

CSE 443 – PATTERN RECOGNITION HOMEWORK 3 REPORT

Exercise 1:

$$\begin{aligned}
 \mu^* &= \underset{\mu}{\operatorname{argmin}} \sum_{x \in S} \|x - \mu\|_2^2 \\
 &= \underset{\mu}{\operatorname{argmin}} \sum_{x \in S} \langle x - \mu, x - \mu \rangle \\
 &= \underset{\mu}{\operatorname{argmin}} \sum_{x \in S} (\langle x, x \rangle - 2\langle x, \mu \rangle + \langle \mu, \mu \rangle) \\
 &= \underset{\mu}{\operatorname{argmin}} \langle \mu, \mu \rangle - 2 \sum_{x \in S} \langle x, \mu \rangle \\
 &= \underset{\mu}{\operatorname{argmin}} \langle \mu, \mu \rangle - 2n \left\langle \frac{1}{n} \sum_{x \in S} x, \mu \right\rangle \\
 &= \underset{\mu}{\operatorname{argmin}} \langle \mu, \mu \rangle - 2\langle \bar{x}, \mu \rangle \\
 &= \underset{\mu}{\operatorname{argmin}} \langle \mu, \mu \rangle - 2\langle \bar{x}, \mu \rangle + \langle \bar{x}, \bar{x} \rangle \\
 &= \underset{\mu}{\operatorname{argmin}} \langle \mu - \bar{x}, \mu - \bar{x} \rangle \\
 &= \underset{\mu}{\operatorname{argmin}} \|\mu - \bar{x}\|_2^2
 \end{aligned}$$

The $\|\cdot\|_2^2$ can never be smaller than 0. Therefore choosing $\mu = \bar{x}$ minimizes the expressions, as $\|\mu - \bar{x}\|_2^2$ becomes 0. Hence, $\mu^* = \bar{x}$.

Exercise 2:

X_1	X_2	t
0	0	0
0	1	1
1	0	1
1	1	0

1. $b > 0$

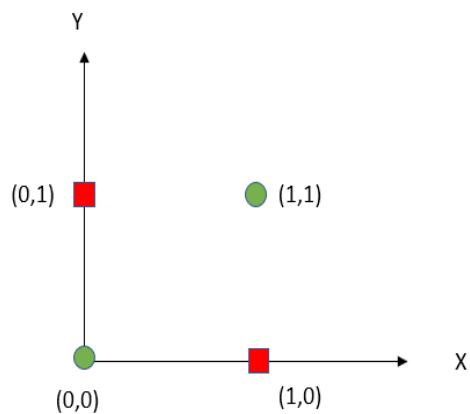
2. $W_2 + b > 0$

3. $W_1 + b > 0$

4. $W_1 + W_2 + b < 0$

if we add 2 and 3, we yield, $w_1 + W_2 + 2b > 0$

According to this inequality, $w_1 + W_2 + b > -b$. So, according to 4. expression, $-b > 0$ and $b < 0$. This expression conflicts 1. expression. And as shown in following figure, there is no line to separate these two class. So, we can say that **logical XOR is not linearly seperable problem.**



Exercise 3:

In this part, trained and tested SVM(Support Vector Machine) classifier with using 5-fold cross validation. Conducted grid search to optimize kernel parameters and compared the performance of classifier with linear, polynomial and RBF kernels.

Dataset

Used the University of Houston dataset provided by the 2013 GRSS Fusion Contest. The dataset has both LiDAR and hyperspectral data, contains 15 classes, 2832 training pixels and 12197 test pixels. There are 144 spectral bands in the data.

PCA(Principal Component Analysis) is applied to hyperspectral data and first three principal components are selected and LiDAR data is added to this data. Therefore, each pixel is represented 4-dimension vector. These vectors are given to SVM classifier as input.

Results

	RBF Kernel C : 1000 gamma : 10^{-5}	Linear Kernel C : 0.1	Poly. Kernel C : 1 degree : 15
Overall Accuracy	53.60 %	53.08 %	45.33 %
Average Accuracy	56.50 %	48.59 %	50.60 %
Kappa	0.5004	0.4949	0.4117