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# University of Brasilia

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University of Brasilia **Data Structures** 

```
set ts=4 sw=4 sta nu rnu sc stl+=%F cindent
imap {<CR> {<CR>}<Esc>0
nmap <F2> 0V$%d
nmap <C-down> :m+1<CR>
nmap < C-up > :m-2 < CR >
vmap < C-c > "+y
nmap <C-a> ggVG
syntax on
alias cmp='g++ -Wall -Wformat=2 -Wshadow -Wconversion -
  fsanitize=address -fsanitize=undefined -fno-sanitize-
  recover -std=c++11'
```

## **Data Structures**

## **Merge Sort Tree**

```
struct MergeTree{
   int n:
   vector<vector<int>> st;
   void build(int p, int L, int R, const int v[]){
      if(L == R){
          st[p].push_back(v[L]);
          return:
      int mid = (L+R)/2;
      build(2*p, L, mid, v);
      build(2*p+1, mid+1, R, v);
      st[p].resize(R-L+1);
      merge(st[2*p].begin(), st[2*p].end(),
              st[2*p+1].begin(), st[2*p+1].end(),
              st[p].begin());
   }
   int query(int p, int L, int R, int i, int j, int x)
     const\{
      if(L > j || R < i) return 0;
      if(L >= i && R <= j){
          int id = lower_bound(st[p].begin(), st[p].end
            (), x) - st[p].begin();
          return int(st[p].size()) - id;
      int mid = (L+R)/2;
      return query(2*p, L, mid, i, j, x) +
          query(2*p+1, mid+1, R, i, j, x);
   }
public:
   MergeTree(int sz, const int v[]): n(sz), st(4*sz){
      build(1, 1, n, v);
   //number of elements >= x on segment [i, j]
   int query(int i, int j, int x) const{
      if(i > j) swap(i, j);
      return query(1, 1, n, i, j, x);
   }
};
Wavelet Tree
template<typename T>
class wavelet{
   T L, R;
   vector<int> 1;
```

```
vector<T> sum; // <<</pre>
wavelet *lef, *rig;
```

```
int r(int i) const{ return i - l[i]; }
public:
   template<typename ITER>
   wavelet(ITER bg, ITER en){
       lef = rig = nullptr;
       L = *bg, R = *bg;
       for(auto it = bg; it != en; it++)
          L = min(L, *it), R = max(R, *it);
       if(L == R) return;
       T mid = L + (R - L)/2;
       1.reserve(std::distance(bg, en) + 1);
       sum.reserve(std::distance(bg, en) + 1);
       1.push_back(0), sum.push_back(0);
       for(auto it = bg; it != en; it++)
          l.push_back(l.back() + (*it <= mid)),</pre>
          sum.push_back(sum.back() + *it);
       auto tmp = stable_partition(bg, en, [mid](T x){
          return x <= mid;</pre>
       });
       if(bg != tmp) lef = new wavelet(bg, tmp);
       if(tmp != en) rig = new wavelet(tmp, en);
   }
    ~wavelet(){
       delete lef;
       delete rig;
   // 1 index, first is 1st
   T kth(int i, int j, int k) const{
       if(L >= R) return L;
       int c = 1[j] - 1[i-1];
       if(c \ge k) return lef \ge kth(l[i-1]+1, l[j], k);
       else return rig->kth(r(i-1)+1, r(j), k - c);
   }
   // # elements > x on [i, j]
   int cnt(int i, int j, T x) const{
       if(L > x) return j - i + 1;
       if(R <= x || L == R) return 0;
       int ans = 0;
       if(lef) ans += lef->cnt(l[i-1]+1, l[j], x);
       if(rig) ans += rig->cnt(r(i-1)+1, r(j), x);
       return ans;
   }
   // sum of elements <= k on [i, j]</pre>
   T sumk(int i, int j, T k){
       if(L == R) return R <= k ? L * (j - i + 1) : 0;
       if(R <= k) return sum[j] - sum[i-1];</pre>
       int ans = 0;
       if(lef) ans += lef->sumk(l[i-1]+1, l[j], k);
       if(rig) ans += rig->sumk(r(i-1)+1, r(j), k);
      return ans;
   // swap (i, i+1) just need to update "array" l[i]
};
Order Set
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
```

