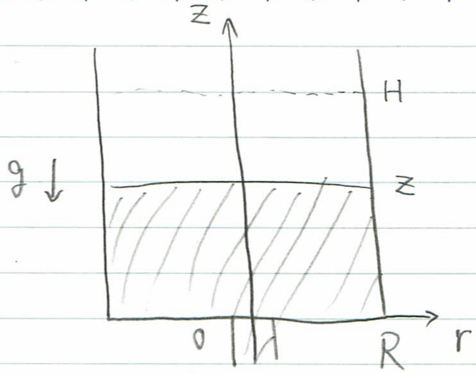
H30 流体力学



[1] (1) ベルヌーイの定理より

$$\frac{1}{2}w^2 = gz$$

$$w = \sqrt{2gz}$$

(2)

$$dV = wa\,dt$$

$$dV = a\sqrt{2gz}\,dt$$

(3)

$$dV = -\pi R^2 dz$$

(4) (2), (3) より

$$a\sqrt{2gz}\,dt = -\pi R^2 dz$$

$$dt = -\frac{\pi R^2}{a\sqrt{2gz}}dz$$

$$T = -\frac{\pi R^2}{a\sqrt{2g}}\int_H^0 \frac{dz}{\sqrt{z}}$$

$$= -\frac{\pi R^2}{a\sqrt{2g}}\left[2\sqrt{z}\right]_H^0 = \frac{\pi R^2}{a}\sqrt{\frac{2H}{g}}$$

[2] $W(z) = -z^2$

(1) $W(z) = -(x+iy)^2$

$$= -(x^2-y^2) - 2ixy$$

よって

$$\Phi = -(x^2-y^2), \quad \psi = -2xy$$

これを使って

$$u = \frac{\partial\Phi}{\partial x} = -2x, \quad v = \frac{\partial\Phi}{\partial y} = 2y$$

したがって $\frac{dx}{u} = \frac{dy}{v}$ より $-\frac{dx}{x} = \frac{dy}{y}$

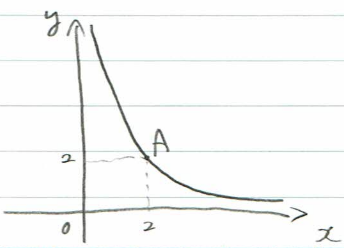
$$-\log x = \log y + C$$

$$xy = C$$

点(2, 2)のとき $C = 4$ より

$$xy = 4$$

(2)



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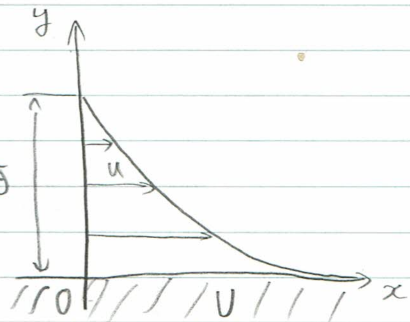
(3) 線分 OA の長さは

$$\sqrt{2^2+2^2} = 2\sqrt{2}$$

$$u = \sqrt{(-2x)^2+(2y)^2} = 2\sqrt{x^2+y^2} = 2\sqrt{x^2+\frac{16}{x^2}} = 2\sqrt{\frac{x^4+16}{x^2}}$$

[3] (1) 連続の方程式

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$



(2) x方向 NS方程式

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

y方向 NS方程式

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

(3)

$$\frac{\partial P}{\partial x}=0,\quad v=0$$

連続の方程式

$$\frac{\partial u}{\partial x}=0$$

x方向 NS方程式

$$\frac{\partial u}{\partial t}=\nu\frac{\partial^2 u}{\partial y^2}$$

y方向 NS方程式

$$\frac{\partial P}{\partial y}=0$$

(4) $u=u(y)$　$P=P(x)$

$t<0$ では $u(t,y)=0$

$t\geq 0$ では $y=0$ で $u=U$

$y\to\infty$ で $u\to 0$

$$\delta = 2\sqrt{\nu t}$$

(5) $\eta=\frac{y}{\delta}$, $u(y,t)=U_0 f(\eta)$

$$\frac{\partial u}{\partial t}=U_0\frac{df}{d\eta}\left(-\frac{y}{\delta(t)^2}\cdot\frac{d\delta(t)}{dt}\right)$$

$$\nu\frac{\partial^2 u}{\partial y^2}=\nu U_0\frac{\partial}{\partial y}\left(\frac{df}{d\eta}\cdot\frac{\partial\eta}{\partial y}\right)$$

$$=\nu U_0\frac{d^2 f}{d\eta^2}\cdot\frac{1}{\delta(t)^2}$$

よって

$$f''+2\eta f'=0$$