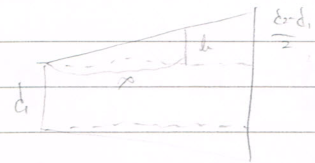
Date . . No.

材料力学　4回目

[1]　(1) $\lambda_1 = \dfrac{P l_1}{E A_1} = \boxed{\dfrac{4 P l_1}{\pi E d_1^2}}$



$h : \dfrac{d_2-d_1}{2} = x : l_2$

$h = \dfrac{d_2-d_1}{2 l_2} x$

$d = d_1 + 2h = d_1 + \dfrac{d_2-d_1}{l_2} x$

$\lambda_2 = \int_0^{l_2} \dfrac{P}{E A_2} dx = \dfrac{4P}{\pi E} \int_0^{l_2} d^{-2} dx = \dfrac{4P}{\pi E} \int_0^{l_2} \left(d_1 + \dfrac{d_2-d_1}{l_2} x \right)^{-2} dx$

$= -\dfrac{4P}{\pi E} \cdot \dfrac{l_2}{d_2-d_1} \left[\left(d_1 + \dfrac{d_2-d_1}{l_2} x \right)^{-1} \right]_0^{l_2}$

$= -\dfrac{4 P l_2}{\pi E} \cdot \dfrac{1}{d_2-d_1} \left\{ (d_1 + d_2 - d_1)^{-1} - (d_1)^{-1} \right\}$

$= -\dfrac{4 P l_2}{\pi E} \cdot \dfrac{1}{d_2-d_1} \left(\dfrac{1}{d_2} - \dfrac{1}{d_1} \right)$

$= -\dfrac{4 P l_2}{\pi E} \cdot \dfrac{1}{d_2-d_1} \cdot \dfrac{d_1-d_2}{d_1 d_2}$

$= \boxed{\dfrac{4 P l_2}{\pi E d_1 d_2}}$

(2) 軸力をP'とすると，2つの棒材の伸びは，

$\lambda_1' = \dfrac{4 P' l_1}{\pi E d_1^2} + \alpha l_1 t$ ，$\lambda_2' = \dfrac{4 P' l_2}{\pi E d_1 d_2} + \alpha l_2 t$

全体の伸びは，

$\lambda' = \lambda_1' + \lambda_2' = \dfrac{4 P' l_1}{\pi E d_1^2} + \alpha l_1 t + \dfrac{4 P' l_2}{\pi E d_1 d_2} + \alpha l_2 t = 0$

$\dfrac{4}{\pi E d_1} \left(\dfrac{l_1}{d_1} + \dfrac{l_2}{d_2} \right) P' = -\alpha t (l_1 + l_2)$

$P' = -\dfrac{\alpha t \pi E d_1^2 d_2 (l_1 + l_2)}{4 (l_1 d_2 + l_2 d_1)}$

$\therefore \sigma = \dfrac{P'}{A_1} = -\dfrac{\alpha t \cancel{\pi} E \cancel{d_1^2} d_2 (l_1 + l_2)}{\cancel{4} (l_1 d_2 + l_2 d_1)} \cdot \dfrac{\cancel{4}}{\cancel{\pi} \cancel{d_1^2}} = \boxed{-\dfrac{\alpha t E d_2 (l_1 + l_2)}{l_1 d_2 + l_2 d_1}}$

[2] (1) $w : w_1 = x : l \to w = \frac{w_1}{l}x$

$M_x = R_A x - \frac{1}{2}wx \cdot \frac{1}{3}x = \boxed{R_A x - \frac{w_1}{6l}x^3}$

(2) $EI\frac{d^2y}{dx^2} = -M_x \to EI\frac{d^2y}{dx^2} = \frac{w_1}{6l}x^3 - R_A x$

