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熱力学 4回目

[1] (1) $0 \to 1$; 定容変化より, $dv = 0$, $v_1 = v_0$

熱力学第一法則より, $dq = du + pdv$

$dq = du = mC_v dT$

$Q = mC_v(T_1 - T_0)$

$\therefore T_1 = \boxed{\frac{Q}{mC_v} + T_0}$

理想気体の状態方程式より, $pv = mRT \to v = mRT/p = const$ より,

$T/p = const$

$T_1/p_1 = T_0/p_0$

$\therefore p_1 = \frac{T_1}{T_0} p_0 = \boxed{\left(\frac{Q}{mC_vT_0} + 1\right) p_0}$

(2) $dq = Tds$ より,

$ds = \frac{dq}{T} = \frac{mC_v dT}{T}$

$\therefore S_{01} = mC_v \ln\left(\frac{T_1}{T_0}\right) = \boxed{mC_v \ln\left(\frac{Q}{mC_vT_0} + 1\right)}$

(3) $1 \to 2$; 断熱変化より, $dq = 0$

$pv^k = const$ より,

$p_0 v_2^k = p_1 v_0^k$

$\therefore v_2 = \left(\frac{p_1}{p_0}\right)^{\frac{1}{k}} v_0 = \boxed{v_0\left(\frac{Q}{mC_vT_0} + 1\right)^{\frac{1}{k}}}$

$pv = mRT \longrightarrow p = mRT/v$, $pv^k = const$ より,

$Tv^{k-1} = const$

$T_1 v_0^{k-1} = T_2 v_2^{k-1}$

$\therefore T_2 = \left(\frac{v_0}{v_2}\right)^{k-1} T_1 = \left(\frac{Q}{mC_vT_0} + 1\right)^{-1+\frac{1}{k}} \cdot \left(\frac{Q}{mC_vT_0} + 1\right) T_0$

$= \boxed{T_0\left(\frac{Q}{mC_vT_0} + 1\right)^{\frac{1}{k}}}$

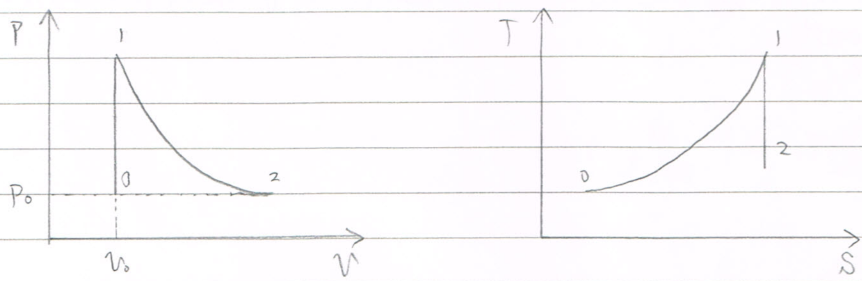
(4) $dq = Tds = 0$ より,

$\therefore S_{12} = \boxed{0}$

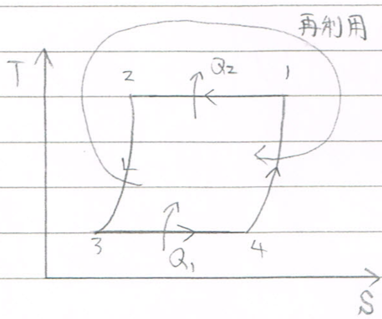
(5) $dq = du + pdv = 0$ より,

$$W_{12} = -mC_v(T_2 - T_1)$$
$$= mC_v\left\{\frac{Q}{mC_v} + T_0 - T_0\left(\frac{Q}{mC_vT_0} + 1\right)^{\frac{1}{\kappa}}\right\}$$
$$= \boxed{mC_vT_0\left\{\frac{Q}{mC_vT_0} + 1 - \left(\frac{Q}{mC_vT_0} + 1\right)^{\frac{1}{\kappa}}\right\}}$$

(6)



2 (1)



(2) $2 \rightarrow 3$: 等容変化より, $dv = 0$

$\therefore W_{23} = \int_2^3 p\,dv = \boxed{0}$

$dq = du + pdv$ より,

$dq = du$

$\therefore Q_{23} = U_3 - U_2 = \boxed{mC_v(T_3 - T_2)}$

(3) $3 \rightarrow 4$: 等温変化より, $dT = 0$

$\therefore W_{34} = \int_3^4 p\,dv = p_3v_3\int_3^4 \frac{dv}{v} = p_3v_3 \ln\left(\frac{v_4}{v_3}\right) = mRT_3 \ln\left(\frac{v_1}{v_2}\right) = \boxed{mRT_3 \ln \varepsilon}$

