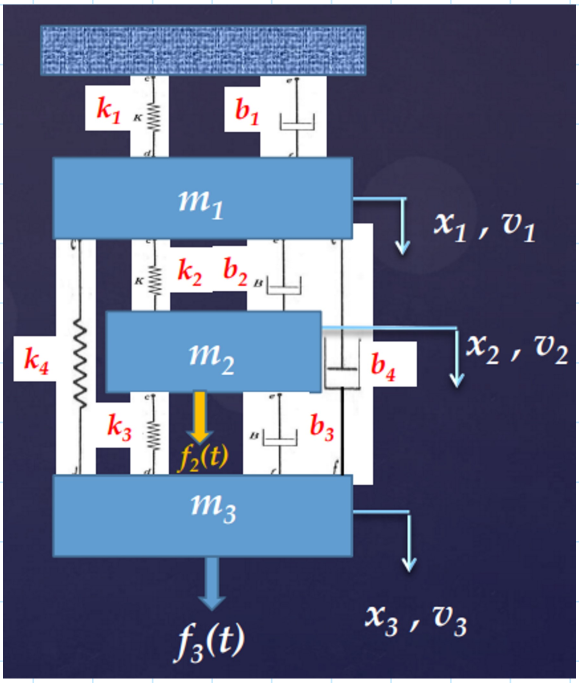
Exercício 2

terça-feira, 21 de setembro de 2021 18:57

Ex.2



2) • Energia cinética: $T = \frac{1}{2}\left(m_1\dot{x}_1^2 + m_2\dot{x}_2^2 + m_3\dot{x}_3^2\right)$

• Energia potencial (Desconsiderando a gravitacional):

$$V = \frac{1}{2}\left[K_1x_1^2 + K_2(x_2-x_1)^2 + K_3(x_3-x_2)^2 + K_4(x_3-x_1)^2\right]$$

• Potencial de Rayleigh:

$$R = \frac{1}{2}\left[b_1\dot{x}_1^2 + b_2(\dot{x}_2-\dot{x}_1)^2 + b_3(\dot{x}_3-\dot{x}_2)^2 + b_4(\dot{x}_3-\dot{x}_1)^2\right]$$

• Lagrangeana: $L = T - V$

• Para x_1: $\rightarrow \frac{\partial L}{\partial \dot{x}_1} = m_1\dot{x}_1 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1\ddot{x}_1$

$\rightarrow \frac{\partial L}{\partial x_1} = -K_1x_1 + K_2(x_2-x_1) + K_4(x_3-x_1)$

$\rightarrow \frac{\partial R}{\partial \dot{x}_1} = b_1\dot{x}_1 - b_2(\dot{x}_2-\dot{x}_1) - b_4(\dot{x}_3-\dot{x}_1)$

$\rightarrow$ Eq. de Lagrange: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial x_1} + \frac{\partial R}{\partial \dot{x}_1} = 0 \Rightarrow$

$$\Rightarrow \boxed{m_1\ddot{x}_1 + (b_1+b_2+b_4)\dot{x}_1 + (K_1+K_2+K_4)\cdot x_1 - b_2\dot{x}_2 - b_4\dot{x}_3 - K_2x_2 - K_4x_3 = 0}$$

• Para x_2: → $\dfrac{\partial L}{\partial \dot{x}_2} = m_2\dot{x}_2 \Rightarrow \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_2}\right) = m_2\ddot{x}_2$

→ $\dfrac{\partial L}{\partial x_2} = -K_2(x_2-x_1) + K_3(x_3-x_2)$

→ $\dfrac{\partial R}{\partial \dot{x}_2} = b_2(\dot{x}_2-\dot{x}_1) - b_3(\dot{x}_3-\dot{x}_2)$

→ Eq. de Lagrange: $\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_2}\right) - \dfrac{\partial L}{\partial x_2} + \dfrac{\partial R}{\partial \dot{x}_2} = f_2(t) \Rightarrow$

$$\Rightarrow \boxed{m_2\ddot{x}_2 + (b_2+b_3)\dot{x}_2 + (K_2+K_3)x_2 - b_2\dot{x}_1 - b_3\dot{x}_3 - K_2x_1 - K_3x_3 = f_2(t)}$$

• Para x_3: → $\dfrac{\partial L}{\partial \dot{x}_3} = m_3\dot{x}_3 \Rightarrow \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_3}\right) = m_3\ddot{x}_3$

