

ICPC Team Reference

University of Brasilia

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```
set ts=4 sw=4 sta nu rnu sc stl+=%F cindent
set bg=dark ruler clipboard=unnamed,unnamedplus
timeoutlen=100
imap {<CR> {<CR>}<Esc>O
nmap <F2> 0V$%d
nmap <C-down> :m+1<CR>
nmap <C-up> :m-2<CR>
vmap <C-c> "+y
nmap <C-a> ggVG
syntax on
alias cmp='g++ -Wall -Wformat=2 -Wshadow -Wconversion -
fsanitize=address -fsanitize=undefined -fno-sanitize-
recover -std=c++14'
```

Data Structures

Merge Sort Tree

```
struct MergeTree{
    int n;
    vector<vector<int>> st;

    void build(int p, int L, int R, const int v[]){
        if(L == R){
            st[p].push_back(v[L]);
            return;
        }
        int mid = (L+R)/2;
        build(2*p, L, mid, v);
        build(2*p+1, mid+1, R, v);
        st[p].resize(R-L+1);
        merge(st[2*p].begin(), st[2*p].end(),
              st[2*p+1].begin(), st[2*p+1].end(),
              st[p].begin());
    }

    int query(int p, int L, int R, int i, int j, int x)
    const{
        if(L > j || R < i) return 0;
        if(L >= i && R <= j){
            int id = lower_bound(st[p].begin(), st[p].end(),
                                x) - st[p].begin();
            return int(st[p].size()) - id;
        }
        int mid = (L+R)/2;
        return query(2*p, L, mid, i, j, x) +
               query(2*p+1, mid+1, R, i, j, x);
    }
};

public:
    MergeTree(int sz, const int v[]): n(sz), st(4*sz){
        build(1, 1, n, v);

        //number of elements >= x on segment [i, j]
        int query(int i, int j, int x) const{
            if(i > j) swap(i, j);
            return query(1, 1, n, i, j, x);
        }
};
```

Wavelet Tree

```
template<typename T>
class wavelet{
    T L, R;
    vector<int> l;
    vector<T> sum; // <<
```

```
    wavelet *lef, *rig;

    int r(int i) const{ return i - l[i]; }

public:
    template<typename ITER>
    wavelet(ITER bg, ITER en){
        lef = rig = nullptr;
        L = *bg, R = *bg;

        for(auto it = bg; it != en; it++){
            L = min(L, *it), R = max(R, *it);
            if(L == R) return;

            T mid = L + (R - L)/2;
            l.reserve(std::distance(bg, en) + 1);
            sum.reserve(std::distance(bg, en) + 1);
            l.push_back(0), sum.push_back(0);
            for(auto it = bg; it != en; it++){
                l.push_back(l.back() + (*it <= mid)),
                sum.push_back(sum.back() + *it);
            }

            auto tmp = stable_partition(bg, en, [mid](T x){
                return x <= mid;
            });

            if(bg != tmp) lef = new wavelet(bg, tmp);
            if(tmp != en) rig = new wavelet(tmp, en);
        }

        ~wavelet(){
            delete lef;
            delete rig;
        }

        // 1 index, first is 1st
        T kth(int i, int j, int k) const{
            if(L >= R) return L;
            int c = l[j] - l[i-1];
            if(c >= k) return lef->kth(l[i-1]+1, l[j], k);
            else return rig->kth(r(i-1)+1, r(j), k - c);
        }

        // # elements > x on [i, j]
        int cnt(int i, int j, T x) const{
            if(L > x) return j - i + 1;
            if(R <= x || L == R) return 0;
            int ans = 0;
            if(lef) ans += lef->cnt(l[i-1]+1, l[j], x);
            if(rig) ans += rig->cnt(r(i-1)+1, r(j), x);
            return ans;
        }

        // sum of elements <= k on [i, j]
        T sumk(int i, int j, T k){
            if(L == R) return R <= k ? L * (j - i + 1) : 0;
            if(R <= k) return sum[j] - sum[i-1];
            int ans = 0;
            if(lef) ans += lef->sumk(l[i-1]+1, l[j], k);
            if(rig) ans += rig->sumk(r(i-1)+1, r(j), k);
            return ans;
        }

        // swap (i, i+1) just need to update "array" l[i]
    };
```

Order Set

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

#include <ext/pb_ds/detail/standard_policies.hpp>

using namespace __gnu_pbds; // or pb_ds;
```

```
template<typename T, typename B = null_type>
using oset = tree<T, B, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
// find_by_order / order_of_key
```

Hash table

```
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;

struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        // http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }

    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM = chrono::
            steady_clock::now().time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};

gp_hash_table<long long, int, custom_hash> table;
unordered_map<long long, int, custom_hash> uhash;
uhash.reserve(1 << 15);
uhash.max_load_factor(0.25);
```

Convex Hull Trick Simple

```
struct Line{
    ll m, b;
    inline ll eval(ll x) const{
        return x * m + b;
    }
};

// min => cht.back().m >= L.m
// max => cht.back().m <= L.m
void push_line(vector<Line> &cht, Line L){
    while((int)cht.size() >= 2){
        int sz = (int)cht.size();
        if((long double)(L.b-cht[sz-1].b)*(cht[sz-2].m-L.m)
            <= (long double)(L.b-cht[sz-2].b)*(cht[sz-1].m-L.m)){
            cht.pop_back();
        }
        else break;
    }
    cht.push_back(L);
}

// x increasing; pos = 0 in first call
ll linear_search(const vector<Line> &cht, ll x, int &pos){
    while(pos+1 < (int)cht.size()){
        /*>>*/ if(cht[pos].eval(x) >= cht[pos+1].eval(x)) pos++;
        else break;
    }
    return cht[pos].eval(x);
}
```

```
ll binary_search(const vector<Line> &cht, ll x){
    int L = 0, R = (int)cht.size()-2;
    int bans = (int)cht.size()-1;
    while(L <= R){
        int mid = (L+R)/2;
        if(cht[mid].eval(x) >= cht[mid+1].eval(x)) // <<<
            L = mid + 1;
        else bans = mid, R = mid - 1;
    }
    return cht[bans].eval(x);
}
```

Convex Hull Trick

```
const ll is_query = -(1LL<<62);
struct Line{
    ll m, b;
    mutable function<const Line*> succ;
    bool operator<(const Line& rhs) const{
        if(rhs.b != is_query) return m < rhs.m;
        const Line* s = succ();
        if(!s) return 0;
        ll x = rhs.m;
        return b - s->b < (s->m - m) * x;
    }
};

struct Cht : public multiset<Line>{ // maintain max
    bool bad(iterator y){
        auto z = next(y);
        if(y == begin()){
            if(z == end()) return 0;
            return y->m == z->m && y->b <= z->b;
        }
        auto x = prev(y);
        if(z == end()) return y->m == x->m && y->b <= x->b;
        return (long double)(x->b - y->b)*(z->m - y->m)
            >= (long double)(y->b - z->b)*(y->m - x->m);
    }

    void insert_line(ll m, ll b){
        auto y = insert({ m, b });
        y->succ = [=]{ return next(y) == end() ? 0 : &*
            next(y); };
        if(bad(y)){ erase(y); return; }
        while(next(y) != end() && bad(next(y))) erase(
            next(y));
        while(y != begin() && bad(prev(y))) erase(prev(y));
    }

    ll eval(ll x){
        auto l = *lower_bound((Line) { x, is_query });
        return l.m * x + l.b;
    }
};
```

Convex Hull Trick

```
/**
 * Author: Simon Lindholm
 * source: https://github.com/kth-competitive-
    programming/kactl/blob/master/content/data-structures
    /LineContainer.h
 * License: CC0
 */

struct Line {
    mutable ll m, b, p;
    bool operator<(const Line& o) const { return m < o.m
```

```

    };
    bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> { // CPP14
    only
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    const ll inf = LLONG_MAX;
    ll div(ll a, ll b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = inf; return false; }
        if (x->m == y->m) x->p = x->b > y->b ? inf : -inf;
        ;
        else x->p = div(y->b - x->b, x->m - y->m);
        return x->p >= y->p;
    }
    void add(ll m, ll b) {
        auto z = insert({m, b, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        assert(!empty());
        auto l = *lower_bound(x);
        return l.m * x + l.b;
    }
};

```

Min queue

```

template<typename T>
class minQ{
    deque<tuple<T, int, int> > p;
    T delta;
    int sz;
public:
    minQ() : delta(0), sz(0) {}
    inline int size() const{ return sz; }
    inline void add(T x){ delta += x; }
    inline void push(T x, int id){
        x -= delta, sz++;
        int t = 1;
        while(p.size() > 0 && get<0>(p.back()) >= x)
            t += get<1>(p.back()), p.pop_back();
        p.emplace_back(x, t, id);
    }
    inline void pop(){
        get<1>(p.front())--, sz--;
        if(!get<1>(p.front())) p.pop_front();
    }
    T getmin() const{ return get<0>(p.front())+delta; }
    int getid() const{ return get<2>(p.front()); }
};

```

Sparse Table

```

const int N = 100005;

int v[N], n;
int dn[N][20];
int fn(int i, int j){
    if(j == 0) return v[i];
    if(~dn[i][j]) return dn[i][j];
    return dn[i][j] = min(fn(i, j-1), fn(i + (1 << (j-1)), j-1));
}

```

```

}

int lg(int x){ return 31 - __builtin_clz(x); }

int getmn(int l, int r){ // [l, r]
    int lz = lg(r - l + 1);
    return min(fn(l, lz), fn(r - (1 << lz) + 1, lz));
}

```

Treap

```

// source: https://github.com/victorsenam/caderno/blob/master/code/treap.cpp
//const int N = ; typedef int num;
num X[N]; int en = 1, Y[N], sz[N], L[N], R[N];
void calc(int u) { // update node given children info
    if(!u) return;
    sz[u] = sz[L[u]] + 1 + sz[R[u]];
    // code here, no recursion
}
void unlaze(int u) {
    if(!u) return;
    // code here, no recursion
}
void split_val(int u, num x, int &l, int &r) { // l gets
    <= x, r gets > x
    unlaze(u); if(!u) return (void) (l = r = 0);
    if(X[u] <= x) { split_val(R[u], x, l, r); R[u] = 1;
        l = u; }
    else { split_val(L[u], x, l, r); L[u] = r; r = u; }
    calc(u);
}
void split_sz(int u, int s, int &l, int &r) { // l gets
    first s, r gets remaining
    unlaze(u); if(!u) return (void) (l = r = 0);
    if(sz[L[u]] < s) { split_sz(R[u], s - sz[L[u]] - 1,
        l, r); R[u] = 1; l = u; }
    else { split_sz(L[u], s, l, r); L[u] = r; r = u; }
    calc(u);
}
int merge(int l, int r) { // els on l <= els on r
    unlaze(l); unlaze(r); if(!l || !r) return l + r; int
        u;
    if(Y[l] > Y[r]) { R[l] = merge(R[l], r); u = 1; }
    else { L[r] = merge(l, L[r]); u = r; }
    calc(u); return u;
}
void init(int n=N-1) { // XXX call before using other
    funcs
    for(int i = en = 1; i <= n; i++) { Y[i] = i; sz[i] =
        1; L[i] = R[i] = 0; }
    random_shuffle(Y + 1, Y + n + 1);
}
void insert(int &u, int it){
    unlaze(u);
    if(!u) u = it;
    else if(Y[it] > Y[u]) split_val(u, X[it], L[it], R[
        it]), u = it;
    else insert(X[it] < X[u] ? L[u] : R[u], it);
    calc(u);
}
void erase(int &u, num key){
    unlaze(u);
    if(!u) return;
    if(X[u] == key) u = merge(L[u], R[u]);
    else erase(key < X[u] ? L[u] : R[u], key);
    calc(u);
}

