材料力学　4回目

[1](1) 温度上昇後の2つの材料の長さは等しいので,

$$\begin{cases} l' = l + \alpha_A l \Delta T_1 \\ l' = (l-\Delta l) + \alpha_B (l-\Delta l)\Delta T_1 \end{cases}$$

$$l + \alpha_A l \Delta T_1 = (l-\Delta l) + \alpha_B (l-\Delta l)\Delta T_1$$

$$\Delta T_1\{(\alpha_B-\alpha_A)l - \alpha_B \Delta l\} = \Delta l$$

$$\therefore \Delta T_1 = \boxed{\frac{\Delta l}{(\alpha_B-\alpha_A)l-\alpha_B\Delta l}}$$

$$l' = l + \alpha_A l \frac{\Delta l}{(\alpha_B-\alpha_A)l-\Delta l} = \boxed{l\left\{1+\frac{\alpha_A \Delta l}{(\alpha_B-\alpha_A)l-\alpha_B\Delta l}\right\}}$$

(2) ~~軸力をPとすると、A・Bそれぞれの伸びは,~~

軸力 : R_A , R_B

$R_A + R_B = 0$　でも可

$$\lambda_A = \frac{R l' \sigma_A}{E_A \cancel{S_A}} + \alpha_A l' \Delta T_2 \quad , \quad \lambda_B = \frac{P l' \sigma_B}{E_B \cancel{S_B}} + \alpha_B l' \Delta T_2$$

$\lambda_A = \lambda_B$ より,

$$\frac{\sigma_A P \cancel{l'}}{E_A \cancel{S_A}} + \alpha_A \cancel{l'} \Delta T_2 = \frac{\sigma_B P \cancel{l'}}{E_B \cancel{S_B}} + \alpha_B \cancel{l'} \Delta T_2$$

$$\left(\frac{1}{E_A \cancel{S_A}} - \frac{1}{E_B \cancel{S_B}}\right)P = (\alpha_B-\alpha_A)\Delta T_2$$

$$P = -\frac{(\alpha_B-\alpha_A)\Delta T_2}{E_A S_A - E_B S_B} E_A S_A E_B S_B$$

$$\therefore \sigma_A = \frac{P}{S_A} = \boxed{-\frac{(\alpha_B-\alpha_A)E_A E_B S_B \Delta T_2}{E_A S_A - E_B S_B}}$$

$$\sigma_B = \frac{P}{S_B} = \boxed{-\frac{(\alpha_B-\alpha_A)E_A S_A E_B \Delta T_2}{E_A S_A - E_B S_B}}$$

力のつりあいより,

$$\sigma_A S_A + \sigma_B S_B = 0 \rightarrow \sigma_B = -\frac{S_A}{S_B}\sigma_A$$

$$\left(\frac{1}{E_A} + \frac{S_A}{E_B S_B}\right)\sigma_A = (\alpha_B-\alpha_A)\Delta T_2$$

$$\sigma_A = \boxed{\frac{(\alpha_B-\alpha_A)E_A E_B S_B \Delta T_2}{E_A S_A + E_B S_B}}$$

$$\sigma_B = -\frac{S_A}{S_B}\sigma_A = \boxed{-\frac{(\alpha_B-\alpha_A)E_A S_A E_B \Delta T_2}{E_A S_A + E_B S_B}}$$

$$\lambda_A = \frac{\sigma_A l'}{E_A} + \alpha_A l' \Delta T_2 = \frac{(\alpha_B-\alpha_A)E_B S_B \Delta T_2}{E_A S_A + E_B S_B} l' + \alpha_A l' \Delta T_2$$

$$= l'\Delta T_2\left\{\alpha_A + \frac{(\alpha_B-\alpha_A)E_B S_B}{E_A S_A + E_B S_B}\right\}$$

$$= l\Delta T_2\left\{1+\frac{\alpha_A \Delta l}{(\alpha_B-\alpha_A)l-\alpha_B\Delta l}\right\}\left\{\frac{\alpha_A(E_A S_A + \cancel{E_B S_B}) + (\alpha_B - \cancel{\alpha_A})E_B S_B}{E_A S_A + E_B S_B}\right\}$$

$$= \boxed{l\Delta T_2\left\{1+\frac{\alpha_A \Delta l}{(\alpha_B-\alpha_A)l-\alpha_B\Delta l}\right\}\left(\frac{\alpha_A E_A S_A + \alpha_B E_B S_B}{E_A S_A + E_B S_B}\right)}$$

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[2] (1) $0 \leqq x \leqq l$ のとき,

$$M_x = \boxed{R_A x - \frac{1}{2}wx^2 - M_A}$$

(2) $\frac{d^2y}{dx^2} = -\frac{M_x}{EI} \to EI\frac{d^2y}{dx^2} = \frac{1}{2}wx^2 - R_A x + M_A$

$$EI\frac{dy}{dx} = \frac{1}{6}wx^3 - \frac{1}{2}R_A x^2 + M_A x + C_1$$

$$EIy = \frac{1}{24}wx^4 - \frac{1}{6}R_A x^3 + \frac{1}{2}M_A x^2 + C_1 x + C_2$$

