Date . . No.

数学 4回目

[1] $\int_{-1}^{1} \frac{dx}{\sqrt[3]{(x-a)^2}}$

$x - a = t$ とおく

$dx = dt$

x	$-1 \to 1$
t	$-1-a \to 1-a$

$$(与式) = \int_{-1-a}^{1-a} t^{-\frac{2}{3}} dt = 3\left[t^{\frac{1}{3}}\right]_{-1-a}^{1-a} = \boxed{3\left(\sqrt[3]{1-a} - \sqrt[3]{-1-a}\right)}$$

[2] $\frac{du}{dt} = g - ku^2$

$$\frac{du}{dt} = -k\left(u^2 - \frac{g}{k}\right)$$

$$\frac{du}{u^2 - \frac{g}{k}} = -k\,dt$$

$$\frac{1}{2}\sqrt{\frac{k}{g}}\left(-\frac{1}{u+\sqrt{\frac{g}{k}}} + \frac{1}{u-\sqrt{\frac{g}{k}}}\right)du = -k\,dt$$

$$\log\left|\frac{u-\sqrt{\frac{g}{k}}}{u+\sqrt{\frac{g}{k}}}\right| = -2\sqrt{gk}\,t + C$$

$$\frac{u-\sqrt{\frac{g}{k}}}{u+\sqrt{\frac{g}{k}}} = Ce^{-2\sqrt{gk}\,t}$$

$$u - \sqrt{\frac{g}{k}} = Ce^{-2\sqrt{gk}\,t}\left(u + \sqrt{\frac{g}{k}}\right)$$

$$\left(1 - Ce^{-2\sqrt{gk}\,t}\right)u = \sqrt{\frac{g}{k}}\left(1 + Ce^{-2\sqrt{gk}\,t}\right)$$

$$u(t) = \sqrt{\frac{g}{k}}\,\frac{1+Ce^{-2\sqrt{gk}\,t}}{1-Ce^{-2\sqrt{gk}\,t}}$$

$$u(0) = \sqrt{\frac{g}{k}}\,\frac{1+C}{1-C} = 0 \to C = -1$$

$$u(t) = \boxed{\sqrt{\frac{g}{k}}\,\frac{1-e^{-2\sqrt{gk}\,t}}{1+e^{-2\sqrt{gk}\,t}}}$$

$$\lim_{t\to\infty} u(t) = \boxed{\sqrt{\frac{g}{k}}}$$

[3]

$$\begin{vmatrix} 1 & x^3 & x^2 & x \\ x^3 & x^2 & x & 1 \\ x^2 & x & 1 & x^3 \\ x & 1 & x^3 & x^2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 & x^2 & x \\ 1 & x^2 & x & 1 \\ x^2 & x & 1 & 1 \\ x & 1 & 1 & x^2 \end{vmatrix} \longrightarrow \begin{vmatrix} 1 & 1 & x^2 & x \\ 0 & x^2-1 & x-x^2 & 1-x \\ 0 & x-x^2 & 1-x^4 & 1-x^3 \\ 0 & 1-x & 1-x^3 & 0 \end{vmatrix}$$

$$\rightarrow \begin{vmatrix} x^2-1 & x-x^2 & 1-x \\ x-x^2 & 1-x^4 & 1-x^3 \\ 1-x & 1-x^3 & 0 \end{vmatrix} \longrightarrow \begin{vmatrix} x^2-1 & x-x^2 & 1-x \\ x-x^2 & 1-x & 0 \\ 1-x & 0 & 0 \end{vmatrix}$$

$$= -(1-x)^3 = -(1-3x+3x^2-x^3) = x^3-3x^2+3x-1$$

$$= \boxed{-3x^2+3x}$$

[4] $\frac{dy}{dt} + 3\int_0^t y\,dt = \sin t$

ラプラス変換して,

$$sY(s) - f(0) + 3\left(\frac{1}{s}Y(s) + \frac{1}{s}\int_0^0 y\,dt\right) = \frac{1}{s^2+1}$$

$$sY(s) + \frac{3}{s}Y(s) = \frac{1}{s^2+1}$$

$$\frac{s^2+3}{s}Y(s) = \frac{1}{s^2+1}$$

$$Y(s) = \frac{s}{(s^2+1)(s^2+3)}$$

$$\frac{s}{(s^2+1)(s^2+3)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+3} = \frac{(As+B)(s^2+3)+(Cs+D)(s^2+1)}{(s^2+1)(s^2+3)}$$

$$= \frac{As^3+3As+Bs^2+3B+Cs^3+Cs+Ds^2+D}{(s^2+1)(s^2+3)}$$

$$\rightarrow \begin{cases} A+C=0 \rightarrow C=-A \\ B+D=0 \\ 3A+C=1 \rightarrow A=\frac{1}{2},\ C=-\frac{1}{2},\ B=0,\ D=0 \\ 3B+D=0 \end{cases}$$

$$Y(s) = \frac{1}{2}\frac{s}{s^2+1} - \frac{1}{2}\frac{s}{s^2+3}$$

$$\therefore y(t) = \boxed{\frac{1}{2}(\cos t - \cos\sqrt{3}t)}$$