```

```

int create_node(num key){
    X[en] = key;
    sz[en] = 1;
    L[en] = R[en] = 0;
    return en++;
}
int query(int u, int l, int r){//0 index
    unlaze(u);
    if(u! or r < 0 or l >= sz[u]) return
        identity_element;
    if(l <= 0 and r >= sz[u] - 1) return subt_data[u];
    int ans = query(L[u], l, r);
    if(l <= sz[ L[u] ] and sz[ L[u] ] <= r)
        ans = max(ans, st[u]);
    ans = max(ans, query(R[u], l-sz[L[u]]-1, r-sz[L[u]
        ]-1));
    return ans;
}

```

ColorUpdate

// source: <https://github.com/tfg50/Competitive-Programming/tree/master/Biblioteca/Data%20Structures>

```

#include <set>
#include <vector>

template <class Info = int>
class ColorUpdate {
public:
    struct Range {
        Range(int l = 0) { this->l = l; }
        Range(int l, int r, Info v) {
            this->l = l;
            this->r = r;
            this->v = v;
        }
        int l, r;
        Info v;

        bool operator < (const Range &b) const { return l
            < b.l; }
    };

    std::vector<Range> upd(int l, int r, Info v) {
        std::vector<Range> ans;
        if(l >= r) return ans;
        auto it = ranges.lower_bound(l);
        if(it != ranges.begin()) {
            it--;
            if(it->r > l) {
                auto cur = *it;
                ranges.erase(it);
                ranges.insert(Range(cur.l, l, cur.v));
                ranges.insert(Range(l, cur.r, cur.v));
            }
        }
        it = ranges.lower_bound(r);
        if(it != ranges.begin()) {
            it--;
            if(it->r > r) {
                auto cur = *it;
                ranges.erase(it);
                ranges.insert(Range(cur.l, r, cur.v));
                ranges.insert(Range(r, cur.r, cur.v));
            }
        }
        for(it = ranges.lower_bound(l); it != ranges.end

```

```

        ) && it->l < r; it++) {
            ans.push_back(*it);
        }
        ranges.erase(ranges.lower_bound(l), ranges.
            lower_bound(r));
        ranges.insert(Range(l, r, v));
        return ans;
    }
private:
    std::set<Range> ranges;
};

```

Heavy Light Decomposition

```

void dfs_sz(int u){
    sz[u] = 1;

    for(auto &v : g[u]) if(v == p[u]){
        swap(v, g[u].back());
        g[u].pop_back();
        break;
    }

    for(auto &v : g[u]){
        p[v] = u;
        dfs_sz(v);
        sz[u] += sz[v];
        if(sz[v] > sz[ g[u][0] ])
            swap(v, g[u][0]);
    }
}

// nxt[u] = start of path with u
// set nxt[root] = root beforehand
void dfs_hld(int u){
    in[u] = t++;
    rin[in[u]] = u;
    for(auto v : g[u]){
        nxt[v] = (v == g[u][0] ? nxt[u] : v);
        dfs_hld(v);
    }
    out[u] = t;
}

// subtree of u => [ in[u], out[u] )
// path from nxt[u] to u => [ in[ nxt[u] ], in[u] ]

```

Iterative Segtree

```

T query(int l, int r, int &pos){ // [l, r]
    T rl, rr;
    for(l += n, r += n+1; l < r; l >>= 1, r >>= 1){
        if(l & 1) rl = merge(rl, st[l++]);
        if(r & 1) rr = merge(st[--r], rr);
    }
    return merge(rl, rr);
}

// initially save v[i] in st[n+i] for all i in [0, n)
void build(){
    for(int p = n-1; p > 0; p--){
        st[p] = merge(st[2*p], st[2*p+1]);
    }
}

void update(int p, T val){
    st[p += n] = val;
    while(p >>= 1) st[p] = merge(st[2*p], st[2*p+1]);
}

```

LiChao's Segtree

```
void add_line(line nw, int v = 1, int l = 0, int r =
maxn) { // [l, r)
    int m = (l + r) / 2;
    bool lef = nw.eval(l) < st[v].eval(l);
    bool mid = nw.eval(m) < st[v].eval(m);
    if(mid) swap(st[v], nw);
    if(r - l == 1) {
        return;
    } else if(lef != mid) {
        add_line(nw, 2 * v, l, m);
    } else {
        add_line(nw, 2 * v + 1, m, r);
    }
}

int get(int x, int v = 1, int l = 0, int r = maxn) {
    int m = (l + r) / 2;
    if(r - l == 1) {
        return st[v].eval(x);
    } else if(x < m) {
        return min(st[v].eval(x), get(x, 2*v, l, m));
    } else {
        return min(st[v].eval(x), get(x, 2*v+1, m, r));
    }
}
```

Palindromic tree

```
#include <bits/stdc++.h>

using namespace std;

const int maxn = 3e5 + 1, sigma = 26;
int len[maxn], link[maxn], to[maxn][sigma];
int slink[maxn], diff[maxn], series_ans[maxn];
int sz, last, n;
char s[maxn];

void init()
{
    s[n++] = -1;
    link[0] = 1;
    len[1] = -1;
    sz = 2;
}

int get_link(int v)
{
    while(s[n - len[v] - 2] != s[n - 1]) v = link[v];
    return v;
}

void add_letter(char c)
{
    s[n++] = c - 'a';
    last = get_link(last);
    if(!to[last][c])
    {
        len[sz] = len[last] + 2;
        link[sz] = to[get_link(link[last])][c];
        diff[sz] = len[sz] - len[link[sz]];
        if(diff[sz] == diff[link[sz]])
            slink[sz] = slink[link[sz]];
        else
            slink[sz] = link[sz];
        to[last][c] = sz++;
    }
```

```

    }
    last = to[last][c];
}

int main()
{
    ios::sync_with_stdio(0);
    cin.tie(0);
    init();
    string s;
    cin >> s;
    int n = s.size();
    int ans[n + 1];
    memset(ans, 63, sizeof(ans));
    ans[0] = 0;
    for(int i = 1; i <= n; i++)
    {
        add_letter(s[i - 1]);
        for(int v = last; len[v] > 0; v = slink[v])
        {
            series_ans[v] = ans[i - (len[slink[v]] + diff[v])];
            if(diff[v] == diff[link[v]])
                series_ans[v] = min(series_ans[v],
                    series_ans[link[v]]);
            ans[i] = min(ans[i], series_ans[v] + 1);
        }
        cout << ans[i] << "\n";
    }
    return 0;
}
```

Math

Extended Euclidean Algorithm

```
// a*x + b*y = gcd(a, b), <gcd, x, y>
tuple<int, int, int> gcd(int a, int b) {
    if(b == 0) return make_tuple(a, 1, 0);
    int q, w, e;
    tie(q, w, e) = gcd(b, a % b);
    return make_tuple(q, e, w - e * (a / b));
}
```

Chinese Remainder Theorem

```
// x = vet[i].first (mod vet[i].second)
ll crt(vector<pair<ll, ll>> vet){

    ll ans = vet[0].first, lcm = vet[0].second;
    ll a, b, g, x, y;

    for(int i = 1; i < (int)vet.size(); i++){
        tie(a, b) = vet[i];
        tie(g, x, y) = gcd(lcm, b);
        ans = ans + x * (a - ans) / g % (b / g) * lcm;
        lcm = lcm * b / g;
        ans = (ans % lcm + lcm) % lcm;
    }

    return ans;
}
```

Prefix inverse

```
inv[1] = 1;
for(int i = 2; i < p; i++)
    inv[i] = (p - (p/i) * inv[p/i] % p) % p;
```

Pollard Rho

```
ll rho(ll n){
    if(n % 2 == 0) return 2;

    ll d, c, x, y;
    do{
        c = llrand() % n, x = llrand() % n, y = x;
        do{
            x = add(mul(x, x, n), c, n);
            y = add(mul(y, y, n), c, n);
            y = add(mul(y, y, n), c, n);
            d = __gcd(abs(x - y), n);
        }while(d == 1);
    }while(d == n);
    return d;
}

ll pollard_rho(ll n){
    ll x, c, y, d, k;
    int i;
    do{
        i = 1;
        x = llrand() % n, c = llrand() % n;
        y = x, k = 4;
        do{
            if(++i == k) y = x, k *= 2;
            x = add(mul(x, x, n), c, n);
            d = __gcd(abs(x - y), n);
        }while(d == 1);
    }while(d == n);
    return d;
}

void factorize(ll val, map<ll, int> &fac){
    if(rabin(val)) fac[ val ]++;
    else{
        ll d = pollard_rho(val);
        factorize(d, fac);
        factorize(val / d, fac);
    }
}

map<ll, int> factor(ll val){
    map<ll, int> fac;
    if(val > 1) factorize(val, fac);
    return fac;
}
```

Miller Rabin

```
bool rabin(ll n){
    if(n <= 1) return 0;
    if(n <= 3) return 1;
    ll s = 0, d = n - 1;
    while(d % 2 == 0) d /= 2, s++;
    for(int k = 0; k < 64; k++){
        ll a = (llrand() % (n - 3)) + 2;
        ll x = fexp(a, d, n);
        if(x != 1 && x != n-1){
            for(int r = 1; r < s; r++){
                x = mul(x, x, n);
                if(x == 1) return 0;
                if(x == n-1) break;
            }
            if(x != n-1) return 0;
        }
    }
}
```

```
return 1;
}
```

Totiente

```
ll totiente(ll n){
    ll ans = n;
    for(ll i = 2; i*i <= n; i++){
        if(n % i == 0){
            ans = ans / i * (i - 1);
            while(n % i == 0) n /= i;
        }
    }

    if(n > 1) ans = ans / n * (n - 1);
    return ans;
}
```

Primitive root

// a primitive root modulo n is any number g such that any c coprime to n is congruent to a power of g modulo n.

```
bool exists_root(ll n){
    if(n == 1 || n == 2 || n == 4) return true;
    if(n % 2 == 0) n /= 2;
    if(n % 2 == 0) return false;

    // test if n is a power of only one prime
    for(ll i = 3; i * i <= n; i += 2) if(n % i == 0){
        while(n % i == 0) n /= i;
        return n == 1;
    }
    return true;
}

ll primitive_root(ll n){
    if(n == 1 || n == 2 || n == 4) return n - 1;
    if(not exists_root(n)) return -1;
    ll x = phi(n);
    auto pr = factorize(x);
    auto check = [x, n, pr](ll m){
        for(ll p : pr) if(fexp(m, x / p, n) == 1)
            return false;
        return true;
    };
    for(ll m = 2; ; m++) if(__gcd(m, n) == 1)
        if(check(m)) return m;
}
```

// Let's denote $R(n)$ as the set of primitive roots modulo n, p is prime
 $g \in R(p) \Rightarrow (pow(g, p-1, p * p) == 1 ? g+p : g) \in R(pow(p, k))$, for all $k > 1$
 $g \in R(pow(p, k)) \Rightarrow (g \% 2 == 1 ? g : g + pow(p, k)) \in R(2*pow(p, k))$

