H29 流体力学

[1] (1)

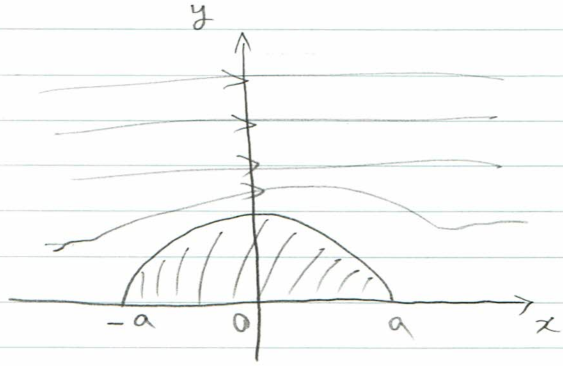
$$W(z) = Uz - \left(-\frac{\mu}{z}\right)$$

$$W(z) = U(\cos\theta + i\sin\theta) + \frac{\mu}{z}$$

$z = a$ のとき $W(z) = 0$ となる

$$0 = Ua + \frac{\mu}{a}$$

$$\therefore \mu = -Ua^2$$



(2) $$W(z) = U\left(z + \frac{a^2}{z}\right) = U\left(re^{i\theta} + \frac{a^2}{r}e^{-i\theta}\right)$$

$$= U\left(r + \frac{a^2}{r}\right)\cos\theta + iU\left(r - \frac{a^2}{r}\right)\sin\theta$$

$$\Phi = U\left(r + \frac{a^2}{r}\right)\cos\theta, \quad \Psi = U\left(r - \frac{a^2}{r}\right)\sin\theta$$

よって

$$u = \frac{1}{r}\frac{\partial \Phi}{\partial \theta} = -U\left(1 + \frac{a^2}{r^2}\right)\sin\theta$$

ベルヌーイの定理より

$$P(\theta) = \frac{1}{2}\rho U^2 + p_0 - \frac{1}{2}\rho u_\theta^2$$

$$= \frac{1}{2}\rho U^2 + p_0 - \frac{1}{2}\rho U^2\left(1 + \frac{a^2}{r^2}\right)^2\sin^2\theta$$

$$= \frac{1}{2}\rho U^2\left\{1 - \left(1 + \frac{a^2}{r^2}\right)^2\sin^2\theta\right\} + p_0$$

したがって

$$F_y = -\int_0^\pi aP(\theta)\sin\theta\, d\theta$$

$$\cancel{= -a\int_0^\pi \left(\frac{1}{2}\rho U^2 + p_0\right)\sin\theta\, d\theta + \frac{1}{2}\rho U^2 a\left(1 + \frac{a^2}{r^2}\right)^2\int_0^\pi \sin^3\theta\, d\theta}$$

$$\cancel{= -\left(\frac{1}{2}\rho U^2 + p_0\right)\left[-\cos\theta\right]_0^\pi + \frac{1}{2}\rho U^2\left(1 + \frac{a^2}{r^2}\right)^2\left[x - \frac{x^3}{3}\right]_1^{-1}}$$

$$\cancel{= -a(\rho U^2 + 2p_0) - \frac{2}{3}\rho U^2 a}$$

$$= -a\left(\frac{5}{3}\rho U^2\right.$$

$-\frac{4}{3}$

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[2] (1) 連続の式より

$$Q = V_a A_a = V_d A_d$$

ここで $A_a = \frac{\pi d_1^2}{4}$, $A_d = \frac{\pi d_2^2}{4}$

よって

$$V_d = \frac{d_1^2}{d_2^2} V_a$$

(2) 運動量保存則より

$$p_a\left(\frac{\pi d_2^2}{4} - \frac{\pi d_1^2}{4}\right)Dt + p_a \frac{\pi d_1^2}{4} Dt - p_d \frac{\pi d_2^2}{4} Dt + m_1 V_a - m_2 V_d = 0$$

$$(P_a - P_d)\cdot\frac{\pi d_2^2}{4} Dt + \rho\cdot\frac{\pi d_2^2}{4} V_a V_d Dt - \rho\cdot\frac{\pi d_2^2}{4} V_d^2 = 0$$

$$P_a - P_d = -\rho\left(\frac{d_1^2}{d_2^2} V_a^2 - \frac{d_1^4}{d_2^4} V_a^2\right)$$

$$-P_d = -P_a - \frac{\rho V_a^2 d_1^2}{d_2^4}(d_2^2 - d_1^2)$$

$$P_d = P_a + \frac{\rho V_a^2 d_1^2}{d_2^4}(d_2^2 - d_1^2)$$

(3) ベルヌーイの定理より

$$P_g = P_d + \frac{1}{2}\rho(V_d^2 - V_a^2)$$

$$= P_a + \frac{\rho V_a^2 d_1^2}{d_2^4}(d_2^2 - d_1^2) + \frac{1}{2}\rho V_a^2\left(\frac{d_1^4}{d_2^4} - 1\right)$$

$$= P_a + \frac{\rho V_a^2 d_1^2}{d_2^4}(d_2^2 - d_1^2) + \frac{\rho V_a^2}{2 d_2^4}(d_1^4 - d_2^4)$$

$$= P_a + \frac{\rho V_a^2}{2 d_2^4}(-d_1^4 + 2 d_1^2 d_2^2 - d_2^4)$$

$$= P_a - \frac{\rho V_a^2}{2 d_2^4}(d_1^2 - d_2^2)^2$$

[3] (1) $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(2) x方向

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

y方向

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

(3) $\frac{\partial P}{\partial x} = -G$, $t = 0$, $v = 0$ より　また $\nu = \frac{\mu}{\rho}$

$$\frac{\partial u}{\partial x} = 0$$

x: $$\frac{\partial^2 u}{\partial y^2} = -\frac{G}{\nu\rho} = -\frac{G}{\mu}$$

y: $$\frac{\partial P}{\partial y} = 0$$

(4) (3) より　$u = u(y)$, $P = P(x)$

$$u(y) = -\frac{G}{2\mu}y^2 + C_1 y + C_2$$

ここで

$$u(2H) = -\frac{2G}{\mu}H^2 + 2C_1 H + C_2 = 0$$

$$u(-2H) = -\frac{2G}{\mu}H^2 - 2C_1 H + C_2 = 0$$

$$\therefore C_1 = 0, \quad C_2 = \frac{2G}{\mu}H^2$$

よって $$u(y) = -\frac{G}{2\mu}y^2 + \frac{2G}{\mu}H^2 = -\frac{G}{2\mu}\left\{y^2 - (2H)^2\right\}$$

(5) $H < y \leq 2H$

$$u(y) = -\frac{G}{2\mu_0}\left\{y^2 - (2H)^2\right\}$$

$y \leq H$ のとき

$$u(y) = -\frac{G}{4\mu_0}y^2 + C_1 y + C_2$$

$$u(H) = -\frac{G}{4\mu_0}H^2 + C_1 H + C_2 = \frac{3G}{4\mu_0}H^2$$

$$u(-H) = -\frac{G}{4\mu_0}H^2 - C_1 H + C_2 = \frac{3G}{4\mu_0}H^2$$

$$\therefore C_1 = 0, \quad C_2 = \frac{7G}{4\mu_0}H^2$$

したがって $$u(y) = -\frac{G}{4\mu_0}\left(y^2 - 7H^2\right)$$