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流体力学 4回目

[1] (1) (a) 流線 … 流体の各点における接線ベクトルが、速度方向と一致するような流体の曲線

(b) 流跡線 … 流体粒子の軌跡

(2) $\frac{dx}{u} = \frac{dy}{v}$

$\frac{dx}{x} = -\frac{dy}{y}$

$\log x = -\log y + C$

$\therefore \boxed{xy = C}$ (Cは任意定数)

(3) $u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$

$\partial \psi = x dy$, $\partial \psi = y dx$

$\psi = xy + f(x)$, $\psi = xy + f(y)$

2式は一致するので、$f(x) = f(y) = C$

$\therefore \psi = \boxed{xy + C}$ (Cは積分定数)

(4) $\frac{dx}{dt} = u = x$ $\frac{dy}{dt} = v = -y$

$\frac{dx}{x} = dt$ $\frac{dy}{y} = -dt$

$\log|x| = t + C$ $\log|y| = -t + C$

$x = C_1 e^t$ $y = C_2 e^{-t}$

$(x, y) = (C_1 e^t, C_2 e^{-t})$

$t=0$ のとき、$(x, y) = (C_1, C_2) = (1, 1) \longrightarrow C_1 = C_2 = 1$

$(x, y) = (e^t, e^{-t})$

$t = \alpha$ のとき、 $\therefore (x, y) = \boxed{(e^{\alpha}, e^{-\alpha})}$

[2] (1) $u = -V_t \sin\theta = -\frac{C}{r} \cdot \frac{y}{r} = \boxed{-\frac{Cy}{x^2+y^2}}$

$v = V_t \cos\theta = \frac{C}{r} \cdot \frac{x}{r} = \boxed{\frac{Cx}{x^2+y^2}}$

(2) $\omega = \nabla \times V = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix} \begin{bmatrix} -\frac{Cy}{x^2+y^2} \\ \frac{Cx}{x^2+y^2} \end{bmatrix} = \frac{C(x^2+y^2) - Cx \cdot 2x}{(x^2+y^2)^2} + \frac{C(x^2+y^2) - Cy \cdot 2y}{(x^2+y^2)^2} = \boxed{0}$

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(3) 自由渦，ポテンシャル渦

(4) $\Gamma = \int_0^{2\pi} V_t dA = \int_0^{2\pi} \frac{c}{r} \cdot r d\theta = \boxed{2\pi c}$

(5) $w = \phi + i\psi$

(6) $dw = \frac{\partial w}{\partial x} dx + \frac{\partial w}{\partial y} dy = \left(\frac{\partial \phi}{\partial x} + i\frac{\partial \psi}{\partial x}\right) dx + \left(\frac{\partial \phi}{\partial y} + i\frac{\partial \psi}{\partial y}\right)$

$= (u - iv) dx + (v + iu) dy \quad = (u - iv) dz$

$\therefore \frac{dw}{dz} = \boxed{u - iv}$

(7) $u - iv = -\frac{cy}{x^2+y^2} - i\frac{cx}{x^2+y^2} = -\frac{c(xi + y)}{x^2+y^2} = -\frac{ci(x - iy)}{(x+iy)(x-iy)} = -\frac{ci}{x+iy} = -i\frac{c}{z}$

$\therefore \boxed{u = 0,\ v = +\frac{c}{z}}$

(8) $\frac{dw}{dz} = u - iv$

$w = \int (u - iv) dz = -i\int \frac{c}{z} dz = \boxed{-iC\log|z| + D}$ （Dは積分定数）

[3] (1) $\int_0^a u(r)\, 2\pi r dr = 2\pi A \int_0^a (r - r^3/a^2) dr = 2\pi A \left[\frac{1}{2}r^2 - r^4/4a^2\right]_0^a$

$= 2\pi A \left(\frac{1}{2}a^2 - \frac{1}{4}a^2\right) = \boxed{\frac{1}{2}\pi a^2 A}$

(2) $Q = \frac{1}{2}\pi a^2 A$ ，平均流速；u_m ，断面積；S とすると，

$Q = u_m S$ より，

$u_m = \frac{Q}{S} = \frac{\frac{1}{2}\pi a^2 A}{\pi a^2} = \frac{1}{2}A$

$Re = \frac{u_m \cdot 2a}{\nu} = \boxed{\frac{aA}{\nu}}$

(3) $\tau = \left|\rho\nu \frac{du}{dr}\right|_{r=a}\Big| = \left|\rho\nu \cdot \left(-2A \cdot \frac{r}{a^2}\right)\right|_{r=a}\Big| = \boxed{\frac{2\rho\nu A}{a}}$

(4) $\Delta P \cdot S = \tau \cdot 2\pi a L$

$$\therefore \Delta P = \frac{2\pi a L \tau}{S} = \frac{2\pi a L \cdot 2\rho \nu A}{\pi a^2 \cdot a} = \boxed{\frac{4\rho \nu A L}{a^2}}$$

(5)
$$\lambda = \frac{\Delta P}{\frac{1}{2}\rho U^2 \left(\frac{L}{2a}\right)} = \frac{4\rho \nu A L / a^2}{\frac{1}{2}\rho U \cdot \frac{1}{2}A \cdot \frac{L}{2a}} = \frac{32\nu}{Ua}$$

$$Re = \frac{2aU}{\nu} \rightarrow U = \frac{Re\nu}{2a}$$

$$\lambda = \frac{32\nu}{a} \cdot \frac{2a}{Re\nu} = \boxed{\frac{64}{Re}}$$

(6)
$$Re = \frac{2aU_m}{\nu} = \frac{2 \cdot 5 \cdot 10^{-3} \cdot 10 \cdot 10^{-2}}{1 \cdot 10^{-6}} = \boxed{1.0 \cdot 10^3}$$

$$\lambda = \frac{64}{Re} = \boxed{6.4 \cdot 10^{-2}}$$

$$\Delta P = \frac{4\rho \nu A L}{a^2} = \frac{4\rho\nu \cdot 2U_m \cdot L}{a^2} = \frac{4 \cdot 1 \cdot 10^3 \cdot 1 \cdot 10^{-6} \cdot 2 \cdot 10 \cdot 10^{-2} \cdot 1}{(5 \cdot 10^{-3})^2}$$

$$= \frac{4 \cdot 10^2 \cdot 2}{25} = \boxed{32\,(\mathrm{Pa})}$$

$$U_x = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}, \quad U_y = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

$$U_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$$

$$U_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$$