Mobius Function

```
memset(mu, 0, sizeof mu);
mu[1] = 1;
for(int i = 1; i < N; i++){
    for(int j = i + i; j < N; j += i)
        mu[j] -= mu[i];
}
// g(n) = sum{f(d)} => f(n) = sum{mu(d)*g(n/d)}
```

Mulmod TOP

```
constexpr uint64_t mod = (1ull<<61) - 1;
uint64_t modmul(uint64_t a, uint64_t b){
```

```

uint64_t l1 = (uint32_t)a, h1 = a>>32, l2 = (
    uint32_t)b, h2 = b>>32;
uint64_t l = l1*l2, m = l1*h2 + l2*h1, h = h1*h2;
uint64_t ret = (l&mod) + (l>>61) + (h << 3) + (m >>
    29) + (m << 35 >> 3) + 1;
ret = (ret & mod) + (ret>>61);
ret = (ret & mod) + (ret>>61);
return ret-1;
}

```

Matrix Determinant

```

int n;
long double a[n][n];

long double gauss(){
    long double det = 1;
    for(int i = 0; i < n; i++){
        int q = i;
        for(int j = i+1; j < n; j++){
            if(abs(a[j][i]) > abs(a[q][i]))
                q = j;
        }
        if(abs(a[q][i]) < EPS){
            det = 0;
            break;
        }
        if(i != q){
            for(int w = 0; w < n; w++){
                swap(a[i][w], a[q][w]);
                det = -det;
            }
            det *= a[i][i];
            for(int j = i+1; j < n; j++) a[i][j] /= a[i][i];

            for(int j = 0; j < n; j++) if(j != i){
                if(abs(a[j][i]) > EPS)
                    for(int k = i+1; k < n; k++)
                        a[j][k] -= a[i][k] * a[j][i];
            }
        }

        return det;
    }
}

```

Simplex Method

```

typedef long double dbl;
const dbl eps = 1e-6;
const int N = , M = ;

mt19937 rng(chrono::steady_clock::now().time_since_epoch
    ().count());
struct simplex {
    int X[N], Y[M];
    dbl A[M][N], b[M], c[N];
    dbl ans;
    int n, m;
    dbl sol[N];

    void pivot(int x, int y){
        swap(X[y], Y[x]);
        b[x] /= A[x][y];
        for(int i = 0; i < n; i++){
            if(i != y)
                A[x][i] /= A[x][y];
        }
        A[x][y] = 1. / A[x][y];
        for(int i = 0; i < m; i++){
            if(i != x && abs(A[i][y]) > eps) {

```

```

                b[i] -= A[i][y] * b[x];
                for(int j = 0; j < n; j++) if(j != y)
                    A[i][j] -= A[i][y] * A[x][j];
                A[i][y] = -A[i][y] * A[x][y];
            }
            ans += c[y] * b[x];
        }
        for(int i = 0; i < n; i++){
            if(i != y)
                c[i] -= c[y] * A[x][i];
            c[y] = -c[y] * A[x][y];
        }

        // maximiza sum(x[i] * c[i])
        // sujeito a
        // sum(a[i][j] * x[j]) <= b[i] para 0 <= i < m (Ax
        // <= b)
        // x[i] >= 0 para 0 <= i < n (x >= 0)
        // (n variaveis, m restricoes)
        // guarda a resposta em ans e retorna o valor otimo
        dbl solve(int _n, int _m) {
            this->n = _n; this->m = _m;

            for(int i = 1; i < m; i++){
                int id = uniform_int_distribution<int>(0, i)(
                    rng);
                swap(b[i], b[id]);
                for(int j = 0; j < n; j++)
                    swap(A[i][j], A[id][j]);
            }

            ans = 0.;
            for(int i = 0; i < n; i++) X[i] = i;
            for(int i = 0; i < m; i++) Y[i] = i + n;
            while(true) {
                int x = min_element(b, b + m) - b;
                if(b[x] >= -eps)
                    break;
                int y = find_if(A[x], A[x] + n, [](dbl d) {
                    return d < -eps; }) - A[x];
                if(y == n) throw 1; // no solution
                pivot(x, y);
            }
            while(true) {
                int y = max_element(c, c + n) - c;
                if(c[y] <= eps) break;
                int x = -1;
                dbl mn = 1. / 0.;
                for(int i = 0; i < m; i++){
                    if(A[i][y] > eps && b[i] / A[i][y] < mn)
                        mn = b[i] / A[i][y], x = i;
                }
                if(x == -1) throw 2; // unbounded
                pivot(x, y);
            }
            memset(sol, 0, sizeof(dbl) * n);
            for(int i = 0; i < m; i++){
                if(Y[i] < n)
                    sol[Y[i]] = b[i];
            }
            return ans;
        }
    }
};

```

FFT

```

struct base{
    double r, i;
    base(double _r = 0, double _i = 0) : r(_r), i(_i) {}
    base operator*(base &o) const{
        return {r*o.r - i*o.i, r*o.i + o.r*i};
    }
};

```



```

    }
    double real() const { return r; }
    void operator*=(const base &o){
        (*this) = {r*o.r-i*o.i, r*o.i+o.r*i};
    }
    void operator+=(const base &o){r += o.r, i += o.i; }
    void operator/=(const double &o){ r /= o, i /= o; }
    void operator-=(const base &o){r -= o.r, i -= o.i; }
    base operator+(const base &o){return {r+o.r,i+o.i};}
    base operator-(const base &o){return {r-o.r,i-o.i};}
};

double PI = acos(-1);

void fft(vector<base> &a, bool inv){
    int n = (int)a.size();

    for(int i = 1, j = 0; i < n; i++){
        int bit = n >> 1;
        for(; j >= bit; bit >>= 1) j -= bit;
        j += bit;
        if(i < j) swap(a[i], a[j]);
    }

    for(int sz = 2; sz <= n; sz <= 1) {
        double ang = 2*PI/sz * (inv ? -1 : 1);
        base wlen(cos(ang), sin(ang));
        for(int i = 0; i < n; i += sz){
            base w(1);
            for(int j = 0; j < sz/2; j++){
                base u = a[i+j], v = a[i+j+sz/2] * w;
                a[i+j] = u + v;
                a[i+j+sz/2] = u - v;
                w *= wlen;
            }
        }
    }
    if(inv) for(int i = 0; i < n; i++) a[i] /= 1.0 * n;
}

void multiply(const vector<int> &a, const vector<int> &b
, vector<int> &res){
    vector<base> fa(a.begin(), a.end());
    vector<base> fb(b.begin(), b.end());
    size_t n = 1;
    while(n < a.size()) n <= 1;
    while(n < b.size()) n <= 1;
    n <= 1;
    fa.resize(n), fb.resize(n);

    fft(fa, false), fft(fb, false);
    for(size_t i = 0; i < n; i++)
        fa[i] *= fb[i];
    fft(fa, true);

    res.resize(n);
    for(size_t i = 0; i < n; ++i)
        res[i] = (int)(fa[i].real() + 0.5);
}

```

NTT

```

const int mod = 7340033;
const int root = 5;
const int root_1 = 4404020;
const int root_pw = 1<<20;

void fft (vector<int> &a, bool invert) {

```

```

    int n = (int) a.size();

    for (int i=1, j=0; i<n; ++i) {
        int bit = n >> 1;
        for (; j>=bit; bit>>=1)
            j -= bit;
        j += bit;
        if (i < j)
            swap (a[i], a[j]);
    }

    for (int len=2; len<=n; len<=1) {
        int wlen = invert ? root_1 : root;
        for (int i=len; i<root_pw; i<=1)
            wlen = (int) (wlen * 1ll * wlen % mod);
        for (int i=0; i<n; i+=len) {
            int w = 1;
            for (int j=0; j<len/2; ++j) {
                int u = a[i+j], v = (int) (a[i+j+len/2] * 1
                    ll * w % mod);
                a[i+j] = u+v < mod ? u+v : u+v-mod;
                a[i+j+len/2] = u-v >= 0 ? u-v : u-v+mod;
                w = (int) (w * 1ll * wlen % mod);
            }
        }
    }
    if (invert) {
        int nrev = reverse (n, mod);
        for (int i=0; i<n; ++i)
            a[i] = (int) (a[i] * 1ll * nrev % mod);
    }
}

```

Gauss

// Solves systems of linear equations.
 // To use, build a matrix of coefficients and call run(
 mat, R, C). If the i-th variable is free, row[i] will
 be -1, otherwise it's value will be ans[i].

```

namespace Gauss {
    const int MAXC = 1001;
    int row[MAXC];
    double ans[MAXC];

    void run(double mat[][MAXC], int R, int C) {
        REP(i, C) row[i] = -1;

        int r = 0;
        REP(c, C) {
            int k = r;
            FOR(i, r, R) if(fabs(mat[i][c]) > fabs(mat[k][c]))
                k = i;
            if(fabs(mat[k][c]) < eps) continue;

            REP(j, C+1) swap(mat[r][j], mat[k][j]);
            REP(i, R) if (i != r) {
                double w = mat[i][c] / mat[r][c];
                REP(j, C+1) mat[i][j] -= mat[r][j] * w;
            }
            row[c] = r++;
        }

        REP(i, C) {
            int r = row[i];
            ans[i] = r == -1 ? 0 : mat[r][C] / mat[r][i];
        }
    }
}

```

```
}
```

Gauss Xor

```
const ll MAX = 1e9;
const ll LOG_MAX = 64 - __builtin_clzll((ll)MAX);

struct gauss{
    vector<ll> vet;
    gauss(){}
    gauss(ll val){
        if(val) vet.push_back(val);
    }
    void add(ll val){
        int sig = LOG_MAX;
        for(int i = 0; i < (int)vet.size(); i++){
            while(!(vet[i] & (1LL << sig))) sig--;
            if(val & (1LL << sig)) val ^= vet[i];
        }
        if(!val) return;
        sig = LOG_MAX;
        while(!(val & (1 << sig))) sig--;
        for(auto &x : vet) if(x & (1LL << sig)) x ^= val;
        vet.push_back(val);
        for(int i = (int)vet.size() - 2; i >= 0 && vet[i]
            < vet[i + 1]; i--){
            swap(vet[i], vet[i + 1]);
        }
    }
};
```

Graphs

Dinic

```
const int N = 100005;
const int E = 2000006;
vector<int> g[N];

int ne;
struct Edge{
    int from, to;
    ll flow, cap;
} edge[E];

int lvl[N], vis[N], pass, start = N-2, target = N-1;
int qu[N], qt, px[N];

ll run(int s, int sink, ll minE){
    if(s == sink) return minE;

    ll ans = 0;

    for(; px[s] < (int)g[s].size(); px[s]++){
        int e = g[s][ px[s] ];
        auto &v = edge[e], &rev = edge[e^1];
        if(lvl[v.to] != lvl[s]+1 || v.flow >= v.cap)
            continue; // v.cap - v.flow < lim
        ll tmp = run(v.to, sink, min(minE, v.cap-v.flow));
        v.flow += tmp, rev.flow -= tmp;
        ans += tmp, minE -= tmp;
        if(minE == 0) break;
    }
    return ans;
}

bool bfs(int source, int sink){
    qt = 0;
    qu[qt++] = source;
```

```
lvl[source] = 1;
vis[source] = ++pass;

for(int i = 0; i < qt; i++){
    int u = qu[i];
    px[u] = 0;
    if(u == sink) return true;

    for(int e : g[u]){
        auto v = edge[e];
        if(v.flow >= v.cap || vis[v.to] == pass)
            continue; // v.cap - v.flow < lim
        vis[v.to] = pass;
        lvl[v.to] = lvl[u]+1;
        qu[qt++] = v.to;
    }
}
return false;
}

ll flow(int source = start, int sink = target){
    ll ans = 0;
    //for(lim = (1LL << 62); lim >= 1; lim /= 2)
    while(bfs(source, sink))
        ans += run(source, sink, oo);
    return ans;
}
```

```
void addEdge(int u, int v, ll c = 1, ll rc = 0){
    edge[ne] = {u, v, 0, c};
    g[u].push_back(ne++);
    edge[ne] = {v, u, 0, rc};
    g[v].push_back(ne++);
}

void reset_flow(){
    for(int i = 0; i < ne; i++)
        edge[i].flow = 0;
}
```

