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流体力学 4回目

[1] (1) $P + \rho g h + \frac{1}{2}\rho u^2 = const$

両辺をρgで割って,

$$\underbrace{\frac{P}{\rho g}}_{\text{圧力ヘッド}} + \underbrace{h}_{\text{位置ヘッド}} + \underbrace{\frac{u^2}{2g}}_{\text{速度ヘッド}} = \underbrace{H}_{\text{全ヘッド}}$$

(2) ベルヌーイの定理より,

$$P_A + \rho g h_A + \frac{1}{2}\rho u_A^2 = P_B + \rho g h_B + \frac{1}{2}\rho u_B^2$$

液面では$u_A = 0$, $P_A = P_B$, $h_B = 0$より,

$$\rho g h = \frac{1}{2}\rho u^2$$

$$\therefore u = \boxed{\sqrt{2gh}}$$

(3) $P_B - \Delta P = P_A \rightarrow P_A - P_B = -\Delta P$

ベルヌーイの定理より,

$$P_A + \rho g h = P_B + \frac{1}{2}\rho u^2$$

$$\frac{1}{2}\rho u^2 = \rho g h + P_A - P_B$$

$$= \rho g h - \Delta P$$

$$\therefore u = \boxed{\sqrt{2gh - \frac{2\Delta P}{\rho}}}$$

[2] (1) 非圧縮性流体より,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{U}{L} + \frac{V}{\delta} = 0 \rightarrow \therefore |V| = \boxed{\frac{\delta U}{L}}$$

(2) $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = 0 + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

定常より, $\frac{\partial}{\partial t} = 0$

$$0 + U\frac{U}{L} + \frac{\delta U}{L}\cdot\frac{U}{\delta} = \nu\left(\frac{1}{L}\frac{U}{L} + \frac{1}{\delta}\frac{U}{\delta}\right)$$

$$\boxed{\frac{2U^2}{L} = \nu\left(\frac{U}{L^2} + \frac{U}{\delta^2}\right)}$$

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(3) $L \gg \delta$ のとき, $v \fallingdotseq 0$ より, $v\frac{\partial u}{\partial y} = 0$

また, $\frac{U}{L^2} = 0$

以上より,

$$U\frac{U}{L} + 0 = \nu\left(\frac{U}{L^2} + \frac{U}{\delta^2}\right)$$

$$\frac{U^2}{L} = \nu\frac{U}{\delta^2}$$

$$\therefore \delta = \boxed{\sqrt{\frac{\nu L}{U}}}$$

(4) $\delta = \sqrt{\frac{\nu L}{U}} = \sqrt{\frac{1.5 \cdot 10^{-5} \cdot 1}{10 \cdot 10^3/3600}} \fallingdotseq 2.32\,(\mathrm{mm}) \rightarrow$ 10(mm) より小さい

[3] (1) $Re \ll 1$

(2) $dA = 2\pi a\sin\theta \cdot a\,d\theta$, $\tau = \frac{3}{2}\frac{\mu U}{a}\sin\theta$ より,

$$D_f = \int_0^\pi \tau\sin\theta\,dA = \int_0^\pi \frac{3}{\cancel{2}}\frac{\mu U}{\cancel{a}}\sin^2\theta \cdot \cancel{2}\pi a\sin\theta \cdot a\,d\theta = 3\pi\mu aU\int_0^\pi \sin^3\theta\,d\theta$$

$$= 3\pi\mu aU\int_0^\pi (1-\cos^2\theta)\sin\theta\,d\theta$$

$\cos\theta = t$ とおくと

$-\sin\theta\,d\theta = dt$

$d\theta = -dt/\sin\theta$

θ	$0 \rightarrow \pi$
t	$1 \rightarrow -1$

$$D_f = 3\pi\mu aU\int_1^{-1}(1-t^2)\cancel{\sin\theta}\cdot\left(-\frac{dt}{\cancel{\sin\theta}}\right) = 3\pi\mu aU\int_{-1}^{1}(1-t^2)\,dt$$

$$= 3\pi\mu aU\left[t - \frac{1}{3}t^3\right]_{-1}^{1} = 3\pi\mu aU\left\{\left(1-\frac{1}{3}\right) - \left(-1+\frac{1}{3}\right)\right\} = \boxed{4\pi\mu aU}$$

(3) 加速度 0 より, $F = ma$ から,

$$F = -\frac{4}{3}\pi\rho a^3 g + \frac{4}{3}\pi\rho_0 a^3 g + 4\pi\mu aU = 0$$

$$4\pi\mu aU = \frac{4}{3}\pi a^3 g(\rho - \rho_0)$$

$$U = \boxed{\frac{a^2 g(\rho - \rho_0)}{3\mu}}$$