H27

Date . . No.

材料力学　3回目

[1] (1) 力のつりあいより, $P - R_A - R_C = 0$ …①

A点を基準にすると, aのハンイでは, $F_1 = -R_A$

全体の伸びは0より, a+bのハンイでは, $F_2 = P - R_A$

~~2つの棒材の伸びは等しいので,~~

$$\frac{F_1 a}{ES} + \frac{F_2 b}{ES} = 0 \rightarrow +R_A a = b(P - R_A) \rightarrow R_A = \boxed{\frac{bP}{a+b}}$$

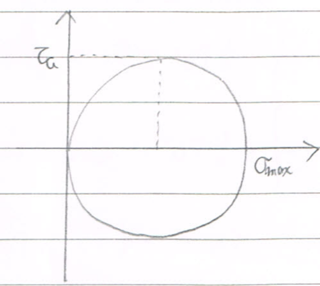
①に代入して,

$$R_C = \boxed{\frac{aP}{a+b}}$$

(2) A点を基準にすると, aのハンイで働いている力は, R_A より,

$$\lambda_B = \frac{R_A a}{ES} = \boxed{\frac{ab}{a+b} \frac{P}{ES}}$$

(3) モールの応力円より,



$$2\tau_a = \sigma_{max} = \frac{R_A}{S} \quad (\because R_A > R_C)$$

$$2\tau_a = \frac{b}{a+b} \frac{P}{S}$$

$$\therefore P = \boxed{\frac{2(a+b)}{b} \cdot \tau_a S}$$

[2] (1) 対称性より, $R_A = R_B = \frac{1}{2} \cdot 2P = P$

(i) $0 \leqq x < a$ のとき,

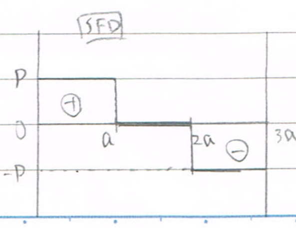
$$M_x = R_A x = Px \quad , \quad V_x = P$$

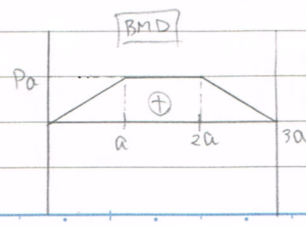
(ii) $a \leqq x < 2a$ のとき,

$$M_x = R_A x - P(x-a) = Pa \quad , \quad V_x = 0$$

(iii) $2a \leqq x < 3a$ のとき,

$$M_x = R_A x - P(x-2a) - P(x-a) = -Px + 3a = -P(x-3a) \quad , \quad V_x = -P$$





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(2) $\frac{d^2y}{dx^2} = -\frac{M_x}{EI}$ を用いて，(1)の(i)，(ii)より，

$$EI\frac{d^2y_1}{dx^2} = -Px \quad , \quad EI\frac{d^2y_2}{dx^2} = -Pa$$

$$EI\frac{dy_1}{dx} = -\frac{1}{2}Px^2 + C_1 \quad , \quad EI\frac{dy_2}{dx} = -Pax + C_3$$

$$EIy_1 = -\frac{1}{6}Px^3 + C_1x + C_2 \quad , \quad EIy_2 = -\frac{1}{2}Pax^2 + C_3x + C_4$$

境界条件より，　$x = 0$ のとき　，$y_1 = 0$

　　　　　　　$x = \frac{3}{2}a$ のとき，　$\frac{dy_2}{dx} = 0$

$$EIy_{1,x=0} = 0 + 0 + C_2 = 0 \longrightarrow C_2 = 0$$

$$EI\left(\frac{dy_2}{dx}\right)_{x=\frac{3}{2}a} = -\frac{3}{2}Pa^2 + C_3 = 0 \longrightarrow C_3 = \frac{3}{2}Pa^2$$

また，C点におけるたわみとたわみ角は等しいから，$x = a$のとき，

$$\begin{cases} -\frac{1}{2}Pa^2 + C_1 = -Pa^2 + \frac{3}{2}Pa^2 \longrightarrow C_1 = Pa^2 \\ -\frac{1}{6}Pa^3 + C_1a = -\frac{1}{2}Pa^3 + \frac{3}{2}Pa^3 + C_4 \end{cases}$$

$$C_4 = -\frac{1}{6}Pa^3 + Pa^3 + \frac{1}{2}Pa^3 - \frac{3}{2}Pa^3 = \frac{-1+6+3-9}{6}Pa^3 = -\frac{1}{6}Pa^3$$

$$y_2 = \frac{P}{EI}\left(-\frac{1}{2}ax^2 + \frac{3}{2}a^2x - \frac{1}{6}a^3\right)$$

$$\therefore y_{max} = (y_2)_{x=\frac{3}{2}a} = \frac{P}{EI}\left(-\frac{9}{8}a^3 + \frac{9}{4}a^3 - \frac{1}{6}a^3\right) = \boxed{\frac{3Pa^3}{24EI}, \quad x = \frac{3}{2}a \text{ のとき}}$$

(3) $I_p = \int r^2 dA = \int_{\frac{d}{2}}^{d} r^2 \cdot 2\pi r dr = \frac{\pi}{2}\left[r^4\right]_{\frac{d}{2}}^{d} = \frac{\pi}{2}\left(d^4 - \frac{d^4}{16}\right) = \frac{15}{32}\pi d^4$

$$\therefore I = \frac{1}{2}I_p = \boxed{\frac{15}{64}\pi d^4}$$

(2)より，　$M_{max} = Pa$

$$Z = \frac{1}{y}I = \frac{1}{d}\frac{15}{64}\pi d^4 = \frac{15}{64}\pi d^3$$

$$\therefore \sigma_{max} = \frac{M_{max}}{Z} = \boxed{\frac{64Pa}{15\pi d^3}}$$

(4) 長さ 3aの棒材に等分布荷重 wがかかっているので,

$$M'_x = -\frac{1}{2} w x^2$$

$$M'_{max} = M'_{x=\frac{3}{2}a} = -\frac{9}{8} w a^2$$

$$\Delta \sigma_{max} = \frac{|M'_{max}|}{Z} = \frac{\cancel{9}^{3}}{\cancel{8}} w a^2 \cdot \frac{\cancel{65}\;8}{{}_{5}\cancel{45}\pi d^3} = \boxed{\frac{24 w a^2}{5 \pi d^3}}$$

[3] (1) $P_{cr} = C \dfrac{\pi^2 E I}{l^2}$

上端自由，下端固定より, $C = \frac{1}{4}$

$$\therefore\ P_{cr} = \boxed{\frac{\pi^2 E I}{4 l^2}}$$

$$y = \boxed{\delta\left(1 - \cos \frac{\pi x}{2l}\right)}$$