Min Cost Max Flow

```
const ll oo = 1e18;
const int N = 222, E = 2 * 1000006;

vector<int> g[N];
int ne;
struct Edge{
    int from, to;
    ll cap, cost;
} edge[E];

int start = N-1, target = N-2, p[N];
int inqueue[N];
ll d[N];

bool spfa(int source, int sink){
    for(int i = 0; i < N; i++) d[i] = oo, inqueue[i] =
        0;

    d[source] = 0;
    queue<int> q;
    q.push(source);
    inqueue[source] = 1;

    while(!q.empty()){
        int u = q.front(); q.pop();
        inqueue[u] = 0;
```

```

    for(int e : g[u]){
        auto v = edge[e];
        if(v.cap > 0 and d[u] + v.cost < d[v.to]){
            d[v.to] = d[u] + v.cost;
            p[v.to] = e;
            if(!inqueue[v.to]){
                q.push(v.to);
                inqueue[v.to] = 1;
            }
        }
    }
}

return d[sink] != oo;
}

// <max flow, min cost>
pair<ll, ll> mincost(int source = start, int sink =
target){
    ll ans = 0, mf = 0;
    while(spfa(source, sink)){
        ll f = oo;

        for(int u = sink; u != source; u = edge[ p[u] ].
            from)
            f = min(f, edge[ p[u] ].cap);

        for(int u = sink; u != source; u = edge[ p[u] ].
            from){
            edge[ p[u] ].cap -= f;
            edge[ p[u] ^ 1 ].cap += f;
        }

        mf += f;
        ans += f * d[sink];
    }
    return {mf, ans};
}

void addEdge(int u, int v, ll c, ll cost){
    edge[ne] = {u, v, c, cost};
    g[u].push_back(ne++);
    edge[ne] = {v, u, 0, -cost};
    g[v].push_back(ne++);
}

```

Small to Large

```

void cnt_sz(int u, int p = -1){
    sz[u] = 1;

    for(int v : g[u]) if(v != p)
        cnt_sz(v, u), sz[u] += sz[v];
}

void add(int u, int p, int big = -1){
    // Update info about this vx in global answer

    for(int v : g[u]) if(v != p && v != big)
        add(v, u);
}

void dfs(int u, int p, int keep){

    int big = -1, mmx = -1;

    for(int v : g[u]) if(v != p && sz[v] > mmx)
        mmx = sz[v], big = v;
}

```

```

    for(int v : g[u]) if(v != p && v != big)
        dfs(v, u, 0);

    if(big != -1) dfs(big, u, 1);

    add(u, p, big);

    for(auto x : q[u]){
        // answer all queries for this vx
    }

    if(!keep){
        // Remove data from this subtree
    }
}

```

Junior e Falta de Ideias

```

#include <bits/stdc++.h>

#define ff first
#define ss second
#define mp make_pair

using namespace std;

typedef long long ll;

vector<pair<int,int>> G[500005];
int subtree[500005], treesize, k;
bool vis[500005];
ll dist[500005], ans;

int dfs(int v, int p){
    subtree[v] = 1;
    for(pair<int,int> x : G[v]){
        if(x.ff != p && !vis[x.ff]) subtree[v] += dfs(x.
            ff,v);
    }
    return subtree[v];
}

int centroid(int v, int p){
    for(pair<int,int> x : G[v]){
        if(x.ff == p || vis[x.ff]) continue;
        if(subtree[x.ff]*2 > treesize) return centroid(x.
            ff,v);
    }
    return v;
}

void procurar_ans(int v, int p, int d_atual, ll custo){
    ans = min(ans, dist[k-d_atual] + custo);
    if(d_atual == k) return;
    for(pair<int,int> x : G[v]){
        if(!vis[x.ff] && x.ff != p)
            procurar_ans(x.ff,v,d_atual+1,custo+x.ss);
    }
}

void atualiza_distancia(int v, int p, int d_atual, ll
custo){
    dist[d_atual] = min(dist[d_atual], custo);
    if(d_atual == k) return;
    for(pair<int,int> x : G[v]){
        if(!vis[x.ff] && x.ff != p)
            atualiza_distancia(x.ff,v,d_atual+1,custo+x.
                ss);
    }
}

```

```

    }
}

void decomp(int v, int p){
    treesize = dfs(v,v);
    // if(treesize < k) return;
    int cent = centroid(v,v);
    vis[cent] = 1;

    for(int i = 1; i <= treesize; i++){
        dist[i] = 1e18;
    }

    for(pair<int,int> x : G[cent]){
        if(!vis[x.ff]){
            procurar_ans(x.ff,cent,1,x.ss);
            atualiza_distancia(x.ff,cent,1,x.ss);
        }
    }

    for(pair<int,int> x : G[cent]){
        if(!vis[x.ff])
            decomp(x.ff, cent);
    }
}

int main(){
    int n,i,a,b;

    scanf("%d%d", &n,&k);
    for(i = 2; i <= n; i++){
        scanf("%d%d", &a,&b);
        G[i].push_back(mp(a,b));
        G[a].push_back(mp(i,b));
    }
    ans = 1e18;
    decomp(1,-1);

    printf("%lld\n", ans == 1e18 ? -1 : ans);

    return 0;
}

```

Kosaraju

```

vector<int> g[N], gt[N], S;

int vis[N], cor[N], tempo = 1;

void dfs(int u){
    vis[u] = 1;
    for(int v : g[u]) if(!vis[v]) dfs(v);
    S.push_back(u);
}

int e;
void dfst(int u){
    cor[u] = e;
    for(int v : gt[u]) if(!cor[v]) dfst(v);
}

int main(){

    for(int i = 1; i <= n; i++) if(!vis[i]) dfs(i);

    e = 0;
    reverse(S.begin(), S.end());
    for(int u : S) if(!cor[u])
        e++, dfst(u);
}

```

```

    return 0;
}

Tarjan

int cnt = 0, root;
void dfs(int u, int p = -1){
    low[u] = num[u] = ++t;
    for(int v : g[u]){
        if(!num[v]){
            dfs(v, u);
            if(u == root) cnt++;
            if(low[v] >= num[u]) u PONTO DE ARTICULACAO;
            if(low[v] > num[u]) ARESTA u->v PONTE;
            low[u] = min(low[u], low[v]);
        }
        else if(v != p) low[u] = min(low[u], num[v]);
    }
}

```

root PONTO DE ARTICULACAO <=> cnt > 1

```

void tarjanSCC(int u){
    low[u] = num[u] = ++cnt;
    vis[u] = 1;
    S.push_back(u);
    for(int v : g[u]){
        if(!num[v]) tarjanSCC(v);
        if(vis[v]) low[u] = min(low[u], low[v]);
    }
    if(low[u] == num[u]){
        ssc[u] = ++ssc_cnt; int v;
        do{
            v = S.back(); S.pop_back(); vis[v] = 0;
            ssc[v] = ssc_cnt;
        }while(u != v);
    }
}

```

Max Clique

```

long long adj[N], dp[N];

for(int i = 0; i < n; i++){
    for(int j = 0; j < n; j++){
        int x;
        scanf("%d",&x);
        if(x || i == j)
            adj[i] |= 1LL << j;
    }
}

int resto = n - n/2;
int C = n/2;
for(int i = 1; i < (1 << resto); i++){
    int x = i;
    for(int j = 0; j < resto; j++){
        if(i & (1 << j))
            x &= adj[j + C] >> C;
    }
    if(x == i){
        dp[i] = __builtin_popcount(i);
    }
}

for(int i = 1; i < (1 << resto); i++){
    for(int j = 0; j < resto; j++){
        if(i & (1 << j))
            dp[i] = max(dp[i], dp[i ^ (1 << j)]);
    }
}

```

```

int maxCliq = 0;
for(int i = 0; i < (1 << C); i++){
    int x = i, y = (1 << resto) - 1;
    for(int j = 0; j < C; j++){
        if(i & (1 << j))
            x &= adj[j] & ((1 << C) - 1), y &= adj[j] >>
                C;
        if(x != i) continue;
        maxCliq = max(maxCliq, __builtin_popcount(i) + dp[y
            ]));
    }
}

```

Dominator Tree

```

vector<int> g[N], gt[N], T[N];
vector<int> S;
int dsu[N], label[N];
int sdom[N], idom[N], dfs_time, id[N];

vector<int> bucket[N];
vector<int> down[N];

void prep(int u){
    S.push_back(u);
    id[u] = ++dfs_time;
    label[u] = sdom[u] = dsu[u] = u;

    for(int v : g[u]){
        if(!id[v])
            prep(v), down[u].push_back(v);
        gt[v].push_back(u);
    }
}

int fnd(int u, int flag = 0){
    if(u == dsu[u]) return u;
    int v = fnd(dsu[u], 1), b = label[ dsu[u] ];
    if(id[ sdom[b] ] < id[ sdom[ label[u] ] ] )
        label[u] = b;
    dsu[u] = v;
    return flag ? v : label[u];
}

void build_dominator_tree(int root, int sz){
    // memset(id, 0, sizeof(int) * (sz + 1));
    // for(int i = 0; i <= sz; i++) T[i].clear();
    prep(root);
    reverse(S.begin(), S.end());

    int w;
    for(int u : S){
        for(int v : gt[u]){
            w = fnd(v);
            if(id[ sdom[w] ] < id[ sdom[u] ] )
                sdom[u] = sdom[w];
        }
        gt[u].clear();

        if(u != root) bucket[ sdom[u] ].push_back(u);

        for(int v : bucket[u]){
            w = fnd(v);
            if(sdom[w] == sdom[v]) idom[v] = sdom[v];
            else idom[v] = w;
        }
        bucket[u].clear();

        for(int v : down[u]) dsu[v] = u;
    }
}

```

```

        down[u].clear();
    }

    reverse(S.begin(), S.end());
    for(int u : S) if(u != root){
        if(idom[u] != sdom[u]) idom[u] = idom[ idom[u] ];
        T[ idom[u] ].push_back(u);
    }
    S.clear();
}