境界条件より, $x=0$ のとき, $\frac{dy}{dx}=0$, $y=0$

$x=l$ のとき, $\frac{dy}{dx}=0$, $y=0$

$$EI\left(\frac{dy}{dx}\right)_{x=0} = 0-0+0+C_1 \to C_1=0$$

$$EIy_{x=0} = 0-0+0+0+C_2=0 \to C_2=0$$

$$EI\left(\frac{dy}{dx}\right)_{x=l} = \frac{1}{6}wl^3 - \frac{1}{2}R_A l^2 + M_A l = 0 \to M_A = \frac{1}{2}R_A l - \frac{1}{6}wl^2$$

$$EIy_{x=l} = \frac{1}{24}wl^4 - \frac{1}{6}R_A l^3 + \frac{1}{2}M_A l^2 = 0$$

$$\frac{1}{24}wl^2 - \frac{1}{6}R_A l + \frac{1}{2}\left(\frac{1}{2}R_A l - \frac{1}{6}wl^2\right) = 0$$

$$\frac{1}{12}R_A l = \frac{1}{24}wl^2 \to \therefore R_A = \boxed{\frac{1}{2}wl}$$

$$M_A = \frac{1}{4}wl^2 - \frac{1}{6}wl^2 = \boxed{\frac{1}{12}wl^2}$$

(3)

$$M_C = R_A l - M_A - \frac{1}{2}wl^2 = \frac{1}{2}wl^2 - \frac{1}{12}wl^2 - \frac{1}{2}wl^2 = \boxed{-\frac{1}{12}wl^2}$$

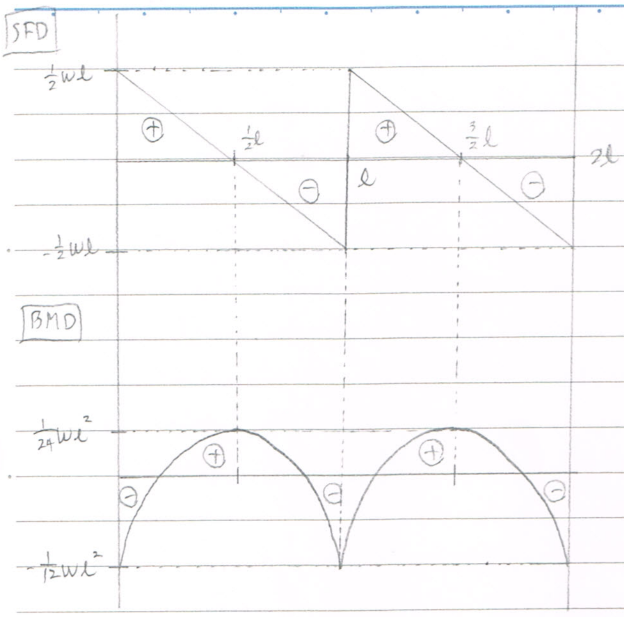
対称性より, $R_B = R_A = \frac{1}{2}wl$, $M_B = M_A = \frac{1}{12}wl^2$

力のつりあいより,

$$R_A + R_B + R_C = 2wl \to R_C = 2wl - wl = \boxed{wl}$$

$$V_x - R_A + wx = 0 \to V_x = \frac{1}{2}wl - wx = w\left(\frac{1}{2}l - x\right)$$

$$M_x = -\frac{1}{2}wx^2 + \frac{1}{2}wl - \frac{1}{12}wl^2 = -\frac{1}{2}w(x^2 - l) - \frac{1}{12}wl^2 = -\frac{1}{2}w\left(x - \frac{l}{2}\right)^2 + \frac{1}{24}wl^2$$



[3] (1) $\lambda_A = \dfrac{P\ell}{EA} = \boxed{\dfrac{4P\ell}{\pi E d^2}}$

(2) $A_B = \dfrac{\pi}{4}d^2 - \dfrac{\pi}{4}\left(\dfrac{d}{2}\right)^2 = \dfrac{3}{16}\pi d^2$

$\lambda_B = \dfrac{P\ell}{EA_B} = \boxed{\dfrac{16P\ell}{3\pi E d^2}}$

(3) $\dfrac{\lambda_B}{\lambda_A} = \dfrac{\frac{16}{3}}{4} = \boxed{\dfrac{4}{3}}$

(4) $T = GI_{PA}\theta_A$, $I_{PA} = \int r^2 dA = \int_0^{\frac{d}{2}} r^2 \cdot 2\pi r\,dr = \dfrac{\pi}{2}\left[r^4\right]_0^{\frac{d}{2}} = \dfrac{\pi}{32}d^4$ より,

$\therefore T = \boxed{\dfrac{\pi d^4 G\theta_A}{32}}$

(5) $I_{PB} = \int r^2 dA = \dfrac{\pi}{2}\left[r^4\right]_{\frac{d}{4}}^{\frac{d}{2}} = \dfrac{\pi}{2}\left(\dfrac{\pi}{16}d^4 - \dfrac{\pi}{256}d^4\right) = \dfrac{15}{512}\pi d^4$

$\therefore T = GI_{PB}\theta_B = \boxed{\dfrac{15\pi d^4 G\theta_B}{512}}$

(6) $\theta_A = 32T/\pi d^4 G$, $\theta_B = 512T/15\pi d^4 G$ より,

$\dfrac{\theta_B}{\theta_A} = \dfrac{512/15}{32} = \dfrac{16\cdot 2}{15\cdot 2} = \boxed{\dfrac{16}{15}}$