H24

数学　4回目

[1] (1) $y = \cos^{-1}(\sin x)$

$\cos y = \sin x$

$-\sin y\,dy = \cos x\,dx$

$$\frac{dx}{dy} = -\frac{\sin y}{\cos x}$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = -\frac{\cos x}{\sin y} = -\frac{\cos x}{\sqrt{1-\cos^2 y}} = -\frac{\cos x}{\sqrt{1-\sin^2 x}} = \boxed{-1}$$

(2) $\begin{cases} x = a(\theta - \sin\theta) \rightarrow \frac{dx}{d\theta} = a(1-\cos\theta) \\ y = a(1-\cos\theta) \rightarrow \frac{dy}{d\theta} = a\sin\theta \end{cases}$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1-\cos\theta)} = \boxed{\frac{\sin\theta}{1-\cos\theta}}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx} = \frac{\cos\theta(1-\cos\theta)-\sin^2\theta}{(1-\cos\theta)^2}\cdot\frac{1}{a(1-\cos\theta)}$$

$$= \frac{-\cancel{1-\cos\theta}}{(1-\cos\theta)^2}\cdot\frac{1}{a(\cancel{1-\cos\theta})} = \boxed{-\frac{1}{a(1-\cos\theta)^2}}$$

[2] (1) $\frac{dy}{dx} + \left(\frac{y}{x} - e^x\right) = 0$

$dy + \left(\frac{y}{x} - e^x\right)dx = 0$

$P(x,y) = \frac{y}{x} - e^x$, $Q(x,y) = 1$ とおく

$\frac{\partial P}{\partial y} = \frac{1}{x}$, $\frac{\partial Q}{\partial x} = 0$

$\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$ より，完全微分形ではない

$$\frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = \frac{1}{x} \equiv \lambda(x)$$

積分因子：$\mu(x) = e^{\int\lambda(x)dx} = e^{\log x} = \boxed{x}$

(2) $\mu(x)P(x,y) = y - xe^x \longrightarrow P(x,y)$

$\mu(x)Q(x,y) = x \longrightarrow Q(x,y)$

一般解：$u(x,y) = \int_{x_0}^{x} P(x,y)\,dx + \int_{y_0}^{y} Q(x_0,y)\,dy$

$= \int_{x_0}^{x}(y - xe^x)dx + \int_{y_0}^{y} x_0\,dy$

$= \int_{x_0}^{x} y\,dx - \int_{x_0}^{x} x(e^x)'dx + x_0(y-y_0)$

$= y(x-x_0) - [xe^x]_{x_0}^{x} + \int_{x_0}^{x} e^x + x_0(y-y_0)$

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$= xy - \cancel{x_0 y} - xe^x + x_0 e^{x_0} + e^x - e^{x_0} + \cancel{x_0 y} - x_0 y_0 = C_0$

$\therefore \boxed{xy - xe^x + e^x = C}$ $(\because C = C_0 - x_0 e^{x_0} + e^{x_0} + x_0 y_0$ は任意定数$)$

[3] $x_1 = {}^t[1, 2, 3]$ , $x_2 = {}^t[3, 1, 2]$ , $x_3 = {}^t[5, 0, k]$

(1) $x_1, x_2, x_3$は一次従属より，

$c_1 x_1 + c_2 x_2 + c_3 x_3 = 0$

$$\begin{cases} c_1 + 3c_2 + 5c_3 = 0 & \rightarrow c_3 = c_1 \\ 2c_1 + c_2 = 0 & \rightarrow c_2 = -2c_1 \\ 3c_1 + 2c_2 + kc_3 = 0 & \rightarrow 3c_1 - 4c_1 + kc_1 = 0 \end{cases}$$

$(k-1)c_1 = 0 \rightarrow \therefore k = \boxed{1}$

(2) $x_3 = \alpha x_1 + \beta x_2$

$$\begin{cases} \alpha + 3\beta = 5 & \rightarrow \alpha = -1 \rightarrow \beta = 2 \\ 2\alpha + \beta = 0 & \rightarrow \beta = -2\alpha \\ 3\alpha + 2\beta = 1 \end{cases}$$

$\therefore \boxed{x_3 = -x_1 + 2x_2}$

(3)

$$x_1 \times x_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{vmatrix} = 4i + k + 9j - 6k - 2j - 3i = i + 4j - 5k = \begin{bmatrix} 1 \\ 7 \\ -5 \end{bmatrix}$$

$$x_1 \times x_3 = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 5 & 0 & 1 \end{vmatrix} = 2i + 15j - 10k - j = 2i + 14j - 10k = 2\begin{bmatrix} 1 \\ 7 \\ -5 \end{bmatrix}$$

