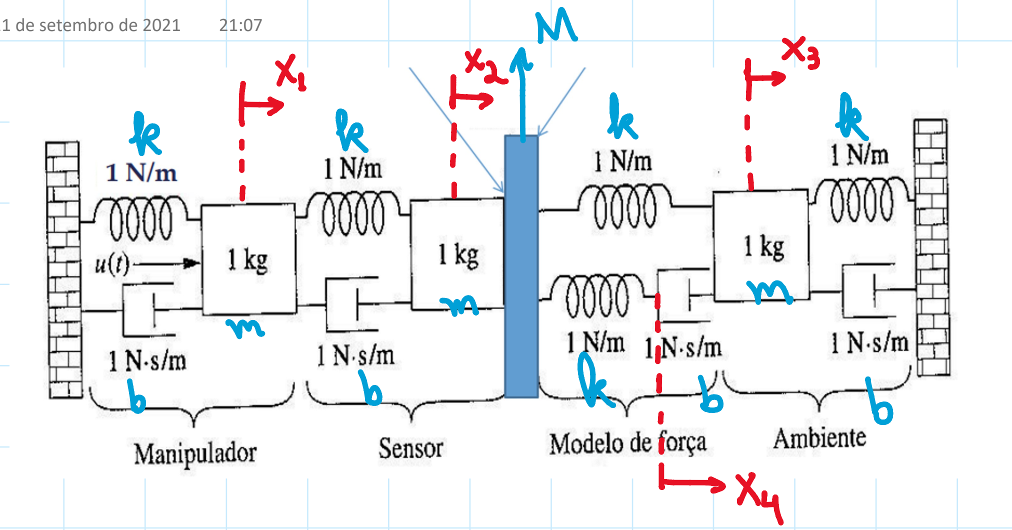
Exercício 3

terça-feira, 21 de setembro de 2021 21:07

Ex3|



a) • Energia cinética: $T = \frac{m\dot{x}_1^2}{2} + \frac{(m+M)\dot{x}_2^2}{2} + \frac{m\dot{x}_3^2}{2}$

• Energia potencial: $V = \frac{k}{2}.\left[x_1^2 + (x_2-x_1)^2 + (x_3-x_2)^2 + x_3^2 + (x_4-x_2)^2\right]$

• Potencial de Rayleigh: $R = \frac{b}{2}.\left[\dot{x}_1^2 + (\dot{x}_2-\dot{x}_1)^2 + \dot{x}_3^2 + (\dot{x}_3-\dot{x}_4)^2\right]$

• Lagrangeano: $L = T - V$

• Para x_1: → $\frac{\partial L}{\partial \dot{x}_1} = m\dot{x}_1 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m\ddot{x}_1$

→ $\frac{\partial L}{\partial x_1} = -k.\left[x_1 - (x_2 - x_1)\right]$

→ $\frac{\partial R}{\partial \dot{x}_1} = b\left[\dot{x}_1 - (\dot{x}_2 - \dot{x}_1)\right]$

→ Eq de Lagrange: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial x_1} + \frac{\partial R}{\partial \dot{x}_1} = u(t) \Rightarrow$

$\Rightarrow \boxed{m\ddot{x}_1 + 2b\dot{x}_1 + 2kx_1 - b\dot{x}_2 - kx_2 = u(t)}$

• Para x_2: $\rightarrow \frac{\partial L}{\partial \dot{x}_2} = (M+m)\dot{x}_2 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = (M+m)\ddot{x}_2$

$\rightarrow \frac{\partial L}{\partial x_2} = -k\cdot[(x_2-x_1)-(x_3-x_2)-(x_4-x_2)]$

$\rightarrow \frac{\partial R}{\partial \dot{x}_2} = b(\dot{x}_2-\dot{x}_1)$

$\rightarrow$ Eq. de Lagrange: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) - \frac{\partial L}{\partial x_2} + \frac{\partial R}{\partial \dot{x}_2} = 0 \Rightarrow$

$$\Rightarrow \boxed{(M+m)\ddot{x}_2 + b\dot{x}_2 + 3kx_2 - b\dot{x}_1 - k(x_1+x_3+x_4) = 0}$$

