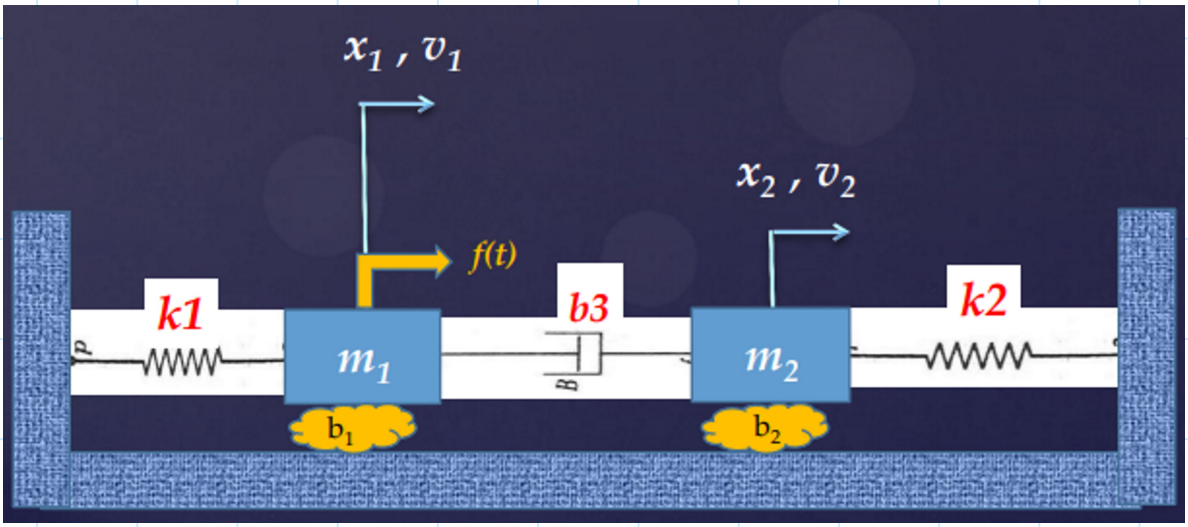
Exercício 5

quarta-feira, 22 de setembro de 2021 18:20

Ex. 5



a) • Energia cinética: $T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2}$

• Energia potencial: $V = \frac{k_1 x_1^2}{2} + \frac{k_2 x_2^2}{2}$

• Potencial de Rayleigh: $R = \frac{b_1 \dot{x}_1^2}{2} + \frac{b_2 \dot{x}_2^2}{2} + \frac{b_3 (\dot{x}_2 - \dot{x}_1)^2}{2}$

• Lagrangeano: $L = T - V$

• Para x_1: $\rightarrow \frac{\partial L}{\partial \dot{x}_1} = m_1 \dot{x}_1 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1$

$\rightarrow \frac{\partial L}{\partial x_1} = -k_1 x_1$

$\rightarrow \frac{\partial R}{\partial \dot{x}_1} = b_1 \dot{x}_1 - b_3 (\dot{x}_2 - \dot{x}_1)$

$$\frac{\partial R}{\partial \dot{x}_1} = b_1 \dot{x}_1 - b_3(\dot{x}_2 - \dot{x}_1)$$

• Eq. de Lagrange: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial x_1} + \frac{\partial R}{\partial \dot{x}_1} = f(t) \Rightarrow$

$$\Rightarrow \boxed{m_1 \ddot{x}_1 + (b_1 + b_3)\dot{x}_1 + k_1 x_1 - b_3 \dot{x}_2 = f(t)}$$

- Para x_2: → $\frac{\partial L}{\partial \dot{x}_2} = m_2 \dot{x}_2 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2$