```

Min Cost Matching

```

// Min cost matching
// O(n^2 * m)
// n == nro de linhas
// m == nro de colunas
// n <= m | flow == n
// a[i][j] = custo pra conectar i a j
vector<int> u(n + 1), v(m + 1), p(m + 1), way(m + 1);
for(int i = 1; i <= n; ++i){
    p[0] = i;
    int j0 = 0;
    vector<int> minv(m + 1, oo);
    vector<char> used(m + 1, false);
    do{
        used[j0] = true;
        int i0 = p[j0], delta = oo, j1;
        for(int j = 1; j <= m; ++j)
            if(!used[j]){
                int cur = a[i0][j] - u[i0] - v[j];
                if(cur < minv[j])
                    minv[j] = cur, way[j] = j0;
                if(minv[j] < delta)
                    delta = minv[j], j1 = j;
            }
        for(int j = 0; j <= m; ++j)
            if(used[j])
                u[p[j]] += delta, v[j] -= delta;
            else
                minv[j] -= delta;
        j0 = j1;
    }while(p[j0] != 0);

    do{
        int j1 = way[j0];
        p[j0] = p[j1];
        j0 = j1;
    }while(j0);
}

```

```

// match[i] = coluna escolhida para linha i
vector<int> match(n + 1);
for(int j = 1; j <= m; ++j)
    match[p[j]] = j;

```

```
int cost = -v[0];
```

Strings

Aho Corasick

```

map<char, int> to[N];
int ne = 1, term[N], fail[N];

void add_string(char *str){
    int p = 0;

    for(int i = 0; str[i]; i++){

```

```

        if(!to[p][ str[i] ]) to[p][ str[i] ] = ne++;
        p = to[p][ str[i] ];
    }
    term[p] = 1;
}

int go(int s, char c){
    while(s && !to[s].count(c)) s = fail[s];
    if(to[s].count(c)) return to[s][c];
    return s;
}

void init(){
    queue<int> q;
    q.push(0);

    int u, v; char c;
    while(!q.empty()){
        u = q.front(); q.pop();

        for(auto w : to[u]){
            tie(c, v) = w;
            q.push(v);
            if(u){
                fail[v] = go(fail[u], c);
                term[v] |= term[ fail[v] ];
            }
        }
    }
}

```

Suffix Array

```

char s[N];
int n, sa[N], tsa[N], lcp[N], r[N], nr[N], c[N];

void sort(int k, int mx){
    mx += 2;
    memset(c, 0, sizeof(int) * mx);
    for(int i = 0; i < n; i++) c[i + k < n ? r[i+k]+2 : 1]++;
    partial_sum(c, c+mx, c);
    int t;
    for(int i = 0; i < n; i++){
        t = sa[i+k < n ? r[ sa[i]+k ]+1 : 0,
        tsa[ c[t]++ ] = sa[i];
        memcpy(sa, tsa, sizeof(int) * n);
    }
}

void build_sa(){
    for(int i = 0; i < n; i++) sa[i] = i, r[i] = s[i];

    int t = 300, a, b;
    for(int sz = 1; sz < n; sz *= 2){
        sort(sz, t), sort(0, t);
        t = nr[ sa[0] ] = 0;
        for(int i = 1; i < n; i++){
            a = sa[i]+sz < n ? r[ sa[i]+sz ] : -1;
            b = sa[i-1]+sz < n ? r[ sa[i-1]+sz ] : -1;
            nr[ sa[i] ] = r[ sa[i] ] == r[ sa[i-1] ] && a == b ? t : ++t;
        }
        if(t == n-1) break;
        memcpy(r, nr, sizeof(int) * n);
    }
}

```

```

}

void build_lcp(){ // lcp[i] = lcp(s[:i], s[:i+1])
    int k = 0;
    for(int i = 0; i < n; i++) r[ sa[i] ] = i;

    for(int i = 0; i < n; i++){
        if(r[i] == n-1) k = 0;
        else{
            int j = sa[r[i]+1];
            while(i+k < n && j+k < n && s[i+k] == s[j+k])
                k++;
        }
        lcp[r[i]] = k;
        if(k) k--;
    }
}

int comp_lcp(int i, int j){
    if(i == j) return n - i;
    if(r[i] > r[j]) swap(i, j);
    return min(lcp[k] for k in [r[i], r[j]-1]);
}

```

Z Algorithm

```

vector<int> z_algo(const string &s) {
    int n = s.size(), L = 0, R = 0;
    vector<int> z(n, 0);
    for(int i = 1; i < n; i++){
        if(i <= R) z[i] = min(z[i-L], R - i + 1);
        while(z[i]+i < n && s[ z[i]+i ] == s[ z[i] ])
            z[i]++;
        if(i+z[i]-1 > R) L = i, R = i + z[i] - 1;
    }
    return z;
}

```

Prefix function/KMP

```

vector<int> prefix_function(const string &s){
    int n = s.size();
    vector<int> b(n+1);
    b[0] = -1;
    int i = 0, j = -1;
    while(i < n){
        while(j >= 0 && s[i] != s[j]) j = b[j];
        b[++i] = ++j;
    }
    return b;
}

void kmp(const string &t, const string &p){
    vector<int> b = prefix_function(p);
    int n = t.size(), m = p.size();
    int j = 0;
    for(int i = 0; i < n; i++){
        while(j >= 0 && t[i] != p[j]) j = b[j];
        j++;
        if(j == m){
            //patern of p found on t
            j = b[j];
        }
    }
}

```

Min rotation

```

int min_rotation(int *s, int N) {
    REP(i, N) s[N+i] = s[i];
}

```

```

int a = 0;
REP(b, N) REP(i, N) {
    if (a+i == b || s[a+i] < s[b+i]) { b += max(0, i-1);
        break; }
    if (s[a+i] > s[b+i]) { a = b; break; }
}
return a;
}

```

All palindrome

```

void manacher(char *s, int N, int *rad) {
    static char t[2*MAXN];
    int m = 2*N - 1;

    REP(i, m) t[i] = -1;
    REP(i, N) t[2*i] = s[i];

    int x = 0;
    FOR(i, 1, m) {
        int &r = rad[i] = 0;
        if (i <= x+rad[x]) r = min(rad[x+x-i], x+rad[x]-i);
        while (i-r-1 >= 0 && i+r+1 < m && t[i-r-1] == t[i+r+1]) ++r;
        if (i+r >= x+rad[x]) x = i;
    }

    REP(i, m) if (i-rad[i] == 0 || i+rad[i] == m-1) ++rad[i];
    REP(i, m) rad[i] /= 2;
}

```

Manacher

```

// odd[i] = largest palindrome with center in character
// i(0-based)
// even[i] = largest palindrome with center between
// chars i-1 and i(0-based)
// pair<even, odd>
pair<vector<int>, vector<int>> build(const string &s){
    // O(n)
    int n = (int)s.size();
    vector<int> vet[2];

    for(int add = 0; add < 2; add++){
        vet[add].resize(n);
        for(int i = 0, l = 0, r = -1; i < n; i++){
            int k = i > r ? add : min(vet[add][l + r - i + !add], r - i + !add);
            while(i - k - !add >= 0 && i + k < n && s[i - k - !add] == s[i + k]) k++;
            vet[add][i] = k--;
            if(i + k > r){
                r = i + k;
                l = i - k - !add;
            }
        }
    }

    return make_pair(vet[0], vet[1]);
}

```

Palindromic Tree

```
const int MAXN = 105000;
```

```

struct node {
    int next[26];
    int len;
}

```

```

    int sufflink;
    int num;
};

int len;
char s[MAXN];
node tree[MAXN];
int num; // node 1 - root with len -1, node 2 - root
           with len 0
int suff; // max suffix palindrome
long long ans;

bool addLetter(int pos) {
    int cur = suff, curlen = 0;
    int let = s[pos] - 'a';

    while(true){
        curlen = tree[cur].len;
        if (pos-1 - curlen >= 0 && s[pos-1 - curlen] == s[pos])
            break;
        cur = tree[cur].sufflink;
    }
    if (tree[cur].next[let]) {
        suff = tree[cur].next[let];
        return false;
    }

    num++;
    suff = num;
    tree[num].len = tree[cur].len + 2;
    tree[cur].next[let] = num;

    if (tree[num].len == 1){
        tree[num].sufflink = 2;
        tree[num].num = 1;
        return true;
    }

    while (true){
        cur = tree[cur].sufflink;
        curlen = tree[cur].len;
        if(pos-1 - curlen >= 0 && s[pos-1 - curlen] == s[pos]){
            tree[num].sufflink = tree[cur].next[let];
            break;
        }
    }

    tree[num].num = 1 + tree[tree[num].sufflink].num;

    return true;
}

void initTree() {
    num = 2; suff = 2;
    tree[1].len = -1; tree[1].sufflink = 1;
    tree[2].len = 0; tree[2].sufflink = 1;
}

int main() {

    initTree();

    for (int i = 0; i < len; i++) {
        addLetter(i);
    }
}

```

```
    return 0;
}
```

Suffix Automaton

```
map<char, int> to[2*N];
int link[2*N], len[2*N], last = 0, sz = 1;

void add_letter(char c){
    int p = last;
    last = sz++;
    len[last] = len[p] + 1;
    for(; !to[p][c]; p = link[p]) to[p][c] = last;
    if(to[p][c] == last){
        link[last] = 0;
        return;
    }
    int u = to[p][c];
    if(len[u] == len[p]+1){
        link[last] = u;
        return;
    }
    int c1 = sz++;
    to[c1] = to[u];
    link[c1] = link[u];
    len[c1] = len[p]+1;
    link[last] = link[u] = c1;
    for(; to[p][c] == u; p = link[p]) to[p][c] = c1;
}
```

Suffix Tree

```
namespace sf {
// const int NS = ; const int N = * 2;
int cn, cd, ns, en = 1, lst;
string S[NS]; int si = -1;
vector<int> sufn[N]; // sufn[si][i] no do sufixo S[si][i]
...
struct node {
    int l, r, si, p, suf;
    map<char, int> adj;
    node() : l(0), r(-1), suf(0), p(0) {}
    node(int L, int R, int S, int P) : l(L), r(R), si(S), p(P) {}
    inline int len() { return r - l + 1; }
    inline int operator[](int i) { return S[si][l + i]; }
    inline int& operator()(char c) { return adj[c]; }
} t[N];
inline int new_node(int L, int R, int S, int P) { t[en] = node(L, R, S, P); return en++; }
void add_string(string s) {
    s += '$'; S[++si] = s; sufn[si].resize(s.size() + 1);
    ; cn = cd = 0;
    int i = 0; const int n = s.size();
    for(int j = 0; j < n; j++)
        for(; i <= j; i++) {
            if(cd == t[cn].len() && t[cn][s[j]]) { cn = t[cn][s[j]]; cd = 0; }
            if(cd < t[cn].len() && t[cn][cd] == s[j]) {
                cd++;
                if(j < s.size() - 1) break;
            }
            else {
                if(i) t[lst].suf = cn;
                for(; i <= j; i++) { sufn[si][i] = cn;
                    cn = t[cn].suf; }
            }
        }
    } else if(cd == t[cn].len()) {
```

```
    sufn[si][i] = en;
    if(i) t[lst].suf = en; lst = en;
    t[cn][s[j]] = new_node(j, n - 1, si, cn);
    cn = t[cn].suf; cd = t[cn].len();
} else {
    int mid = new_node(t[cn].l, t[cn].l + cd - 1, t[cn].si, t[cn].p);
    t[t[cn].p][t[cn][0]] = mid;
    if(ns) t[ns].suf = mid;
    if(i) t[lst].suf = en; lst = en;
    sufn[si][i] = en;
    t[mid][s[j]] = new_node(j, n - 1, si, mid);
    ;
    t[mid][t[cn][cd]] = cn;
    t[cn].p = mid; t[cn].l += cd; cn = t[mid].p;
    int g = cn? j - cd : i + 1; cn = t[cn].suf;
    ;
    while(g < j && g + t[t[cn][S[si][g]]].len() <= j) {
        cn = t[cn][S[si][g]]; g += t[cn].len();
    }
    if(g == j) { ns = 0; t[mid].suf = cn; cd = t[cn].len(); }
    else { ns = mid; cn = t[cn][S[si][g]]; cd = j - g; }
}
}
};
```

