H26

材料力学　4回目

[1] (1) $\sigma = \alpha E \Delta T$ より，壁から圧縮されるので，

$\sigma_1 = \boxed{-\alpha_1 E_1 \Delta T}$

(2) 軸力をPとおくと，それぞれの伸びは，

$$\lambda_1 = \frac{P l_1}{E_1 S_1} + \alpha_1 l_1 \Delta T \quad , \quad \lambda_2 = \frac{P l_2}{E_2 S_2}$$

全体の伸びは，

$$\lambda = \lambda_1 + \lambda_2 = \left(\frac{P l_1}{E_1 S_1} + \alpha_1 l_1 \Delta T \right) + \frac{P l_2}{E_2 S_2} = 0$$

$$\left(\frac{l_1}{E_1 S_1} + \frac{l_2}{E_2 S_2} \right) P = -\alpha_1 l_1 \Delta T$$

$$P = -\frac{E_1 E_2 S_1 S_2 \alpha_1 l_1 \Delta T}{l_1 E_2 S_2 + l_2 E_1 S_1}$$

$$\therefore \sigma_1 = \frac{P}{S_1} = \boxed{-\frac{E_1 E_2 S_2 \alpha_1 l_1 \Delta T}{l_1 E_2 S_2 + l_2 E_1 S_1}}$$

$$\sigma_2 = \frac{P}{S_2} = \boxed{-\frac{E_1 E_2 S_1 \alpha_1 l_1 \Delta T}{l_1 E_2 S_2 + l_2 E_1 S_1}}$$

(3) $$\delta_B = \frac{P l_2}{E_2 S_2} = \boxed{-\frac{\alpha_1 l_1 l_2 E_1 S_1 \Delta T}{l_1 E_2 S_2 + l_2 E_1 S_1}}$$

[2] (1) 力のつりあいより，$R_A = P_1$

点Aのまわりのモーメントのつりあいより，

$M_A = -P_1 l$

$M_x = R_A x + M_A = P_1 x - P_1 l = P_1 (x - l)$

$$\frac{d^2 y}{dx^2} = -\frac{M_x}{EI} \rightarrow EI \frac{d^2 y}{dx^2} = -P_1 (x - l)$$

$$EI \frac{dy}{dx} = -P_1 \left(\frac{1}{2} x^2 - l x + C_1 \right)$$

$$EI y = -P_1 \left(\frac{1}{6} x^3 - \frac{1}{2} l x^2 + C_1 x + C_2 \right)$$

境界条件より，$x = 0$ のとき，$\frac{dy}{dx} = 0$

$x = 0$ のとき，$y = 0$

$$EI \left(\frac{dy}{dx} \right)_{x=0} = -P_1 (0 - 0 + C_1) = 0 \rightarrow C_1 = 0$$

$$EIy_{x=0} = -P_1(0-0+0+C_2) = 0 \rightarrow C_2 = 0$$

$$y = -\frac{P_1}{EI}\left(\frac{1}{6}x^3 - \frac{1}{2}lx^2\right)$$

$$\therefore \delta_{B1} = y_{x=l} = -\frac{P_1}{EI}\left(\frac{1}{6}l^3 - \frac{1}{2}l^3\right) = \boxed{\frac{P_1 l^3}{3EI}}$$

(2) フックの法則より,

$$P_2 = k\delta_{B2}$$

$$\therefore \delta_{B2} = \boxed{\frac{P_2}{k}}$$

(3) $\delta_B = \delta_{B1} - \delta_{B2}$

$$= \frac{P_1 l^3}{3EI} - \frac{P_2 l^3}{3EI} = \frac{l^3}{3EI}(P_1 - P_2) \quad —①$$

