Lista 6 - PME3380

quarta-feira, 20 de outubro de 2021 14:12

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Ex 1 · $f(x) = \cos(x)$

· Linearização em $\pi/4$:

→ $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

→ $\left.\frac{df}{dx}\right|_{x=\frac{\pi}{4}} = -\operatorname{sen}\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

→ $f(x) \cong f\left(\frac{\pi}{4}\right) + \left.\frac{df}{dx}\right|_{x=\frac{\pi}{4}} \cdot \left(x - \frac{\pi}{4}\right) \Rightarrow$

$\Rightarrow \cos(x) \cong \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) \Rightarrow$

$\Rightarrow \boxed{\cos(x) \cong \frac{\sqrt{2}}{2} \cdot \left(1 + \frac{\pi}{4} - x\right)}$

Ex. 2

$$\begin{cases} \ddot{x}_1 + \frac{K}{M_1}(x_1 - x_2) = \frac{u_1}{M_1} & (I) \\ \ddot{x}_2 + \frac{K}{M_2} \cdot (x_2 - x_1) = \frac{u_2}{M_2} & (II) \\ \bar{x} = \frac{M_1 x_1 + M_2 \cdot x_2}{M} & (III) \\ \delta = x_1 - x_2 & (IV) \end{cases}$$

· Substituindo a eq. (IV) em (I) e (II), além de rearranjar:

$$\begin{cases} \ddot{x}_1 = \frac{-K \cdot \delta + u_1}{M_1} & (V) \\ \ddot{x}_2 = \frac{K\delta + u_2}{M_2} & (VI) \end{cases}$$

• Derivando (III) e (IV):

$$\ddot{\bar{x}} = \frac{M_1 \cdot \ddot{x}_1 + M_2 \cdot \ddot{x}_2}{M} \quad \text{(VII)}$$

$$\ddot{\delta} = \ddot{x}_1 - \ddot{x}_2 \quad \text{(VIII)}$$

• Substituindo (V) e (VI) em (VII):

$$\ddot{\bar{x}} = \frac{\cancel{M_1} \cdot \left(\frac{-\cancel{K\delta} + u_1}{\cancel{M_1}} \right) + \cancel{M_2} \cdot \left(\frac{\cancel{K\delta} + u_2}{\cancel{M_2}} \right)}{M} \Rightarrow$$

$$\Rightarrow \boxed{\ddot{\bar{x}} = \frac{u_1 + u_2}{M}}$$

• Substituindo (V) e (VI) em (VIII):

$$\ddot{\delta} = \frac{-K\delta + u_1}{M_1} - \frac{K\delta + u_2}{M_2} \Rightarrow$$

$$\Rightarrow \ddot{\delta} = \frac{-K \overbrace{(M_1 + M_2)}^{=M} \delta}{M_1 \cdot M_2} + \frac{u_1}{M_1} - \frac{u_2}{M_2} \Rightarrow$$

$$\Rightarrow \boxed{\ddot{\delta} = \frac{-KM}{M_1 \cdot M_2} \cdot \delta + \frac{u_1}{M_1} - \frac{u_2}{M_2}}$$

Ex.3 • Define-se: $z = [\underbrace{\bar{x}}_{z_1} \; \underbrace{\delta}_{z_2} \; \underbrace{\dot{\bar{x}}}_{z_3} \; \underbrace{\dot{\delta}}_{z_4}]^T$, $u = [u_1 \;\; u_2]^T$

• Logo: $\dot{z} = [\dot{\bar{x}} \;\; \dot{\delta} \;\; \ddot{\bar{x}} \;\; \ddot{\delta}]^T$

• $\dot{z} = \bar{A} \cdot z + \bar{B} u$

• Seja: $\dot{\bar{x}} = f_1(z, u, t) = z_3$; $\ddot{\bar{x}} = f_3(z, u, t) = \frac{u_1 + u_2}{M}$

$$\dot{\delta} = f_2(z,u,t) = z_4 \quad ; \quad \ddot{\delta} = f_4(z,u,t) = \frac{-KM}{M_1 \cdot M_2} \cdot z_2 + \frac{u_1}{M_1} - \frac{u_2}{M_2}$$

