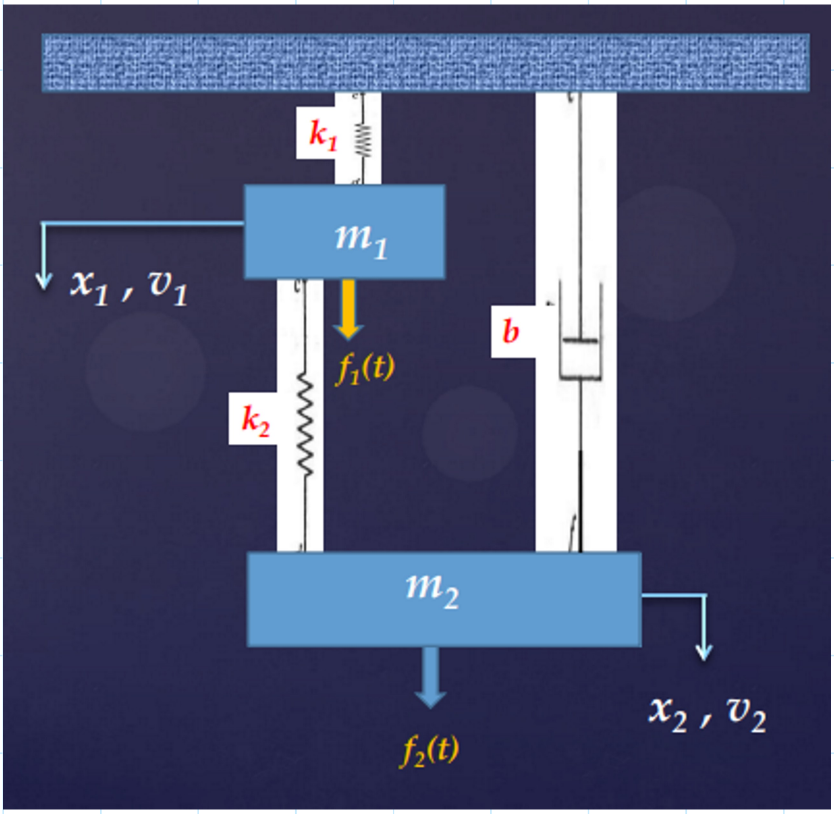
Exercício 4

quarta-feira, 22 de setembro de 2021 17:41

Ex. 4



a) • Energia cinética: $T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2}$

• Energia potencial: $V = \frac{k_1 \cdot x_1^2}{2} + \frac{k_2 (x_2 - x_1)^2}{2}$

• Potencial de Rayleigh: $R = \frac{b \dot{x}_2^2}{2}$

• Lagrangeano: $L = T - V$

• Para x_1: $\rightarrow \frac{\partial L}{\partial \dot{x}_1} = m_1 \dot{x}_1 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1$

$\rightarrow \frac{\partial L}{\partial x_1} = -k_1 x_1 + k_2 (x_2 - x_1)$

$\rightarrow \dfrac{\partial R}{\partial x_1} = 0$

$\rightarrow$ Eq de Lagrange: $\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_1}\right) - \dfrac{\partial L}{\partial x_1} + \dfrac{\partial R}{\partial \dot{x}_1} = f_1(t) \Rightarrow$

$$\Rightarrow \boxed{m_1\ddot{x}_1 + (k_1+k_2)x_1 - k_2 x_2 = f_1(t)}$$

• Para x_2: $\rightarrow \dfrac{\partial L}{\partial \dot{x}_2} = m_2\dot{x}_2 \Rightarrow \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_2}\right) = m_2\ddot{x}_2$