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```
next(v)):
#include <ext/pb_ds/detail/standard_policies.hpp>
                                                                  while(y != begin() && bad(prev(y))) erase(prev(y)
                                                                    );
using namespace __gnu_pbds; // or pb_ds;
                                                               ll eval(ll x){
template<typename T, typename B = null_type>
                                                                   auto 1 = *lower_bound((Line) { x, is_query });
using oset = tree<T, B, less<T>, rb_tree_tag,
                                                                  return 1.m * x + 1.b;
  tree_order_statistics_node_update>;
                                                               }
// find_by_order / order_of_key
                                                           };
Hash table
                                                           Min queue
                                                           template<typename T>
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
                                                           class minQ{
                                                               deque<tuple<T, int, int> > p;
                                                               T delta;
struct custom_hash {
                                                               int sz;
   static uint64_t splitmix64(uint64_t x) {
                                                           public:
       // http://xorshift.di.unimi.it/splitmix64.c
                                                               minQ() : delta(0), sz(0) {}
      x += 0x9e3779b97f4a7c15;
                                                               inline int size() const{ return sz; }
      x = (x \hat{ } (x >> 30)) * 0xbf58476d1ce4e5b9;
                                                               inline void add(T x){ delta += x; }
      x = (x ^(x >> 27)) * 0x94d049bb133111eb;
                                                               inline void push(T x, int id){
      return x \hat{ } (x \gg 31);
                                                                   x -= delta, sz++;
   }
                                                                  int t = 1;
                                                                  while(p.size() > 0 \& get<0>(p.back()) >= x)
   size_t operator()(uint64_t x) const {
                                                                      t += get<1>(p.back()), p.pop_back();
       static const uint64_t FIXED_RANDOM = chrono::
                                                                  p.emplace_back(x, t, id);
         steady_clock::now().time_since_epoch().count();
       return splitmix64(x + FIXED_RANDOM);
                                                               inline void pop(){
   }
                                                                   get<1>(p.front())--, sz--;
                                                                   if(!get<1>(p.front())) p.pop_front();
gp_hash_table<long long, int, custom_hash> table;
                                                               T getmin() const{ return get<0>(p.front())+delta; }
unordered_map<long long, int, custom_hash> uhash;
                                                               int getid() const{ return get<2>(p.front()); }
uhash.reserve(1 << 15);</pre>
                                                           };
uhash.max_load_factor(0.25);
                                                           Sparse Table
Convex Hull Trick
                                                           const int N = 100005;
const ll is_query = -(1LL<<62);</pre>
struct Line{
                                                           int v[N], n;
   11 m. b:
                                                           int dn[N][20];
   mutable function<const Line*()> succ;
                                                           int fn(int i, int j){
   bool operator<(const Line& rhs) const{</pre>
                                                               if(j == 0) return v[i];
       if(rhs.b != is_query) return m < rhs.m;</pre>
                                                               if(~dn[i][j]) return dn[i][j];
       const Line* s = succ();
                                                               return dn[i][j] = min(fn(i, j-1), fn(i + (1 << (j-1)))
       if(!s) return 0;
                                                                 ), j-1));
       11 x = rhs.m;
                                                           }
       return b - s->b < (s->m - m) * x;
                                                           int lg(int x){ return 31 - __builtin_clz(x); }
};
struct Cht : public multiset<Line>{ // maintain max
                                                           int getmn(int 1, int r){ // [1, r]
   bool bad(iterator y){
                                                               int 1z = 1g(r - 1 + 1);
       auto z = next(y);
                                                               return min(fn(1, 1z), fn(r - (1 << 1z) + 1, 1z));
      if(y == begin()){
                                                           }
          if(z == end()) return 0;
                                                           Treap
          return y->m == z->m \&\& y->b <= z->b;
                                                           // source: https://github.com/victorsenam/caderno/blob/
       auto x = prev(y);
      if(z == end()) return y->m == x->m && y->b <= x->
                                                             master/code/treap.cpp
                                                           //const int N = ; typedef int num;
       return (x->b - y->b)*(z->m - y->m) >= (y->b - z->
                                                           num X[N]; int en = 1, Y[N], sz[N], L[N], R[N];
         b)*(y->m - x->m);
                                                           void calc (int u) { // update node given children info
                                                               sz[u] = sz[L[u]] + 1 + sz[R[u]];
   void insert_line(ll m, ll b){
                                                               // code here, no recursion
      auto y = insert({ m, b });
                                                           }
      y->succ = [=]{ return next(y) == end() ? 0 : &*
                                                           void unlaze (int u) {
         next(y); };
                                                               if(!u) return;
       if(bad(y)){ erase(y); return; }
                                                               // code here, no recursion
       while(next(y) != end() && bad(next(y))) erase(
```

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```
void split_val(int u, num x, int &l, int &r) { // l gets
                                                                  it = ranges.lower_bound(r);
   <= x, r gets > x
                                                                  if(it != ranges.begin()) {
   unlaze(u); if(!u) return (void) (1 = r = 0);
                                                                      it--;
   if(X[u] \le x) \{ split_val(R[u], x, 1, r); R[u] = 1;
                                                                      if(it->r>r) {
                                                                         auto cur = *it;
     1 = u; }
   else { split_val(L[u], x, 1, r); L[u] = r; r = u; }
                                                                         ranges.erase(it);
   calc(u);
                                                                         ranges.insert(Range(cur.1, r, cur.v));
                                                                         ranges.insert(Range(r, cur.r, cur.v));
void split_sz(int u, int s, int &l, int &r) { // l gets
                                                                      }
  first s, r gets remaining
   unlaze(u); if(!u) return (void) (1 = r = 0);
                                                                  for(it = ranges.lower_bound(1); it != ranges.end
   if(sz[L[u]] < s) { split_sz(R[u], s - sz[L[u]] - 1,
                                                                    () && it->l < r; it++) {
     1, r); R[u] = 1; 1 = u; }
                                                                      ans.push_back(*it);
   else { split_sz(L[u], s, l, r); L[u] = r; r = u; }
                                                                  ranges.erase(ranges.lower_bound(1), ranges.
   calc(u);
                                                                    lower_bound(r));
int merge(int 1, int r) { // els on 1 <= els on r
                                                                  ranges.insert(Range(1, r, v));
   unlaze(l); unlaze(r); if(!l || !r) return l + r; int
                                                                  return ans;
                                                               }
                                                           private:
   if(Y[1] > Y[r]) { R[1] = merge(R[1], r); u = 1; }
   else { L[r] = merge(1, L[r]); u = r; }
                                                               std::set<Range> ranges;
   calc(u); return u;
                                                           };
                                                           Heavy Light Decomposition
void init(int n=N-1) { // XXX call before using other
                                                           void dfs_sz(int u){
   for(int i = en = 1; i \le n; i++) { Y[i] = i; sz[i] =
                                                               sz[u] = 1;
      1; L[i] = R[i] = 0; }
   random\_shuffle(Y + 1, Y + n + 1);
                                                               for(auto &v : g[u]) if(v == p[u]){
                                                                  swap(v, g[u].back());
                                                                  g[u].pop_back();
ColorUpdate
                                                                  break;
                                                               }
// source: https://github.com/tfg50/Competitive-
  Programming/tree/master/Biblioteca/Data%20Structures
                                                               for(auto &v : g[u]){
                                                                  p[v] = u;
#include <set>
                                                                  dfs_sz(v);
#include <vector>
                                                                  sz[u] += sz[v];
                                                                  if(sz[v] > sz[g[u][0]])
template <class Info = int>
                                                                      swap(v, g[u][0]);
class ColorUpdate {
                                                               }
public:
                                                           }
   struct Range {
       Range(int l = 0) { this->l = 1; }
                                                           // nxt[u] = start of path with u
       Range(int 1, int r, Info v) {
                                                           // set nxt[root] beforehand
          this->1 = 1;
                                                           void dfs_hld(int u){
          this -> r = r;
                                                               in[u] = t++;
          this->v = v;
                                                               rin[in[u]] = u;
                                                               for(auto v : g[u]){
      int 1, r;
                                                                  nxt[v] = (v == g[u][0] ? nxt[u] : v);
      Info v:
                                                                  dfs_hld(v);
                                                               }
      bool operator < (const Range &b) const { return 1</pre>
                                                               out[u] = t;
          < b.1: }
   }:
                                                           // subtree of u => [ in[u], out[u] )
   std::vector<Range> upd(int 1, int r, Info v) {
                                                           // path from nxt[u] to u => [ in[ nxt[u] ], in[u] ]
       std::vector<Range> ans;
      if(1 >= r) return ans;
                                                           Iterative Segtree
       auto it = ranges.lower_bound(1);
      if(it != ranges.begin()) {
                                                           T query(int 1, int r, int &pos){ // [1, r]
          it--;
                                                              T rl, rr;
          if(it->r>1) {
                                                               for(1 += n, r += n+1; 1 < r; 1 >>= 1, r >>= 1){
                                                                  if(1 & 1) rl = merge(rl, st[1++]);
              auto cur = *it;
              ranges.erase(it);
                                                                  if(r & 1) rr = merge(st[--r], rr);
              ranges.insert(Range(cur.1, 1, cur.v));
              ranges.insert(Range(1, cur.r, cur.v));
                                                              return merge(rl, rr);
                                                           }
          }
       }
```