• Para x_3: $\rightarrow \frac{\partial L}{\partial \dot{x}_3} = m\dot{x}_3 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) = m\ddot{x}_3$

$\rightarrow \frac{\partial L}{\partial x_3} = -k[(x_3-x_2)+x_3]$

$\rightarrow \frac{\partial R}{\partial \dot{x}_3} = b\dot{x}_3$

$\rightarrow$ Eq de Lagrange: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_3}\right) - \frac{\partial L}{\partial x_3} + \frac{\partial R}{\partial \dot{x}_3} = 0 \Rightarrow$

$$\Rightarrow \boxed{m\ddot{x}_3 + b\dot{x}_3 + 2kx_3 - kx_2 = 0}$$

• Para x_4: $\rightarrow \frac{\partial L}{\partial \dot{x}_4} = 0 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_4}\right) = 0$

$\rightarrow \frac{\partial L}{\partial x_4} = -k(x_4-x_2)$

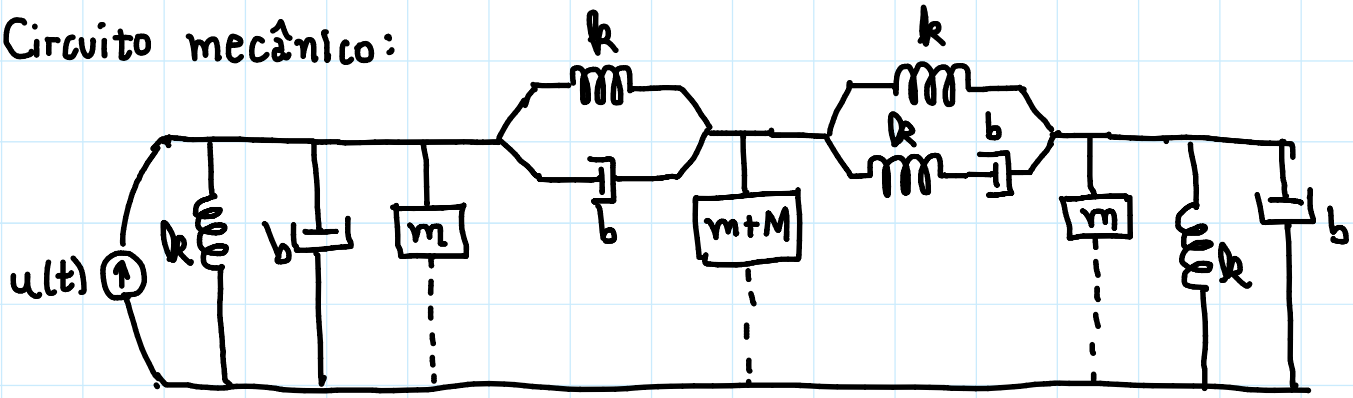
$\rightarrow \frac{\partial R}{\partial \dot{x}_4} = -b(\dot{x}_3-\dot{x}_4)$

$$\rightarrow \frac{\partial R}{\partial \dot{x}_4} = -b(\dot{x}_3 - \dot{x}_4)$$