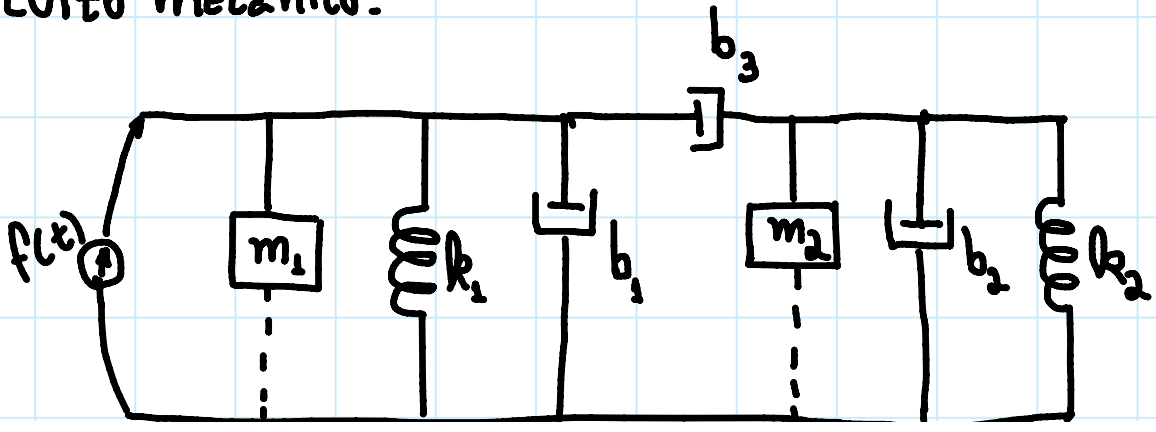
→ $\frac{\partial L}{\partial x_2} = -k_2 x_2$

→ $\frac{\partial R}{\partial \dot{x}_2} = b_2 \dot{x}_2 + b_3(\dot{x}_2 - \dot{x}_1)$

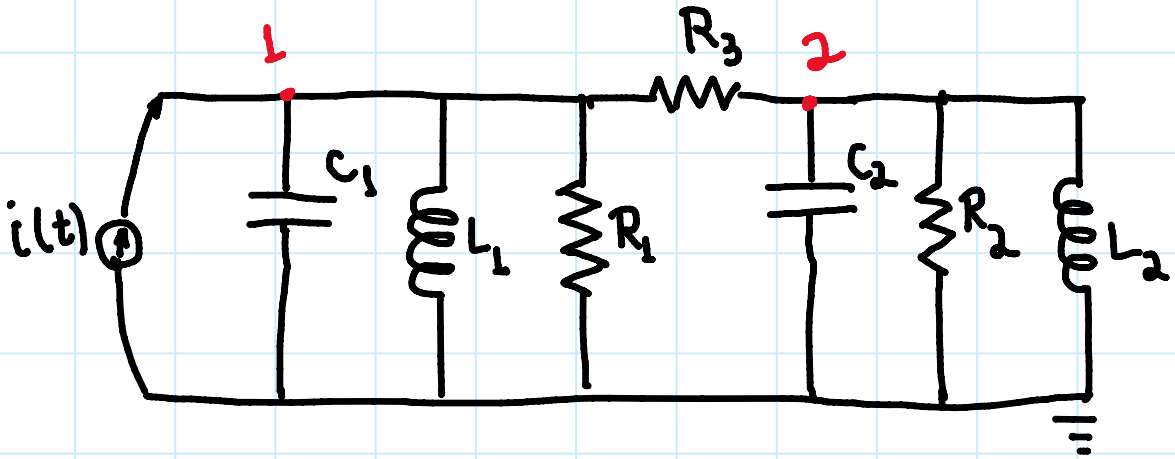
→ Eq. de Lagrange: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) - \frac{\partial L}{\partial x_2} + \frac{\partial R}{\partial \dot{x}_2} = 0 \Rightarrow$

$$\Rightarrow m_2 \ddot{x}_2 + (b_2 + b_3)\dot{x}_2 + k_2 x_2 - b_3 \dot{x}_1 = 0$$

b) • Circuito mecânico:



c) · Circuito elétrico equivalente:



d) · Nó 1: $$V_1\left[C_1D + \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{L_1D}\right] - V_2\frac{1}{R_3} = i(t) \quad (I)$$

· Nó 2: $$V_2\left[C_2D + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{L_2D}\right] - V_1\frac{1}{R_3} = 0 \quad (II)$$

e) · Da analogia do tipo 2:

→ Eq(I): $$\dot{x}_1\left[m_1D + b_1 + b_3 + \frac{k_1}{D}\right] - \dot{x}_2 b_3 = f(t) \Rightarrow$$

$$\Rightarrow \boxed{m\ddot{x}_1 + (b_1+b_2)\dot{x}_1 + kx_1 - b_3\dot{x}_2 = f(t)}$$

$$\rightarrow \text{Eq. (II)}: \dot{X}_2 \left[m_2 D + b_2 + b_3 + \frac{k_2}{D} \right] - \dot{x}_1 b_3 = 0 \Rightarrow$$

$$\Rightarrow \boxed{m_2 \ddot{x}_2 + (b_2 + b_3)\dot{x}_2 + k_2 x_2 - b_3 \dot{x}_1 = 0}$$