· Matriz Jacobiana $\bar{A}$:

$$\bar{A} = \begin{bmatrix} \frac{\partial f_1}{\partial z_1} & \frac{\partial f_1}{\partial z_2} & \frac{\partial f_1}{\partial z_3} & \frac{\partial f_1}{\partial z_4} \\ \frac{\partial f_2}{\partial z_1} & \frac{\partial f_2}{\partial z_2} & \frac{\partial f_2}{\partial z_3} & \frac{\partial f_2}{\partial z_4} \\ \frac{\partial f_3}{\partial z_1} & \frac{\partial f_3}{\partial z_2} & \frac{\partial f_3}{\partial z_3} & \frac{\partial f_3}{\partial z_4} \\ \frac{\partial f_4}{\partial z_1} & \frac{\partial f_4}{\partial z_2} & \frac{\partial f_4}{\partial z_3} & \frac{\partial f_4}{\partial z_4} \end{bmatrix} \Rightarrow \boxed{\bar{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{-KM}{M_1 \cdot M_2} & 0 & 0 \end{bmatrix}}$$

· Matriz Jacobiana $\bar{B}$:

$$\bar{B} = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} \\ \frac{\partial f_3}{\partial u_1} & \frac{\partial f_3}{\partial u_2} \\ \frac{\partial f_4}{\partial u_1} & \frac{\partial f_4}{\partial u_2} \end{bmatrix} \Rightarrow \boxed{\bar{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{M} & \frac{1}{M} \\ \frac{1}{M_1} & \frac{-1}{M_2} \end{bmatrix}}$$

· A partir das equações (III) e (IV):

$$\bar{x} + \frac{M_2}{M}\delta = \frac{M_1 x_1 + M_2 \cdot \cancel{x_2}}{M} + \frac{M_2}{M} \cdot (x_1 - \cancel{x_2}) = \frac{\overbrace{(M_1 + M_2)}^{=\cancel{M}}}{\cancel{M}} x_1 \Rightarrow$$

$$\Rightarrow \quad x_1 = \bar{x} + \frac{M_2}{M}\delta = g_1(z,u,t)$$

$$\bar{x} - \underline{M_1}\,\delta = \underline{M_1 \cancel{x_1} + M_2 \cdot x_2} - \underline{M_1} \cdot (\cancel{x_1} - x_2) = \overbrace{(M_2 + M_1)}^{=\cancel{M}} x_2 \Rightarrow$$

$$\bar{x} - \frac{M_1}{M}\delta = \frac{M_1 \cancel{x_1} + M_2 \cdot x_2}{M} - \frac{M_1}{M}\cdot(\cancel{x_1} - x_2) = \frac{\overbrace{(M_2 + M_1)}}{\cancel{M}} x_2 \Rightarrow$$

$$\Rightarrow x_2 = \bar{x} - \frac{M_1}{M}\delta = g_2(z, u, t)$$

• Define-se: $\begin{cases} y = [\,x_1 \quad x_2\,]^T \\ y = \bar{C}.z + \bar{D}u \end{cases}$

• Matriz Jacobiana $\bar{C}$:

$$\bar{C} = \begin{bmatrix} \frac{\partial g_1}{\partial z_1} & \frac{\partial g_1}{\partial z_2} & \frac{\partial g_1}{\partial z_3} & \frac{\partial g_1}{\partial z_4} \\ \frac{\partial g_2}{\partial z_1} & \frac{\partial g_2}{\partial z_2} & \frac{\partial g_2}{\partial z_3} & \frac{\partial g_2}{\partial z_4} \end{bmatrix} \Rightarrow \boxed{\bar{C} = \begin{bmatrix} 1 & \frac{M_2}{M} & 0 & 0 \\ 1 & \frac{-M_1}{M} & 0 & 0 \end{bmatrix}}$$

• Matriz Jacobiana $\bar{D}$:

$$\bar{D} = \begin{bmatrix} \frac{\partial g_1}{\partial u_1} & \frac{\partial g_1}{\partial u_2} \\ \frac{\partial g_2}{\partial u_1} & \frac{\partial g_2}{\partial u_2} \end{bmatrix} \Rightarrow \boxed{\bar{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}$$