H22

Date . . No.

材料力学 4回目

[1] (1) $\sigma_1 = \frac{P}{A_1} = \boxed{\frac{4P}{\pi d_1^2}}$, $\sigma_2 = \frac{P}{A_2} = \boxed{\frac{4P}{\pi d_2^4}}$

$$\lambda = \frac{Pl}{EA_1} + \frac{Pl}{EA_2} = \boxed{\frac{4Pl}{\pi E}\left(\frac{1}{d_1^2} + \frac{1}{d_2^2}\right)}$$

(2) 最大応力が生じるのは，上端において生じるので，

$$(P_{max})_2 = P + \rho A_1 l g + \rho A_2 l g$$

$$\therefore (\sigma_{max})_2 = \frac{(P_{max})_2}{A_2} = \frac{4Pl}{\pi d_2^2} + \rho l g \cdot \frac{d_1^2}{d_2^2} + \rho l g = \boxed{\frac{4Pl}{\pi d_2^2} + \rho l g\left(1 + \frac{d_1^2}{d_2^2}\right)}$$

(3) $P_1(x) = P + \rho A_1 x g$

$$\sigma = \frac{P}{A} = E\frac{d\lambda}{dx} \rightarrow d\lambda = \frac{P}{EA}dx$$

$$\lambda_1 = \int_0^l \frac{P_1(x)}{EA_1}dx = \frac{1}{EA_1}\int_0^l (P + \rho A_1 g x)\,dx = \frac{1}{EA_1}\left[Px + \frac{1}{2}\rho A_1 g x^2\right]_0^l$$

$$= \frac{1}{EA_1}\left(Pl + \frac{1}{2}\rho A_1 g l^2\right) = \frac{Pl}{EA_1} + \frac{\rho l^2 g}{2E} = \frac{4Pl}{\pi E d_1^2} + \frac{\rho g l^2}{2E}$$

$$P_2(x) = P + \rho A_1 l g + \rho A_2 (x - l) g$$

$$\lambda_2 = \int_l^{2l} \frac{1}{EA_2}\{P + \rho A_1 l g + \rho A_2 g (x - l)\}dx$$

$$= \frac{1}{EA_2}\left[Px + \rho A_1 l g x + \frac{1}{2}\rho A_2 g (x-l)^2\right]_l^{2l}$$

$$= \frac{1}{EA_2}\left\{\left(2Pl + 2\rho A_1 g l^2 + \frac{1}{2}\rho A_2 g l^2\right) - \left(Pl + \rho A g l^2 + 0\right)\right\}$$

$$= \frac{1}{EA_2}\left(Pl + \rho A_1 g l^2 + \frac{1}{2}\rho A_2 g l^2\right)$$

$$= \frac{4Pl}{\pi E d_2^2} + \frac{\rho g l^2}{E}\cdot\frac{d_1^2}{d_2^2} + \frac{\rho g l^2}{2E}$$

$$\therefore \lambda = \lambda_1 + \lambda_2 = \frac{4Pl}{\pi E d_1^2} + \frac{\rho g l^2}{2E} + \frac{4Pl}{\pi E d_2^2} + \frac{\rho g l^2}{E}\cdot\frac{d_1^2}{d_2^2} + \frac{\rho g l^2}{2E}$$

$$= \frac{4Pl}{\pi E}\left(\frac{1}{d_1^2} + \frac{1}{d_2^2}\right) + \frac{\rho g l^2}{E} + \frac{\rho g l^2}{E}\cdot\frac{d_1^2}{d_2^2}$$

$$= \boxed{\frac{4Pl}{\pi E}\left(\frac{1}{d_1^2} + \frac{1}{d_2^2}\right) + \frac{\rho g l^2}{E}\left(1 + \frac{d_1^2}{d_2^2}\right)}$$

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[2] (1) $M_B = -w\cdot\frac{l}{2}\cdot\frac{l}{4} = \boxed{-\frac{1}{8}wl^2}$

