Calculus III - MATH 2210 SP2021

Week 1

Equation for a sphere: $(x-h)^2+(y-j)^2+(z-k)^2=r^2$

Equation of a plane: $a(x-x_p)+b(y-y_p)+c(z-z_p)=d$

Midpoint: $m_x=\frac{x_1-x_2}{2}$

Magnitude: $|u|=\sqrt{u_1^2+u_2^2+u_3^2}$

Projection of U onto V:

$pr_v u=(\frac{u\cdot v}{||v||})$

Sphere eq.: $(x-j)^2+(y-k)^2+(z-l)^2=r^2$

$r=\sqrt{m_1^2+m_2^2+m_3^2}$

$u\times v=< u_2v_3-u_3v_2, u_3v_1-u_1v_3, u_1v_2-u_2v_1 >$

$||u\times v||=||u||||v||\sin\theta$

$a\cdot b=||a||||b||\cos\theta$

The equation for a plane with normal vector <a,b,c> is: $ax+by+cz=d$ - parallel planes have same normal vectors.

Find equation of plane containing three points P, Q, R

$\overrightarrow{\mathrm{PQ}}=\overrightarrow{\mathrm{P}}-\overrightarrow{\mathrm{Q}}$, $\overrightarrow{\mathrm{PR}}=\overrightarrow{\mathrm{P}}-\overrightarrow{\mathrm{R}}$, $< a,b,c >=\overrightarrow{\mathrm{PQ}}\times\overrightarrow{\mathrm{PR}}$

Week 2

$a_t=T\cdot a$

$a_n=\sqrt{||r''(t)||^2-a_t^2}$

$A_t=a_tT(t)+a_nN(t)$

$T(t)=\frac{1}{||r'(t)||}\cdot r'(t)$

$N(t)=\frac{1}{||T'(t)||}\cdot T'(t)$

$K(t)=\frac{||r'(t)\times r''(t)||}{||r'(t)||^3}$

$B(t)=T(t)\times N(t)$

Week 3

Cartesian »Cylindrical

$r=\sqrt{x^2+y^2}$

$\theta=\arctan(\frac{y}{x})$

$z=z$

Cartesian »Spherical

$r=\sqrt{x^2+y^2+z^2}$

$\theta=\arctan(\frac{y}{x})$

$\phi=\arccos(\frac{z}{r})$

Cylindrical »Cartesian

$x=r\cos\theta$

$y=r\sin\theta$

$z=z$

Cylindrical »Spherical

$r=\sqrt{\rho^2+z^2}$

$\phi=\arctan(\frac{\rho}{z})$

$\theta=\theta$

Spherical »Cylindrical

$\rho=r\sin\theta$

$\theta=\theta$

$z=\cos\theta$

Spherical »Cartesian

$x=r\sin\phi\cos\theta$

$y=r\sin\phi\sin\theta$

$z=r\cos\phi$

Equations

Ellipsoid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Elliptic Paraboloid: $z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

Hyperbolic Paraboloid: $z=\frac{x^2}{a^2}-\frac{y^2}{b^2}$

Hyperboloid of One Sheet: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

Hyperboloid of Two Sheets: $\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

Elliptic Cone: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}$

Circles: $(x-h)^2+(y-k)^2=r^2$

Cylindrical coordinate system: (ρ,ϕ,z)

Spherical coordinate system: (r,θ,ϕ)

Ellipse: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Hyperbola: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Parabola: $y=ax^2+bx+c$

Examples

Let L be determined by the equations $y=2$ and $x=6z$.

If we rotate around the X axis, we get an equation

$Ax^2+By^2+Cz^2=1$, find A, B, and C.

$y^2+z^2=2^2$ »$\frac{1}{4}y^2+\frac{1}{4}z^2=1(B,C)$

Find a second point, this case it will be $< 6,2,1 >$

$A(6)^2+\frac{1}{4}(2)^2+\frac{1}{4}(1)^2=1$ »$A(6)^2+\frac{1}{4}(1)^2=0$

$A36=-\frac{1}{4}$ »$A=-\frac{1}{4*36}$

Find an equation of the ellipsoid passing through the points

$(\pm3,0,0),(0,\pm1,0),(0,0,\pm6)$

Use formula of ellipsoid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}$, with $a=\pm3, b=\pm1, c=\pm6$

A gun has a muzzle speed of 90 meters per second. What angle of elevation θ should be used to hit an object 170 meters away? use $g = 9.8\frac{m}{s^2}$

$u = 90\frac{m}{s},\ s = 170m,\ g = 9.8\frac{m}{s^2},\ u_x = \cos\theta,\ u_y = \sin\theta$

$s = u\cos\theta t$ »$170 = 90\cos\theta t$ »$t = \frac{17}{9\cos\theta}$

$s_y = u_y + \frac{1}{2}at^2$ »$0 = 90\sin\theta\frac{17}{9\cos\theta} + 4.9(\frac{17}{9\cos\theta})^2$

$170\tan\theta = \frac{4.9\cdot289}{81}\cdot\frac{1}{\cos^2\theta}$ »$170\frac{\sin}{\cos}\theta = \frac{4.9\cdot289}{81}$

$\frac{170}{85}\sin\theta\cos\theta = \frac{4.9\cdot289}{81\cdot85}$ »$\sin2\theta = \frac{4.9\cdot289}{81\cdot85}$

$2\theta = \arcsin(\frac{4.9\cdot289}{81\cdot85})$