Geometry

2D basics

```
typedef double cod;
double eps = 1e-7;
bool eq(cod a, cod b){ return abs(a - b) <= eps; }

struct vec{
    cod x, y; int id;
    vec(cod a = 0, cod b = 0) : x(a), y(b) {}
    vec operator+(const vec &o) const{
        return {x + o.x, y + o.y};
    }
    vec operator-(const vec &o) const{
        return {x - o.x, y - o.y};
    }
    vec operator*(cod t) const{
        return {x * t, y * t};
    }
    vec operator/(cod t) const{
        return {x / t, y / t};
    }
    cod operator*(const vec &o) const{ // cos
        return x * o.x + y * o.y;
    }
    cod operator^(const vec &o) const{ // sin
        return x * o.y - y * o.x;
    }
    bool operator==(const vec &o) const{
        return eq(x, o.x) && eq(y, o.y);
    }
    bool operator<(const vec &o) const{
        if(!eq(x, o.x)) return x < o.x;
        return y < o.y;
    }
    cod cross(const vec &a, const vec &b) const{
```



```

    return (a-(*this)) ^ (b-(*this));
}
int ccw(const vec &a, const vec &b) const{
    cod tmp = cross(a, b);
    return (tmp > eps) - (tmp < -eps);
}
cod dot(const vec &a, const vec &b) const{
    return (a-(*this)) * (b-(*this));
}
cod len() const{
    return sqrt(x * x + y * y); // <
}
double angle(const vec &a, const vec &b) const{
    return atan2(cross(a, b), dot(a, b));
}
double tan(const vec &a, const vec &b) const{
    return cross(a, b) / dot(a, b);
}
vec unit() const{
    return operator/(len());
}
int quad() const{
    if(x > 0 && y >=0) return 0;
    if(x <=0 && y > 0) return 1;
    if(x < 0 && y <=0) return 2;
    return 3;
}
bool comp(const vec &a, const vec &b) const{
    return (a - *this).comp(b - *this);
}
bool comp(vec b){
    if(quad() != b.quad()) return quad() < b.quad();
    if(!eq(operator^(b), 0)) return operator^(b) > 0;
    return (*this) * (*this) < b * b;
}
template<class T>
void sort_by_angle(T first, T last) const{
    std::sort(first, last, [=](const vec &a, const
        vec &b){
            return comp(a, b);
        });
}
vec rot90() const{ return {-y, x}; }
vec rot(double a) const{
    return {cos(a)*x -sin(a)*y, sin(a)*x +cos(a)*y};
}
vec proj(const vec &b) const{ // proj of *this onto
    b
    cod k = operator*(b) / (b * b);
    return b * k;
}
// proj of (*this) onto the plane orthogonal to b
vec rejection(vec b) const{
    return (*this) - proj(b);
}
};

struct line{
    cod a, b, c; vec n;
    line(vec q, vec w){ // q.cross(w, (x, y)) = 0
        a = -(w.y-q.y);
        b = w.x-q.x;
        c = -(a * q.x + b * q.y);
        n = {a, b};
    }
    cod dist(const vec &o) const{
        return abs(eval(o)) / n.len();
    }
};

```

```

}
bool contains(const vec &o) const{
    return eq(a * o.x + b * o.y + c, 0);
}
cod dist(const line &o) const{
    if(!parallel(o)) return 0;
    if(!eq(o.a * b, o.b * a)) return 0;
    if(!eq(a, 0))
        return abs(c - o.c * a / o.a) / n.len();
    if(!eq(b, 0))
        return abs(c - o.c * b / o.b) / n.len();
    return abs(c - o.c);
}
bool parallel(const line &o) const{
    return eq(n ^ o.n, 0);
}
bool operator==(const line &o) const{
    if(!eq(a*o.b, b*o.a)) return false;
    if(!eq(a*o.c, c*o.a)) return false;
    if(!eq(c*o.b, b*o.c)) return false;
    return true;
}
bool intersect(const line &o) const{
    return !parallel(o) || *this == o;
}
vec inter(const line &o) const{
    if(parallel(o)){
        if(*this == o){ }
        else{ /* dont intersect */ }
    }

    auto tmp = n ^ o.n;
    return {(o.c*b -c*o.b)/tmp, (o.a*c -a*o.c)/tmp};
}
vec at_x(cod x) const{
    return {x, (-c-a*x)/b};
}
vec at_y(cod y) const{
    return {(-c-b*y)/a, y};
}
cod eval(const vec &o) const{
    return a * o.x + b * o.y + c;
}
};

struct segment{
    vec p, q;
    segment(vec a = vec(), vec b = vec()): p(a), q(b) {}
    bool onstrip(const vec &o) const{ // onstrip strip
        return p.dot(o, q) >= -eps && q.dot(o, p) >= -eps
            ;
    }
    cod len() const{
        return (p-q).len();
    }
    cod dist(const vec &o) const{
        if(onstrip(o)) return line(p, q).dist(o);
        return min((o-q).len(), (o-p).len());
    }
    bool contains(const vec &o) const{
        return eq(p.cross(q, o), 0) && onstrip(o);
    }
    bool intersect(const segment &o) const{
        if(contains(o.p)) return true;
        if(contains(o.q)) return true;
        if(o.contains(q)) return true;
        if(o.contains(p)) return true;
    }
};

```

```

        return p.ccw(q, o.p) * p.ccw(q, o.q) == -1
        && o.p.ccw(o.q, q) * o.p.ccw(o.q, p) == -1;
    }
    bool intersect(const line &o) const{
        return o.eval(p) * o.eval(q) <= 0;
    }
    cod dist(const segment &o) const{
        if(line(p, q).parallel(line(o.p, o.q))){
            if(onstrip(o.p) || onstrip(o.q)
            || o.onstrip(p) || o.onstrip(q))
                return line(p, q).dist(line(o.p, o.q));
        }
        else if(intersect(o)) return 0;
        return min(min(dist(o.p), dist(o.q)),
            min(o.dist(p), o.dist(q)));
    }
    cod dist(const line &o) const{
        if(line(p, q).parallel(o))
            return line(p, q).dist(o);
        else if(intersect(o)) return 0;
        return min(o.dist(p), o.dist(q));
    }
};

struct hray{
    vec p, q;
    hray(vec a = vec(), vec b = vec()): p(a), q(b){}
    bool onstrip(const vec &o) const{ // onstrip strip
        return p.dot(q, o) >= -eps;
    }
    cod dist(const vec &o) const{
        if(onstrip(o)) return line(p, q).dist(o);
        return (o-p).len();
    }
    bool intersect(const segment &o) const{
        if(!o.intersect(line(p,q))) return false;
        if(line(o.p, o.q).parallel(line(p,q)))
            return contains(o.p) || contains(o.q);
        return contains(line(p,q).inter(line(o.p,o.q)));
    }
    bool contains(const vec &o) const{
        return eq(line(p, q).eval(o), 0) && onstrip(o);
    }
    cod dist(const segment &o) const{
        if(line(p, q).parallel(line(o.p, o.q))){
            if(onstrip(o.p) || onstrip(o.q))
                return line(p, q).dist(line(o.p, o.q));
            return o.dist(p);
        }
        else if(intersect(o)) return 0;
        return min(min(dist(o.p), dist(o.q)),
            o.dist(p));
    }
    bool intersect(const hray &o) const{
        if(!line(p, q).parallel(line(o.p, o.q)))
            return false;
        auto pt = line(p, q).inter(line(o.p, o.q));
        return contains(pt) && o.contains(pt); // <<
    }
    bool intersect(const line &o) const{
        if(line(p, q).parallel(o)) return line(p, q) == o;
        if(o.contains(p) || o.contains(q)) return true;
        return (o.eval(p) >= -eps)^(o.eval(p)<o.eval(q));
        return contains(o.inter(line(p, q)));
    }
    cod dist(const line &o) const{
        if(line(p,q).parallel(o))

```

```

        return line(p,q).dist(o);
        else if(intersect(o)) return 0;
        return o.dist(p);
    }
    cod dist(const hray &o) const{
        if(line(p, q).parallel(line(o.p, o.q))){
            if(onstrip(o.p) || o.onstrip(p))
                return line(p,q).dist(line(o.p, o.q));
            return (p-o.p).len();
        }
        else if(intersect(o)) return 0;
        return min(dist(o.p), o.dist(p));
    }
};

double heron(cod a, cod b, cod c){
    cod s = (a + b + c) / 2;
    return sqrt(s * (s - a) * (s - b) * (s - c));
}

```

Circle line intersection

```

// intersection of line a * x + b * y + c = 0
// and circle centered at the origin with radius r
double r, a, b, c; // given as input
double x0 = -a*c/(a*a+b*b), y0 = -b*c/(a*a+b*b);
if(c*c > r*r*(a*a+b*b)+EPS)
    puts("no points");
else if(abs(c*c - r*r*(a*a+b*b)) < EPS){
    puts("1 point");
    cout << x0 << ' ' << y0 << '\n';
}
else {
    double d = r*r - c*c/(a*a+b*b);
    double mult = sqrt(d / (a*a+b*b));
    double ax, ay, bx, by;
    ax = x0 + b * mult;
    bx = x0 - b * mult;
    ay = y0 - a * mult;
    by = y0 + a * mult;
    puts ("2 points");
    cout<<ax<< ' ' <<ay<<'\n'<<bx<< ' ' <<by<<'\n';
}

```

Circle Circle intersection

Assume that the first circle is centered at the origin and second at (x_2, y_2) . Find circle line intersection of first circle and line $Ax + By + C = 0$, where $A = -2x_2$, $B = -2y_2$, $C = x_2^2 + y_2^2 + r_1^2 - r_2^2$.

Be aware of corner case with two circles centered at the same point.