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```
// initially save v[i] in st[n+i] for all i
                                                           }
void build(){
   for(int p = n-1; p > 0; p--)
       st[p] = merge(st[2*p], st[2*p+1]);
                                                           map<ll, int> factor(ll val){
                                                               map<ll, int> fac;
                                                               if(val > 1) factorize(val, fac);
void update(int p, T val){
                                                               return fac:
   st[p += n] = val;
   while(p >>= 1) st[p] = merge(st[2*p], st[2*p+1]);
                                                           Miller Rabin
                                                           bool rabin(ll n){
Math
                                                               if(n \ll 1) return 0;
                                                               if(n <= 3) return 1;
Euclides Extendido
                                                               11 s = 0, d = n - 1;
                                                               while(d % 2 == 0) d /= 2, s++;
// a*x + b*y = gcd(a, b), < gcd, x, y>
                                                               for(int k = 0; k < 64; k++){
tuple<int, int, int> euclidesExt(int a, int b) {
                                                                  11 a = (11rand() \% (n - 3)) + 2;
   if(b == 0) return make_tuple(a, 1, 0);
                                                                  11 x = fexp(a, d, n);
   int q, w, e;
                                                                  if(x != 1 \&\& x != n-1){
   tie(q, w, e) = euclidesExt(b, a % b);
                                                                      for(int r = 1; r < s; r++){
   return make_tuple(q, e, w - e * (a / b));
                                                                         x = mul(x, x, n);
                                                                          if(x == 1) return 0;
Preffix inverse
                                                                          if(x == n-1) break;
                                                                      if(x != n-1) return 0;
inv[1] = 1;
for(int i = 2; i < p; i++)
   inv[i] = (p - (p/i) * inv[p%i] % p) % p;
                                                               return 1;
Pollard Rho
                                                           Totiente
ll rho(ll n){
   if(n % 2 == 0) return 2;
                                                           11 totiente(ll n){
                                                               11 \text{ ans} = n;
   11 d, c, x, y;
                                                               for(11 i = 2; i*i <= n; i++){</pre>
   do{
                                                                  if(n \% i == 0){
       c = 11rand() % n, x = 11rand() % n, y = x;
                                                                      ans = ans / i * (i - 1);
                                                                      while(n \% i == 0) n /= i;
          x = add(mul(x, x, n), c, n);
                                                                  }
          y = add(mul(y, y, n), c, n);
          y = add(mul(y, y, n), c, n);
          d = \_gcd(abs(x - y), n);
                                                               if(n > 1) ans = ans / n * (n - 1);
       \}while(d == 1);
                                                               return ans:
   }while(d == n);
   return d;
                                                           Mobius Function
ll pollard_rho(ll n){
                                                           memset(mu, 0, sizeof mu);
   11 x, c, y, d, k;
                                                           mu[1] = 1;
   int i;
                                                           for(int i = 1; i < N; i++)
   do{
                                                               for(int j = i + i; j < N; j += i)
       i = 1:
                                                                  mu[j] -= mu[i];
       x = 11rand() % n, c = 11rand() % n;
                                                           // g(n) = sum{f(d)} => f(n) = sum{mu(d)*g(n/d)}
       y = x, k = 4;
                                                           Mulmod TOP
       do{
          if(++i == k) y = x, k *= 2;
                                                           constexpr uint64_t mod = (1ull<<61) - 1;</pre>
          x = add(mul(x, x, n), c, n);
                                                           uint64_t modmul(uint64_t a, uint64_t b){
          d = \_gcd(abs(x - y), n);
                                                               uint64_t 11 = (uint32_t)a, h1 = a>>32, 12 = (
       \}while(d == 1);
                                                                 uint32_t)b, h2 = b>>32;
   \}while(d == n);
                                                               uint64_t 1 = 11*12, m = 11*h2 + 12*h1, h = h1*h2;
   return d:
                                                               uint64_t ret = (1\&mod) + (1>>61) + (h << 3) + (m >>
}
                                                                 29) + (m << 35 >> 3) + 1;
                                                               ret = (ret & mod) + (ret>>61);
void factorize(ll val, map<ll, int> &fac){
                                                               ret = (ret \& mod) + (ret >> 61);
   if(rabin(val)) fac[ val ]++;
                                                               return ret-1;
   else{
                                                           }
       11 d = pollard_rho(val);
                                                           Determinant
       factorize(d, fac);
```

factorize(val / d, fac);

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```
const double EPS = 1E-9;
int n;
vector < vector<double> > a (n, vector<double> (n));
double det = 1;
for (int i=0; i<n; ++i) {</pre>
   int k = i;
   for (int j=i+1; j<n; ++j)
       if (abs (a[j][i]) > abs (a[k][i]))
          k = j;
   if (abs (a[k][i]) < EPS) {
       det = 0;
      break;
   swap (a[i], a[k]);
   if (i != k)
      det = -det;
   det *= a[i][i];
   for (int j=i+1; j< n; ++j)
      a[i][j] /= a[i][i];
   for (int j=0; j<n; ++j)
      if (j != i && abs (a[j][i]) > EPS)
          for (int k=i+1; k<n; ++k)
              a[j][k] -= a[i][k] * a[j][i];
}
cout << det;</pre>
FFT
struct base{
   double r. i:
   base(double _r = 0, double _i = 0) : r(_r), i(_i) {}
   base operator*(base &o) const{
      return {r*o.r - i*o.i, r*o.i + o.r*i};
   double real() const{ return r; }
   void operator*=(const base &o){
       (*this) = \{r*o.r-i*o.i, r*o.i+o.r*i\};
   void operator+=(const base &o)\{r \neq o.r, i \neq o.i; \}
   void operator/=(const double &o){ r /= o, i /= o; }
   void operator-=(const base &o){r -= o.r, i -= o.i; }
   base operator+(const base &o){return {r+o.r,i+o.i};}
   base operator-(const base &o){return {r-o.r,i-o.i};}
double PI = acos(-1);
void fft(vector<base> &a, bool inv){
   int n = (int)a.size();
   for(int i = 1, j = 0; i < n; i++){
       int bit = n \gg 1;
       for(; j >= bit; bit >>= 1) j -= bit;
       j += bit;
       if(i < j) swap(a[i], a[j]);
   for(int sz = 2; sz <= n; sz <<= 1) {
       double ang = 2*PI/sz * (inv ? -1 : 1);
       base wlen(cos(ang), sin(ang));
       for(int i = 0; i < n; i += sz){
          base w(1);
          for(int j = 0; j < sz/2; j++){
              base u = a[i+j], v = a[i+j+sz/2] * w;
              a[i+j] = u + v;
              a[i+j+sz/2] = u - v;
```

```
w *= wlen;
           }
       }
   if(inv) for(int i = 0; i < n; i++) a[i] /= 1.0 * n;
}
void multiply(const vector<int> &a, const vector<int> &b
  , vector<int> &res){
   vector<base> fa(a.begin(), a.end());
   vector<base> fb(b.begin(), b.end());
   size_t n = 1;
   while(n < a.size()) n <<= 1;
   while(n < b.size()) n <<= 1;
   n <<= 1:
   fa.resize(n), fb.resize(n);
   fft(fa, false), fft(fb, false);
   for(size_t i = 0; i < n; i++)
       fa[i] *= fb[i];
   fft(fa, true);
   res.resize (n);
   for(size_t i = 0; i < n; ++i)
       res[i] = int(fa[i].real() + 0.5);
}
NTT
const int mod = 7340033;
const int root = 5;
const int root_1 = 4404020;
const int root_pw = 1<<20;</pre>
void fft (vector<int> & a, bool invert) {
   int n = (int) a.size();
   for (int i=1, j=0; i<n; ++i) {</pre>
       int bit = n \gg 1;
       for (; j>=bit; bit>>=1)
           j -= bit;
       j += bit;
       if (i < j)
           swap (a[i], a[j]);
   }
   for (int len=2; len<=n; len<<=1) {</pre>
       int wlen = invert ? root_1 : root;
       for (int i=len; i<root_pw; i<<=1)</pre>
           wlen = int (wlen * 111 * wlen % mod);
       for (int i=0; i<n; i+=len) {</pre>
           int w = 1;
           for (int j=0; j<len/2; ++j) {</pre>
              int u = a[i+j], v = int (a[i+j+len/2] * 1
                11 * w % mod);
              a[i+j] = u+v < mod ? u+v : u+v-mod;
              a[i+j+len/2] = u-v >= 0 ? u-v : u-v+mod;
              w = int (w * 111 * wlen % mod);
          }
       }
   if (invert) {
       int nrev = reverse (n, mod);
       for (int i=0; i<n; ++i)</pre>
           a[i] = int (a[i] * 111 * nrev % mod);
   }
}
```

University of Brasilia Graphs

# Graphs

## Dinic

```
const int N = 100005;
const int E = 2000006;
vector<int> g[N];
int ne;
struct Edge{
   int from, to:
   11 flow, cap;
} edge[E];
int lvl[N], vis[N], pass, start = N-2, target = N-1;
int qu[N], qt, px[N];
11 run(int s, int sink, ll minE){
   if(s == sink) return minE;
   11 \text{ ans} = 0:
   for(; px[s] < (int)g[s].size(); px[s]++){</pre>
       int e = g[s][ px[s] ];
       auto &v = edge[e], &rev = edge[e^1];
       if(lvl[v.to] != lvl[s]+1 || v.flow >= v.cap)
         continue;
       11 tmp = run(v.to, sink,min(minE, v.cap-v.flow));
       v.flow += tmp, rev.flow -= tmp;
       ans += tmp, minE -= tmp;
       if(minE == 0) break;
   }
   return ans;
}
bool bfs(int source, int sink){
   qt = 0;
   qu[qt++] = source;
   lvl[source] = 1;
   vis[source] = ++pass;
   for(int i = 0; i < qt; i++){
       int u = qu[i];
       px[u] = 0;
       if(u == sink) return true;
       for(int e : g[u]){
          auto v = edge[e];
          if(v.flow >= v.cap || vis[v.to] == pass)
             continue:
          vis[v.to] = pass;
          lvl[v.to] = lvl[u]+1;
           qu[qt++] = v.to;
   return false;
11 flow(int source = start, int sink = target){
   11 \text{ ans} = 0;
   while(bfs(source, sink))
       ans += run(source, sink, oo);
   return ans;
}
void addEdge(int u, int v, ll c = 1, ll rc = 0){
   edge[ne] = \{u, v, 0, c\};
```

```
g[u].push_back(ne++);
   edge[ne] = \{v, u, 0, rc\};
   g[v].push_back(ne++);
void reset_flow(){
   for(int i = 0; i < ne; i++)
       edge[i].flow = 0;
Min Cost Max Flow
const 11 oo = 1e18;
const int N = 505;
const int E = 30006;
vector<int> g[N];
int ne;
struct Edge{
   int from, to;
   11 cap, cost;
} edge[E];
int lvl[N], vis[N], pass, source, target, p[N], px[N];
11 d[N];
11 back(int s, ll minE){
   if(s == source) return minE;
   int e = p[s];
   11 f = back(edge[e].from, min(minE, edge[e].cap));
   edge[e].cap -= f;
   edge[e^1].cap += f;
   return f;
}
int dijkstra(){
   forn(i, N) d[i] = oo;
   priority_queue<pair<ll, int> > q;
   d[source] = 0;
   q.emplace(0, source);
   while(!q.empty()){
       11 dis = -q.top().ff;
       int u = q.top().ss; q.pop();
       if(dis > d[u]) continue;
       for(int e : g[u]){
          auto v = edge[e];
          if(v.cap <= 0) continue;</pre>
          if(d[u] + v.cost < d[v.to]){
              d[v.to] = d[u] + v.cost;
              p[v.to] = e;
              q.emplace(-d[v.to], v.to);
          }
      }
   }
   return d[target] != oo;
}
```

