Numerical Analysis – Homework02/Qestion2

$$\boldsymbol{A}=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

For coorinates in image **B(1,1)** and coordinates in image **F(2,2)**.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$$a_{11}+2a_{12}+a_{13}=2$$

$$a_{21}+2a_{22}+a_{23}=2$$

For coorinates in image **B(2,1)** and coordinates in image **F(-1,4)**.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$$

$$2a_{11}+a_{12}+a_{13}=-1$$

$$2a_{21}+1a_{22}+a_{23}=4$$

For coorinates in image **B(3,1)** and coordinates in image **F(-4,4)**.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \\ 1 \end{bmatrix}$$

$$3a_{11}+a_{12}+a_{13}=-4$$

$$3a_{21}+a_{22}+a_{23}=4$$

And we have two linear equation system.

First

$$a_{11}+2a_{12}+a_{13}=2$$

$$2a_{11}+a_{12}+a_{13}=-1$$

$$3a_{11}+a_{12}+a_{13}=-4$$

Second

$$a_{21}+2a_{22}+a_{23}=2$$

$$2a_{21}+1a_{22}+a_{23}=4$$

$$3a_{21}+a_{22}+a_{23}=4$$

We can solve these equation system by using LU factorization.Because ,two equation systems have same matrix.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}$$

A = LU = $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 5/3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & -1 \\ 0 & 0 & -1/3 \end{bmatrix}$

Ax = LUx = b
Ux = y (we must calculate y vector to solve x vector.)
Ly = b(we can calculate y vector by using tis equation).

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 5/3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 5/3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}$$

$y_1 = 2$ $\qquad$ $y_1 = 2$

$y_2 = -5$ $\qquad$ $y_2 = 0$

$y_3 = -5/3$ $\qquad$ $y_3 = -2$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & -1 \\ 0 & 0 & -1/3 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{21} \\ a_{22} \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ -5/3 \end{bmatrix} \text{ and}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & -1 \\ 0 & 0 & -1/3 \end{bmatrix} \begin{bmatrix} a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$$

$a_{11}=-3$ $a_{21}=0$

$a_{12}=0$ $a_{22}=-2$

$a_{13}=5$ $a_{23}=6$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 0 & 5 \\ 0 & -2 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

A is transformation matrix that convert image B into image F. But we must find matrix that convert image F into image B.
So we must calculate A^{-1} .

We can calculate A^{-1} by using A. A^{-1} = I.

$$\begin{bmatrix} -3 & 0 & 5 \\ 0 & -2 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -1/3 & 0 & 5/3 \\ 0 & -1/2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$