$EI\frac{dy}{dx} = \frac{w_1}{24l}x^4 - \frac{1}{2}R_A x^2 + C_1$

$EIy = \frac{w_1}{120l}x^5 - \frac{1}{6}R_A x^3 + C_1 x + C_2$

境界条件より, $x = 0$ のとき, $y = 0$　　$x = l$ のとき, $\frac{dy}{dx} = 0$

$x = l$ のとき, $y = 0$

$EIy_{x=0} = 0 - 0 + 0 + C_2 = 0 \to C_2 = 0$

$EI\left(\frac{dy}{dx}\right)_{x=l} = \frac{w_1}{24}l^3 - \frac{1}{2}R_A l^2 + C_1 = 0 \to C_1 = \frac{1}{2}R_A l^2 - \frac{w_1}{24}l^3$

$EIy_{x=l} = \frac{w_1}{120}l^4 - \frac{1}{6}R_A l^3 + C_1 l = 0$

$\frac{w_1}{120}l^4 - \frac{1}{6}R_A l^3 + l\left(\frac{1}{2}R_A l^2 - \frac{w_1}{24}l^3\right) = 0$

$\frac{1}{3}R_A l^2 = \frac{w_1}{30}l^3$

$\therefore R_A = \boxed{\frac{1}{10}w_1 l}$　　$M_{x=\frac{l}{\sqrt{5}}} = \frac{w_1}{10\sqrt{5}}l^2 - \frac{w_1}{30\sqrt{5}}l^2 = \frac{w_1}{15\sqrt{5}}l^2$

$\frac{l^3}{5\sqrt{5}}$

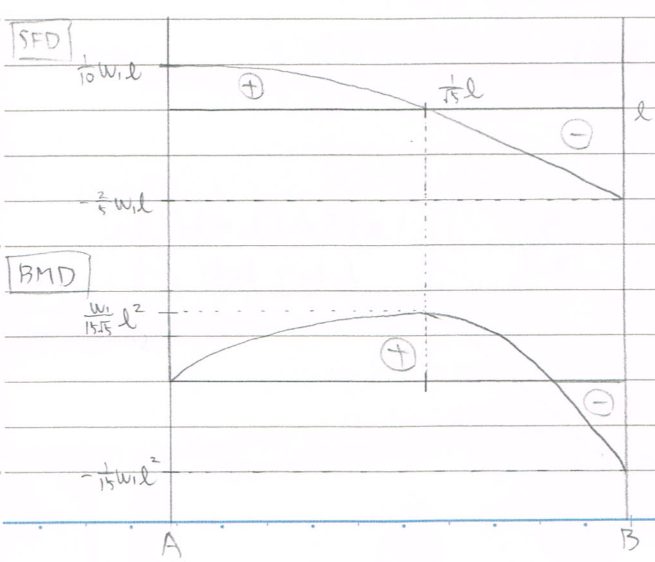
(3) 力のつりあいより,

$R_A + R_B = \frac{1}{2}w_1 l \to \therefore R_B = \frac{1}{2}w_1 l - \frac{1}{10}w_1 l = \boxed{\frac{2}{5}w_1 l}$

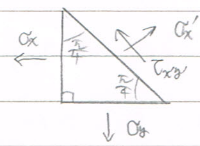
$M_B = R_A \cdot l - \frac{1}{2}w_1 l \cdot \frac{1}{3}l = \frac{1}{10}w_1 l^2 - \frac{1}{6}w_1 l^2 = \boxed{-\frac{1}{15}w_1 l^2}$

$V_x - R_A + \frac{1}{2}wx = 0 \to V_x = \frac{1}{10}w_1 l - \frac{w_1}{2l}x^2$　　$\frac{w_1}{2l}x^2 = \frac{1}{10}w_1 l$

$M_x = \frac{1}{10}w_1 l x - \frac{w_1}{6l}x^3$　　$x^2 = \frac{1}{5}l^2 \to x = \frac{l}{\sqrt{5}}$