$x + 7y - 5z = 0$

平面の式で一致したので，$\boxed{x_1 + 7x_2 - 5x_3 = 0}$

(4) $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ , $b = \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}$ , $x = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ とおく

拡大係数行列

$$[A \ \ b] = \left[\begin{array}{ccc|c} 1 & 3 & 5 & -1 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 4 \end{array}\right] = B$$

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$$P_{31}(-3)P_{21}(-2)B = \left[\begin{array}{ccc|c} 1 & 3 & 5 & -1 \\ 0 & -5 & -10 & 5 \\ 0 & -7 & -14 & 7 \end{array}\right] = B_1$$

$$P_{32}(7)P_{12}(-3)P_2(-\tfrac{1}{5})B_1 = \left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \begin{cases} a - c = 0 \\ b + 2c = 0 \end{cases}$$

$c = k$ とおくと, $a = k$, $b = -2k$

$$\therefore \boxed{\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + k \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}} \quad (k\text{は任意定数})$$

[4] (1) $f(t) = \boxed{\frac{1}{a}t\{u(t) - u(t-a)\}}$

(2)
$$\begin{aligned}
\mathcal{L}\{f(t)\} &= \mathcal{L}\left[\frac{1}{a}t\{u(t) - u(t-a)\}\right] \\
&= \frac{1}{a}\int_0^\infty e^{-st}t\{u(t) - u(t-a)\}dt \\
&= \frac{1}{a}\int_0^\infty te^{-st}dt - \frac{1}{a}\int_0^\infty te^{-st}u(t-a)dt \\
&= \frac{1}{a}\int_0^\infty t(-\frac{1}{s}e^{-st})'dt - \frac{1}{a}\int_a^\infty t(-\frac{1}{s}e^{-st})'dt \\
&= -\frac{1}{sa}\left[te^{-st}\right]_0^\infty + \frac{1}{sa}\int_0^\infty e^{-st}dt + \frac{1}{sa}\left[te^{-st}\right]_a^\infty - \frac{1}{sa}\int_a^\infty e^{-st}dt \\
&= 0 - \frac{1}{s^2a}\left[e^{-st}\right]_0^\infty + \frac{1}{sa}(0 - ae^{-sa}) + \frac{1}{s^2a}\left[e^{-st}\right]_a^\infty \\
&= +\frac{1}{s^2a} - \frac{1}{s}e^{-sa} - \frac{1}{s^2a}e^{-sa} \\
&= \boxed{-\frac{1}{s}e^{-sa} - \frac{1}{s^2a}(e^{-sa} - 1)}
\end{aligned}$$

(3) $y' + y = f(t)$

ラプラス変換して,

$$sY(s) - y(0) + Y(s) = -\frac{1}{s}e^{-sa} - \frac{1}{s^2a}(e^{-sa} - 1)$$

$$(s+1)Y(s) = -\frac{1}{s}e^{-sa} - \frac{1}{s^2a}e^{-sa} + \frac{1}{s^2a}$$

$$Y(s) = -\frac{e^{-sa}}{s(s+1)} - \frac{e^{-sa}}{as^2(s+1)} + \frac{1}{as^2(s+1)}$$

$\frac{1}{s^2(s+1)}$ について,

$s+1$ の因子について, $\left.\frac{1}{s^2}\right|_{s=-1} = 1$

$s^2$ の因子について, $\left.\frac{1}{s+1}\right|_{s=0} = 1$

$s$ の因子について, $\left.\frac{d}{ds}\left(\frac{1}{s+1}\right)\right|_{s=0} = \left.\frac{-1}{(s+1)^2}\right|_{s=0} = -1$

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$$Y(s) = -e^{-sa}\left(\frac{1}{s} - \frac{1}{s+1}\right) - \frac{e^{-sa}}{a^2}\left(\frac{1}{s+1} + \frac{1}{s^2} - \frac{1}{s}\right) + \frac{1}{a}\left(\frac{1}{s+1} + \frac{1}{s^2} - \frac{1}{s}\right)$$

$$y(t) = \boxed{-\{1 - e^{-(t-a)}\}\mathcal{U}(t-a) - \frac{1}{a}\left(e^{-(t-a)} + t - a - 1\right)\mathcal{U}(t-a) + \frac{1}{a}\left(e^{-t} + t - 1\right)}$$