

Constant Acceleration

$$d = d_0 + v_0 t + \frac{1}{2} a t^2$$

$$v = v_0 + a t$$

$$v^2 = v_0^2 + 2a(\Delta x)$$

Projectile Motion

$$v_x(t) = v_{x0} = v_0 \cos \theta_0$$

$$x(t) = x_0 + v_0 \cos(\theta_i) t$$

$$v_y(t) = v_0 \sin \theta_i - g t$$

$$y(t) = y_0 + v_0 \sin(\theta_i) t + \frac{1}{2} a t^2$$

$$v = v_{0y} - g t$$

$$v^2 = v_y^2 - 2 a d$$

Vectors with angles

$$A_x = A \cos \theta$$

$$A_y = A \sin \theta$$

$$\tan \theta = \frac{A_y}{A_x}$$

$$\theta = \arctan \frac{A_y}{A_x}$$

Angular Velocity

$$s = r \theta$$

$$v = r \omega$$

$$v = \frac{2\pi r}{T}$$

$$\omega = \frac{v}{r}$$

$$\omega = \frac{d\theta}{dt} = \frac{2\pi}{T}$$

$$T = \frac{2\pi r}{v}$$

Angular Acc.

$$r = \text{radius}$$

$$\alpha = \text{acceleration } \left(\frac{\text{rad}}{\text{s}^2}\right)$$

$$\omega = \text{velocity } \left(\frac{\text{rad}}{\text{s}}\right)$$

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

$$s = \text{arc length}$$

$$s = s_0 + v_0 t + \frac{1}{2} a t^2$$

$$s = r \Delta \theta$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\Delta \theta = \frac{w_f^2 - w_0^2}{2\alpha}$$

$$v_f = r \omega$$

$$v_f^2 = v_0^2 + 2a(\Delta s)$$

$$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta \theta)$$

$$\omega_f = \omega_0 + \alpha \Delta t$$

$$a_c = \frac{v_f^2}{r} \text{ (centripetal)}$$

$$a_r = \omega^2 r$$

$$a_t = r \alpha \text{ (tangential)}$$

$$v_{ang} = \frac{r}{T}$$

$$a_{ang} = \frac{v_{ang}}{T}$$

$$a_{total} = \sqrt{a_t^2 + a_c^2}$$

$$\sum F_r = m a_c = \frac{1}{2} m v^2$$

Work/Energy

$$K = \text{Kinetic Energy}$$

$$U = \text{Potential Energy}$$

$$\text{Master equation}$$

$$\Delta E^m = -f_k d$$

$$K(U) = \frac{1}{2} m v^2$$

$$PE = m g d$$

$$W = PE + KE$$

$$\text{Conservation of Energy}$$

$$K_1 + U_1 = K_2 + U_2$$

$$U_{sp} = \frac{1}{2} k x^2$$

Momentum

$$p = \text{Momentum}$$

$$J = \text{Joules}$$

$$p = m v$$

$$F_{net} = \frac{dp}{dt}, m \frac{v}{dt}$$

$$F_{avg} = \frac{\Delta p}{\Delta t}$$

$$\text{Impulse} = \int F dt = J$$

Collisions/Explosions

General Eq. (for inelastic, share common final velocity.):

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

- During inelastic collision, some KE goes to thermal.
- If KE is conserved, called perfectly elastic.
- Momentum is always conserved

KE in elastic collision (general, 2D):

$$m_1 \frac{v_{1i}^2}{2} + m_2 \frac{v_{2i}^2}{2} = m_1 \frac{v_{1f}^2}{2} + m_2 \frac{v_{2f}^2}{2}$$

KE in 1D: $v_{1i} - v_{2i} = v_{2f} - v_{1f}$

If needed to split to x/y components

$$\theta = \tan^{-1} \left(\frac{v_y}{v_x} \right)$$

$$v_f = \sqrt{v_y^2 + v_x^2}$$

Friction

$$f_k = \mu_k m g$$

$$\sum F_x = m a$$

$$\sum F_y = n - m g = 0$$

$$\vec{F}_{net} = \sum \vec{F}_x + \sum \vec{F}_y$$

$$a = \frac{f_{net}}{m}$$

$$\mu m g = m a$$

Newton's Laws

$$F = m a$$

$$\sum F = T - m g = m a$$

$$\sum F = m a + m g$$

$$\sum F = \frac{T - f_k}{m}$$

$$\sum F_{m_a + m_b} = (m_a + m_b) a$$

Center of Mass

$$x_{cm} = \frac{m_1 x_1 \dots (+m_2 x_2)}{m_1 \dots (+m_2)}$$

$$y_{cm} = \frac{m_1 y_1 \dots (+m_2 y_2)}{m_1 \dots (+m_2)}$$

$$r_{cm} = \frac{1}{M} \sum m_i x_i$$

$$K_{rot} = \frac{\omega^2}{2} \sum m_i r_i^2$$

Rotation of a Rigid Body

$$K_{sys} = \frac{1}{2} L \omega^2$$

$$I = \sum m_i r_i^2, I = \int r^2 dm$$

$$\frac{1}{2} I \omega_i^2 + M g \cdot y_{cm,i} = \frac{1}{2} I \omega_f^2 + M g \cdot y_{cm,f}$$

Parallel-Axis Theorem

$$I_{cm} = \frac{1}{12} M L^2 \text{ (if at center.)}$$

$$I_A = I_{cm} + M d^2 \equiv \frac{1}{3} M L^2$$

$$I_{cyl} = \frac{1}{2} \cdot M R^2$$

Torque

$$\tau = r F \sin \varphi$$

$$\tau = m g x$$

$$(-, \text{cw} : +, \text{ccw})$$

$$d = r \sin \varphi$$

$$\theta = \varphi - 90$$

$$\tau_g = M g d \equiv \tau_g = M g x_{cm}$$

$$\tau_{net} = I \alpha \equiv (\sum m_i r_i^2) \alpha$$

$$\alpha = \frac{\tau}{I}$$

Misc. Equations

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$rpm \rightarrow rad = \frac{rev}{min} \times \frac{2\pi rad \cdot min}{rev \cdot 60s}$$

$$rad \rightarrow rev = \frac{rad}{2\pi}$$

Rolling Motion

$$x_{cm} = R\theta, v_{cm} = R\omega, a_{cm} = R\alpha$$

$$K_{total} = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2$$

Simple Harmonic Motion

ω = angular freq.

A = amplitude,

equilibrium = $x(0)$

f = Frequency

T = Period

\tilde{v} = Wave number ($\frac{rad}{m}$)

ϕ_0 = Phase shift

λ = Wave length

$$x(t) = A \cos(\omega t + \phi_0 \tilde{v})$$

$$v(t) = -A\omega \sin(\omega t + \phi_0 \tilde{v})$$

$$a(t) = -A\omega^2 \cos(\omega t + \phi_0 \tilde{v})$$

$$\omega = 2\pi f$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\omega = \sqrt{\frac{g}{A}}$$

$$\phi = \omega t$$

$$T = \frac{2\pi}{\omega}$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$T = \frac{1}{f}$$

$$\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

$$E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$\tilde{v} = \frac{1}{\omega} = \frac{2\pi}{\lambda}$$

$$v = \lambda f = \frac{\lambda \omega}{2\pi} = \frac{\lambda}{T}$$

$$x_i = A \cos \phi_0$$

$$v_i = -A\omega \sin \phi_0$$

$$v_{max} = \omega A$$

$$a_{max} = \omega^2 A$$