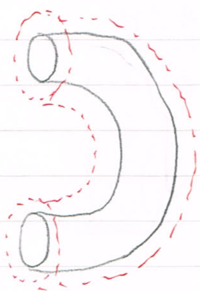
流体力学　3回目

[1]　(1) $mv = \rho SU \cdot U$

圧力の影響を含めると、

$\boxed{\rho SU^2 + P_0 S}$

(2)



(3) 微小時間Δtに流入・流出する運動量は、

流入；$\rho SU^2 \Delta t$

流出；$\rho SU(-U)\Delta t$

運動量の和；$\rho SU^2\Delta t - \rho SU(-U)\Delta t$

力積；$-F\Delta t$

運動量保存則より、圧力の影響を含めると、

$\rho SU^2\Delta t - \rho SU(-U)\Delta t - F\Delta t + 2P_0 S = 0$

$\therefore F = \boxed{2S(\rho U^2 + P_0)}$

[2]　(1) $W_1 = \boxed{Uz}$

(2) $W_2 = m\{\log(z \pm d) - \log z\}$

吹き出し；$W_2' = m\{\log(z+d) - \log z\}$

吸い込み；$W_2'' = -m\{\log(z-d) - \log z\}$

$\therefore W_2 = W_2' + W_2'' = m\{\log(z+d) - \cancel{\log z}\} - m\{\log(z-d) - \cancel{\log z}\}$

$= \boxed{m\log\frac{z+d}{z-d}}$

(3) $W = W_1 + W_2 = Uz + m\log\frac{z+d}{z-d} = Uz + m\{\log(z+d) - \log(z-d)\}$

よどみ点では流速0より、

$\frac{\partial W}{\partial z} = U + m\left(\frac{1}{z+d} - \frac{1}{z-d}\right) = 0$

$$\frac{U}{m} + \frac{1}{z+d} - \frac{1}{z-d} = 0$$

$$\frac{U}{m}(z^2 - d^2) + \cancel{z} - d - \cancel{z} - d = 0$$

$$\frac{U}{m}(z^2 - d^2) = 2d$$

$$z^2 - d^2 = \frac{2md}{U}$$

$$\therefore z = \pm\sqrt{\frac{2md}{U} + d^2} = x + iy$$

以上より，よどみ点は

$$\boxed{C\left(\sqrt{\frac{2md}{U} + d^2},\ 0\right) \quad D\left(-\sqrt{\frac{2md}{U} + d^2},\ 0\right)}$$

(4) (i) 吹き出し

$$W_2 = \lim_{d \to 0}\left[m\left\{\log(z+d) - \log z\right\}\right]$$

$$= m\lim_{d \to 0} \log\left(1 + \frac{d}{z}\right) = \frac{q}{z}$$

(ii) 吸い込み

$$W_2 = \lim_{d \to 0}\left[m\left\{\log(z-d) - \log z\right\}\right]$$

$$= m\lim_{d \to 0} \log\left(1 - \frac{d}{z}\right) = \frac{d}{z}$$

(i), (ii) より，

$$W_2 = \frac{q}{z} + \frac{q}{z} = \boxed{\frac{2q}{z}}$$

(5) $W = Uz + \dfrac{2q}{z}$

よどみ点では流速は0より，

$$\frac{\partial W}{\partial z} = U - \frac{2q}{z^2} = 0 \longrightarrow z = \sqrt{\frac{2q}{U}} = x + iy$$

$z = x + iy$ とおく

$$W = U(x+iy) + \frac{2q}{x+iy}$$

$$= U(x+iy) + \frac{2q}{x^2+y^2}(x - iy)$$

$$= \left(Ux + \frac{2qx}{x^2+y^2}\right) + i\left(Uy - \frac{2qy}{x^2+y^2}\right) = \phi + i\psi$$

$$\phi = Ux + \frac{2q x}{x^2+y^2}, \quad \psi = Uy - \frac{2q y}{x^2+y^2}$$

よどみ点では流速0より,

$$\psi = Uy - \frac{2q y}{x^2+y^2} = 0$$

$$y\left(U - \frac{2q}{x^2+y^2}\right) = 0$$

$$\begin{cases} y = 0 \\ U - \dfrac{2q}{x^2+y^2} = 0 \longrightarrow x^2+y^2 = \dfrac{2q}{U} \end{cases}$$

