Question 2: Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $3x^4-3x^2-3=0$ on $[1,2]$. Use p_0 = 1.And calculate the the theoretical number of iterations required accoding to Coorollary 2.5.

Solution:

For x = g(x), $g(x)=(3x^2+3)^{(1/4)}$

g(x) is contiunuous on the interval [1,2] and $1\leqslant g(x)\leqslant 2$

$$g'(x)=\frac{3x}{2(3x^2+3)^{(3/4)}}$$

$p_1=g(p_0)$ $p_0=1$ and $p_1=g(1)=1,565$

$g'(1)=0,3912$ and $g'(2)=0,3935$

$|g'(x)|\leqslant 0,392\leqslant k<1$ Let k = 0,392

$$|p_n-p|\leqslant\frac{k^n}{(1-k)}|p_0-p_1|$$

$$|p_n-p|\leqslant\frac{(0,393)^n}{(1-0,393)}|1-1,565|$$

$$|p_n-p|\leqslant 0,93\times(0,393)^n$$

n > 4,859

n = 5 (The theoretical number of iterations required.)

$p_1 = g(p_0) = g(1) = 1{,}565$ $\qquad$ $p_1 - p_0 = 0{,}565 > 0{,}01$

$p_2 = g(p_1) = g(1{,}565) = 1{,}793$ $\qquad$ $p_2 - p_1 = 0{,}228 > 0{,}01$

$p_3 = g(p_2) = g(1{,}793) = 1{,}885$ $\qquad$ $p_3 - p_2 = 0{,}0{,}092 > 0{,}01$

$p_4 = g(p_3) = g(1{,}885) = 1{,}922$ $\qquad$ $p_4 - p_3 = 0{,}036 > 0{,}01$

$p_5 = g(p_4) = g(1{,}922) = 1{,}937$ $\qquad$ $p_5 - p_4 = 0{,}014 > 0{,}01$

$p_6 = g(p_5) = g(1{,}937) = 1{,}943$ $\qquad$ $p_6 - p_5 = 0{,}005 < 0{,}01$

Approximate root is 1,943.

Question 3:

Exercise Set 2.3/question 4. Let $f(x) = -x^3 - cosx$, $p_0 = -1, p_1 = 0$, find p_3 .

a) Use secant method.
b) Use the method of false position.

Solution:

a) Formula = $p_n = p_{n-1} - \frac{(p_{n-1} - p_{n-2})(\cos p_{n-1} - p_{n-1})}{(\cos p_{n-1} - p_{n-1}) - (\cos p_{n-2} - p_{n-2})}$

$$P_2 = 0 - \frac{(0-(-1))(\cos 0 - 0)}{(\cos 0 - 0) - (\cos(-1) - (-1))} = -0{,}999$$

$$P_3 = -0{,}999 - \frac{(-0{,}999 - 0)(\cos(-0{,}999) - (-0{,}999))}{(\cos(-0{,}999) - (-0{,}999)) - (\cos 0 - 0)} = -1{,}25208$$

Exercise Set 2.3/question 5. Use Newton's method to find solutions accurate to within 10 −4 for the following problems.

a) $x^3-2x^2-5=0,[1,4]$

b) $x^3+3x^2-1=0,[-3,-2]$

c) $x-cosx=0,[0,\pi/2]$

d) $x-0{,}8-0{,}2\,sinx=0,[0,\pi/2]$

Solution:

Formula : $p_n=p_{(n-1)}-\dfrac{f(p_{n-1})}{f^{'}(p_{n-1})}$

a) $f(x)=x^3-2x^2-5$ $\qquad$ $f^{'}(x)=3x^2-4x$

Let $p_0=2$.

$p_1=2-\dfrac{f(2)}{f^{'}(2)}=3{,}25000$ $\qquad$ $p_1-p_0=1{,}25000>0{,}0001$

$p_2=3{,}250-\dfrac{f(3{,}250)}{f^{'}(3{,}250)}=2{,}81103$ $\qquad$ $p_2-p_1=0{,}43896>0{,}0001$

$p_3=2{,}811-\dfrac{f(2{,}811)}{f^{'}(2{,}811)}=2{,}69799$ $\qquad$ $p_3-p_2=0{,}11304>0{,}0001$

$$p_4 = 2,697 - \frac{f(2,697)}{f'(2,697)} = 2,69067 \qquad p_4 - p_3 = 0,00731 > 0,0001$$

$$p_5 = 2,690 - \frac{f(2,690)}{f'(2,690)} = 2,69064 \qquad p_5 - p_4 = 0,00003 < 0,0001$$

Approximate root is 2,690.

b) $f(x) = x^3 + 3x^2 - 1 \qquad f'(x) = 3x^2 + 6x$

Let $p_0 = -3$

$$p_1 = -3 - \frac{f(-3)}{f'(-3)} = -2,88888 \qquad p_1 - p_0 = 1,11111 > 0,0001$$

$$p_2 = 2,88888 - \frac{f(2,88888)}{f'(2,88888)} = -2,87945$$

$$p_2 - p_1 = 0,00943 > 0,0001$$

$$p_3 = -2,87945 - \frac{f(-2,87945)}{f'(-2,87945)} = -2,87938$$

$$p_3 - p_2 = 0,00006 < 0,0001$$

Approximate root is -2,87938.

c) $f(x)=x-cosx$ $f^{'}(x)=1+sinx$

Let $p_0=0$.

$$p_1=0-\frac{f(0)}{f^{'}(0)}=1,00000 \qquad p_1-p_0=1,00000>0,0001$$

$$p_2=1,00000-\frac{f(1,00000)}{f^{'}(1,00000)}=0,75036$$

$$p_2-p_1=0,24963>0,0001$$

$$p_3=0,75036-\frac{f(0,75036)}{f^{'}(0,75036)}=0,73911$$

$$p_3-p_2=0,01125>0,0001$$

$$p_4=0,73911-\frac{f(0,73911)}{f^{'}(0,73911)}=0,73908$$

$$p_4-p_3=0,00002<0,0001$$

Approximate root is 0,73908.

d) $f(x) = x - 0{,}8 - 0{,}2\,sinx \qquad f^{'}(x) = 1 - 0{,}2\,cosx$

Let $p_0 = 0$.

$$p_1 = 0 - \frac{f(0)}{f^{'}(0)} = -2{,}88888 \qquad p_1 - p_0 = 1{,}00000 > 0{,}0001$$

$$p_2 = -1{,}00000 - \frac{f(1{,}00000)}{f^{'}(1{,}00000)} = 0{,}96445$$

$$p_2 - p_1 = 0{,}03554 > 0{,}0001$$

$$p_1 = -3 - \frac{f(-3)}{f^{'}(-3)} = -2{,}88888$$

$$p_3 - p_2 = 0{,}00011 > 0{,}0001$$

$$p_4 = 0{,}96433 - \frac{f(0{,}96433)}{f^{'}(0{,}96433)} = 0{,}96433$$

$$p_4 - p_3 = 0{,}00000 < 0{,}0001$$

Approximate root is 0,96433.