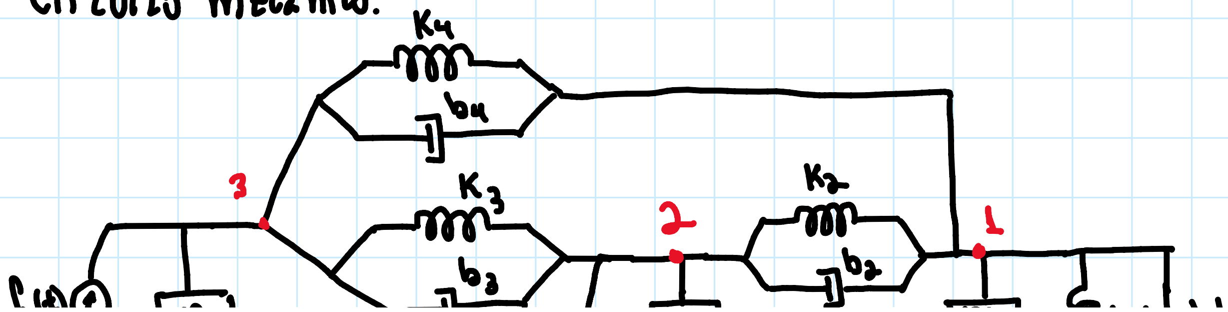
→ $\dfrac{\partial L}{\partial x_3} = -K_3(x_3-x_2) - K_4(x_3-x_1)$

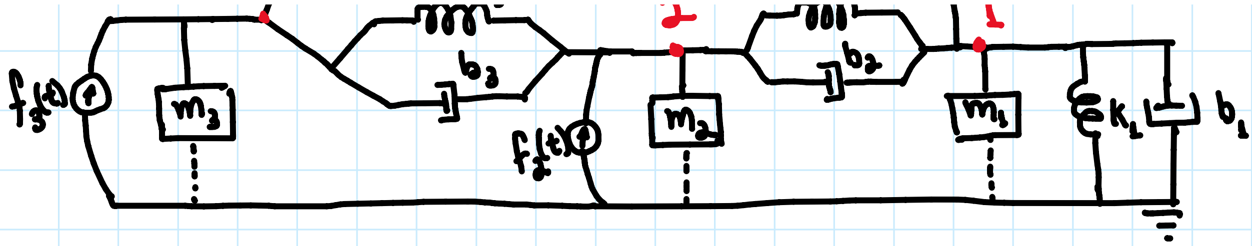
→ $\dfrac{\partial R}{\partial \dot{x}_3} = b_3(\dot{x}_3-\dot{x}_2) + b_4(\dot{x}_3-\dot{x}_1)$

→ Eq. de Lagrange: $\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_3}\right) - \dfrac{\partial L}{\partial x_3} + \dfrac{\partial R}{\partial \dot{x}_3} = f_3(t) \Rightarrow$

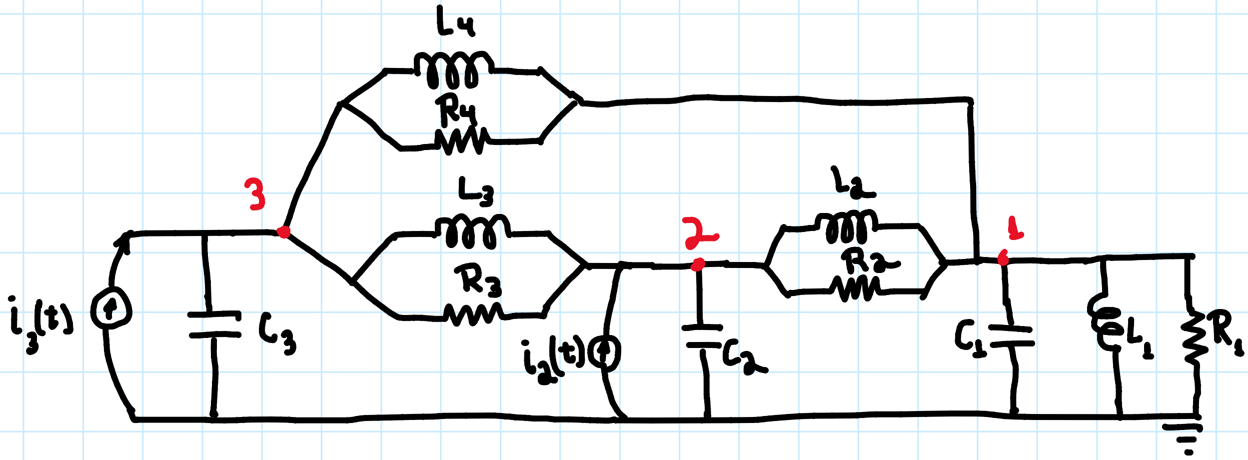
$$\Rightarrow \boxed{m_3\ddot{x}_3 + (b_3+b_4)\dot{x}_3 + (K_3+K_4)x_3 - b_4\dot{x}_1 - b_3\dot{x}_2 - K_4x_1 - b_3x_2 = f_3(t)}$$

