

# Calculus III - MATH 2210 FA2021

## 1 Week 1

Equation for a sphere:  $(x - h)^2 + (y - j)^2 + (z - k)^2 = r^2$

Midpoint:  $m_x = \frac{x_1 + x_2}{2}$

Magnitude:  $|u| = \sqrt{u_1^2 + u_2^2 + u_3^2}$

Projection of U onto V:

$$pr_v u = \left( \frac{u \cdot v}{\|v\|} \right)$$

$$u \times v = \langle u_2 v_3 - u_3 v_2, u_3 v_1 - u_1 v_3, u_1 v_2 - u_2 v_1 \rangle$$

## 2 Week 2

$$a_t = T \cdot a$$

$$a_n = \sqrt{\|r''(t)\|^2 - a_t^2}$$

$$A_t = a_t T(t) + a_n N(t)$$

$$T(t) = \frac{1}{\|r'(t)\|} \cdot r'(t)$$

$$N(t) = \frac{1}{\|T'(t)\|} \cdot T'(t)$$

$$K(t) = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3}$$

$$B(t) = T(t) \times N(t)$$

## 3 Week 3

### 3.1 Cartesian » Cylindrical

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \arctan\left(\frac{y}{x}\right)$$

$$z = z$$

### 3.2 Cartesian » Spherical

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \arctan\left(\frac{y}{x}\right)$$

$$\varphi = \arccos\left(\frac{z}{r}\right)$$

### 3.3 Cylindrical » Cartesian

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$z = z$$

### 3.4 Spherical » Cylindrical

$$P = r \sin \theta$$

$$\theta = \theta$$

$$z = r \cos \theta$$

## Spherical » Cartesian

$$x = r \sin \phi \cos \theta$$

$$y = r \sin \phi \sin \theta$$

$$z = r \cos \phi$$

## Examples

### 0.1

Let L be determined by the equations  $y = 2$  and  $x = 6z$ . If we rotate around the X axis, we get an equation

$Ax^2 + By^2 + Cz^2 = 1$ , find A, B, and C.

$$y^2 + z^2 = 2^2$$

$$\frac{1}{4}y^2 + \frac{1}{4}z^2 = 1(B, C)$$

Find a second point, this case it will be  $\langle 6, 2, 1 \rangle$

$$A(6)^2 + \frac{1}{4}(2)^2 + \frac{1}{4}(1)^2 = 1$$

$$A(6)^2 + \frac{1}{4}(1)^2 = 0$$

$$A36 = -\frac{1}{4}$$

$$A = -\frac{1}{4 \cdot 36}$$

### 0.2

Find an equation of the ellipsoid passing through the points  $(\pm 3, 0, 0), (0, \pm 1, 0), (0, 0, \pm 6)$

Use formula of ellipsoid:  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$ , with  $a = \pm 3, b = \pm 1, c = \pm 6$