材料力学　5回目

[1] (1) $R_A = P$,　$M_A = -Pl$

$M_x = Px - Pl = P(x-l)$

$EI\frac{d^2y}{dx^2} = -P(x-l)$

$EI\frac{dy}{dx} = -P(\frac{1}{2}x^2 - lx + C_1)$

$EIy = -P(\frac{1}{6}x^3 - \frac{1}{2}lx^2 + C_1x + C_2)$

境　$x=0 \to \frac{dy}{dx} = 0$,　$x=0 \to y=0$　$\Rightarrow C_1 = C_2 = 0$

$y = -\frac{P}{6EI}(x^3 - 3lx^2)$

$\therefore \delta_{B1} = y_{x=l} = \boxed{\frac{Pl^3}{3EI}}$

(3) $R_A = Q$,　$M_A = Qa$

$M_x = -Qx + Qa = Q(a-x)$

$EI\frac{d^2y}{dx^2} = Q(x-a)$

$EI\frac{dy}{dx} = Q(\frac{1}{2}x^2 - ax + C_1)$

$EIy = Q(\frac{1}{6}x^3 - \frac{1}{2}ax^2 + C_1x + C_2)$

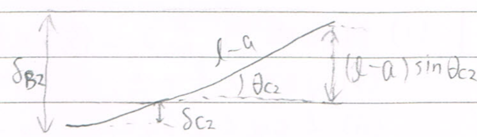
境　$x=0 \to \frac{dy}{dx} = 0$,　$x=0 \to y=0 \Rightarrow C_1 = C_2 = 0$

$y = \frac{Q}{6EI}(x^3 - 3ax^2)$

$\therefore \delta_{C2} = y_{x=a} = \boxed{-\frac{Qa^3}{3EI}}$

(2) $\delta_{C1} = y_{x=a} = \boxed{\frac{Pa^2}{6EI}(3l-a)}$

(4) θ_{C2}は微小より, $(l-\theta)\sin\theta_{C2} = \theta_{C2}(l-a)$



$\theta_{C2} = \left(\frac{dy}{dx}\right)_{x=a} = \frac{Q}{EI}(\frac{1}{2}a^2 - a^2) = -\frac{Qa^2}{2EI}$

よって,

$\delta_{B2} = \delta_{C2} + \theta_{C2}(l-a)$

$= -\frac{Qa^3}{3EI} - \frac{Qa^2}{2EI}(l-a)$

$= -\frac{Qa^2}{6EI}\{2a + 3(l-a)\}$

$= -\frac{Qa^2}{6EI}(3l-a)$

$\delta_B = \delta_{B1} + \delta_{B2}$

$= \frac{Pl^3}{3EI} - \frac{Qa^2}{6EI}(3l-a)$　―①

ここで, (C)の状態では点Cでのたわみは0より,

$\delta_C = \delta_{C1} + \delta_{C2} = \frac{Pa^2}{6EI}(3l-a) - \frac{Qa^3}{3EI} = 0 \to Q = \frac{P}{2a}(3l-a)$

①に代入

$\delta_B = \frac{Pl^3}{3EI} - \frac{Pa}{12EI}(3l-a)^2 = \boxed{\frac{P}{12EI}\{4l^3 - a(3l-a)^2\}}$

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[2] (1) $\lambda_T = \alpha_1 a \Delta T + \alpha_2 b \Delta T = \boxed{(\alpha_1 a + \alpha_2 b)\Delta T}$

(2) 軸力をPとおくと、断面積をA それぞれの棒材の伸びは、

$\lambda_1 = \frac{Pa}{E_1 A} + \alpha_1 a \Delta T, \quad \lambda_2 = \frac{Pb}{E_2 A} + \alpha_2 b \Delta T$

全体の伸びは、

$\lambda = \lambda_1 + \lambda_2 = \left(\frac{Pa}{E_1 A} + \alpha_1 a \Delta T\right) + \left(\frac{Pb}{E_2 A} + \alpha_2 b \Delta T\right) = 0$

$\left(\frac{a}{E_1} + \frac{b}{E_2}\right)\frac{P}{A} = -(\alpha_1 a + \alpha_2 b)\Delta T$

$P = -\frac{(\alpha_1 a + \alpha_2 b)E_1 E_2 \Delta T}{aE_2 + bE_1} \cdot A$

$\therefore \sigma_c = -\frac{P}{A} = \boxed{\frac{(\alpha_1 a + \alpha_2 b)E_1 E_2 \Delta T}{aE_2 + bE_1}}$