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```
pair<11, 11> mincost(){
   11 ans = 0, mf = 0;
   while(dijkstra()){
       11 f = back(target, oo);
       mf += f;
       ans += f * d[target];
   return {mf, ans};
}
void addEdge(int u, int v, 11 c, 11 cost){
   edge[ne] = \{u, v, c, cost\};
   g[u].pb(ne++);
Small to Large
void cnt_sz(int u, int p = -1){
   sz[u] = 1;
   for(int v : g[u]) if(v != p)
       cnt_sz(v, u), sz[u] += sz[v];
}
void add(int u, int p, int big = -1){
   // Update info about this vx in global answer
   for(int v : g[u]) if(v != p && v != big)
       add(v, u);
void dfs(int u, int p, int keep){
   int big = -1, mmx = -1;
   for(int v : g[u]) if(v != p \&\& sz[v] > mmx)
       mmx = sz[v], big = v;
   for(int v : g[u]) if(v != p && v != big)
       dfs(v, u, 0);
   if(big != -1) dfs(big, u, 1);
   add(u, p, big);
   \quad \text{for}(\text{auto } x \,:\, q[u])\{
       // answer all queries for this vx
   if(!keep){
       // Remove data from this subtree
Iunior e Falta de Ideias
#include <bits/stdc++.h>
#define ff first
#define ss second
#define mp make_pair
using namespace std;
typedef long long 11;
vector<pair<int,int>> G[500005];
int subtree[500005], treesize, k;
bool vis[500005];
```

```
ll dist[500005], ans;
int dfs(int v, int p){
   subtree[v] = 1;
   for(pair<int,int> x : G[v])
       if(x.ff != p \&\& !vis[x.ff]) subtree[v] += dfs(x.
         ff.v):
   return subtree[v];
int centroid(int v, int p){
   for(pair<int,int> x : G[v]){
       if(x.ff == p || vis[x.ff]) continue;
       if(subtree[x.ff]*2 > treesize) return centroid(x.
   }
   return v;
}
void procurar_ans(int v, int p, int d_atual, ll custo){
   ans = min(ans, dist[k-d_atual] + custo);
   if(d_atual == k) return;
   for(pair<int,int> x : G[v]){
       if(!vis[x.ff] && x.ff != p)
          procurar_ans(x.ff,v,d_atual+1,custo+x.ss);
   }
}
void atualiza_distancia(int v, int p, int d_atual, 11
  custo){
   dist[d_atual] = min(dist[d_atual], custo);
   if(d_atual == k) return;
   for(pair<int,int> x : G[v]){
       if(!vis[x.ff] && x.ff != p)
          atualiza_distancia(x.ff,v,d_atual+1,custo+x.
             ss);
   }
}
void decomp(int v, int p){
   treesize = dfs(v,v);
   // if(treesize < k) return;</pre>
   int cent = centroid(v,v);
   vis[cent] = 1;
   for(int i = 1; i <= treesize; i++)
       dist[i] = 1e18;
   for(pair<int,int> x : G[cent]){
       if(!vis[x.ff]){
          procurar_ans(x.ff,cent,1,x.ss);
          atualiza_distancia(x.ff,cent,1,x.ss);
   for(pair<int,int> x : G[cent]){
       if(!vis[x.ff])
          decomp(x.ff, cent);
}
int main(){
   int n,i,a,b;
   scanf("%d%d", &n,&k);
   for(i = 2; i \le n; i++){
       scanf("%d%d", &a,&b);
```

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```
G[i].push_back(mp(a,b));
                                                                   do{
                                                                       v = S.back(); S.pop_back(); vis[v] = 0;
       G[a].push_back(mp(i,b));
   }
                                                                       ssc[v] = ssc_cnt;
                                                                   }while(u != v);
   ans = 1e18;
   decomp(1,-1);
   printf("%lld\n", ans == 1e18 ? -1 : ans);
                                                            Max Clique
   return 0;
                                                            long long adj[N], dp[N];
Kosaraju
                                                            for(int i = 0; i < n; i++){
                                                               for(int j = 0; j < n; j++){
vector<int> g[N], gt[N], S;
                                                                   int x;
                                                                   scanf("%d",&x);
int vis[N], cor[N], tempo = 1;
                                                                   if(x \mid \mid i == j)
                                                                       adj[i] |= 1LL << j;
void dfs(int u){
                                                               }
   vis[u] = 1;
                                                            }
   for(int v : g[u]) if(!vis[v]) dfs(v);
   S.push_back(u);
                                                            int resto = n - n/2;
                                                            int C = n/2;
                                                            for(int i = 1; i < (1 << resto); i++){}
int e;
void dfst(int u){
                                                               int x = i;
                                                                for(int j = 0; j < resto; j++)
   cor[u] = e;
   for(int v : gt[u]) if(!cor[v]) dfst(v);
                                                                   if(i & (1 << j))
                                                                      x \&= adj[j + C] >> C;
                                                                if(x == i){
int main(){
                                                                   dp[i] = __builtin_popcount(i);
   for(int i = 1; i <= n; i++) if(!vis[i]) dfs(i);</pre>
                                                            }
   e = 0;
                                                            for(int i = 1; i < (1 << resto); i++)</pre>
   reverse(S.begin(), S.end());
                                                               for(int j = 0; j < resto; j++)
   for(int u : S) if(!cor[u])
                                                                   if(i & (1 << j))
       e++, dfst(u);
                                                                       dp[i] = max(dp[i], dp[i ^ (1 << j)]);
   return 0;
                                                            int maxCliq = 0;
                                                            for(int i = 0; i < (1 << C); i++){}
                                                                int x = i, y = (1 << resto) - 1;
Tarjan
                                                                for(int j = 0; j < C; j++)
                                                                   if(i & (1 << j))
int cnt = 0, root;
                                                                      x \&= adj[j] \& ((1 << C) - 1), y \&= adj[j] >>
void dfs(int u, int p = -1){
                                                                        С;
   low[u] = num[u] = ++t;
                                                               if(x != i) continue;
   for(int v : g[u]){
                                                               maxCliq = max(maxCliq, __builtin_popcount(i) + dp[y
       if(!num[v]){
                                                                  ]);
          dfs(v, u);
          if(v == root) cnt++;
                                                            Dominator Tree
          if(low[v] >= num[u]) u PONTO DE ARTICULAÇÃO;
          if(low[v] > num[u]) ARESTA u->v PONTE;
          low[u] = min(low[u], low[v]);
                                                            vector<int> g[N], gt[N], T[N];
                                                            vector<int> S;
       else if(v != p) low[u] = min(low[u], num[v]);
                                                            int dsu[N], label[N];
                                                            int sdom[N], idom[N], dfs_time, id[N];
   }
}
                                                            vector<int> bucket[N];
root PONTO DE ARTICULAÇÃO <=> cnt > 1
                                                            vector<int> down[N];
void tarjanSCC(int u){
                                                            void prep(int u){
   low[u] = num[u] = cnt++;
                                                               S.push_back(u);
   vis[u] = 1;
                                                               id[u] = ++dfs_time;
   S.push_back(u);
                                                               label[u] = sdom[u] = dsu[u] = u;
   for(int v : g[u]){
       if(!num[v]) tarjanSCC(v);
                                                               for(int v : g[u]){
       if(vis[v]) low[u] = min(low[u], low[v]);
                                                                   if(!id[v])
                                                                      prep(v), down[u].push_back(v);
   if(low[u] == num[u]){
                                                                   gt[v].push_back(u);
       ssc[u] = ++ssc_cnt; int v;
```

