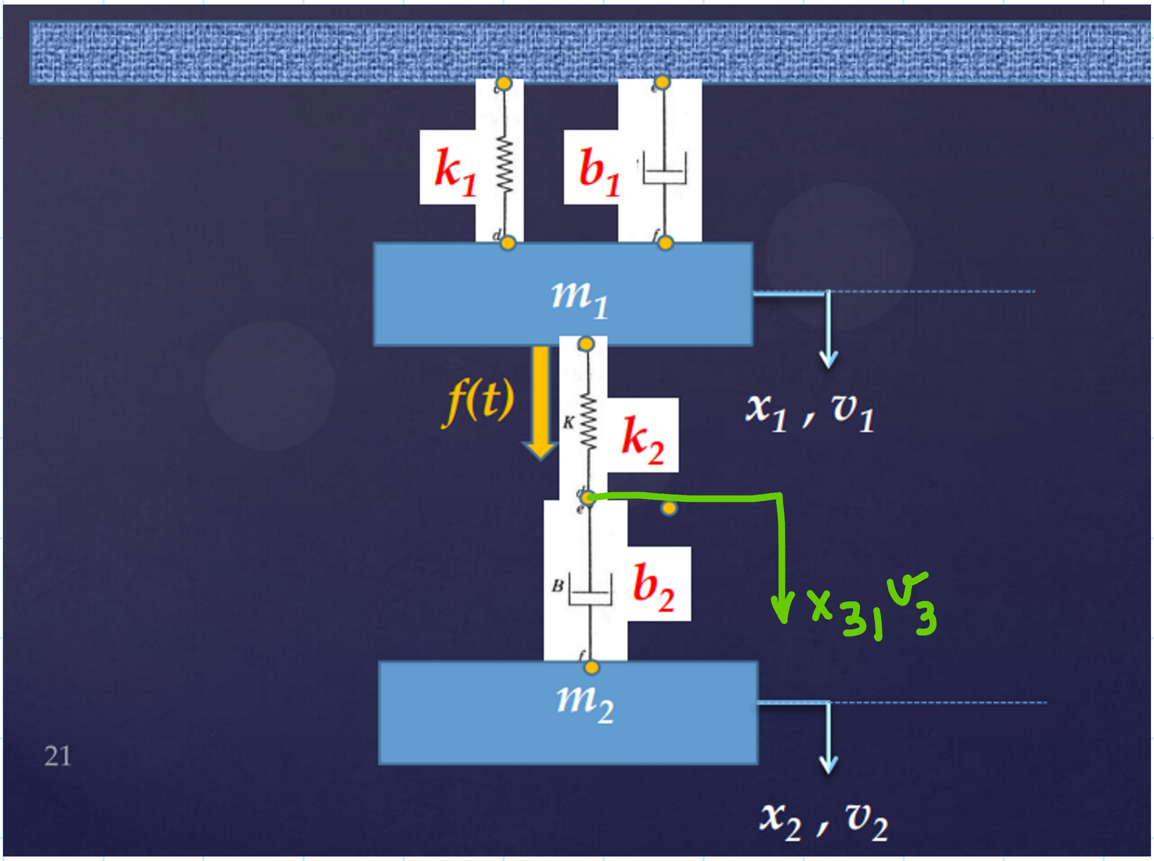
Exercício 6

quarta-feira, 22 de setembro de 2021 19:25

Ex.6



a) • Energia cinética: $T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2}$

• Energia potencial: $V = \frac{k_1 x_1^2}{2} + \frac{k_2 (x_3 - x_1)^2}{2}$

• Potencial de Rayleigh: $R = \frac{b_1 \dot{x}_1^2}{2} + \frac{b_2 (\dot{x}_3 - \dot{x}_2)^2}{2}$

• Lagrangeana: $L = T - V$

• Para x_1: $\frac{\partial L}{\partial \dot{x}_1} = m_1 \dot{x}_1 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1$