(3) $\delta_B = \frac{Pa}{E_1 A} + \alpha_1 a \Delta T$

$= -\frac{(\alpha_1 a + \alpha_2 b)aE_2 \Delta T}{aE_2 + bE_1} + \alpha_1 a \Delta T$

$= \frac{-(\cancel{\alpha_1 a} + \alpha_2 b)aE_2 + (\cancel{aE_2} + bE_1)\alpha_1 a}{aE_2 + bE_1}\Delta T$

$= \boxed{\frac{(\alpha_1 E_1 - \alpha_2 E_2)ab\Delta T}{aE_2 + bE_1}}$

[3] (1) 対称性より、点Oに働く反力は、 $R_0 = \frac{1}{2}P$

(i) $0 \leqq x < \frac{l}{2}$ のとき、 $M_x = R_0 x = \boxed{\frac{1}{2}Px}$

(ii) $\frac{l}{2} \leqq x < l$ のとき、 $M_x = R_0 x - P\left(x - \frac{l}{2}\right) = -\frac{1}{2}Px + \frac{1}{2}Pl = \boxed{\frac{1}{2}P(l-x)}$

(2) $U = \frac{1}{2EI}\left[\int_0^{\frac{l}{2}}\left(\frac{1}{2}Px\right)^2 dx + \int_{\frac{l}{2}}^{l}\left\{\frac{1}{2}P(l-x)\right\}^2 dx\right]$

$= \frac{P^2}{8EI}\left\{\int_0^{\frac{l}{2}} x^2 dx + \int_{\frac{l}{2}}^{l}(l-x)^2 dx\right\}$

$= \frac{P^2}{8EI}\left\{\left[\frac{1}{3}x^3\right]_0^{\frac{l}{2}} - \frac{1}{3}\left[(l-x)^3\right]_{\frac{l}{2}}^{l}\right\}$

$= \frac{P^2}{8EI}\left\{\frac{1}{3}\cdot\frac{l^3}{8} - \frac{1}{3}\left(0 - \frac{l^3}{8}\right)\right\}$

$= \boxed{\frac{P^2 l^3}{96EI}}$

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(3) 物体のもっていた位置エネルギーとはりに蓄えられた弾性ひずみエネルギーは等しいので、

$U = P^2 l^3 / 96EI = mgh$

$\therefore P = \boxed{\frac{4}{l}\sqrt{\frac{6mghEI}{l}}}$

2) 96
2) 48
2) 24
2) 12
6

(4) 物体がした仕事と物体がもっていた位置エネルギーは等しいので、

$\frac{1}{2}P\delta = mgh$

$\therefore \delta = \frac{2mgh}{P} = 2mgh \cdot \frac{l}{4}\sqrt{\frac{l}{6mghEI}} = \boxed{\frac{l}{2}\sqrt{\frac{mghl}{6EI}}}$

(5) $I = \int_{-\frac{a}{2}}^{\frac{a}{2}} y^2 \cdot a\,dy = \frac{a}{3}\left[y^3\right]_{-\frac{a}{2}}^{\frac{a}{2}} = \frac{a}{3}\left(\frac{a^3}{8} + \frac{a^3}{8}\right) = \frac{a^4}{12}$

$Z = \frac{1}{y}I = \frac{2}{a} \cdot \frac{a^4}{12} = \frac{a^3}{6}$

$M_{max} = M_{x=\frac{l}{2}} = \frac{1}{4}Pl$

$\sigma_{max} = \frac{M_{max}}{Z} = \frac{1}{4}Pl \cdot \frac{6}{a^3} = \frac{3l}{2a^3} \cdot \frac{4}{l}\sqrt{\frac{6mghEI}{l}} = \frac{6}{a^3}\sqrt{\frac{6mghEI}{l}}$

$I = \frac{a^4}{12}$ より、

$\sigma_{max} = \frac{6}{a^3} \cdot \sqrt{\frac{a^4}{12}} \cdot \sqrt{\frac{6mghE}{l}} = \frac{3}{\sqrt{3}a} \cdot \sqrt{\frac{6mghE}{l}} = \boxed{\frac{3}{a}\sqrt{\frac{2mghE}{l}}}$