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```
}
                                                                   for(int j = 1; j \le m; ++j)
                                                                      if(! used[j]){
int fnd(int u, int flag = 0){
                                                                          int cur = a[i0][j] - u[i0] - v[j];
   if(u == dsu[u]) return u;
                                                                          if(cur < minv[j])</pre>
   int v = fnd(dsu[u], 1), b = label[ dsu[u] ];
                                                                             minv[j] = cur, way[j] = j0;
   if(id[ sdom[b] ] < id[ sdom[ label[u] ] ])
                                                                          if(minv[j] < delta)</pre>
       label[u] = b;
                                                                              delta = minv[j] , j1 = j;
   dsu[u] = v;
                                                                      }
                                                                   for(int j = 0; j \le m; ++j)
   return flag ? v : label[u];
}
                                                                      if(used[j])
                                                                          u[p[j]] += delta, v[j] -= delta;
void build_dominator_tree(int root, int sz){
   // memset(id, 0, sizeof(int) * (sz + 1));
                                                                          minv[j] -= delta;
   // for(int i = 0; i <= sz; i++) T[i].clear();
                                                                   j0 = j1;
                                                               }while(p[j0] != 0);
   prep(root);
   reverse(S.begin(), S.end());
                                                               do{
                                                                   int j1 = way[j0];
   int w:
                                                                  p[j0] = p[j1];
   for(int u : S){
                                                                   j0 = j1;
                                                               }while(j0);
       for(int v : gt[u]){
                                                            }
          w = fnd(v);
          if(id[ sdom[w] ] < id[ sdom[u] ])</pre>
                                                            // match[i] = coluna escolhida para linha i
              sdom[u] = sdom[w];
                                                            vector<int> match(n + 1);
                                                            for(int j = 1; j \le m; ++j)
                                                               match[p[j]] = j;
      gt[u].clear();
       if(u != root) bucket[ sdom[u] ].push_back(u);
                                                            int cost = -v[0];
                                                            Strings
       for(int v : bucket[u]){
          w = fnd(v);
                                                            Aho Corasick
          if(sdom[w] == sdom[v]) idom[v] = sdom[v];
          else idom[v] = w;
                                                            map<char, int> to[N];
                                                            int ne = 1, term[N], fail[N];
      bucket[u].clear();
                                                            void add_string(char *str){
       for(int v : down[u]) dsu[v] = u;
                                                               int p = 0;
      down[u].clear();
                                                               for(int i = 0; str[i]; i++){
                                                                   if(!to[p][ str[i] ]) to[p][ str[i] ] = ne++;
   reverse(S.begin(), S.end());
                                                                  p = to[p][ str[i] ];
   for(int u : S) if(u != root){
                                                               term[p] = 1;
       if(idom[u] != sdom[u]) idom[u] = idom[ idom[u] ];
      T[ idom[u] ].push_back(u);
   }
                                                            int go(int s, char c){
                                                               while(s && !to[s].count(c)) s = fail[s];
   S.clear();
                                                               if(to[s].count(c)) return to[s][c];
                                                               return s:
Min Cost Matching
                                                            }
// Min cost matching
                                                            void init(){
// O(n^2 * m)
// n == nro de linhas
                                                               queue<int> q;
// m == nro de colunas
                                                               q.push(0);
// n <= m | flow == n
// a[i][j] = custo pra conectar i a j
                                                               int u, v; char c;
vector < int > u(n + 1), v(m + 1), p(m + 1), way(m + 1);
                                                               while(!q.empty()){
for(int i = 1; i \le n; ++i){
                                                                  u = q.front(); q.pop();
   p[0] = i;
   int j0 = 0;
                                                                   for(auto w : to[u]){
   vector<int> minv(m + 1 , oo);
                                                                      tie(c, v) = w;
                                                                      q.push(v);
   vector<char> used(m + 1 , false);
                                                                      if(u){}
                                                                          fail[v] = go(fail[u], c);
      used[j0] = true;
                                                                          term[v] |= term[ fail[v] ];
       int i0 = p[j0] , delta = oo, j1;
```

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```
vector<int> z(n, 0);
          }
       }
                                                               for(int i = 1; i < n; i++){
   }
                                                                   if(i \le R) z[i] = min(z[i-L], R - i + 1);
}
                                                                  while(z[i]+i < n \& s[z[i]+i] == s[z[i]])
Suffix Array
                                                                  if(i+z[i]-1 > R) L = i, R = i + z[i] - 1;
                                                               return z;
char s[N];
int n, sa[N], tsa[N], lcp[N], r[N], nr[N], c[N];
                                                           Prefix function/KMP
void sort(int k, int mx){
                                                           vector<int> preffix_function(const string &s){
   mx += 2:
   memset(c, 0, sizeof(int) * mx);
                                                               int n = s.size();
                                                               vector<int> b(n+1);
   for(int i = 0; i < n; i++) c[i + k < n ? r[i+k]+2 :
                                                               b[0] = -1;
                                                               int i = 0, j = -1;
   partial_sum(c, c+mx, c);
                                                               while(i < n){
   int t:
                                                                  while(j >= 0 \&\& s[i] != s[j]) j = b[j];
   for(int i = 0; i < n; i++)
                                                                  b[++i] = ++j;
       t = sa[i]+k < n ? r[ sa[i]+k ]+1 : 0,
       tsa[c[t]++] = sa[i];
                                                               return b;
   memcpy(sa, tsa, sizeof(int) * n);
                                                           }
}
                                                           void kmp(const string &t, const string &p){
void build_sa(){
                                                               vector<int> b = preffix_function(p);
                                                               int n = t.size(), m = p.size();
   for(int i = 0; i < n; i++) sa[i] = i, r[i] = s[i];
                                                               int j = 0;
                                                               for(int i = 0; i < n; i++){
   int t = 300, a, b;
                                                                  while(j \ge 0 \& t[i] != p[j]) j = b[j];
   for(int sz = 1; sz < n; sz *= 2){
                                                                   i++:
       sort(sz, t), sort(0, t);
                                                                   if(j == m){
       t = nr[sa[0]] = 0;
       for(int i = 1; i < n; i++){
                                                                      //patern of p found on t
          a = sa[i]+sz < n ? r[ sa[i]+sz ] : -1;
                                                                      j = b[j];
          b = sa[i-1]+sz < n ? r[ sa[i-1]+sz ] : -1;
                                                               }
          nr[ sa[i] ] = r[ sa[i] ] == r[ sa[i-1] ] \&\& a
             == b ? t : ++t;
                                                           Min rotation
       if(t == n-1) break;
       memcpy(r, nr, sizeof(int) * n);
                                                           int min_rotation(int *s, int N) {
   }
                                                             REP(i, N) s[N+i] = s[i];
}
                                                             int a = 0;
void build_lcp(){ // lcp[i] = lcp(s[:i], s[:i+1])
                                                             REP(b, N) REP(i, N) {
   int k = 0;
                                                               if (a+i == b \mid \mid s[a+i] < s[b+i]) { b += max(0, i-1);}
   for(int i = 0; i < n; i++) r[ sa[i] ] = i;
                                                               if (s[a+i] > s[b+i]) { a = b; break; }
   for(int i = 0; i < n; i++){
       if(r[i] == n-1) k = 0;
                                                             return a;
       else{
          int j = sa[r[i]+1];
                                                            All palindrome
          while(i+k < n \&\& j+k < n \&\& s[i+k] == s[j+k])
             k++;
                                                           void manacher(char *s, int N, int *rad) {
                                                             static char t[2*MAX];
       lcp[r[i]] = k;
                                                             int m = 2*N - 1;
       if(k) k--;
   }
                                                             REP(i, m) t[i] = -1;
}
                                                             REP(i, N) t[2*i] = s[i];
int comp_lcp(int i, int j){
                                                             int x = 0;
   if(i == j) return n - i;
                                                             FOR(i, 1, m) {
   if(r[i] > r[j]) swap(i, j);
                                                               int &r = rad[i] = 0;
   return min(lcp[k]  for k in [r[i], r[j]-1]);
                                                               if (i <= x+rad[x]) r = min(rad[x+x-i], x+rad[x]-i);</pre>
                                                               while (i-r-1 >= 0 \& i+r+1 < m \& t[i-r-1] == t[i+r]
Z Algorithm
                                                                 +11) ++r:
                                                               if (i+r >= x+rad[x]) x = i;
vector<int> z_algo(const string &s) {
   int n = s.size(), L = 0, R = 0;
```