また,

$\delta_B = \dfrac{P_2}{k}$ より,

$$\frac{P_2}{k} = \frac{l^3}{3EI}(P_1 - P_2) \longrightarrow P_2 = \frac{kl^3}{3EI}(P_1 - P_2)$$

$$\left(1 + \frac{kl^3}{3EI}\right)P_2 = \frac{kl^3}{3EI}P_1$$

$$P_2 = \frac{3EI}{3EI + kl^3} \cdot \frac{kl^3}{3EI}P_1 = \frac{kl^3}{3EI + kl^3}P_1$$

$P = P_1 + P_2$ より,

$$P = P_1 + \frac{kl^3}{3EI + kl^3}P_1 = \left(1 + \frac{kl^3}{3EI + kl^3}\right)P_1$$

$$P_1 = \frac{3EI + kl^3}{3EI + 2kl^3}P$$

$$P_2 = \frac{kl^3}{\cancel{3EI + kl^3}} \cdot \frac{\cancel{3EI + kl^3}}{3EI + 2kl^3}P = \frac{kl^3}{3EI + 2kl^3}P$$

① に代入して,

$$\therefore \delta_B = \frac{l^3}{3EI} \cdot \frac{1}{3EI + 2kl^3}\left\{(3EI + \cancel{kl^3})P - \cancel{kl^3 P}\right\}$$

$$= \boxed{\frac{Pl^3}{3EI + 2kl^3}}$$

[3] (1) $I_p = \int r^2 dA = \int_0^{\frac{d}{2}} r^2 \cdot 2\pi r\, dr = \frac{\pi}{2}\left[r^4\right]_0^{\frac{d}{2}} = \frac{\pi}{32}d^4$

$Z_p = \frac{1}{\frac{d}{2}} I_p = \frac{2}{d}\cdot\frac{\pi}{32}d^4 = \frac{\pi}{16}d^3$

$\therefore\ \tau_1 = \frac{T}{Z_p} = \boxed{\frac{16T}{\pi d^3}}$

(2) $2Z = Z_p$ より，$Z = \frac{1}{2}Z_p = \frac{\pi}{32}d^3$

$\therefore\ \sigma = \frac{M}{Z}\sin\theta = \boxed{\frac{32M}{\pi d^3}\sin\theta}$

(3) 相当曲げモーメントをM_e，相当ねじりトルクをT_eとおくと，

$T_e = \sqrt{M^2+T^2}$ ，$M_e = \frac{1}{2}\left(M + \sqrt{M^2+T^2}\right)$

$\therefore\ \sigma_{max} = \frac{M_e}{Z} = \boxed{\frac{16}{\pi d^3}\left(M + \sqrt{M^2+T^2}\right)}$

$\therefore\ \tau_{max} = \frac{T_e}{Z_p} = \boxed{\frac{16}{\pi d^3}\sqrt{M^2+T^2}}$

別 $\sigma_1 = \frac{32M}{\pi d^3}$ ，$\sigma_2 = 0$，$\tau_{12} = \frac{16T}{\pi d^3}$ とおくと，

$\therefore\ \tau_{max} = \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau_{12}^{\ 2}} = \sqrt{\left(\frac{16M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2} = \boxed{\frac{16}{\pi d^3}\sqrt{M^2+T^2}}$

$\therefore\ \sigma_{max} = \frac{\sigma_1+\sigma_2}{2} + \tau_{max} = \frac{16M}{\pi d^3} + \frac{16}{\pi d^3}\sqrt{M^2+T^2}$

$= \boxed{\frac{16}{\pi d^3}\left(M + \sqrt{M^2+T^2}\right)}$

(4) 中心 $\left(\frac{\sigma_{max}+\sigma_{min}}{2}, 0\right)$，半径：$\frac{16}{\pi d^3}\sqrt{M^2+T^2}$

$\tan 2\varphi = \frac{T}{M}$ より，　$\sigma_1 = \frac{32M}{\pi d^3}$

$\therefore\ \varphi = \boxed{\frac{1}{2}\tan^{-1}\frac{T}{M}}$

$\sigma_{min} = \frac{16}{\pi d^3}\left(M - \sqrt{M^2+T^2}\right)$