

Calculus III - MATH 2210 SP2021

Week 1

Equation for a sphere: $(x-h)^2 + (y-j)^2 + (z-k)^2 = r^2$

Equation of a plane: $a(x-x_p) + b(y-y_p) + c(z-z_p) = d$

Midpoint: $m_x = \frac{x_1+x_2}{2}$

Magnitude: $|u| = \sqrt{u_1^2 + u_2^2 + u_3^2}$

Projection of U onto V:

$$pr_v u = \left(\frac{u \cdot v}{||v||} \right)$$

Sphere eq.: $(x-j)^2 + (y-k)^2 + (z-l)^2 = r^2$

$$r = \sqrt{m_1^2 + m_2^2 + m_3^2}$$

$$u \times v = \langle u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1 \rangle$$

$$||u \times v|| = ||u|| ||v|| \sin \theta$$

$$a \cdot b = ||a|| ||b|| \cos \theta$$

The equation for a plane with normal vector $\langle a, b, c \rangle$ is:

$ax + by + cz = d$ - parallel planes have same normal vectors.

Find equation of plane containing three points P, Q, R

$$\overrightarrow{PQ} = \overrightarrow{P} - \overrightarrow{Q}, \overrightarrow{PR} = \overrightarrow{P} - \overrightarrow{R}, \langle a, b, c \rangle = \overrightarrow{PQ} \times \overrightarrow{PR}$$

Week 2

$$a_t = T \cdot a$$

$$a_n = \sqrt{||r''(t)||^2 - a_t^2}$$

$$A_t = a_t T(t) + a_n N(t)$$

$$T(t) = \frac{1}{||r'(t)||} \cdot r'(t)$$

$$N(t) = \frac{1}{||T'(t)||} \cdot T'(t)$$

$$K(t) = \frac{||r'(t) \times r''(t)||}{||r'(t)||^3}$$

$$B(t) = T(t) \times N(t)$$

Week 3

Cartesian » Cylindrical

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \arctan\left(\frac{y}{x}\right)$$

$$z = z$$

Cartesian » Spherical

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \arctan\left(\frac{y}{x}\right)$$

$$\phi = \arccos\left(\frac{z}{r}\right)$$

Cylindrical » Cartesian

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$z = z$$

Cylindrical » Spherical

$$r = \sqrt{\rho^2 + z^2}$$

$$\phi = \arctan\left(\frac{\rho}{z}\right)$$

$$\theta = \theta$$

Spherical » Cylindrical

$$\rho = r \sin \theta$$

$$\theta = \theta$$

$$z = r \cos \theta$$

Spherical » Cartesian

$$x = r \sin \phi \cos \theta$$

$$y = r \sin \phi \sin \theta$$

$$z = r \cos \phi$$

Equations

$$\text{Ellipsoid: } \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$\text{Elliptic Paraboloid: } z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$\text{Hyperbolic Paraboloid: } z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

$$\text{Hyperboloid of One Sheet: } \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\text{Hyperboloid of Two Sheets: } \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\text{Elliptic Cone: } \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

$$\text{Circles: } (x-h)^2 + (y-k)^2 = r^2$$

$$\text{Cylindrical coordinate system: } (\rho, \phi, z)$$

$$\text{Spherical coordinate system: } (r, \theta, \phi)$$

$$\text{Ellipse: } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{Hyperbola: } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\text{Parabola: } y = ax^2 + bx + c$$

Examples

Let L be determined by the equations $y = 2$ and $x = 6z$.

If we rotate around the X axis, we get an equation

$$Ax^2 + By^2 + Cz^2 = 1, \text{ find A, B, and C.}$$

$$y^2 + z^2 = 2^2 \gg \frac{1}{4}y^2 + \frac{1}{4}z^2 = 1(B, C)$$

Find a second point, this case it will be $\langle 6, 2, 1 \rangle$

$$A(6)^2 + \frac{1}{4}(2)^2 + \frac{1}{4}(1)^2 = 1 \gg A(6)^2 + \frac{1}{4}(1)^2 = 0$$

$$A36 = -\frac{1}{4} \gg A = -\frac{1}{4 \cdot 36}$$

Find an equation of the ellipsoid passing through the points

$$(\pm 3, 0, 0), (0, \pm 1, 0), (0, 0, \pm 6)$$

Use formula of ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, with $a = \pm 3, b = \pm 1, c = \pm 6$

A gun has a muzzle speed of 90 meters per second.
 What angle of elevation θ should be used to hit an object
 170 meters away? use $g = 9.8 \frac{m}{s^2}$

$$u = 90 \frac{m}{s}, s = 170m, g = 9.8 \frac{m}{s^2}, u_x = \cos \theta, u_y = \sin \theta$$

$$s = u \cos \theta t \gg 170 = 90 \cos \theta t \gg t = \frac{17}{9 \cos \theta}$$

$$s_y = u_y + \frac{1}{2}at^2 \gg 0 = 90 \sin \theta \frac{17}{9 \cos \theta} + 4.9 \left(\frac{17}{9 \cos \theta} \right)^2$$

$$170 \tan \theta = \frac{4.9 \cdot 289}{81} \cdot \frac{1}{\cos^2 \theta} \gg 170 \frac{\sin \theta}{\cos \theta} = \frac{4.9 \cdot 289}{81}$$

$$\frac{170}{85} \sin \theta \cos \theta = \frac{4.9 \cdot 289}{81 \cdot 85} \gg \sin 2\theta = \frac{4.9 \cdot 289}{81 \cdot 85}$$

$$2\theta = \arcsin\left(\frac{4.9 \cdot 289}{81 \cdot 85}\right)$$

$$\rho = 4 = \text{Sphere}$$

$$\theta = \frac{\pi}{3} = \text{Half plane}$$

$$r = 4 = \text{Circular cylinder}$$

$$\phi = \frac{\pi}{3} = \text{Cone}$$

$$r^2 + z^2 = 16 = \text{Sphere}$$

$$z = r^2 = \text{Elliptic or Circular Paraboloid}$$

$$r = 2 \cos \theta = \text{Circular Cylinder}$$

$$\rho = 2 \cos \phi = \text{Sphere}$$

$$\rho \cos \phi = 4 = \text{Plane}$$