$dq = du + pdv$ より, $dq = pdv$

$\therefore Q_{34} = W_{34} = \boxed{mRT_3 \ln \varepsilon}$

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(4) $1 \to 2$；等温変化より，$dT=0$

$dq = du + pdv$ より，

$dq = pdv$

$$Q_{12} = \int_1^2 pdv = p_1 v_1 \ln\left(\frac{v_2}{v_1}\right) = -mRT_1 \ln\left(\frac{v_1}{v_2}\right) = -mRT_1 \ln \varepsilon$$

$$\varepsilon = \frac{Q_2}{w} = \frac{Q_{12}}{Q_{12} - Q_{34}} = \frac{mRT_1 \ln\varepsilon}{mRT_1 \ln\varepsilon - mRT_3 \ln\varepsilon} = \boxed{\frac{T_1}{T_1 - T_3}}$$

[3] (1) 断熱変化より，$pv^k = \text{const}$

$$p_1 v_1^k = p_2 v_2^k$$

$$\therefore p_2 = \boxed{\left(\frac{v_1}{v_2}\right)^k p_1}$$

(2) $pv = RT \to v = RT/p$ ，$pv^k = \text{const}$ より，

$$T/p^{\frac{k-1}{k}} = \text{const}$$

$$\frac{T_1}{p_1^{\frac{k-1}{k}}} = \frac{T_c}{p_c^{\frac{k-1}{k}}}$$

$$\therefore p_c = p_1\left(\frac{T_c}{T_1}\right)^{\frac{k}{k-1}} = \boxed{p_1\left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}} \quad \left(\because \frac{T_c}{T_1} = \frac{2}{k+1}\right)$$

(3) 開いた系のエネルギ式より，位置エネルギー・仕事・熱の移動は0より，

$$m\left(h_1 + \frac{1}{2}w_1^2\right) = m\left(h_2 + \frac{1}{2}w_2^2\right)$$

$$w_2 = \sqrt{2(h_1 - h_2)}$$

$$= \sqrt{2c_p(T_1 - T_2)}$$

$$= \sqrt{2\frac{k}{k-1}R(T_1 - T_2)}$$

$$= \sqrt{\frac{2k}{k-1}(p_1 v_1 - p_2 v_2)}$$

$$= \sqrt{\frac{2k}{k-1}p_1 v_1\left\{1 - \frac{p_2}{p_1}\cdot\frac{v_2}{v_1}\right\}}$$

$p_1 v_1^k = p_2 v_2^k$ より，

$$p_2/p_1 = (v_1/v_2)^k$$

$$\therefore w_2 = \sqrt{\frac{2k}{k-1}p_1 v_1\left\{1 - \left(\frac{v_1}{v_2}\right)^{k-1}\right\}} \quad \text{—①}$$

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臨界流れでは，$P_2=P_c$，$T_2=T_c$

$Tv^{k-1}=\mathrm{const}$ より，

$T_1 v_1^{k-1}=T_c v_2^{k-1}$

$$\frac{v_1}{v_2}=\left(\frac{T_c}{T_1}\right)^{\frac{1}{k-1}}=\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}$$

①に代入

$$w_2=\sqrt{\frac{2k}{k-1}P_1v_1\left\{1-\frac{2}{k+1}\right)}=\sqrt{\frac{2k}{k-1}P_1v_1\cdot\frac{k-1}{k+1}}=\boxed{\sqrt{\frac{2k}{k+1}P_1v_1}}$$

(4) $P_1v_1^k=P_2v_2^k$ より，

$$P_1v_1=P_2\frac{v_2^k}{v_1^{k-1}}=P_2v_2\left(\frac{v_2}{v_1}\right)^{k-1} \quad ②$$

臨界流れにおいて，$T_1v_1^{k-1}=T_cv_2^{k-1}$

$$\left(\frac{v_2}{v_1}\right)^{k-1}=\frac{T_1}{T_c}=\frac{k+1}{2}$$

②に代入

$$P_1v_1=P_2v_2\cdot\frac{k+1}{2}$$

(3)より，

$$w_2=\sqrt{\frac{2k}{k+1}P_1v_1}=\sqrt{\frac{2k}{\cancel{k+1}}\cdot\frac{\cancel{k+1}}{\cancel{2}}P_2v_2}=\sqrt{kP_2v_2}\longrightarrow \text{音速と一致}$$

(5) 臨界圧力比：$\frac{P_c}{P_1}$ より，

(a) $k=1.40$ のとき，

$$\frac{P_c}{P_1}=0.5283\longrightarrow P_c=0.5283\cdot0.18=0.095094\,(\mathrm{MPa})$$

$P_2>P_c$ より，$\boxed{\text{先細ノズル}}$

(b) $k=1.30$ のとき，

$$P_c=0.5457\cdot4.00=2.1828\,(\mathrm{MPa})$$

$P_c>P_2$ より，$\boxed{\text{末広ノズル}}$