Calculus III - MATH 2210 SP2021

Week 1

Equation for a sphere: $(x-h)^2+(y-j)^2+(z-k)^2 = r^2$
Midpoint: $m_x = \frac{x_1-x_2}{2}$
Magnitude: $|u| = \sqrt{u_1^2+u_2^2+u_3^2}$
Projection of U onto V:
$pr_v u = (\frac{u \cdot v}{||v||})$ Sphere eq.: $(x-j)^2+(y-k)^2+(z-l)^2 = r^2$, $r = \frac{\sqrt{m_1^2+m_2^2+m_3^2}}{2}$
$u \times v =< u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1 >$
$||u \times v|| = ||u||||v|| \sin\theta$ $a \cdot b = ||a||||b|| \cos\theta$
The equation for a plane with normal vector <a,b,c> is: $ax + by + cz = d$ - parallel planes have same normal vectors.
Find equation of plane containing three points P, Q, R
$\overrightarrow{\mathrm{PQ}} = \overrightarrow{\mathrm{P}} - \overrightarrow{\mathrm{Q}}$, $\overrightarrow{\mathrm{PR}} = \overrightarrow{\mathrm{P}} - \overrightarrow{\mathrm{R}}$, $n = \overrightarrow{\mathrm{PQ}} \times \overrightarrow{\mathrm{PR}}$

Week 2

$a_t = T \cdot a$
$a_n = \sqrt{||r''(t)||^2 - a_t^2}$
$A_t = a_tT(t) + a_nN(t)$
$T(t) = \frac{1}{||r'(t)||} \cdot r'(t)$
$N(t) = \frac{1}{||T'(t)||} \cdot T'(t)$
$K(t) = \frac{||r'(t) \times r''(t)||}{||r'(t)||^3}$
$B(t) = T(t) \times N(t)$

Week 3

Cartesian »Cylindrical

$r = \sqrt{x^2+y^2}$
$\theta = \arctan(\frac{y}{x})$
$z = z$

Cartesian »Spherical

$r = \sqrt{x^2+y^2+z^2}$
$\theta = \arctan(\frac{y}{x})$
$\phi = \arccos(\frac{z}{r})$

Cylindrical »Cartesian

$x = r\cos\theta$
$y = r\sin\theta$
$z = z$

Cylindrical »Spherical

$r = \sqrt{\rho^2+z^2}$
$\phi = \arctan(\frac{\rho}{z})$
$\theta = \theta$

Spherical »Cylindrical

$\rho = r\sin\theta$
$\theta = \theta$
$z = \cos\theta$

Spherical »Cartesian

$x = r\sin\phi\cos\theta$
$y = r\sin\phi\sin\theta$
$z = r\cos\phi$

Equations

Ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
Elliptic Paraboloid: $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$
Hyperbolic Paraboloid: $z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$
Hyperboloid of One Sheet: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
Hyperboloid of Two Sheets: $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
Elliptic Cone: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$
Circles: $(x-h)^2 + (y-k)^2 = r^2$
Cylindrical coordinate system: (ρ, ϕ, z)
Spherical coordinate system: (r, θ, ϕ)

Examples

Let L be determined by the equations $y = 2$ and $x = 6z$. If we rotate around the X axis, we get an equation $Ax^2 + By^2 + Cz^2 = 1$, find A, B, and C.
$y^2 + z^2 = 2^2$
$\frac{1}{4}y^2 + \frac{1}{4}z^2 = 1(B, C)$
Find a second point, this case it will be $< 6, 2, 1 >$
$A(6)^2 + \frac{1}{4}(2)^2 + \frac{1}{4}(1)^2 = 1$
$A(6)^2 + \frac{1}{4}(1)^2 = 0$
$A36 = -\frac{1}{4}$
$A = -\frac{1}{4*36}$

Find an equation of the ellipsoid passing through the points
$(\pm 3, 0, 0), (0, \pm 1, 0), (0, 0, \pm 6)$
Use formula of ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$, with $a = \pm 3, b = \pm 1, c = \pm 6$