b) • Circuito mecânico:





c) • Circuito elétrico equivalente:



d) • Nó 3: $V_3 C_3 D + (V_3 - V_2)\cdot\left(\frac{1}{R_3}+\frac{1}{L_3 D}\right) + (V_3 - V_1)\left(\frac{1}{R_4}+\frac{1}{L_4 D}\right) = i_3(t) \Rightarrow$

$$\Rightarrow \boxed{V_3\cdot\left(C_3D+\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{L_3D}+\frac{1}{L_4D}\right) - V_1\left(\frac{1}{R_4}+\frac{1}{L_4D}\right) - V_2\left(\frac{1}{R_3}+\frac{1}{L_3D}\right) = i_3(t)} \quad (I)$$

• Nó 2: $-(V_3-V_2)\cdot\left(\frac{1}{R_3}+\frac{1}{L_3D}\right) + V_2 C_2 D + (V_2-V_1)\cdot\left(\frac{1}{R_2}+\frac{1}{L_2D}\right) = i_2(t) \Rightarrow$

$$\Rightarrow \boxed{V_2\left(C_2D+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{L_2D}+\frac{1}{L_3D}\right) - V_1\left(\frac{1}{R_2}+\frac{1}{L_2D}\right) - V_3\left(\frac{1}{R_3}+\frac{1}{L_3D}\right) = i_2(t)} \quad (II)$$

• Nó 1: $-(V_2-V_1)\cdot\left(\frac{1}{R_2}+\frac{1}{L_2D}\right) - (V_3-V_1)\left(\frac{1}{R_4}+\frac{1}{L_4D}\right) + V_1\left(C_1D+\frac{1}{R_1}+\frac{1}{L_1D}\right) = 0 \Rightarrow$

$$\Rightarrow \boxed{V_1\left(C_1D+\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{L_1D}+\frac{1}{L_2D}+\frac{1}{L_4D}\right) - V_2\left(\frac{1}{R_2}+\frac{1}{L_2D}\right) - V_3\left(\frac{1}{R_4}+\frac{1}{L_4D}\right) = 0} \quad (III)$$

e) • Pela analogia do tipo 2: $(v = \dot{x})$

→ Eq.(III): $v_1\left[m_1D + b_1 + b_2 + b_4 + (K_1+K_2+K_4)\frac{1}{D}\right] - v_2\left(b_2 + \frac{K_2}{D}\right)$

$$-v_3\left(b_4+\frac{K_4}{D}\right)=0 \Rightarrow$$

$$\Rightarrow \boxed{m_1\ddot{x}_1+(b_1+b_2+b_3)\dot{x}_1+(K_1+K_2+K_3)x_1-b_2\dot{x}_2-K_2x_2-b_4\dot{x}_3-K_4x_3=0}$$

→ Eq. (II): $v_2\left[m_2D+b_2+b_3+(K_2+K_3)\frac{1}{D}\right]-v_1\left(b_2+\frac{K_2}{D}\right)-v_3\left(b_3+\frac{K_3}{D}\right)=f_2(t)\Rightarrow$

$$\Rightarrow \boxed{m_2\ddot{x}_2+(b_2+b_3)\dot{x}_2+(K_2+K_3)x_2-b_2\dot{x}_1-K_2x_1-b_3\dot{x}_3-K_3x_3=f_2(t)}$$

→ Eq. (I): $v_3\left[m_3D+b_3+b_4+(K_3+K_4)\frac{1}{D}\right]-v_1\left(b_4+\frac{K_4}{D}\right)-v_2\left(b_3+\frac{K_3}{D}\right)=f_3(t)\Rightarrow$

$$\Rightarrow \boxed{m_3\ddot{x}_3+(b_3+b_4)\dot{x}_3+(K_3+K_4)x_3-b_4\dot{x}_1-K_4x_1-b_3\dot{x}_2-K_3x_2=f_3(t)}$$