University of Brasilia Geometry

```
REP(i, m) if (i-rad[i] == 0 || i+rad[i] == m-1) ++rad[
                                                               tree[1].len = -1; tree[1].sufflink = 1;
                                                               tree[2].len = 0; tree[2].sufflink = 1;
 REP(i, m) rad[i] /= 2;
                                                           }
                                                           int main() {
Palindromic Tree
                                                               initTree();
const int MAXN = 105000;
                                                               for (int i = 0; i < len; i++) {
struct node {
                                                                   addLetter(i);
   int next[26]:
   int len;
   int sufflink;
                                                               return 0;
   int num;
                                                            Suffix Automaton
int len;
                                                           map<char, int> to[2*N];
char s[MAXN];
                                                           int link[2*N], len[2*N], last = 0, sz = 1;
node tree[MAXN];
int num; // node 1 - root with len -1, node 2 - root
                                                           void add_letter(char c){
  with len 0
                                                               int p = last;
int suff; // max suffix palindrome
                                                               last = sz++;
long long ans;
                                                               len[last] = len[p] + 1;
                                                               for(; !to[p][c]; p = link[p]) to[p][c] = last;
bool addLetter(int pos) {
                                                               if(to[p][c] == last){
   int cur = suff, curlen = 0;
                                                                   link[last] = 0;
   int let = s[pos] - 'a';
                                                                  return;
   while(true){
                                                               int u = to[p][c];
       curlen = tree[cur].len;
                                                               if(len[u] == len[p]+1){
       if (pos-1 - curlen >= 0 \& s[pos-1 - curlen] == s
                                                                   link[last] = u;
         [pos])
                                                                   return:
          break;
       cur = tree[cur].sufflink;
                                                               int c1 = sz++;
                                                               to[c1] = to[u];
   if (tree[cur].next[let]) {
                                                               link[c1] = link[u];
       suff = tree[cur].next[let];
                                                               len[c1] = len[p]+1;
       return false;
                                                               link[last] = link[u] = c1;
   }
                                                               for(; to[p][c] == u; p = link[p]) to[p][c] = c1;
   num++:
                                                            Geometry
   suff = num:
   tree[num].len = tree[cur].len + 2;
                                                            2D basics
   tree[cur].next[let] = num;
   if (tree[num].len == 1){
                                                            typedef double coord;
       tree[num].sufflink = 2;
                                                            double eps = 1e-7;
       tree[num].num = 1;
                                                           bool eq(coord a, coord b){ return abs(a - b) <= eps; }</pre>
       return true;
   }
                                                           struct vec{
                                                               coord x, y; int id;
   while (true){
                                                               vec(coord \ a = 0, \ coord \ b = 0) : x(a), y(b) \{\}
       cur = tree[cur].sufflink;
                                                               vec operator+(const vec &o) const{
       curlen = tree[cur].len;
                                                                   return \{x + o.x, y + o.y\};
       if(pos-1 - curlen) = 0 \&\& s[pos-1 - curlen] == s[
                                                               vec operator-(const vec &o) const{
          tree[num].sufflink = tree[cur].next[let];
                                                                  return \{x - o.x, y - o.y\};
          break;
       }
                                                               vec operator*(coord t) const{
   }
                                                                   return {x * t, y * t};
   tree[num].num = 1 + tree[tree[num].sufflink].num;
                                                               vec operator/(coord t) const{
                                                                  return {x / t, y / t};
   return true;
}
                                                               coord operator*(const vec &o) const{ // cos
                                                                   return x * o.x + y * o.y;
void initTree() {
   num = 2; suff = 2;
                                                               coord operator^(const vec &o) const{ // sin
```

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```
return x * o.y - y * o.x;
   }
   bool operator==(const vec &o) const{
      return eq(x, o.x) \& eq(y, o.y);
   bool operator<(const vec &o) const{</pre>
      if(!eq(x, o.x)) return x < o.x;
      return y < o.y;
   coord cross(const vec &a, const vec &b) const{
       return (a-(*this)) ^ (b-(*this));
   int ccw(const vec &a, const vec &b) const{
       coord tmp = cross(a, b);
      return (tmp > eps) - (tmp < -eps);</pre>
   }
   coord dot(const vec &a, const vec &b) const{
      return (a-(*this)) * (b-(*this));
   coord len() const{
      return sqrt(x * x + y * y); // <
   double angle(const vec &a, const vec &b) const{
      return atan2(cross(a, b), dot(a, b));
   }
   double tan(const vec &a, const vec &b) const{
      return cross(a, b) / dot(a, b);
   vec unit() const{
      return operator/(len());
   int quad() const{
      if(x > 0 \& y >= 0) return 0;
      if(x \le 0 \& y > 0) return 1;
      if(x < 0 \& y <=0) return 2;
      return 3;
   }
   bool comp(const vec &a, const vec &b) const{
      return (a - *this).comp(b - *this);
   bool comp(vec b){
      if(quad() != b.quad()) return quad() < b.quad();</pre>
       if(!eq(operator^(b), 0)) return operator^(b) > 0;
      return (*this) * (*this) < b * b;
   template<class T>
   void sort_by_angle(T first, T last) const{
       std::sort(first, last, [=](const vec &a, const
         vec &b){
          return comp(a, b);
      });
   vec rot90() const{ return {-y, x}; }
   vec rot(double a) const{
       return \{\cos(a)*x - \sin(a)*y, \sin(a)*x + \cos(a)*y\};
};
struct line{
   coord a, b, c; vec n;
   line(vec q, vec w){ // q.cross(w, (x, y)) = 0
      a = -(w.y-q.y);
      b = w.x-q.x;
       c = -(a * q.x + b * q.y);
       n = \{a, b\};
   coord dist(const vec &o) const{
```

