H22

Date . . No.

数学　4回目

[1] (1) $z = \tan^{-1}(ax)$

$ax = \tan z$

$a\,dx = dz/\cos^2 z$

$$\frac{dz}{dx} = a\cos^2 z = \frac{a}{1+\tan^2 z} = \boxed{\frac{a}{1+a^2x^2}}$$

(2) $z = \tan^{-1}(xy)$

$xy = \tan z$

$y = \tan z / x \rightarrow \partial y = \partial z / x\cos^2 z \rightarrow \frac{\partial z}{\partial y} = x\cos^2 z$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}(x\cos^2 z) = \frac{\partial}{\partial x}\left(\frac{x}{1+\tan^2 z}\right) = \frac{\partial}{\partial x}\left(\frac{x}{1+x^2y^2}\right)$$

$$= \frac{1+x^2y^2 - x\cdot 2xy^2}{(1+x^2y^2)^2} = \boxed{\frac{1-x^2y^2}{(1+x^2y^2)^2}}$$

[2] $dT_l = 2\pi \cdot \frac{1}{2}\frac{dl}{\sqrt{gl}} = \frac{\pi dl}{\sqrt{gl}}$, $dT_g = 2\pi\sqrt{l}\cdot\left(-\frac{1}{2}\frac{dg}{g\sqrt{g}}\right) = -\frac{\pi}{g}\sqrt{\frac{l}{g}}\,dg$

$$dT = dT_l + dT_g = \frac{\pi dl}{\sqrt{gl}} - \frac{\pi}{g}\sqrt{\frac{l}{g}}\,dg$$

$$= \pi\sqrt{\frac{l}{g}}\left(\frac{dl}{l} - \frac{dg}{g}\right) = \frac{T}{2}\left(\frac{dl}{l} - \frac{dg}{g}\right)$$

$$\therefore \boxed{\frac{dT}{T} = \frac{1}{2}\left(\frac{dl}{l} - \frac{dg}{g}\right)}$$

[3] $$x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 3y = (\log x)^2$$

$x = e^t$ とおくと,

$\frac{dx}{dt} = e^t$

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = \frac{dy}{dt}e^{-t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx} = \frac{d}{dt}\left(\frac{dy}{dt}e^{-t}\right)\frac{dt}{dx} = \left(\frac{d^2y}{dt^2}e^{-t} - \frac{dy}{dt}e^{-t}\right)\cdot e^{-t}$$

$$= \frac{d^2y}{dt^2}e^{-2t} - \frac{dy}{dt}e^{-2t}$$

与式へ代入

$$e^{2t}\left(\frac{d^2y}{dt^2}e^{-2t}-\frac{dy}{dt}e^{-2t}\right)+5e^{t}\cdot\frac{dy}{dt}e^{-t}+3y=t^2$$

$$\frac{d^2y}{dt^2}+4\frac{dy}{dt}+3y=t^2 \quad \text{—} ①$$

(左辺) = 0とおき，特性方程式を解くと，

$\lambda^2+4\lambda+3=0 \rightarrow (\lambda+1)(\lambda+3)=0 \rightarrow \lambda=-1,-3$

一般解；$y=C_1e^{-t}+C_2e^{-3t}$

特殊解；$y_0=A_2t^2+A_1t+A_0$ とおく

$y_0'=2A_2t+A_1 \quad , \quad y_0''=2A_2$

①に代入

$2A_2+4(2A_2t+A_1)+3(A_2t^2+A_1t+A_0)=t^2$

$3A_2t^2+(8A_2+3A_1)t+(2A_2+4A_1+3A_0)=t^2$

$$\rightarrow \begin{cases} 3A_2=1 \rightarrow A_2=1/3 \\ 8A_2+3A_1=0 \rightarrow A_1=-8/9 \\ 2A_2+4A_1+3A_0=0 \rightarrow A_0=(-2/3+32/9)/3 = 26/27 \end{cases}$$

$y_0=\frac{1}{3}t^2-\frac{8}{9}t+\frac{26}{27}$

$$\therefore y=C_1e^{-t}+C_2e^{-3t}+\frac{1}{3}t^2-\frac{8}{9}t+\frac{26}{27}$$

$$=\boxed{\frac{C_1}{x}+\frac{C_2}{x^3}+\frac{1}{3}(\log x)^2-\frac{3}{9}\log x+\frac{26}{27}}$$