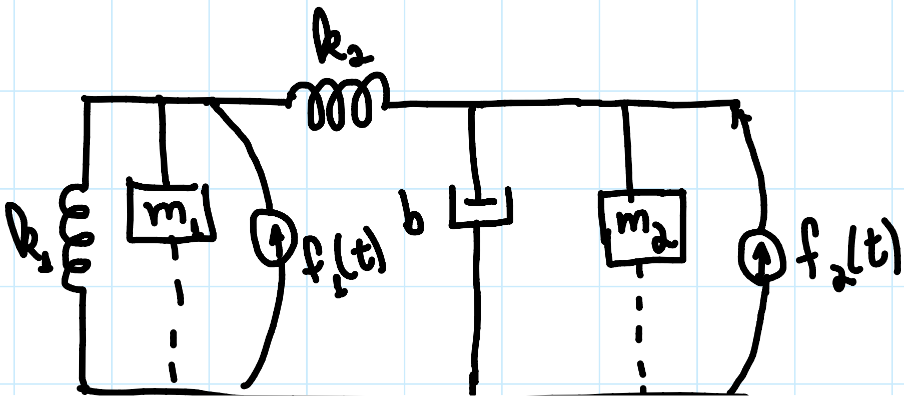
$\rightarrow \dfrac{\partial L}{\partial x_2} = -k_2(x_2 - x_1)$

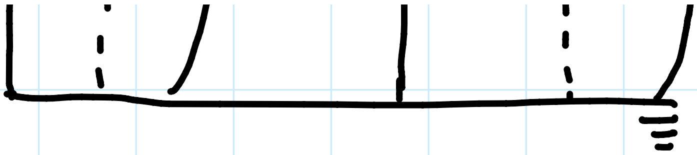
$\rightarrow \dfrac{\partial R}{\partial \dot{x}_2} = b\dot{x}_2$

$\rightarrow$ Eq. de Lagrange: $\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_2}\right) - \dfrac{\partial L}{\partial x_2} + \dfrac{\partial R}{\partial \dot{x}_2} = f_2(t) \Rightarrow$

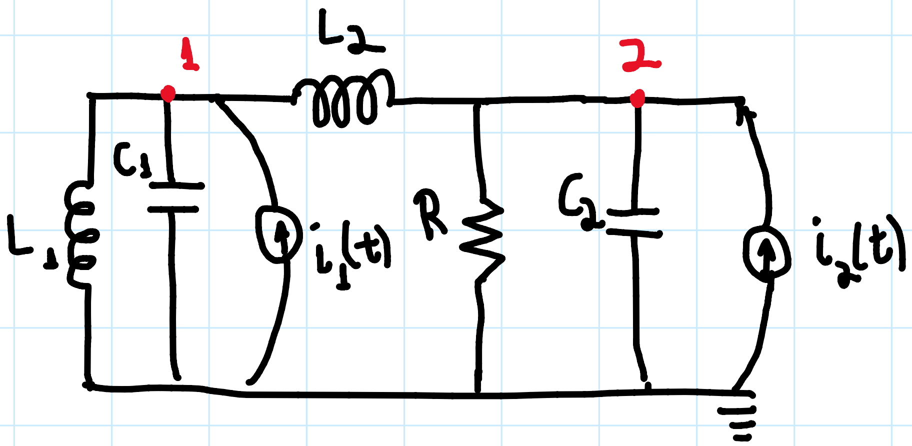
$$\Rightarrow \boxed{m_2\ddot{x}_2 + b\dot{x}_2 + k_2 x_2 - k_2 x_1 = f_2(t)}$$

b) • Circuito mecânico:





c) · Circuito elétrico equivalente:



d) · Nó 1: $\boxed{V_1 \cdot \left[C_1 D + \frac{1}{L_1 D} + \frac{1}{L_2 D}\right] - V_2 \frac{1}{L_2 D} = i_1(t)}$ (I)

· Nó 2: $\boxed{V_2 \left[C_2 D + \frac{1}{R} + \frac{1}{L_2 D}\right] - V_1 \frac{1}{L_2 D} = i_2(t)}$ (II)

e) · Da analogia do tipo 2:

→ Eq (I): $\dot{x}_1 \left[m_1 D + \frac{k_1}{D} + \frac{k_2}{D}\right] - \dot{x}_2 \frac{k_2}{D} = f_1(t) \Rightarrow$

$\Rightarrow \boxed{m_1 \ddot{x}_1 + (k_1 + k_2) x_1 - k_2 x_2 = f_1(t)}$

→ Eq (II): $\ddot{x}_2 \left[m_2 D + b + \frac{k_2}{D}\right] - \dot{x}_1 \frac{k_2}{D} = f_2(t) \Rightarrow$

$$\Rightarrow \boxed{m_2 \ddot{x}_2 + b\dot{x}_2 + k_2 x_2 - k_2 x_1 = f_2(t)}$$