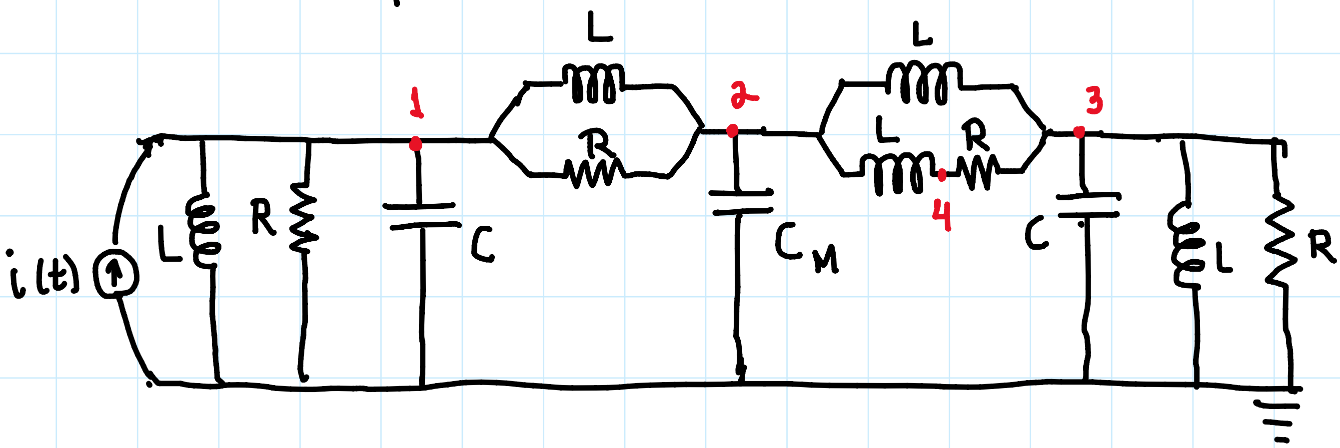
$$\rightarrow \text{Eq. de Lagrange: } \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_4}\right) - \frac{\partial L}{\partial x_4} + \frac{\partial R}{\partial \dot{x}_4} = 0 \Rightarrow$$

$$\Rightarrow \boxed{b\dot{x}_4 + k x_4 - b\dot{x}_3 - k x_2 = 0}$$

b) • Circuito mecânico:



c) • Circuito elétrico equivalente:



d) • Nó 1: $\boxed{V_1\left[CD + \frac{2}{R} + \frac{2}{LD}\right] - V_2\left[\frac{1}{R} + \frac{1}{LD}\right] = i(t)}$ (I)

• Nó 2: $\boxed{V_2\left[C_M D + \frac{1}{R} + \frac{3}{LD}\right] - V_1\left[\frac{1}{R} + \frac{1}{LD}\right] - (V_3 + V_4)\frac{1}{LD} = 0}$ (II)

• Nó 3: $\boxed{V_3\left[CD + \frac{2}{R} + \frac{2}{LD}\right] - V_1\frac{1}{R} - V_2 \cdot \frac{1}{LD} = 0}$ (III)

- Nó 4: $\boxed{V_4\left[\frac{1}{R}+\frac{1}{LD}\right]-V_2\frac{1}{LD}-V_3\frac{1}{R}=0}$ (IV)

e) · Da analogia do tipo 2:

→ Eq (I): $\dot{x}_1\left[mD+2b+\frac{2k}{D}\right]-\dot{x}_2\left[b+\frac{k}{D}\right]=u(t) \Rightarrow$

$$\Rightarrow \boxed{m\ddot{x}_1+2b\dot{x}_1+2kx_1-b\dot{x}_2-kx_2=u(t)}$$

→ Eq (II): $\dot{x}_2\left[(m+M)D+b+\frac{3k}{D}\right]-\dot{x}_1\left[b+\frac{k}{D}\right]-(\dot{x}_3+\dot{x}_4)\frac{k}{D}=0 \Rightarrow$

$$\Rightarrow \boxed{(m+M)\ddot{x}_2+b\dot{x}_2+3kx_2-b\dot{x}_1-k(x_1+x_3+x_4)=0}$$

→ Eq (III): $\dot{x}_3\left[mD+2b+\frac{2k}{D}\right]-\dot{x}_1 b-\dot{x}_2\frac{k}{D}=0 \Rightarrow$

$$\Rightarrow \boxed{m\ddot{x}_3+2b\dot{x}_3+2kx_3-b\dot{x}_1-kx_2=0}$$

→ Eq (IV): $\dot{x}_4\left[b+\frac{k}{D}\right]-\dot{x}_2\frac{k}{D}-\dot{x}_3 b=0 \Rightarrow$

$$\Rightarrow \boxed{b\dot{x}_4+kx_4-b\dot{x}_3-kx_2=0}$$