[4] (1) $\tilde{a}_{11}=-a^2 \quad , \quad \tilde{a}_{12}=a^2 \quad , \quad \tilde{a}_{13}=a^2$

$\tilde{a}_{21}=a^2 \quad , \quad \tilde{a}_{22}=-a^2 \quad , \quad \tilde{a}_{23}=a^2$

$\tilde{a}_{31}=a^2 \quad , \quad \tilde{a}_{32}=a^2 \quad , \quad \tilde{a}_{33}=-a^2$

$$\tilde{A}=\begin{bmatrix}-a^2 & a^2 & a^2\\ a^2 & -a^2 & a^2\\ a^2 & a^2 & -a^2\end{bmatrix}=a^2\begin{bmatrix}-1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 1 & -1\end{bmatrix}$$

$$|A|=a^3+a^3=2a^3$$

$$\therefore A^{-1}=\frac{\tilde{A}}{|A|}=\boxed{\frac{1}{2}\begin{bmatrix}-1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 1 & -1\end{bmatrix}}$$

(2)

$$\det(A-kE) = \det\begin{bmatrix} -k & a & a \\ a & -k & a \\ a & a & -k \end{bmatrix} = -k^3 + a^3 + a^3 + a^2k + a^2k + a^2k$$
$$= -k^3 + 3a^2k + 2a^3 = 0$$

$$k^3 - 3a^2k - 2a^3 = 0$$
$$(k+a)(k^2 - ak - 2a^2) = 0$$
$$(k+a)(k+a)(k-2a) = 0$$

∴ 固有値：$k = -a$(重解), $2a$

1	0	$-3a^2$	$-2a^3$	$\lfloor -a$
↓	$-a$	a^2	$2a^3$	
1	$-a$	$-2a^2$	‖ 0	

求める行列は、

$$\boxed{P = \begin{bmatrix} -a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & 2a \end{bmatrix}}$$

[5]

$$\begin{cases} f(x) = 0 & (x < -a,\ a < x) \\ f(x) = \frac{b}{a}x + b & (-a \leqq x \leqq 0) \\ f(x) = -\frac{b}{a}x + b & (0 \leqq x \leqq a) \end{cases}$$

$$\hat{f}(\omega) = \frac{1}{2\pi}\left\{\int_{-a}^{0}\left(\frac{b}{a}x + b\right)e^{-i\omega x}dx + \int_{0}^{a}\left(-\frac{b}{a}x + b\right)e^{-i\omega x}dx\right\}$$

$$\cancel{= \frac{1}{2\pi}\left\{\int_{-a}^{0}\left(\frac{b}{a}x + b\right)\cdot\left(-\frac{1}{i\omega}e^{-i\omega x}\right)'dx + \int_{0}^{a}\left(-\frac{b}{a}x + b\right)\cdot\left(-\frac{1}{i\omega}e^{-i\omega x}\right)'dx\right\}}$$

$$= \frac{1}{2\pi}\left\{\left[-\frac{1}{i\omega}\left(\frac{b}{a}x + b\right)e^{-i\omega x}\right]_{-a}^{0} + \frac{b}{i\omega a}\int_{-a}^{0}e^{-i\omega x}dx\right.$$
$$\left. + \left[-\frac{1}{i\omega}\left(-\frac{b}{a}x + b\right)e^{-i\omega x}\right]_{0}^{a} - \frac{b}{i\omega a}\int_{0}^{a}e^{-i\omega x}dx\right\}$$

$$= \frac{1}{2\pi}\left\{\cancel{-\frac{1}{i\omega}\cdot b} + \frac{b}{a\omega^2}\left[e^{-i\omega x}\right]_{-a}^{0} \cancel{- \frac{1}{i\omega}(0 - b)} - \frac{b}{a\omega^2}\left[e^{-i\omega x}\right]_{0}^{a}\right\}$$

$$= \frac{1}{2\pi}\left\{\frac{b}{a\omega^2}\left(1 - e^{ia\omega}\right) - \frac{b}{a\omega^2}\left(e^{-ia\omega} - 1\right)\right\}$$

$$= \frac{1}{2\pi}\left\{\frac{b}{a\omega^2}\left(1 - \cos a\omega - \cancel{i\sin a\omega}\right) - \frac{b}{a\omega^2}\left(\cos a\omega - \cancel{i\sin a\omega} - 1\right)\right\}$$

$$= \boxed{\frac{b}{\pi a\omega^2}\left(1 - \cos a\omega\right)}$$