H24

材料力学　5回目

[1] (1) $R_{B1} = \boxed{P}$, $M_{B1} = \boxed{-Pl}$

(2) $M_x = -Px$

$\frac{d^2y}{dx^2} = -\frac{M_x}{EI} \rightarrow EI\frac{d^2y}{dx^2} = Px$

$EI\frac{dy}{dx} = \frac{1}{2}Px^2 + C_1$

$EIy = \frac{1}{6}Px^3 + C_1x + C_2$

境 $x = l \rightarrow \frac{dy}{dx} = 0$, $x = l \rightarrow y = 0$

$EI\left(\frac{dy}{dx}\right)_{x=l} = \frac{1}{2}Pl^2 + C_1 = 0 \rightarrow C_1 = -\frac{1}{2}Pl^2$

$EIy_{x=l} = \frac{1}{6}Pl^3 - \frac{1}{2}Pl^3 + C_2 = 0 \rightarrow C_2 = \frac{1}{3}Pl^3$

$y = \frac{P}{6EI}(x^3 - 3l^2x + 2l^3)$

$\frac{dy}{dx} = \frac{P}{2EI}(x^2 - l^2)$

$\therefore \theta_{A1} = \left(\frac{dy}{dx}\right)_{x=0} = \boxed{-\frac{Pl^2}{2EI}}$

$y_{A1} = y_{x=0} = \boxed{\frac{Pl^3}{3EI}}$

(3) $R_{B2} = \boxed{0}$, $M_{B2} = \boxed{M}$

(4) $M_x = M \rightarrow EI\frac{d^2y}{dx^2} = -M$

$EI\frac{dy}{dx} = -Mx + C_1$

$EIy = -\frac{1}{2}Mx^2 + C_1x + C_2$

境 $x = l \rightarrow \frac{dy}{dx} = 0$, $x = l \rightarrow y = 0$

$EI\left(\frac{dy}{dx}\right)_{x=l} = -Ml + C_1 = 0 \rightarrow C_1 = Ml$

$EIy_{x=l} = -\frac{1}{2}Ml^2 + Ml^2 + C_2 = 0 \rightarrow C_2 = -\frac{1}{2}Ml^2$

$\frac{dy}{dx} = -\frac{M}{EI}(x - l)$

$y = -\frac{M}{2EI}(x^2 - 2lx + l^2)$

$\therefore \theta_{A2} = \left(\frac{dy}{dx}\right)_{x=0} = \boxed{\frac{Ml}{EI}}$

$y_{A2} = y_{x=0} = \boxed{-\frac{Ml^2}{2EI}}$

(5) A端でのたわみとたわみ角は、

$y_A = y_{A1} + y_{A2} = \frac{Pl^3}{3EI} - \frac{Ml^2}{2EI} = \frac{l^2}{6EI}(2Pl - 3M) = \delta$ — ①

$\theta_A = \theta_{A1} + \theta_{A2} = -\frac{Pl^2}{2EI} + \frac{Ml}{EI} = -\frac{l}{2EI}(Pl - 2M) = 0 \rightarrow M = \frac{1}{2}Pl$

①に代入

$\frac{l^2}{6EI}\cdot\left(2Pl - \frac{3}{2}Pl\right) = \delta$

$\frac{l^2}{12EI}P = \delta$

$P = 12EI\delta/l^2 \rightarrow \therefore M_A = M = \frac{1}{2}l\cdot 12EI\delta/l^2 = \boxed{\frac{6EI\delta}{l}}$

図1(c)より、R_Aによって曲げが生じたとすると、

$\therefore R_A = -P = \boxed{-\frac{12EI\delta}{l^2}}$

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[2] (1) $P_1 = \gamma A_1 x$

$\therefore \sigma_1(x) = \frac{P_1}{A_1} = \boxed{\gamma x}$

(2) $\sigma_{1max} = \sigma_1(l_1) = \boxed{\gamma l_1}$

(3) $P_2 = W + \gamma A_2 x$

$\therefore \sigma_2(x) = \frac{P_2}{A_2} = \boxed{\frac{W}{A_2} + \gamma x}$

(4) $\sigma_{2max} = \sigma_2(l_2) = \boxed{\frac{W}{A_2} + \gamma l_2}$

(5) $P_1' = \gamma A_1 l_1$, $P_2' = \gamma A_1 l_1 + \gamma A_2 l_2$

$\sigma_B = \frac{P_1'}{A_1} = \gamma l_1$ — ①

$\sigma_D = \frac{P_2'}{A_2} = \gamma k l_1 + \gamma l_2$ — ②

① + ② より,

$2\sigma_B = \gamma(l_1 + l_2) + \gamma k l_1 = \gamma l + \gamma k l_1 = \gamma(l + k l_1)$ $(\because l = l_1 + l_2)$

① より, $l_1 = \frac{\sigma_B}{\gamma}$

$2\sigma_B = \gamma\left(l + k\frac{\sigma_B}{\gamma}\right) = \gamma l + k\sigma_B$

$\gamma l = (2-k)\sigma_B$

$\therefore l_{max} = \boxed{\frac{2-k}{\gamma}\sigma_B}$

[3] (1) $M_C = \boxed{-Pa}$

(2) $M_D = \boxed{P(h-l)}$, $T_D = \boxed{Pw}$

(3) $I_p = \int r^2 dA = \frac{\pi}{32}d^4$

$Z_p = \frac{1}{y}I_p = \frac{2}{d}\cdot\frac{\pi}{32}d^4 = \frac{\pi}{16}d^3$

$Z = \frac{1}{2}Z_p = \frac{\pi}{32}d^3$

$\therefore \sigma_D = \frac{M_D}{Z} = \boxed{\frac{32P}{\pi d^3}(h-l)}$

$\tau_D = \frac{T_D}{Z_p} = \boxed{\frac{16Pw}{\pi d^3}}$

(4) $\sigma_1 = \sigma_D$, $\sigma_2 = 0$, $\tau_{12} = \tau_D$ とおくと,

$\sigma_{max} = \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau_{12}^2} = \frac{16P}{\pi d^3}(h-l) + \sqrt{\left\{\frac{16P}{\pi d^3}(h-l)\right\}^2 + \left(\frac{16Pw}{\pi d^3}\right)^2}$

$$= \boxed{\frac{16P}{\pi d^3}\left\{(h-l)+\sqrt{(h-l)^2+W^2}\right\}}$$

(5) $\sigma_a = \sigma_{max} = \frac{16P}{\pi d^3}\{\bigcirc\}$

$$d = \sqrt[3]{\frac{16P}{\sigma_a \pi}\left\{(h-l)+\sqrt{(h-l)^2+W^2}\right\}}$$

最大主応力は点Oからhの面で発生するので、$(h-l)$をhに変えて、

$$\therefore\ d = \boxed{\sqrt[3]{\frac{16P}{\sigma_a \pi}\left\{h+\sqrt{h^2+W^2}\right\}}}$$