```
return abs(eval(o)) / n.len();
   }
   bool contains(const vec &o) const{
      return eq(a * o.x + b * o.y + c, \emptyset);
   coord dist(const line &o) const{
       if(!parallel(o)) return 0;
       if(!eq(o.a * b, o.b * a)) return 0;
       if(!eq(a, 0))
          return abs(c - o.c * a / o.a) / n.len();
       if(!ea(b. 0))
          return abs(c - o.c * b / o.b) / n.len();
       return abs(c - o.c);
   bool parallel(const line &o) const{
      return eq(n ^ o.n, 0);
   bool operator==(const line &o) const{
       if(!eq(a*o.b, b*o.a)) return false;
       if(!eq(a*o.c, c*o.a)) return false;
       if(!eq(c*o.b, b*o.c)) return false;
       return true;
   bool intersect(const line &o) const{
      return !parallel(o) || *this == o;
   vec inter(const line &o) const{
       if(parallel(o)){
          if(*this == o){ }
          else{ /* dont intersect */ }
       auto tmp = n \hat{o}.n;
       return {(o.c*b -c*o.b)/tmp, (o.a*c -a*o.c)/tmp};
   vec at_x(coord x) const{
      return \{x, (-c-a*x)/b\};
   }
   vec at_y(coord y) const{
      return \{(-c-b*y)/a, y\};
   coord eval(const vec &o) const{
       return a * o.x + b * o.y + c;
};
struct segment{
   vec p, q;
   segment(vec a = vec(), vec b = vec()): p(a), q(b) {}
   bool onstrip(const vec &o) const{ // onstrip strip
       return p.dot(o, q) >= -eps && q.dot(o, p) >= -eps
   }
   coord len() const{
       return (p-q).len();
   coord dist(const vec &o) const{
       if(onstrip(o)) return line(p, q).dist(o);
       return min((o-q).len(), (o-p).len());
   bool contains(const vec &o) const{
       return eq(p.cross(q, o), 0) && onstrip(o);
   bool intersect(const segment &o) const{
       if(contains(o.p)) return true;
       if(contains(o.q)) return true;
       if(o.contains(q)) return true;
```

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```
if(o.contains(p)) return true;
       return p.ccw(q, o.p) * p.ccw(q, o.q) == -1
       && o.p.ccw(o.q, q) * o.p.ccw(o.q, p) == -1;
   bool intersect(const line &o) const{
       return o.eval(p) * o.eval(q) <= 0;</pre>
   coord dist(const segment &o) const{
       if(line(p, q).parallel(line(o.p, o.q))){
          if(onstrip(o.p) || onstrip(o.q)
          || o.onstrip(p) || o.onstrip(q))
              return line(p, q).dist(line(o.p, o.q));
       else if(intersect(o)) return 0;
       return min(min(dist(o.p), dist(o.q)),
                min(o.dist(p), o.dist(q)));
   coord dist(const line &o) const{
       if(line(p, q).parallel(o))
          return line(p, q).dist(o);
       else if(intersect(o)) return 0;
       return min(o.dist(p), o.dist(q));
   }
};
struct hray{
   vec p, q;
   hray(vec a = vec(), vec b = vec()): p(a), q(b){}
   bool onstrip(const vec &o) const{ // onstrip strip
       return p.dot(q, o) >= -eps;
   coord dist(const vec &o) const{
       if(onstrip(o)) return line(p, q).dist(o);
       return (o-p).len();
   bool intersect(const segment &o) const{
       if(!o.intersect(line(p,q))) return false;
       if(line(o.p, o.q).parallel(line(p,q)))
          return contains(o.p) || contains(o.q);
       return contains(line(p,q).inter(line(o.p,o.q)));
   bool contains(const vec &o) const{
       return eq(line(p, q).eval(o), 0) && onstrip(o);
   coord dist(const segment &o) const{
       if(line(p, q).parallel(line(o.p, o.q))){
          if(onstrip(o.p) || onstrip(o.q))
              return line(p, q).dist(line(o.p, o.q));
          return o.dist(p);
       else if(intersect(o)) return 0;
       return min(min(dist(o.p), dist(o.q)),
                o.dist(p));
   bool intersect(const hray &o) const{
       if(!line(p, q).parallel(line(o.p, o.q)))
          return false;
       auto pt = line(p, q).inter(line(o.p, o.q));
       return contains(pt) && o.contains(pt); // <<</pre>
   bool intersect(const line &o) const{
       if(line(p, q).parallel(o)) return line(p, q)== o;
       if(o.contains(p) || o.contains(q)) return true;
       return (o.eval(p) >= -eps)^(o.eval(p)<o.eval(q));</pre>
       return contains(o.inter(line(p, q)));
   coord dist(const line &o) const{
```

```
if(line(p,q).parallel(o))
          return line(p,q).dist(o);
       else if(intersect(o)) return 0;
      return o.dist(p);
   coord dist(const hray &o) const{
       if(line(p, q).parallel(line(o.p, o.q))){
          if(onstrip(o.p) || o.onstrip(p))
              return line(p,q).dist(line(o.p, o.q));
          return (p-o.p).len();
       else if(intersect(o)) return 0;
       return min(dist(o.p), o.dist(p));
};
double heron(coord a, coord b, coord c){
   coord s = (a + b + c) / 2;
   return sqrt(s * (s - a) * (s - b) * (s - c));
Nearest Points
struct pt {
   int x, y, id;
};
inline bool cmp_x (const pt & a, const pt & b) {
   return a.x < b.x | | a.x == b.x && a.y < b.y;
inline bool cmp_y (const pt & a, const pt & b) {
   return a.y < b.y;</pre>
pt a[MAXN];
double mindist;
int ansa, ansb;
inline void upd_ans (const pt & a, const pt & b) {
   double dist = sqrt((a.x-b.x)*(a.x-b.x) + (a.y-b.y)
      *(a.y-b.y) + .0);
   if (dist < mindist)</pre>
       mindist = dist, ansa = a.id, ansb = b.id;
void rec (int 1, int r) {
   if (r - 1 \le 3) {
       for (int i=1; i<=r; ++i)
          for (int j=i+1; j<=r; ++j)
              upd_ans (a[i], a[j]);
       sort (a+1, a+r+1, &cmp_y);
       return:
   }
   int m = (1 + r) >> 1;
   int midx = a[m].x;
   rec (1, m), rec (m+1, r);
   static pt t[MAXN];
   merge (a+1, a+m+1, a+m+1, a+r+1, t, \&cmp_y);
   copy (t, t+r-l+1, a+l);
   int tsz = 0;
   for (int i=1; i<=r; ++i)
       if (abs (a[i].x - midx) < mindist) {</pre>
          for (int j=tsz-1; j>=0 && a[i].y - t[j].y <</pre>
            mindist; --j)
```

```
upd_ans (a[i], t[j]);
                                                              if(v[0].ccw(v.back(), p) > 0) return 0;
          t[tsz++] = a[i];
                                                              if(v[0].ccw(v.back(), p) == 0)
      }
                                                                  return v[0].dot(p, v.back()) >= 0
}
                                                                     && v.back().dot(p, v[0]) >= 0;
                                                              int L = 1, R = (int)v.size() - 1, ans = 1;
sort (a, a+n, &cmp_x);
mindist = 1E20;
rec (0, n-1);
                                                              while(L <= R){</pre>
                                                                  int mid = (L+R)/2;
Convex Hull
                                                                  if(v[0].ccw(v[mid], p) >= 0) ans = mid, L = mid
vector<vec> monotone_chain_ch(vector<vec> P){
                                                                  else R = mid-1;
   sort(P.begin(), P.end());
   vector<vec> L, U;
                                                              return v[ans].ccw(v[(ans+1)%v.size()], p) >= 0;
   for(auto p : P){
      while(L.size() >= 2 && L[L.size() - 2].cross(L.
                                                           Minkowski sum
         back(), p) < 0)
          L.pop_back();
                                                           vector<vec> mk(const vector<vec>&a,const vector<vec>&b){
      L.push_back(p);
                                                              int i = 0, j = 0;
                                                               for(int k = 0; k < (int)a.size(); k++)if(a[k] < a[i])
   }
                                                                  i = k;
   reverse(P.begin(), P.end());
                                                               for(int k = 0; k < (int)b.size(); k++)if(b[k] < b[j])
   for(auto p : P){
      while(U.size() >= 2 && U[U.size() - 2].cross(U.
         back(), p) < 0)
                                                              vector<vec> c;
                                                              c.reserve(a.size() + b.size());
          U.pop_back();
                                                               for(int k = 0; k < int(a.size()+b.size()); k++){</pre>
       U.push_back(p);
                                                                  vec pt{a[i] + b[j]};
   }
                                                                  if((int)c.size() >= 2
                                                                   && c[c.size()-2].ccw(c.back(), pt) == 0)
                                                                      c.pop_back();
   L.pop_back(), U.pop_back();
                                                                  c.push_back(pt);
   L.reserve(L.size() + U.size());
                                                                  int q = i+1, w = j+1;
                                                                  if(q == int(a.size())) q = 0;
   L.insert(L.end(), U.begin(), U.end());
                                                                  if(w == int(b.size())) w = 0;
                                                                  if(c.back().ccw(a[i]+b[w], a[q]+b[j]) < 0) i = q;
   return L;
}
                                                                  else j = w;
Check point inside polygon
                                                              c.shrink_to_fit();
bool below(const vector<vec> &vet, vec p){
                                                              return c;
   auto it = lower_bound(vet.begin(), vet.end(), p);
   if(it == vet.end()) return false;
   if(it == vet.begin()) return *it == p;
                                                           Miscellaneous
   return prev(it)->cross(*it, p) <= 0;</pre>
}
                                                           LIS
bool above(const vector<vec> &vet, vec p){
                                                           multiset<int> S;
   auto it = lower_bound(vet.begin(), vet.end(), p);
                                                           for(int i = 0; i < n; i++){
   if(it == vet.end()) return false;
                                                              auto it = S.upper_bound(a[i]); // low for inc
   if(it == vet.begin()) return *it == p;
                                                              if(it != S.end()) S.erase(it);
   return prev(it)->cross(*it, p) >= 0;
                                                              S.insert(a[i]);
                                                           }
                                                           ans = S.size();
// lowerhull, upperhull and point, borders included
                                                           DSU rollback
bool inside_poly(const vector<vec> &lo, const vector<vec</pre>
  > &hi, vec p){
                                                           #include <bits/stdc++.h>
   return below(hi, p) && above(lo, p);
                                                           using namespace std;
Check point inside polygon without lower/upper
     hull
                                                           struct DSU{
                                                              vector < int > sz, p, change;
// borders included
                                                              vector<tuple<int, int, int>> modifications;
// must not have 3 colinear consecutive points
                                                              vector<size_t> saves;
bool inside_poly(const vector<vec> &v, vec p){
                                                              bool bipartite;
   if(v[0].ccw(v[1], p) < 0) return false;
```

```
DSU(int n): sz(n+1, 1), p(n+1), change(n+1),
     bipartite(true){
       iota(p.begin(), p.end(), 0);
   }
   void add_edge(int u, int v){
       if(!bipartite) return;
       int must_change = get_colour(u) == get_colour(v);
       int a = rep(u), b = rep(v);
       if(sz[a] < sz[b]) swap(a, b);
      if(a != b){
          p[b] = a;
          modifications.emplace_back(b, change[b],
            bipartite);
          change[b] ^= must_change;
          sz[a] += sz[b];
      else if(must_change){
          modifications.emplace_back(0, change[0],
            bipartite);
          bipartite = false;
   }
   int rep(int u){
      return p[u] == u ? u : rep(p[u]);
   }
   int get_colour(int u){
      if(p[u] == u) return change[u];
      return change[u] ^ get_colour(p[u]);
   void reset(){
      modifications.clear();
       saves.clear();
      iota(p.begin(), p.end(), 0);
       fill(sz.begin(), sz.end(), 1);
       fill(change.begin(), change.end(), 0);
      bipartite = true;
   }
   void rollback(){
      int u = get<0>(modifications.back());
       tie(ignore, change[u], bipartite) = modifications
         .back();
       sz[p[u]] -= sz[u];
      p[u] = u;
      modifications.pop_back();
   }
   void reload(){
       while(modifications.size() > saves.back())
          rollback();
       saves.pop_back();
   }
   void save(){
       saves.push_back(modifications.size());
   }
};
const int N = 100005;
const int B = 318;
int n, m, q;
int x[N], y[N], 1[N], r[N], ans[N];
```

```
vector<int> qu[N];
int brute(int lef, int rig, DSU &s){
   s.save();
    for(int i = lef; i <= rig; i++)</pre>
       s.add_edge(x[i], y[i]);
    int ret = s.bipartite;
    s.reload();
   return ret;
}
int main(){
    scanf("%d %d %d", &n, &m, &q);
    for(int i = 1; i <= m; i++)
       scanf("%d %d", x+i, y+i);
   DSU s(n);
    for(int i = 0; i < q; i++){
       scanf("%d %d", l+i, r+i);
       if(r[i] - 1[i] \le B + 10)
           ans[i] = brute(l[i], r[i], s);
       else qu[l[i] / B].push_back(i);
   }
    for(int i = 0; i <= m / B; i++){</pre>
       sort(qu[i].begin(), qu[i].end(),[](int a, int b){
           return r[a] < r[b];
       });
       s.reset();
       int R = (i+1)*B-1;
       for(int id : qu[i]){
           \textbf{while}(\texttt{R} \ < \ \texttt{r[id]}) \ \textit{++}\texttt{R}, \ \texttt{s.add\_edge}(\texttt{x[R]}, \ \texttt{y[R]});
           s.save();
           for(int k = 1[id]; k < (i+1)*B; k++)
               s.add_edge(x[k], y[k]);
           ans[id] = s.bipartite;
           s.reload();
       }
    for(int i = 0; i < q; i++)
       printf("%s\n",ans[i] ? "Possible":"Impossible");
Buildings
// count the number of circular arrays
// of size m, with elements on range
// [1, c**(x*x)]
#include<bits/stdc++.h>
using namespace std;
#define debug(x) cerr << fixed << #x << " = " << x <<
#define 11 long long
const int MOD = 1e9 + 7:
const int MAX = 1e5 + 5;
int dp[MAX];
inline int add(int a, int b) {
 a += b;
```

```
if(a >= MOD) {
   a -= MOD;
 return a;
inline int sub(int a, int b) {
 a -= b;
 if(0 > a) {
   a += MOD;
 return a;
inline int mult(int a, int b) {
 return (1LL * a * b) % MOD;
int f_exp(int x, int exp) {
 if(exp == 0) {
   return 1;
 else if(exp & 1) {
   return mult(x, f_{exp}(x, exp - 1));
 return f_{exp}(mult(x, x), exp / 2);
inline int inv(int x) {
 return f_exp(x, MOD - 2);
int main()
 ios::sync_with_stdio(false);
 cin.tie(NULL); cout.tie(NULL);
 int n. m. c:
 cin >> n >> m >> c;
 int x = f_{exp}(c, n * n);
 int ans = f_{exp}(x, m);
 for(int i = 1; i <= m; i++) {</pre>
   if(m % i == 0) {
     int y = f_{exp}(x, i);
     for(int j = 1; j < i; j++) {
       if(i % j == 0) {
        y = sub(y, mult(j, dp[j]));
     dp[i] = mult(y, inv(i));
     ans = sub(ans, mult(i - 1, dp[i]));
 cout << ans << '\n';</pre>
 return 0;
}
Rand
cout << RAND_MAX << endl;</pre>
mt19937 rng(chrono::steady_clock::now().time_since_epoch
  ().count()):
vector<int> permutation(N);
iota(permutation.begin(), permutation.end(), 0);
```

```
shuffle(permutation.begin(), permutation.end(), rng);
iota(permutation.begin(), permutation.end(), 0);
for(int i = 1; i < N; i++){
   swap(permutation[i], permutation[
     uniform_int_distribution<int>(0, i)(rng)]);
Klondike
// minimum number of moves to make
// all elements equal
// move: change a segment of equal value
// elements to any value
int v[305];
int dp[305][305];
int rec[305][305];
int f(int 1, int r){
 if(r == 1) return 1;
 if(r < 1) return 0;</pre>
 if(dp[l][r] != -1) return dp[l][r];
 int ans = f(1+1, r) + 1;
 for(int i = 1+1; i <= r; i++)
   if(v[i] == v[1])
     ans = min(ans, f(1, i - 1) + f(i+1, r));
 return dp[l][r] = ans;
}
int main() {
 int n. m:
 memset(dp, -1, sizeof dp);
 scanf("%d %d",&n , &m);
 for(int i = 0; i < n; i++){
   scanf("%d",v+i);
   if(i && v[i] == v[i-1]){
     i--;
     n--;
   }
 }
 printf("%d\n",f(0, n-1) - 1);
 // printf("%d\n",rec[0][n-1] );
 // printf("%d\n",rec[1][n-1] );
 // printf("%d\n",rec[2][n-3] );
Hilbert Order
// maybe use B = n / sqrt(q)
inline int64_t hilbertOrder(int x, int y, int pow = 21,
  int rotate = 0) {
   if(pow == 0) return 0;
   int hpow = 1 \ll (pow-1);
   int seg = (x < hpow) ? (
       (y < hpow) ? 0 : 3
   ):(
       (y < hpow) ? 1 : 2
   );
   seg = (seg + rotate) & 3;
   const int rotateDelta[4] = {3, 0, 0, 1};
   int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
   int nrot = (rotate + rotateDelta[seg]) & 3;
   int64_t subSquareSize = int64_t(1) << (2*pow - 2);
   int64_t ans = seg * subSquareSize;
```

```
int64_t add = hilbertOrder(nx, ny, pow-1, nrot);
ans += (seg == 1 || seg == 2) ? add : (subSquareSize
    - add - 1);
return ans;
```