- Para x_1: $\rightarrow \dfrac{\partial L}{\partial \dot{x}_1} = m_1 \dot{x}_1 \Rightarrow \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1$

$$\rightarrow \frac{\partial L}{\partial x_1} = -k_1 x_1 + k_2 (x_3 - x_1)$$

$$\rightarrow \frac{\partial R}{\partial \dot{x}_1} = b_1 \dot{x}_1$$

$\rightarrow$ Eq. de Lagrange: $\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_1}\right) - \dfrac{\partial L}{\partial x_1} + \dfrac{\partial R}{\partial \dot{x}_1} = f(t) \Rightarrow$

$$\Rightarrow \boxed{m_1 \ddot{x}_1 + b_1 \dot{x}_1 + (k_1 + k_2) x_1 - k_2 x_3 = f(t)}$$

- Para x_2: $\rightarrow \dfrac{\partial L}{\partial \dot{x}_2} = m_2 \dot{x}_2 \Rightarrow \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2$

$$\rightarrow \frac{\partial L}{\partial x_2} = 0$$

$$\rightarrow \frac{\partial R}{\partial \dot{x}_2} = -b_2 (\dot{x}_3 - \dot{x}_2)$$

$\rightarrow$ Eq. de Lagrange: $\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_2}\right) - \dfrac{\partial L}{\partial x_2} + \dfrac{\partial R}{\partial \dot{x}_2} = 0 \Rightarrow$

$$\Rightarrow \boxed{m_2 \ddot{x}_2 + b_2 \dot{x}_2 - b_2 \dot{x}_3 = 0}$$

- Para x_3: $\rightarrow \dfrac{\partial L}{\partial \dot{x}_3} = 0 \Rightarrow \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_3}\right) = 0$

• Para x_3: → $\frac{\partial L}{\partial \dot{x}_3} = 0 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) = 0$

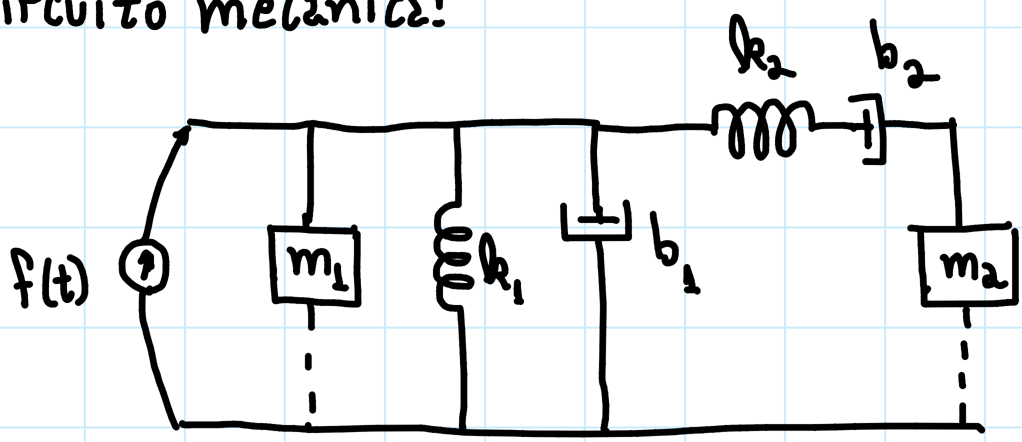
→ $\frac{\partial L}{\partial x_3} = -k_2(x_3 - x_1)$

