

$\omega = \sqrt{\frac{k}{m}} \phi = \omega t$

$T = \frac{2\pi}{\omega}$

$T = 2\pi \sqrt{\frac{m}{k}}$

$T = \frac{1}{f}$

$E = \frac{1}{2} mv^2 + \frac{1}{2} kx^2 = \frac{1}{2} kA^2$

$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$

$f = \frac{1}{T}$

$x_i = A\cos\phi_0$

$v_i = -A\omega \sin\phi_0 v_{max} = \omega A$

$a_{max} = \omega^2 A$