H29 数学（途中あり）

[1] (1) $y = \log \dfrac{\sqrt{x^2+1}}{x^2+1} = \log (x^2+1)^{\frac{1}{2}} - \log (x^2+1)$

$= \dfrac{1}{2} \log (x^2+1) - \log (x^2+1)$

$y' = \dfrac{1}{2} \dfrac{2x}{x^2+1} - \dfrac{2x}{x^2+1}$

$= \dfrac{-x}{x^2+1}$

(2) $\int 3^{\sqrt{2x}} dx$

$\sqrt{2x} = t$ とおく

$2x = t^2$

$2dx = 2t\,dt$

$dx = t\,dt$

よって

$\int 3^t \cdot t\,dt = \int e^{t\log 3} \cdot t\,dt$

$= t \times \dfrac{1}{\log 3} e^{t\log 3} - \int e^{t \log 3} dt + C$

$= \dfrac{t}{\log 3} e^{t\log 3} - \dfrac{1}{(\log 3)^2} e^{t\log 3} + C$

$= \dfrac{1}{\log 3} e^{t\log 3} \left(t - \dfrac{1}{\log 3} \right) + C$

$= \dfrac{3^{\sqrt{2x}}}{\log 3} \left(\sqrt{2x} - \dfrac{1}{\log 3} \right) + C$

[2] $A = \begin{bmatrix} a^2 & 2abc & -b^2 \\ 0 & ac & b \\ 0 & 0 & c^2 \end{bmatrix}$, $B = \begin{bmatrix} a^3 & 3a^2b & 3ab^2 & 3ac^2 & 2b^2c^2 \\ 0 & a^2c & 2ab^2 & 2a^2b & b^2c^2 \\ 0 & 0 & ac^2 & a^2b & bc^2 \\ 0 & 0 & 0 & c^2 & b^2c \\ 0 & 0 & 0 & 0 & c \end{bmatrix}$

(1) $|A| = a^3c^3$, $|B| = a^6c^6$

(2) $c = 0$ のとき rank$A = 2$ になるため

$B = \begin{bmatrix} a^3 & 3a^2b & 3ab^2 & 0 & 0 \\ 0 & 0 & 2ab^2 & 2a^2b & 0 \\ 0 & 0 & 0 & a^2b & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ より rank $B = 3$

(3) $\left[\begin{array}{ccc|ccc} a^2 & 2abc & -b^2 & 1 & 0 & 0 \\ 0 & ac & b & 0 & 1 & 0 \\ 0 & 0 & c^2 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & \frac{2bc}{a} & -\frac{b^2}{a^2} & 1/a^2 & 0 & 0 \\ 0 & 1 & \frac{b}{ac} & 0 & 1/ac & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/c^2 \end{array}\right]$

$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/a^2 & -2b/a^2 & 3b^2/a^2c^2 \\ 0 & 1 & 0 & 0 & 1/ac & -b/ac^3 \\ 0 & 0 & 1 & 0 & 0 & 1/c^2 \end{array}\right]$ よって $A^{-1} = \begin{bmatrix} 1/a^2 & -2b/a^2 & 3b^2/a^2c^2 \\ 0 & 1/ac & -b/ac^3 \\ 0 & 0 & 1/c^2 \end{bmatrix}$

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(4)

$$AA^t = \begin{bmatrix} a^2 & 2abc & -b^2 \\ 0 & ac & b \\ 0 & 0 & c^2 \end{bmatrix}\begin{bmatrix} a^2 & 0 & 0 \\ 2abc & ac & 0 \\ -b^2 & b & c^2 \end{bmatrix}$$

$$= \begin{bmatrix} a^4 & 2a^2c^2b-b^3 & -b^2c^2 \\ 2a^2c^2b-b^3 & a^2c^2 & cb^2 \\ 0 & 0 & c^4 \end{bmatrix}$$

よって

$$a^4 = 1, \quad c^4 = 1, \quad cb^2 = 0$$

したがって

$$a = c = \pm 1, \quad b = 0$$

[3] $(x^2 + 2xy + y)dx + (y^2 + x^2 + x)dy = 0 \quad \cdots ①$

(1) $P(x,y) = x^2 + 2xy + y$, $Q(x,y) = y^2 + x^2 + x$

よって $\dfrac{P}{\partial y} = 2x + 1$, $\dfrac{Q}{\partial x} = 2x + 1$

したがって $\dfrac{P}{\partial y} = \dfrac{Q}{\partial x}$ より 完全微分方程式である

(2)

$$u(x,y) = \int_{x_0}^{x} P(x,y)\,dx + \int_{y_0}^{y} Q(x_0,y)\,dy$$
$$= \int_{x_0}^{x} (x^2 + 2xy + y)\,dx + \int_{y_0}^{y} (y^2 + x_0^2 + x_0)\,dy$$
$$= \left[\frac{1}{3}x^3 + x^2y + xy\right]_{x_0}^{x} + \left[\frac{1}{3}y^3 + x_0^2y + x_0y\right]_{y_0}^{y}$$
$$= \frac{1}{3}x^3 + x^2y + xy - \left(\frac{1}{3}x_0^3 + x_0^2y + x_0y\right) + \frac{1}{3}y^3 + x_0^2y + x_0y - \left(\frac{1}{3}y_0^3 + x_0^2y_0 + x_0y_0\right)$$
$$= \frac{1}{3}x^3 + x^2y + xy + \frac{1}{3}y^3 = C_1$$
$$x^3 + 3x^2y + 3xy + y^3 = C_2$$

[4] (1) $y(t) = e^{at}$

$$\mathcal{L}\{y(t)\} = \frac{1}{s-a}$$

(2) $\begin{cases} x'(t) - 4x(t) + 2y(t) = 0 & -① \\ y'(t) + 2x(t) - y(t) = 0 & -② \end{cases}$ $x(0) = 1$, $y(0) = 0$

①より $s\mathcal{L}\{x(t)\} - x(0) - 4\mathcal{L}\{x(t)\} + 2\mathcal{L}\{y(t)\} = 0$

$(s-4)\mathcal{L}\{x(t)\} + 2\mathcal{L}\{y(t)\} = 1 \quad -③$

②より $s\mathcal{L}\{y(t)\} - y(0) + 2\mathcal{L}\{x(t)\} - \mathcal{L}\{y(t)\} = 0$

$2\mathcal{L}\{x(t)\} + (s-1)\mathcal{L}\{y(t)\} = 0 \quad -④$

③, ④ より $\mathcal{L}\{x(t)\} = \dfrac{1}{5}\left(\dfrac{4}{s-5} + \dfrac{1}{s}\right)$ $\mathcal{L}\{y(t)\} = \dfrac{2}{5}\left(\dfrac{1}{s} - \dfrac{1}{s-5}\right)$

したがって

$x(t) = \dfrac{1}{5}(4e^{5t} + 1)$, $y(t) = \dfrac{2}{5}(1 - e^{5t})$