(2) $0 \leqq x < l$ のとき,

$M_x = R_A x - M_A - wx\cdot\frac{1}{2}x = \boxed{-\frac{1}{2}wx^2 + R_A x - M_A}$

(3) $\frac{d^2y}{dx^2} = -\frac{M_x}{EI} \rightarrow EI\frac{d^2y}{dx^2} = \frac{1}{2}wx^2 - R_A x + M_A$

$EI\frac{dy}{dx} = \frac{1}{6}wx^3 - \frac{1}{2}R_A x^2 + M_A x + C_1$

$EIy = \frac{1}{24}wx^4 - \frac{1}{6}R_A x^3 + \frac{1}{2}M_A x^2 + C_1 x + C_2$

境界条件より, $x=0$ のとき, $\frac{dy}{dx}=0$ $x=0$ のとき, $y=0$

$x = l$ のとき, $y = 0$

$EI\left(\frac{dy}{dx}\right)_{x=0} = 0 - 0 + 0 + C_1 = 0 \rightarrow C_1 = 0$

$EIy_{x=0} = 0 - 0 + 0 + 0 + C_2 = 0 \rightarrow C_2 = 0$

$EIy_{x=l} = \frac{1}{24}wl^4 - \frac{1}{6}R_A l^3 + \frac{1}{2}M_A l^2 = 0 \rightarrow M_A = \frac{1}{3}R_A l - \frac{1}{12}wl^2$ — ①

また, B点のまわりのモーメントのつりあいより,

$M_B = R_A l - M_A - \frac{1}{2}wl^2 = -\frac{1}{8}wl^2$ — ②

②に①を代入

$R_A l - \left(\frac{1}{3}R_A l - \frac{1}{12}wl^2\right) - \frac{1}{2}wl^2 = -\frac{1}{8}wl^2$

$\frac{2}{3}R_A l = \frac{7}{24}wl^2 \rightarrow \therefore R_A = \boxed{\frac{7}{16}wl}$

$\therefore M_A = \frac{7}{48}wl^2 - \frac{1}{12}wl^2 = \boxed{\frac{1}{16}wl^2}$

(4) 力のつりあいより, $R_A + R_B = \frac{3}{2}wl$

$\therefore R_B = \frac{3}{2}wl - \frac{7}{16}wl = \boxed{\frac{17}{16}wl}$

$0 \leqq x < l$ のとき,

$V_x - R_A + wx = 0 \rightarrow V_x = \frac{7}{16}wl - wx = w\left(\frac{7}{16}l - x\right)$

$M_x = -\frac{1}{2}wx^2 + \frac{7}{16}wlx - \frac{1}{16}wl^2$

$= -\frac{1}{2}w\left(x - \frac{7}{16}l\right)^2 + \frac{17}{512}wl^2$

$l \leq x < \frac{3}{2}l$ のとき,

$V_x - R_A + wx - R_B = 0 \rightarrow V_x = \frac{3}{2}wl - wx = w\left(\frac{3}{2}l - x\right)$

$M_x = R_A x - M_A - \frac{1}{2}wx^2 + R_B(x - l)$

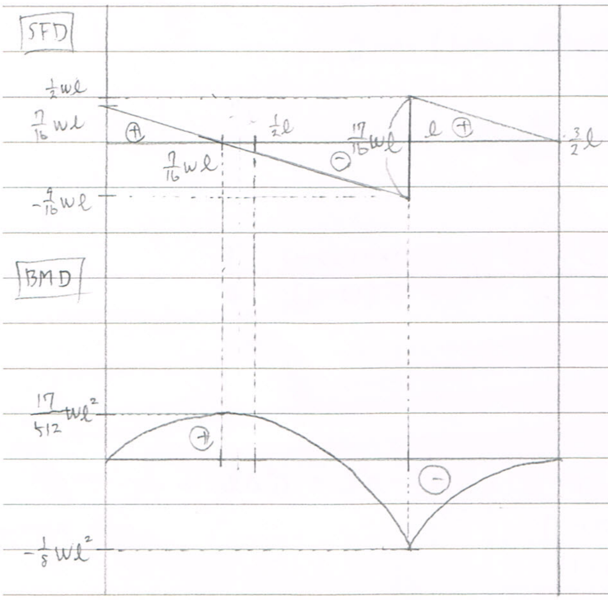
$= \frac{3}{2}wlx - \frac{1}{16}wl^2 - \frac{1}{2}wx^2 - \frac{17}{16}wl^2$