Tangents of two circles

```

// solve first for same circle(and infinitely many
// tangents)
// Find up to four tangents of two circles
void tangents(pt c, double r1, double r2, vector<line> &
ans){
    double r = r2 - r1;
    double z = c.x * c.x + c.y * c.y;
    double d = z - r * r;
    if(d < -EPS) return;
    d = sqrt(abs(d));
    line l;
    l.a = (c.x * r + c.y * d) / z;
    l.b = (c.y * r - c.x * d) / z;
}

```

```

    l.c = r1;
    ans.push_back (l);
}

vector<line> tangents(circle a, circle b){
    vector<line> ans;
    pt aux = a.center - b.center;
    for(int i = -1; i <= 1; i += 2)
        for(int j = -1; j <= 1; j += 2)
            tangents(aux, a.r * i, b.r * j, ans);
    for(size_t i = 0; i < ans.size(); ++i)
        ans[i].c -= ans[i].a * a.x + ans[i].b * a.y;
    return ans;
}

```

Convex Hull

```

vector<vec> monotone_chain_ch(vector<vec> P){
    sort(P.begin(), P.end());

    vector<vec> L, U;
    for(auto p : P){
        while(L.size() >= 2 && L[L.size() - 2].cross(L.
            back(), p) < 0)
            L.pop_back();

        L.push_back(p);
    }

    reverse(P.begin(), P.end());
    for(auto p : P){
        while(U.size() >= 2 && U[U.size() - 2].cross(U.
            back(), p) < 0)
            U.pop_back();

        U.push_back(p);
    }

    L.pop_back(), U.pop_back();

    L.reserve(L.size() + U.size());
    L.insert(L.end(), U.begin(), U.end());

    return L;
}

```

Check point inside polygon

```

bool below(const vector<vec> &vet, vec p){
    auto it = lower_bound(vet.begin(), vet.end(), p);
    if(it == vet.end()) return false;
    if(it == vet.begin()) return *it == p;
    return prev(it)->cross(*it, p) <= 0;
}

bool above(const vector<vec> &vet, vec p){
    auto it = lower_bound(vet.begin(), vet.end(), p);
    if(it == vet.end()) return false;
    if(it == vet.begin()) return *it == p;
    return prev(it)->cross(*it, p) >= 0;
}

// lowerhull, upperhull and point, borders included
bool inside_poly(const vector<vec> &lo, const vector<vec>
    &hi, vec p){
    return below(hi, p) && above(lo, p);
}

```

Check point inside polygon without lower/upper hull

```

// borders included
// must not have 3 colinear consecutive points
bool inside_poly(const vector<vec> &v, vec p){
    if(v[0].ccw(v[1], p) < 0) return false;
    if(v[0].ccw(v.back(), p) > 0) return 0;
    if(v[0].ccw(v.back(), p) == 0)
        return v[0].dot(p, v.back()) >= 0
            && v.back().dot(p, v[0]) >= 0;

    int L = 1, R = (int)v.size() - 1, ans = 1;

    while(L <= R){
        int mid = (L+R)/2;
        if(v[0].ccw(v[mid], p) >= 0) ans = mid, L = mid
            +1;
        else R = mid-1;
    }

    return v[ans].ccw(v[(ans+1)%v.size()], p) >= 0;
}

```

Minkowski sum

```

vector<vec> mk(const vector<vec> &a, const vector<vec> &b){
    int i = 0, j = 0;
    for(int k = 0; k < (int)a.size(); k++){
        if(a[k] < a[i])
            i = k;
        for(int k = 0; k < (int)b.size(); k++){
            if(b[k] < b[j])
                j = k;
        }

        vector<vec> c;
        c.reserve(a.size() + b.size());
        for(int k = 0; k < (int)(a.size()+b.size()); k++){
            vec pt{a[i] + b[j]};
            if((int)c.size() >= 2
                && c[c.size()-2].ccw(c.back(), pt) == 0)
                c.pop_back();
            c.push_back(pt);
            int q = i+1, w = j+1;
            if(q == (int)a.size()) q = 0;
            if(w == (int)b.size()) w = 0;
            if(c.back().ccw(a[i]+b[w], a[q]+b[j]) < 0) i = q;
            else j = w;
        }
        c.shrink_to_fit();

        return c;
    }
}

```

Geo Notes

Center of mass

System of points(2D/3D): Mass weighted average of points.

Frame(2D/3D): Get middle point of each segment solve as previously.

Triangle: Average of vertices.

2D Polygon: Compute **signed** area and center of mass of triangle $((0,0), p_i, p_{i+1})$. Then solve as system of points.

Polyhedron surface: Solve each face as a 2D polygon (be aware of $(0,0)$) then replace each face with its center of mass and solve as system of points.

Tetrahedron(Triangular pyramid): As triangles, its the average of points.

Polyhedron: Can be done as 2D polygon, but with tetrahe-

dralization instead of triangulation.

Pick's Theorem

Given a polygon without self-intersections and all its vertices on integer coordinates in some 2D grid. Let A be its area, I the number of points with integer coordinates strictly inside the polygon and B the number of points with integer coordinates in the border of the polygon. The following formula holds: $A = I + \frac{B}{2} - 1$.

Miscellaneous

LIS

```
multiset<int> S;
for(int i = 0; i < n; i++){
    auto it = S.upper_bound(a[i]); // low for inc
    if(it != S.end()) S.erase(it);
    S.insert(a[i]);
}
ans = S.size();
```

DSU rollback

```
#include <bits/stdc++.h>

using namespace std;

struct DSU{
    vector<int> sz, p, change;
    vector<tuple<int, int, int>> modifications;
    vector<size_t> saves;
    bool bipartite;

    DSU(int n): sz(n+1, 1), p(n+1), change(n+1),
        bipartite(true){
        iota(p.begin(), p.end(), 0);
    }

    void add_edge(int u, int v){
        if(!bipartite) return;
        int must_change = get_colour(u) == get_colour(v);
        int a = rep(u), b = rep(v);
        if(sz[a] < sz[b]) swap(a, b);
        if(a != b){
            p[b] = a;
            modifications.emplace_back(b, change[b], bipartite);
            change[b] ^= must_change;
            sz[a] += sz[b];
        }
        else if(must_change){
            modifications.emplace_back(0, change[0], bipartite);
            bipartite = false;
        }
    }

    int rep(int u){
        return p[u] == u ? u : rep(p[u]);
    }

    int get_colour(int u){
        if(p[u] == u) return change[u];
        return change[u] ^ get_colour(p[u]);
    }
}
```

```
void reset(){
    modifications.clear();
    saves.clear();
    iota(p.begin(), p.end(), 0);
    fill(sz.begin(), sz.end(), 1);
    fill(change.begin(), change.end(), 0);
    bipartite = true;
}

void rollback(){
    int u = get<0>(modifications.back());
    tie(ignore, change[u], bipartite) = modifications.back();
    sz[p[u]] -= sz[u];
    p[u] = u;
    modifications.pop_back();
}

void reload(){
    while(modifications.size() > saves.back())
        rollback();
    saves.pop_back();
}

void save(){
    saves.push_back(modifications.size());
}

};

const int N = 100005;
const int B = 318;

int n, m, q;
int x[N], y[N], l[N], r[N], ans[N];

vector<int> qu[N];

int brute(int lef, int rig, DSU &s){
    s.save();
    for(int i = lef; i <= rig; i++)
        s.add_edge(x[i], y[i]);
    int ret = s.bipartite;
    s.reload();
    return ret;
}

int main(){
    scanf("%d %d %d", &n, &m, &q);

    for(int i = 1; i <= m; i++)
        scanf("%d %d", &x[i], &y[i]);

    DSU s(n);
    for(int i = 0; i < q; i++){
        scanf("%d %d", &l[i], &r[i]);
        if(r[i] - l[i] <= B + 10)
            ans[i] = brute(l[i], r[i], s);
        else qu[l[i] / B].push_back(i);
    }

    for(int i = 0; i <= m / B; i++){
        sort(qu[i].begin(), qu[i].end(), [](int a, int b){
            return r[a] < r[b];
        });
        s.reset();
    }
}
```

```

    int R = (i+1)*B-1;

    for(int id : qu[i]){
        while(R < r[id]) ++R, s.add_edge(x[R], y[R]);
        s.save();
        for(int k = l[id]; k < (i+1)*B; k++)
            s.add_edge(x[k], y[k]);
        ans[id] = s.bipartite;
        s.reload();
    }
}

for(int i = 0; i < q; i++)
    printf("%s\n", ans[i] ? "Possible" : "Impossible");
}

```

Buildings

```

// count the number of circular arrays
// of size m, with elements on range
// [1, c**(x*x)]
#include <bits/stdc++.h>
using namespace std;

#define debug(x) cerr << fixed << #x << " = " << x << endl;

const int MOD = 1e9 + 7, MAX = 1e5 + 5;
int dp[MAX];

inline int add(int a, int b) {
    return a + b >= MOD ? a + b - MOD : a;
}
inline int sub(int a, int b) {
    return a - b < 0 ? a - b + MOD : a;
}
inline int mult(int a, int b) {
    return (1LL * a * b) % MOD;
}
int f_exp(int x, int exp) {
    if(exp == 0) return 1;
    else if(exp & 1) return mult(x, f_exp(x, exp - 1));
    return f_exp(mult(x, x), exp / 2);
}
inline int inv(int x) {
    return f_exp(x, MOD - 2);
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(NULL); cout.tie(NULL);

    int n, m, c;
    cin >> n >> m >> c;

    int x = f_exp(c, n * n);
    int ans = f_exp(x, m);
    for(int i = 1; i <= m; i++) {
        if(m % i == 0) {
            int y = f_exp(x, i);
            for(int j = 1; j < i; j++) {
                if(i % j == 0)
                    y = sub(y, mult(j, dp[j]));
            }
            dp[i] = mult(y, inv(i));
            ans = sub(ans, mult(i - 1, dp[i]));
        }
    }
}

```

```

}

cout << ans << '\n';

return 0;
}

Rand

#include <random>
#include <chrono>

cout << RAND_MAX << endl;
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
vector<int> permutation(N);

iota(permutation.begin(), permutation.end(), 0);

shuffle(permutation.begin(), permutation.end(), rng);

iota(permutation.begin(), permutation.end(), 0);

for(int i = 1; i < N; i++){
    swap(permutation[i], permutation[
        uniform_int_distribution<int>(0, i)(rng)]);
}

Klondike

// minimum number of moves to make
// all elements equal
// move: change a segment of equal value
// elements to any value

int v[305];
int dp[305][305];
int rec[305][305];

int f(int l, int r){
    if(r == l) return 1;
    if(r < l) return 0;
    if(dp[l][r] != -1) return dp[l][r];
    int ans = f(l+1, r) + 1;
    for(int i = l+1; i <= r; i++){
        if(v[i] == v[l])
            ans = min(ans, f(l, i - 1) + f(i+1, r));
    }

    return dp[l][r] = ans;
}

int main() {
    int n, m;
    memset(dp, -1, sizeof dp);
    scanf("%d %d", &n, &m);
    for(int i = 0; i < n; i++){
        scanf("%d", &v[i]);
        if(i && v[i] == v[i-1]){
            i--;
            n--;
        }
    }
    printf("%d\n", f(0, n-1) - 1);
    // printf("%d\n", rec[0][n-1]);
    // printf("%d\n", rec[1][n-1]);
    // printf("%d\n", rec[2][n-3]);
}

```

Hilbert Order

```
// maybe use B = n / sqrt(q)
inline int64_t hilbertOrder(int x, int y, int pow = 21,
    int rotate = 0) {
    if(pow == 0) return 0;
    int hpow = 1 << (pow-1);
    int seg = (x < hpow) ? (
        (y < hpow) ? 0 : 3
    ) : (
        (y < hpow) ? 1 : 2
    );
    seg = (seg + rotate) & 3;
    const int rotateDelta[4] = {3, 0, 0, 1};
    int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
    int nrot = (rotate + rotateDelta[seg]) & 3;
    int64_t subSquareSize = int64_t(1) << (2*pow - 2);
    int64_t ans = seg * subSquareSize;
    int64_t add = hilbertOrder(nx, ny, pow-1, nrot);
    ans += (seg == 1 || seg == 2) ? add : (subSquareSize
        - add - 1);
    return ans;
}
```

