CSE 464 DIGITAL IMAGE PROCESSİNG - HOMEWORK 2

Task 1 :

In this part, I implmented convolution with two methods. First is classical method and second is tactic of Convolution Theorem. When the program starts, ask user to enter image file path and kernel file path. Kernel file must be in the following format

row colum
k00, k011 … k0n
.
.
kn0 …….. kn-1n-1

Example:

3 3
-1 0 1
-1 0 1
-1 0 1

İf kernel file format is not above format and image file path is not found, program does not work.

In this part I implemented Convolution nicely using classical method and to implement Convolution theorem, I implemented Fast Fourier Theorem and tested this implementation but I can not do proper padding for kernel. So this theorem implementation does not give correct result.

Task 2 :

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$ laplacian operator.

$$x' = x\cos(\theta) - y\sin(\theta) \qquad and \qquad y' = x\sin(\theta) + y\cos(\theta)$$

x' and y' represents rotated versions of x and y.

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial x'^2} + \frac{\partial^2 f}{\partial y'^2}$$ we can prove that laplacian operation is not influenced by rotation.

$$x' + y\sin(\theta) = x\cos(\theta) \qquad\qquad y' - x\sin(\theta) = y\cos(\theta)$$

$$x = \frac{x' + y\sin(\theta)}{\cos(\theta)} \qquad\qquad x = \frac{y' - y\cos(\theta)}{\sin(\theta)}$$

$y = \frac{x\cos(\theta) - x'}{\sin(\theta)}$ $\quad$ $y = \frac{y' - x\sin(\theta)}{\cos(\theta)}$

$x = \frac{x' + y\sin(\theta)}{\cos(\theta)} = \frac{y' - y\cos(\theta)}{\sin(\theta)}$ and $y = \frac{x\cos(\theta) - x'}{\sin(\theta)} = \frac{y' - x\sin(\theta)}{\cos(\theta)}$

$y = y'\cos(\theta) - x'\sin(\theta)$ and $x = y'\sin(\theta) + x'\cos(\theta)$

$\frac{\partial f}{\partial x'} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial x'} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial x'}$ we can find $\frac{\partial x}{\partial x'}$ by taking derivative of x according to x' using rotation formula. Therefore $\frac{\partial x}{\partial x'} = \cos\theta$

And we can find $\frac{\partial y}{\partial x'}$ by taking derivative of y according to x' using rotation formula.Therefore $\frac{\partial y}{\partial x'} = -\sin\theta$. So, $\frac{\partial f}{\partial x'} = \frac{\partial f}{\partial x}\cos\theta - \frac{\partial f}{\partial y}\sin\theta$

if we take derivetive of above formula with respect to x'

Equation1: $\frac{\partial^2 f}{\partial x'^2} = \frac{\partial^2 f}{\partial x^2}\cos^2\theta - \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)\sin\theta\cos\theta - \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)\cos\theta\sin\theta + \frac{\partial^2 f}{\partial y^2}\sin^2\theta$

we can find $\frac{\partial x}{\partial y'}$ by taking derivative of x according to y' using rotation formula.

Therefore $\frac{\partial x}{\partial y'} = \sin\theta$

And we can find $\frac{\partial y}{\partial y'}$ by taking derivative of y according to y' using rotation formula.

Therefore $\frac{\partial y}{\partial y'} = \cos\theta$

$\frac{\partial f}{\partial y'} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial y'} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial y'}$ = $\frac{\partial f}{\partial x}\sin\theta + \frac{\partial f}{\partial y}\cos\theta$ if we take derivetive of this formula with respect to x'

Equation2: $\frac{\partial^2 f}{\partial x'^2} = \frac{\partial^2 f}{\partial x^2}\sin^2\theta + \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)\sin\theta\cos\theta + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)\cos\theta\cos\theta + \frac{\partial^2 f}{\partial y^2}\sin^2\theta$

if we add **Equation1** and **Equation2**, we yield:

$\frac{\partial^2 f}{\partial x'^2}+\frac{\partial^2 f}{\partial y'^2}=\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}$ As a result, this formula proves that laplacian operator is independent from rotation.