# **Modular Factorial**

```
// Compute (1*2*...*(p-1)*1*(p+1)*(p+2)*..*n) % p
// in O(p*lg(n))
int factmod(int n, int p){
   int ans = 1;
   while(n > 1){
       for(int i = 2; i <= n % p; i++)</pre>
          ans = (ans * i) % p;
       n \neq p;
       if(n \% 2) ans = p - ans;
   }
   return ans % p;
}
int fac_pow(int n, int p){
   int ans = 0;
   while(n) n \neq p, ans += n;
   return ans;
}
int C(int n, int k, int p){
   if(fac_pow(n, p) > fac_pow(n-k, p) + fac_pow(k, p))
       return 0;
   int tmp = factmod(k, p) * factmod(n-k, p);
   return (f_exp(tmp, p-2, p) * factmod(n, p)) % p;
```

## Enumaratin all submasks of a bitmask

```
// loop through all submask of a given bitmask
// it does not include mask 0
for(int sub = mask; sub; sub = (sub-1)&mask){
}
```

## Burnside's Lemma

Let  $(G, \oplus)$  be a finite group that acts on a set X. It should hold that  $e_g * x = x$  and  $g_1 * (g_2 * x) = (g_1 \oplus g_2) * x$ ,  $\forall x \in X, g_1, g_2 \in G$ . For each  $g \in G$  let  $X^g = \{x \in X \mid g * x = x\}$ . The number of orbits its given by:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

# Wilson's Theorem

 $(n-1)! = -1 \mod n \iff n \text{ is prime}$ 

## **Fibonacci**

- $F_{n-1}F_{n+1} F_n^2 = (-1)^n$
- $\bullet \ F_{n+k} = F_k F_{n+1} + F_{k-1} F_n$
- $GCD(F_n, F_m) = F_{GCD(n,m)}$
- $F_n = \frac{(\frac{1+\sqrt{5}}{2})^n (\frac{1-\sqrt{5}}{2})^n}{\sqrt{5}}$

## **Notes**

When we repeat something and each time we have probability p to succeed then the expected number or tries is  $\frac{1}{p}$ , till we succeed.