$= -\frac{1}{2}w(x^2 - 3lx) - \frac{9}{8}wl^2$

$= -\frac{1}{2}w\left(x - \frac{3}{2}l\right)^2$

$x = \frac{3}{2}l$ のとき, $M_x = 0$

$\frac{1}{2}wl - \frac{17}{16}wl = -\frac{9}{16}wl$



[3] (1) $\sigma = \frac{P}{A} = E\varepsilon$ より, $\varepsilon = \frac{P}{EA}$

$\therefore \varepsilon_L = \boxed{\frac{P}{Ea^2}}$

(2) $\nu = -\frac{\varepsilon_T}{\varepsilon_L} \rightarrow \varepsilon_T = -\nu\varepsilon_L = \boxed{-\frac{P\nu}{Ea^2}}$

(3) $V = a^2 l$

$V + \Delta V = (a + \Delta a)^2(l + \Delta l) = (a^2 + 2a\Delta a + \Delta a^2)(l + \Delta l)$

$= a^2 l + a^2\Delta l + 2al\Delta a + 2a\Delta a\Delta l + l\Delta a^2 + \Delta a^2\Delta l$

長さの増加量ΔaとΔlは微小より, $\Delta a\Delta l = 0$, $\Delta a^2 = 0$ とすると,

$V + \Delta V = a^2 l + a^2\Delta l + 2al\Delta a$

$$\Delta V = (V+\Delta V) - V = a^2 l + a^2 \Delta l + 2al\Delta a - a^2 l = a^2\Delta l + 2al\Delta a$$

$$\frac{\Delta V}{V} = \frac{a^2\Delta l + 2al\Delta a}{a^2 l} = \frac{\Delta l}{l} + 2\frac{\Delta a}{a}$$

ここで，$\varepsilon_L = \frac{\Delta l}{l}$ ，$\varepsilon_T = \frac{\Delta a}{a}$ より

$$\frac{\Delta V}{V} = \varepsilon_L + 2\varepsilon_T = \frac{P}{Ea^2} - \frac{2P\nu}{Ea^2} = \boxed{\frac{P}{Ea^2}(1-2\nu)}$$

(4) 棒の側面を拘束しながら ⟶ 两軸引張り

$$\varepsilon_L = \frac{1}{E}\{\sigma_L - 2\nu\sigma_T\}, \quad \varepsilon_T = \frac{1}{E}\{\sigma_T - \nu(\sigma_L + \sigma_T)\} = 0$$

$$\varepsilon_T = \frac{1}{E}\{(1-\nu)\sigma_T - \nu\sigma_L\} = 0 \longrightarrow \sigma_T = \frac{\nu}{1-\nu}\sigma_L$$

以上より，

$$\varepsilon_L = \frac{1}{E}\left\{\sigma_L - 2\nu\cdot\frac{\nu}{1-\nu}\sigma_L\right\} = \frac{1}{E}\cdot\frac{1-\nu-2\nu^2}{1-\nu}\sigma_L = \frac{(\nu+1)(2\nu-1)}{E(\nu-1)}\cdot\frac{P}{a^2}$$

$$= \boxed{\frac{(\nu+1)(2\nu-1)}{Ea^2(\nu-1)}P}$$

(5) $V = a^2 l$，$V+\Delta V = a^2(l+\Delta l) \longrightarrow \Delta V = a^2(l+\Delta l) - a^2 l = a^2\Delta l$

$$\therefore \frac{\Delta V}{V} = \frac{a^2\Delta l}{a^2 l} = \frac{\Delta l}{l} = \varepsilon_L = \boxed{\frac{(\nu+1)(2\nu-1)}{Ea^2(\nu-1)}P}$$