Task 3 :

h(x , y) = 3 f (x , y) + 2 f (x−1, y) +2 f (x+1, y) − 17 f (x , y−1) + 99 f (x , y+1)

h(x,y) = w(-1, -1) f(x-1, y-1) + w(-1, 0) f(x-1, y) + w(-1, 1) f(x-1, y+1) + w(0, -1) f(x, y-1) + w(0, 0) f(x, y) + w(0, 1) f(x, y+1) + w(1, -1) f(x+1, y-1) + w(1, 0) f(x+1, y) + w(1, 1) f(x+1, y+1)

$$h(x,y)=\sum_{s=-1}^{1}\sum_{t=-1}^{1} w(s,t) f(x+s, y+t)$$

a)

Linarity condition, h(α.f + β.g) = α.h(f) + β.g(f)

h(α.f + β.g) *= 3 (α.f(x,y) + β.g(x,y)) + 2(α.f(x-1,y) + β.g(x-1,y)) + 2(α.f(x+1,y) + β.g(x+1,y)) - 17(α.f(x,y-1) + β.g(x,y-1)) + 99(α.f(x,y+1) + β.g(x,y+1))*

α.h(f) *= 3 α .f (x , y) + 2 α .f (x−1, y) +2 α .f (x+1, y) − 17 α .f (x , y−1) + 99 α .f (x , y+1)*

β.g(f) *= 3 β .g (x , y) + 2 β.g (x−1, y) +2 β.fg(x+1, y) − 17 β .g (x , y−1) + 99 β .g (x , y+1)*

α.h(f) + β.g(f) *= 3 α .f (x , y) + 2 α .f (x−1, y) +2 α .f (x+1, y) − 17 α .f (x , y−1) + 99 α .f (x , y+1) + 3 β .g (x , y) + 2 β.g (x−1, y) +2 β.fg(x+1, y) − 17 β .g (x , y−1) + 99 β .g (x , y+1)*

h(α.f + β.g) = α.h(f) + β.g(f) . So we can say that **This filter is linear.**

For Example, Assume that f and g are two images and α and β are arbitrary constant.

$$f=\begin{bmatrix}1&2&3&4&5\\2&3&4&5&6\\3&4&5&6&7\\4&5&6&7&8\\5&6&7&8&9\end{bmatrix} \quad \text{and} \quad g=\begin{bmatrix}9&8&7&6&5\\8&7&6&5&4\\7&6&5&4&3\\6&5&4&3&2\\5&4&3&2&1\end{bmatrix}$$

Suppose α = 2 and β = 3

So, $\alpha.f=\begin{bmatrix}2&4&6&8&10\\4&6&8&10&12\\6&8&10&12&14\\8&10&12&14&16\\10&12&14&16&18\end{bmatrix}$ and $\beta.g=\begin{bmatrix}27&24&21&18&15\\24&21&18&15&12\\21&18&15&12&9\\18&15&12&9&6\\15&12&9&6&3\end{bmatrix}$

and $\alpha.f+\beta.g=\begin{bmatrix}29&28&27&26&25\\28&27&26&25&24\\27&26&25&24&23\\26&25&24&23&22\\25&24&23&22&21\end{bmatrix}$

and we apply filter h to `α.f + β.g`

Result is

$$Result=\begin{bmatrix}2287&2198&2109&1974&48\\2198&2109&2020&1887&46\\2109&2020&1931&1800&44\\-257&-247&-237&-267&42\\-408&-391&-374&-357&0\end{bmatrix}$$

$$\alpha.h(f)=\begin{bmatrix}766&944&1122&1272&24\\944&1122&1300&1446&28\\1122&1300&1478&1620&32\\-86&-106&-126&-86&36\\-204&-238&-272&-306&0\end{bmatrix} \qquad \beta.h(g)=\begin{bmatrix}1521&1254&987&702&24\\1254&987&720&441&18\\987&720&453&180&12\\-171&-141&-111&-81&6\\-204&-153&-102&-51&0\end{bmatrix}$$

$$\alpha . h(f) + \beta . h(g) = \begin{bmatrix} 2287 & 2198 & 2109 & 1974 & 48 \\ 2198 & 2109 & 2020 & 1887 & 46 \\ 2109 & 2020 & 1931 & 1800 & 44 \\ -257 & -247 & -237 & -267 & 42 \\ -408 & -391 & -374 & -357 & 0 \end{bmatrix}$$

Result are same, So we can say that **Filter h is linear.**

b)

w(-1, -1) = 0
w(-1, 0) = 2
w(-1, 1) = 0
w(0, -1) = -17
w(0, 0) = 3
w(0, 1) = 99
w(1, -1) = 0
w(1, 0) = 2
w(1, 1) = 0

$$\begin{bmatrix} w(-1,-1) & w(0,-1) & w(1,-1) \\ w(-1,0) & w(0,0) & w(1,0) \\ w(-1,1) & w(0,1) & w(1,1) \end{bmatrix} \text{ so filter is } \begin{bmatrix} 0 & -17 & 0 \\ 2 & 3 & 2 \\ 0 & 99 & 0 \end{bmatrix}$$

Convolution filter is obtained from by flipping original filter horizontally and vertically.

So, Convolution mask is $\begin{bmatrix} 0 & 99 & 0 \\ 2 & 3 & 2 \\ 0 & -17 & 0 \end{bmatrix}$