H25

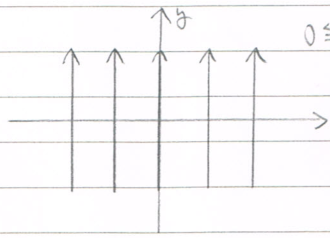
Date　・　・　No.

# 流体力学　4回目

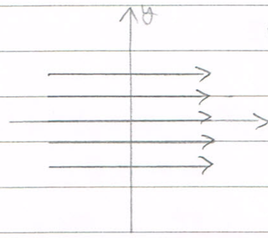
[1] (1) (a) 流体粒子 … 流体の領域における体積を無限に小さくしたもので，力学における質点と同様の扱い.

(b) 流線 … 流体の各点における接線~~ベクトル~~方向が，速度ベクトルと一致するような曲線

(2)

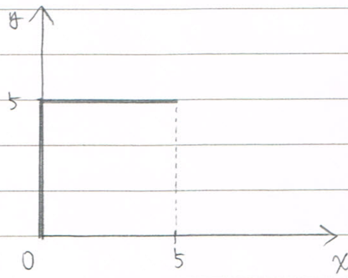


$0 \leq t \leq 5$

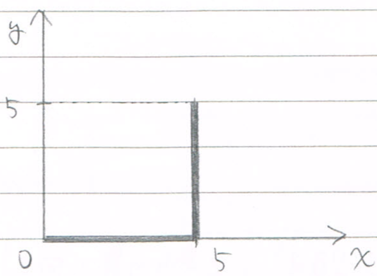


$5 \leq t \leq 10$

(3)



(4)



[2] (1) $w_1 = \boxed{Uz}$

(2) $w_2 = \boxed{m \log z}$

(3) $w = w_1 + w_2 = Uz + m \log z$

$$\frac{dw}{dz} = \boxed{\cancel{U + \frac{m}{z}}}$$

$$\frac{dw}{dz} = U + \frac{m}{z} = U + \frac{m}{x+iy} = U + \frac{m}{x^2+y^2}(x-iy)$$

$$= U + \frac{mx}{x^2+y^2} - i\frac{my}{x^2+y^2} = u - iv$$

$$\therefore \boxed{u = U + \frac{mx}{x^2+y^2}, \quad v = \frac{my}{x^2+y^2}}$$

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よどみ点では流速0より,

$$\frac{dw}{dz} = U + \frac{m}{z} = 0 \rightarrow z = -\frac{m}{U} = x + iy$$

$\therefore$ よどみ点：$(x, y) = \boxed{\left(-\frac{m}{U}, 0\right)}$

(4) $W_2 = m\log z$

$z = re^{i\theta}$ より,

$W_2 = m\log re^{i\theta} = m\log r + im\theta = \phi + i\psi$

$\phi = m\log r$ , $\psi = m\theta$

$$u_\theta = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{m}{r}$$

$Q = 2\pi r \cdot u_\theta = 2\pi r \cdot \frac{m}{r} = \boxed{2\pi m}$

(5) $Q = \boxed{UD}$

(6) $Q = UD = 2\pi m$

$\therefore m = \boxed{\frac{UD}{2\pi}}$

[3] (1) $R_e = \frac{Vd}{\nu} = \boxed{\frac{\rho_0 V d}{\mu}}$

(2) ストークスの抵抗法則

(3) 重力：$G = mg = \rho \cdot \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \cdot g = \boxed{\frac{1}{6}\pi\rho g d^3 \text{, z軸の負の向き}}$

浮力：$B = \boxed{\frac{1}{6}\pi\rho_0 g d^3 \text{, z軸の正の向き}}$

抵抗：$D = \boxed{3\pi\mu V d \text{, z軸の正の向き}}$

(4) 運動方程式より,

$$\underline{\frac{1}{6}\pi\rho d^3 \frac{d^2 z(t)}{dt^2} = \frac{1}{6}\pi g d^3(\rho_0 - \rho) + 3\pi\mu V d}\ //$$

(5) 運動方程式より,

$$-\frac{1}{6}\pi\rho d^3\frac{dV}{dt}=\frac{1}{6}\pi g d^3(\rho_0-\rho)+3\pi\mu V d$$

$$\frac{dV}{dt}=\frac{\rho-\rho_0}{\rho}g-\frac{18\mu V}{\rho d^2}=\left(1-\frac{\rho_0}{\rho}\right)g-\frac{18\mu V}{\rho d^2}$$

$g'=\left(1-\frac{\rho_0}{\rho}\right)g$, $\frac{1}{\tau}=-\frac{18\mu}{\rho d^2}$ とおくと,

$$\frac{dV}{dt}=g'+\frac{1}{\tau}V=g'\left(1+\frac{1}{g'\tau}V\right)$$

$$\frac{dV}{1+\frac{1}{g'\tau}V}=g'dt$$

$$g'\tau\log\left|1+\frac{V}{g'\tau}\right|=g't+C$$

$$\log\left|1+\frac{V}{g'\tau}\right|=\frac{t}{\tau}+C$$

$$1+\frac{V}{g'\tau}=\pm\exp\left[\frac{t}{\tau}+C\right]=Ce^{\frac{t}{\tau}}$$

$$V(t)=g'\tau\left(Ce^{\frac{t}{\tau}}-1\right)$$

$t=0$ のとき, $V=0$ より,

$$V(0)=g'\tau(C\cdot1-1)=0\rightarrow C=1$$

$$V(t)=-\frac{\rho d^2 g}{18\mu}\left(1-\frac{\rho_0}{\rho}\right)\left(e^{-\frac{18\mu}{\rho d^2}t}-1\right)$$

$$\lim_{t\to\infty}V(t)=\boxed{\frac{\rho d^2 g}{18\mu}\left(1-\frac{\rho_0}{\rho}\right)\text{, 終端速度}}$$

$V_{max}=\frac{\rho d^2 g}{18\mu}\left(1-\frac{\rho_0}{\rho_0}\right)$ より,

$$Re=\frac{V_{max}d}{\frac{\mu}{\rho}}=\frac{\rho^2 d^3 g}{18\mu^2}\left(1-\frac{\rho_0}{\rho}\right)<1$$

となるとき, ストークスの抵抗法則は常に成り立つ

(6) $F=\frac{1}{3}\cdot3\pi\mu V d=\pi\mu V d$

$$\Delta P\cdot\cancel{4\pi\left(\frac{d}{2}\right)^2}\ \pi d^2=F$$

$$\therefore\Delta P=\frac{F}{\pi d^2}=\frac{\pi\mu V d}{\pi d^2}=\boxed{\cancel{\frac{\mu V}{d}}}\quad\boxed{\frac{4\pi\mu V}{d}}$$

$\Delta P>0$ より, 前の方が大きい