
ABSTAT17

IGC, April 10-13, 2017

Project: EM algorithm¹

Suppose you are given a sample of m male twin pairs, f female twin pairs, and d opposite sex twin pairs. Estimate the probability p that a twin pair is identical and the probability q that a child is male.

Here $y = (m, f, d)$ is the observed data and $\theta = (p, q)$ is the parameter vector. If we knew exactly which pairs of same-sex twins were identical, then it would be easy to estimate p and q . Thus, we postulate complete data $x = (m_1, m_2, f_1, f_2, d)$, with m_1 representing the number of male identical twin pairs and m_2 the number of male non-identical twin pairs. Note that m_1 and m_2 are observations of the random variables M_1 and M_2 , respectively. f_1 and f_2 are defined similarly.

Note that $m_1 + m_2 = m$, $f_1 + f_2 = f$ e $m + f + d = N$. Algorithm EM should be used in order to estimate p and q .

1. The complete likelihood function is:

$$L(\theta|x) = \frac{N!}{m_1!m_2!f_1!f_2!d!} (pq)^{m_1} [(1-p)q^2]^{m_2} [p(1-q)]^{f_1} [(1-p)(1-q)^2]^{f_2} [(1-p)2q(1-q)]^d,$$

since the identical twins have the same genetic code. Identify the distribution of the population.

2. Present an expression for the logarithm of $L(\theta|x)$.

¹Exercise based on <http://www.leg.ufpr.br/~paulojus/EM/EM-Exemplos-Lange.pdf>

3. Step E

Considering that the expected values for M_1 e M_2 , at iteration k , are:

$$m_1^k = E(M_1|y, \theta^k) = m \frac{p^k q^k}{p^k q^k + (1 - p^k)(q^k)^2}, \quad m_2^k = E(M_2|y, \theta^k) = m - m_1^k,$$

determine f_1^k, f_2^k .

4. Step M

Find the estimators of p and q at iteration $k + 1$.

5. Develop a script for the EM algorithm and find estimates for p and q .
6. Prepare some slides including:
 - Description of the problem;
 - Description of the method (workflow);
 - Eventually some parts of the script;
 - The answer to the problem;