H23

Date . . No.

数学 4回目

[1] $\int_0^\infty e^{-ax}\cos bx\,dx \qquad (a>0)$

$$= \int_0^\infty \left(-\frac{1}{a}e^{-ax}\right)'\cos bx\,dx = \left[-\frac{1}{a}e^{-ax}\cos bx\right]_0^\infty - \frac{b}{a}\int_0^\infty \left(-\frac{1}{a}e^{-ax}\right)'\sin bx\,dx$$

$$= -(0-1)\frac{1}{a} + \frac{b}{a^2}\left[e^{-ax}\sin bx\right]_0^\infty - \frac{b^2}{a^2}\int_0^\infty e^{-ax}\cos bx\,dx$$

$$\left(1+\frac{b^2}{a^2}\right)\int_0^\infty e^{-ax}\cos bx\,dx = \frac{1}{a} + 0$$

$$\therefore \int_0^\infty e^{-ax}\cos bx\,dx = \frac{1}{a}\cdot\frac{a^2}{a^2+b^2} = \boxed{\frac{a}{a^2+b^2}}$$

[2] $y^2 + (2xy + x^2)\frac{dy}{dx} = 0$

両辺をx^2で割って，

$$\left(\frac{y}{x}\right)^2 + \left(2\frac{y}{x}+1\right)\frac{dy}{dx} = 0$$

$u = \frac{y}{x}$ とおく

$$u^2 + (2u+1)\left(x\frac{du}{dx}+u\right) = 0$$

$$x(2u+1)\frac{du}{dx} = -u^2 - u(2u+1) = -3u^2 - u = -u(3u+1)$$

$$\frac{2u+1}{u(3u+1)}du = -\frac{dx}{x}$$

$$\left(\frac{1}{u} - \frac{1}{3u+1}\right)du = -\frac{dx}{x}$$

積分して，

$$\log|u| - \frac{1}{3}\log|3u+1| = -\log|x| + C$$

$$\frac{1}{3}\log\left|\frac{u^3x^3}{3u+1}\right| = C \longrightarrow \frac{u^3x^3}{3u+1} = C$$

$$u^3x^3 = C(3u+1)$$

$$y^3 = 3C\frac{y}{x} + C$$

$\therefore \boxed{xy^3 = 3Cy + Cx}$ （Cは積分定数）

[3] $A=(a_1, a_2, a_3)$, $B=(b_1, b_2, b_3)$, $C=(c_1, c_2, c_3)$ の3点を通る平面内に

$x = (x_1, x_2, x_3)$ をとる

$\vec{AB} = (b_1-a_1,\ b_2-a_2,\ b_3-a_3)$, $\vec{AC} = (c_1-a_1,\ c_2-a_2,\ c_3-a_3)$

$\vec{Ax} = (x_1-a_1,\ x_2-a_2,\ x_3-a_3)$

$\vec{X} = \vec{AB}\times\vec{AC}$ とすると，$\vec{Ax}\cdot\vec{X} = 0$ となるので，スカラー三重積も0となる．

$$\begin{vmatrix} x_1-a_1 & x_2-a_2 & x_3-a_3 \\ b_1-a_1 & b_2-a_2 & b_3-a_3 \\ c_1-a_1 & c_2-a_2 & c_3-a_3 \end{vmatrix} = 0$$

$A\cdot(B\times C) = 0$

$$\rightarrow \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

$$\rightarrow \begin{vmatrix} x-a_1 & x_2-a_2 & x_3-a_3 & 0 \\ a_1 & a_2 & a_3 & 1 \\ b_1-a_1 & b_2-a_2 & b_3-a_3 & 0 \\ c_1-a_1 & c_2-a_2 & c_3-a_3 & 0 \end{vmatrix} = 0 \rightarrow \begin{vmatrix} x_1 & x_2 & x_3 & 1 \\ a_1 & a_2 & a_3 & 1 \\ b_1 & b_2 & b_3 & 1 \\ c_1 & c_2 & c_3 & 1 \end{vmatrix} = 0$$

$\boxed{4}$

$$\begin{cases} \frac{dy(t)}{dt} + y(t) + 2\frac{dz(t)}{dt} + 3z(t) = 6e^{-2t} \\ 3\frac{dy(t)}{dt} - y(t) + 4\frac{dz(t)}{dt} + z(t) = 0 \end{cases}$$

ラプラス変換して,

$$\begin{cases} sY(s) - y(0) + Y(s) + 2sZ(s) - 2Z(0) + 3Z(s) = \frac{6}{s+2} \\ 3sY(s) - 3y(0) - Y(s) + 4sZ(s) - 4Z(0) + Z(s) = 0 \end{cases}$$

$$\begin{cases} (s+1)Y(s) + (2s+3)Z(s) = \frac{6}{s+2} + 2 \\ (3s-1)Y(s) + (4s+1)Z(s) = 4 \end{cases} \longrightarrow Z(s) = \frac{1}{4s+1}\{4-(3s-1)Y(s)\}$$

$$(s+1)Y(s) + \frac{2s+3}{4s+1}\{4-(3s-1)Y(s)\} = \frac{2s+10}{s+2}$$

$$(s+1)(4s+1)Y(s) + 4(2s+3) - (2s+3)(3s-1)Y(s) = \frac{2s+10}{s+2}\cdot(4s+1)$$

$$(4s^2+5s+1)Y(s) - (6s^2+7s-3)Y(s) = (8s^2+42s+10)/s+2 - 8s - 12$$

$$-(2s^2+2s-4)Y(s) = \{8s^2+42s+10 - (8s+12)(s+2)\}/s+2$$

$$-2(s-1)(s+2)Y(s) = \frac{14(s-1)}{s+2}$$

$$Y(s) = -\frac{7}{(s+2)^2}$$

$$y(t) = -7\cdot\frac{1}{1!}e^{-2t}t = \boxed{-7te^{-2t}}$$