[3] (1) σ_x'が働く面をAとおくと，



水平方向の力のつりあいより，

$$\sigma_x' \cancel{\cos\frac{\pi}{4}} \cdot \cancel{A} = \sigma_x \cdot \cancel{A \sin\frac{\pi}{4}} + \tau_{x'y'} \cancel{\cos\frac{\pi}{4}} \cdot \cancel{A}$$

$$\sigma_x' = \sigma_x + \tau_{x'y'} \quad -① $$

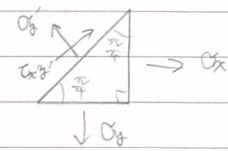
鉛直方向の力のつりあいより，

$$\sigma_x' \cancel{\sin\frac{\pi}{4}} \cdot \cancel{A} + \tau_{x'y'} \cancel{\sin\frac{\pi}{4}} \cdot \cancel{A} = \sigma_y \cdot \cancel{A \cos\frac{\pi}{4}}$$

$$\sigma_x' + \tau_{x'y'} = \sigma_y \quad -②$$

①＋②より，

$$2\sigma_x' = \sigma_x + \sigma_y \qquad \therefore \sigma_x' = \boxed{\frac{1}{2}(\sigma_x + \sigma_y)}$$



σ_y'が働く面をAとおくと，

水平方向の力のつりあいより，

$$\sigma_y' \cancel{\cos\frac{\pi}{4}} \cdot \cancel{A} = \tau_{x'y'} \cancel{\cos\frac{\pi}{4}} \cdot \cancel{A} + \sigma_x \cdot \cancel{A \sin\frac{\pi}{4}}$$

$$\sigma_y' = \tau_{x'y'} + \sigma_x \quad -③$$

鉛直方向の力のつりあいより，

$$\sigma_y' \cancel{\sin\frac{\pi}{4}} \cdot \cancel{A} + \tau_{x'y'} \cancel{\sin\frac{\pi}{4}} \cdot \cancel{A} = \sigma_y \cdot \cancel{A \cos\frac{\pi}{4}}$$

$$\sigma_y' + \tau_{x'y'} = \sigma_y \quad -④$$

③＋④より，

$$2\sigma_y' = \sigma_x + \sigma_y \qquad \therefore \sigma_y' = \boxed{\frac{1}{2}(\sigma_x + \sigma_y)}$$

①より， $\tau_{x'y'} = \sigma_x' - \sigma_x$

$$\therefore \tau_{x'y'} = \boxed{\frac{1}{2}(\sigma_y - \sigma_x)}$$

(2) $\sigma_x' = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(-\sigma + \sigma) = \boxed{0}$

$\sigma_y' = \boxed{0}$

$\tau_{x'y'} = \frac{1}{2}(\sigma_y - \sigma_x) = \frac{1}{2}(\sigma + \sigma) = \boxed{\sigma} \longrightarrow \boxed{\text{純せん断}}$

(3) $$\sigma_{max} = \frac{\sigma_x' + \sigma_y'}{2} + \sqrt{\left(\frac{\sigma_x' - \sigma_y'}{2}\right)^2 + \tau_{x'y'}^2} = 0 + \sqrt{0 + \sigma^2} = \boxed{\sigma}$$

$$\sigma_{min} = \frac{\sigma_x' + \sigma_y'}{2} - \sqrt{\left(\frac{\sigma_x' - \sigma_y'}{2}\right)^2 + \tau_{x'y'}^2} = 0 - \sqrt{0 + \sigma^2} = \boxed{-\sigma}$$

No. Date

(4) 二軸引張りより,

$$\varepsilon_x = \frac{1}{E}\{\sigma_x - \nu\sigma_y\}, \quad \varepsilon_y = \frac{1}{E}\{\sigma_y - \nu\sigma_x\}$$

$\sigma_x = -\sigma, \quad \sigma_y = \sigma$ より,

$$\varepsilon_{max} = \varepsilon_y = \frac{1}{E}(\sigma + \nu\sigma) = \boxed{\frac{\sigma}{E}(1+\nu)}$$

$$\varepsilon_{min} = \varepsilon_x = \frac{1}{E}(-\sigma - \nu\sigma) = \boxed{-\frac{\sigma}{E}(1+\nu)}$$

(5) $\tau_{x'y'} = G|\gamma_{max}|$

$$|\gamma_{max}| = \frac{\tau_{x'y'}}{G}$$

$G = \dfrac{E}{2(1+\nu)}, \quad \tau_{x'y'} = \sigma$ より,

$$\therefore |\gamma_{max}| = \boxed{\frac{2\sigma}{E}(1+\nu)}$$