→ $\frac{\partial R}{\partial \dot{x}_3} = b_2(\dot{x}_3 - \dot{x}_2)$

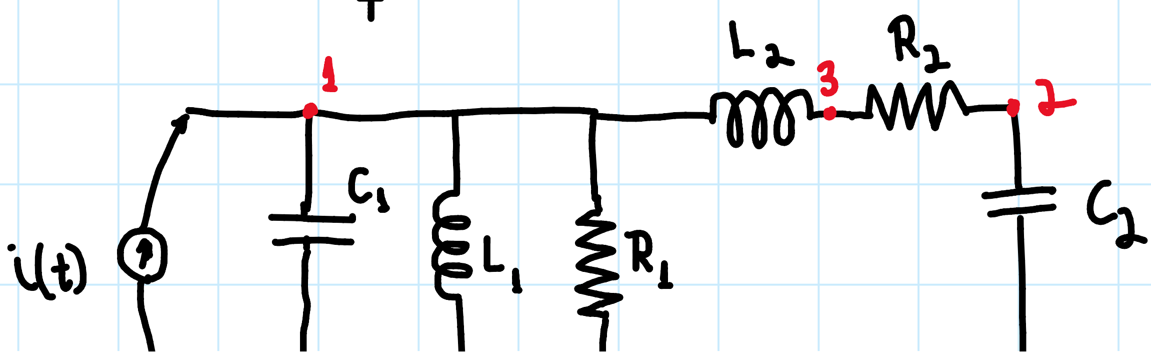
→ Eq. de Lagrange: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) - \frac{\partial L}{\partial x_3} + \frac{\partial R}{\partial \dot{x}_3} = 0 \Rightarrow$

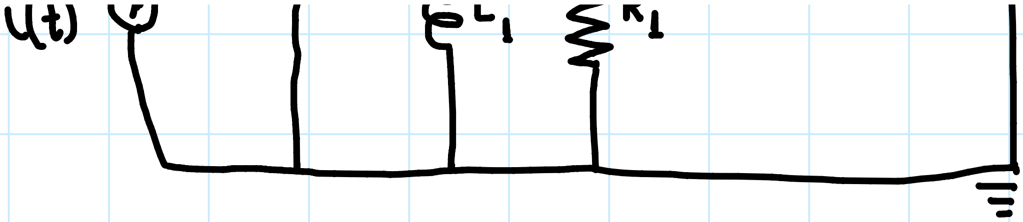
$$\Rightarrow \boxed{b_2\dot{x}_3 + k_2 x_3 - b_2\dot{x}_2 - k_2 x_1 = 0}$$

b) • Circuito mecânico:



c) • Circuito elétrico equivalente:





d) • Nó 1: $\boxed{V_1\left[C_1D+\frac{1}{R_1}+\frac{1}{L_1D}+\frac{1}{L_2D}\right]-V_3\frac{1}{L_2D}=i(t)}$ (I)

• Nó 2: $\boxed{V_2\left[C_2D+\frac{1}{R_2}\right]-V_3\frac{1}{R_2}=0}$ (II)

• Nó 3: $\boxed{V_3\left[\frac{1}{R_2}+\frac{1}{L_2D}\right]-V_1\frac{1}{L_2D}-V_2\frac{1}{R_2}=0}$ (III)

e) • Da analogia do tipo 2:

→ Eq. (I): $\dot{x}_1\left[m_1D+b_1+\frac{k_1}{D}+\frac{k_2}{D}\right]-\dot{x}_3\frac{k_2}{D}=f(t) \Rightarrow$

$\Rightarrow \boxed{m_1\ddot{x}_1+b_1\dot{x}_1+(k_1+k_2)x_1-k_2x_3=f(t)}$

→ Eq (II): $\dot{x}_2\left[m_2D+b_2\right]-\dot{x}_3b_2=0 \Rightarrow$

$\Rightarrow \boxed{m_2\ddot{x}_2+b_2\dot{x}_2-b_2\dot{x}_3=0}$

→ Eq. (III): $\dot{x}_3\left[b_2+\underline{k_2}\right]-\dot{x}_1\underline{k_2}-\dot{x}_2b_2=0 \Rightarrow$

$\rightarrow$ Eq. (III): $\dot{x}_3\left[b_2 + \frac{k_2}{D}\right] - \dot{x}_1 \frac{k_2}{D} - \dot{x}_2 b_2 = 0 \Rightarrow$

$$\Rightarrow \boxed{b_2\dot{x}_3 + k_2x_3 - b_2\dot{x}_2 - k_2x_1 = 0}$$