∴ $\boxed{x^2+y^2 = \frac{2q}{U} \text{ の円 または } y=0 \text{ の直線}}$

[3] (1) $P_B = \rho g h = 1.0\cdot 10^3 \cdot 9.8 \cdot 0.2 = 19.6\cdot 10^2\,(\mathrm{Pa}) = \boxed{1.96\,(\mathrm{kPa})}$

(2) $Q = \bar{u}A$ より,

$$\bar{u} = \frac{Q}{A} = \frac{4Q}{\pi d^2} = \frac{\cancel{4}\cdot \cancel{3.14}\cdot 10^{-6}}{\cancel{\pi}\cdot 4^2}\cdot \cancel{10^6} = \boxed{0.25\,(\mathrm{m/s})}$$

(3) $x^* = \frac{x}{D}$, $u^* = \frac{u}{\bar{u}}$, $t^* = \frac{t}{\frac{D}{\bar{u}}}$, $p^* = \frac{p}{\Delta P}$ より,

$$x = Dx^*, \quad u = \bar{u}u^*, \quad t = \frac{D}{\bar{u}}t^*, \quad p = \Delta P\, p^*$$

$$\frac{\bar{u}}{D/\bar{u}}\frac{\partial u^*}{\partial t^*} + \bar{u}u^*\frac{\bar{u}\,\partial u^*}{D\,\partial x^*} = -\frac{1}{\rho}\frac{\Delta P\,\partial p^*}{L\,\partial x^*} + \nu\frac{\partial}{\partial x}\frac{\partial u}{\partial x}$$

$$\frac{\bar{u}^2}{D}\frac{\partial u^*}{\partial t^*} + \frac{\bar{u}^2}{D}u^*\frac{\partial u^*}{\partial x^*} = -\frac{\Delta P}{\rho L}\frac{\partial p^*}{\partial x^*} + \frac{\nu\bar{u}}{D^2}\frac{\partial^2 u^*}{\partial x^{*2}}$$

$$\boxed{\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} = -\frac{\Delta P}{\rho u^2}\frac{D}{L}\frac{\partial p^*}{\partial x^*} + \frac{\nu}{\bar{u}D}\frac{\partial^2 u^*}{\partial x^{*2}}}$$

(4) $R_e = \boxed{\frac{\bar{u}D}{\nu}}$, $\lambda = \boxed{\frac{2\Delta P}{\rho\bar{u}^2}\frac{D}{L}}$

(5) $Re = \dfrac{\bar{u}D}{\nu} = \dfrac{0.25 \cdot 4.0 \cdot 10^{-3}}{1.0 \cdot 10^{-6}} = \boxed{1.0 \cdot 10^{3}}$

図4より，Reの値から，

$\lambda = \dfrac{64}{Re} = \boxed{6.4 \cdot 10^{-2}}$

(6) $P_A = P_B + \Delta P$

$\lambda = \dfrac{2\Delta P}{\rho \bar{u}^2} \cdot \dfrac{D}{L} = 6.4 \cdot 10^{-2}$ より，

$$\Delta P = \frac{\rho \bar{u}^2 L}{2D} \cdot 6.4 \cdot 10^{-2} = \frac{1.0 \cdot 10^3 \cdot 0.25^2 \cdot 1.6}{\cancel{2 \cdot 4.0 \cdot 10^{-3}}} \cdot \overset{8}{\cancel{6.4 \cdot 10^{-2}}} = 0.8\,(kPa)$$

$P_A = 1.96 + 0.8 = 2.76\,(kPa)$

$P_A = \rho g h_A$ より，

$\therefore h_A = \dfrac{P_A}{\rho g} = \dfrac{2.76 \cdot 10^3}{1.0 \cdot 10^3 \cdot 9.8} = 0.2816\cdots\cdots \fallingdotseq \boxed{0.282\,(m)}$