Modular Factorial

```
// Compute (1*2*...*(p-1)*1*(p+1)*(p+2)*...*n) % p
// in O(p*lg(n))

int factmod(int n, int p){
    int ans = 1;
    while(n > 1){
        for(int i = 2; i <= n % p; i++){
            ans = (ans * i) % p;
        }
        n /= p;
        if(n % 2) ans = p - ans;
    }
    return ans % p;
}

int fac_pow(int n, int p){
    int ans = 0;
    while(n) n /= p, ans += n;
    return ans;
}

int C(int n, int k, int p){
    if(fac_pow(n, p) > fac_pow(n-k, p) + fac_pow(k, p))
        return 0;
    int tmp = factmod(k, p) * factmod(n-k, p);
    return (f_exp(tmp, p-2, p) * factmod(n, p)) % p;
}
```

Enumeration all submasks of a bitmask

```
// loop through all submask of a given bitmask
// it does not include mask 0
for(int sub = mask; sub; sub = (sub-1)&mask){
}
}
```

Slope Trick

```
///By woqja125, contest: Codeforces Round #371 (Div. 1),
    problem: (C) Sonya and Problem Wihtout a Legend,
    Accepted, #

#include <stdio.h>
#include <queue>
```

```
int main() {
    int n, t;
    long long ans = 0;
    std::priority_queue<int> Q;
    scanf("%d%d", &n, &t);
    Q.push(t);
    for(int i = 1; i < n; i++) {
        scanf("%d", &t); t -= i;
        Q.push(t);
        if(Q.top() > t) {
            ans += Q.top() - t;
            Q.pop();
            Q.push(t);
        }
    }
    printf("%lld", ans);
    return 0;
}
```

Fast IO

```
#define pc(x) putchar_unlocked(x)
#define gc(x) getchar_unlocked(x)

inline void scan_int(int &x){
    register int c = gc();
    x = 0;
    int neg = 0;
    for(;; ((c < '0' || c > '9') && c != '-'); c = gc());
    if(c == '-'){
        neg = 1;
        c = gc();
    }
    for(;; c >= '0' && c <= '9'; c = gc())
        x = (x << 1) + (x << 3) + c - '0';
    if(neg) x = -x;
}

inline void print_int(int n){
    int rev = 0, count = 0, neg;
    if(n == 0){
        pc('0');
        return;
    }
    if(n < 0) n = -n, neg = 1;
    while(n % 10 == 0) count++, n /= 10;
    for(rev = 0; n != 0; n /= 10)
        rev = (rev << 3) + (rev << 1) + n % 10;
    if(neg) pc('-');
    while(rev != 0) pc(rev % 10 + '0'), rev /= 10;
    while(count-->0) pc('0');
    pc('\n');
}

inline void print_string(char *str){
    while(*str) pc(*str++);
    pc('\n');
}
```

Knapsack Bounded with Cost

```
// menor custo para conseguir peso ate M usando N tipos
    diferentes de elementos, sendo que o i-esimo elemento
    pode ser usado b[i] vezes, tem peso w[i] e custo c[i]
// O(N * M)

int b[N], w[N], c[N];
MinQueue Q[M];
int d[M] //d[i] = custo minimo para conseguir peso i
```



```

for(int i = 0; i <= M; i++) d[i] = i ? oo : 0;
for(int i = 0; i < N; i++){
    for(int j = 0; j < w[i]; j++){
        Q[j].clear();
        for(int j = 0; j <= M; j++){
            q = Q[j % w[i]];
            if(q.size() >= q) q.pop();
            q.add(c[i]);
            q.push(d[j]);
            d[j] = q.getmin();
        }
    }
}

```

LCA <O(nlgn), O(1)>

```

int start[N], dfs_time;
int tour[2*N], id[2*N];

void dfs(int u){
    start[u] = dfs_time;
    id[dfs_time] = u;
    tour[dfs_time++] = start[u];
    for(int v : g[u]){
        dfs(v);
        id[dfs_time] = u;
        tour[dfs_time++] = start[u];
    }
}

int LCA(int u, int v){
    if(start[u] > start[v]) swap(u, v);
    return id[min(tour[k] for k in [start[u], start[v]])];
}

```

Buffered reader

```

// source: https://github.com/ngthanhtung23/
// ACM_Notebook_new/blob/master/buffered_reader.h
int INP, AM, REACHEOF;
#define BUFSIZE (1<<12)
char BUF[BUFSIZE+1], *inp=BUF;
#define GETCHAR(INP) { \
    if(!*inp && !REACHEOF) { \
        memset(BUF, 0, sizeof BUF); \
        int inpzzz = fread(BUF, 1, BUFSIZE, stdin); \
        if (inpzzz != BUFSIZE) REACHEOF = true; \
        inp=BUF; \
    } \
    INP=*inp++; \
}
#define DIG(a) (((a)>='0')&&((a)<='9'))
#define GN(j) { \
    AM=0; \
    GETCHAR(INP); while(!DIG(INP) && INP!='-') GETCHAR( \
        INP); \
    if (INP=='-') {AM=1; GETCHAR(INP);} \
    j=INP-'0'; GETCHAR(INP); \
    while(DIG(INP)){j=10*j+(INP-'0'); GETCHAR(INP);} \
    if (AM) j=-j; \
}

```

Burnside's Lemma

Let (G, \oplus) be a finite group that acts on a set X . It should hold that $e_g * x = x$ and $g_1 * (g_2 * x) = (g_1 \oplus g_2) * x, \forall x \in X, g_1, g_2 \in G$. For each $g \in G$ let $X^g = \{x \in X \mid g * x = x\}$. The number of orbits its given by:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

Wilson's Theorem

$(n-1)! \equiv -1 \pmod n \iff n$ is prime

Fibonacci

- $F_{n-1}F_{n+1} - F_n^2 = (-1)^n$
- $F_{n+k} = F_kF_{n+1} + F_{k-1}F_n$
- $\text{GCD}(F_n, F_m) = F_{\text{GCD}(n, m)}$
- $F_n = \frac{(\frac{1+\sqrt{5}}{2})^n - (\frac{1-\sqrt{5}}{2})^n}{\sqrt{5}}$

Kirchhoff's Theorem

Laplacian matrix is $L = D - A$, where D is a diagonal matrix with vertex degrees on the diagonals and A is adjacency matrix.

The number of spanning trees is any cofactor of L . i -th cofactor is determinant of the matrix gotten by removing i -th row and column of L .

Multigraphs

In $D[i][i]$ all loops are excluded. $A[i][j]$ = number of edges from i to j .

Directed multigraphs

$D[i][i]$ = indegree of i minus the number of loops at i . $A[i][j]$ = number of edges from i to j .

The number of oriented spanning trees rooted at a vertex i is the determinant of the matrix gotten by removing the i th row and column of L .

Matroid

Let X set of objects, $I \subseteq 2^X$ set of independent sets such that:

1. $\emptyset \in I$
2. $A \in I, B \subseteq A \implies B \in I$
3. Exchange axiom, $A \in I, B \in I, |B| > |A| \implies \exists x \in B \setminus A : A \cup \{x\} \in I$
4. $A \subseteq X$ and I and I' are maximal independent subsets of A then $|I| = |I'|$

Then (X, I) is a matroid. The combinatorial optimization problem associated with it is: Given a weight $w(e) \geq 0 \forall e \in X$, find an independent subset that has the largest possible total weight.

Matroid intersection

```

// Input two matroids (X, I_a) and (X, I_b)
// output set I of maximum size, I \in I_a and I \in I_b
set<> I;
while(1){
    for(e_i : X \ I)
        if(I + e_i \in I_a and I + e_i \in I_b)
            I = I + e_i;
    set<> A, T; queue<> Q;
    for(x : X) label[x] = MARK1;
    for(e_i : X \ I){
        if(I + e_i \in I_a)

```

```

    Q.push(e_i), label[e_i] = MARK2;
else{
    for(x such that I - x + e_i \in I_a)
        A[x].push(e_i);
}
if(I + e_i \in I_b)
    T = T + {e_i}
else{
    for(x such that I - x + e_i \in I_b)
        A[e_i].push(x);
}
}
if(T.empty()) break;
bool found = false;
while(!Q.empty() and !found){
    auto e = Q.front(); Q.pop();
    for(x : A[e]) if(label[x] == MARK1){
        label[x] = e;
        Q.push(x);
        if(x \in T){
            found = true;
            put = 1;
            while(label[x] != MARK2){
                if(put) I = I + x;
                else I = I - x;
                put = 1 - put;
            }
            I = I + x;
            break;
        }
    }
}
if(!found) break;
return I;

```

Where $\text{path}(e) = [e]$ if $\text{label}[e] = \text{MARK2}$, $\text{path}(\text{label}[e]) + [e]$ otherwise.

Matroid Union

Given k matroids over the same set of objects $(X, I_1), (X, I_2), \dots, (X, I_k)$ find $A_1 \in I_1, A_2 \in I_2, \dots, A_k \in I_k$ such that $i \neq j, A_i \cap A_j = \emptyset$ and $|\bigcup_{i=1}^k A_i|$ is maximum. Matroid union can be reduced to matroid intersection as follows.

Let $X' = X \times \{1, 2, \dots, k\}$, ie, k copies of each element of X with different colors. $M1 = (X', Q)$ where $B \in Q \iff \forall 1 \leq i \leq k, \{x \mid (x, i) \in B\} \in I_i$, ie, for each color, B is independent. $M2 = (X', W)$ where $B \in W \iff i \neq j \implies \neg((x, i) \in B \wedge (x, j) \in B)$, ie, each element is picked by at most one color.

Intersection of $M1$ and $M2$ is the answer for the combinatorial problem of matroid union.

Edge coloring

Data: A graph G

Result: A proper coloring c of the edges of G

Let $U := E(G)$;

while $U \neq \emptyset$ **do**

 Let (u, v) be any edge in U ;

 Let $F[1:k]$ be a maximal fan of u starting at $F[1]=v$;

 Let c be a color that is free on u and d be a color that is free on $F[k]$;

 Invert the cd_u path;

 Let $w \in V(G)$ be such that $w \in F, F' = [F[1]...w]$ is a fan and d is free on w ;

 Rotate F' and set $c(u, w)=d$;

$U := U - (u, v)$;

end

Notes

When we repeat something and each time we have probability p to succeed then the expected number of tries is $\frac{1}{p}$, till we succeed.

Small to large

Trick in statement If k sets are given you should note that the amount of different set sizes is $O(\sqrt{s})$ where s is total size of those sets. And no more than \sqrt{s} sets have size greater than \sqrt{s} . For example, a path to the root in Aho-Corasick through suffix links will have at most $O(\sqrt{s})$ vertices.

gcd on subsegment, we have at most $\log(a_i)$ different values in $\{\text{gcd}(a_j, a_{j+1}, \dots, a_i) \text{ for } j < i\}$.

From static set to expandable. To insert, create a new set with the new element. While there are two sets with same size, merge them. There will be at most $\log(n)$ disjoint sets.

Matrix exponentiation optimization. Save binary power of $A_{n \times n}$ and answer q queries $b = A^m x$ in $O((n^3 + qn^2)\log(m))$.

Ternary search on integers into binary search, comparing $f(\text{mid})$ and $f(\text{mid}+1)$, binary search on derivative

Dynamic offline set For each element we will wind segment of time $[a, b]$ such that element is present in the set during this whole segment. Now we can come up with recursive procedure which handles $[l, r]$ time segment considering that all elements such that $[l, r] \subset [a, b]$ are already included into the set. Now, keeping this invariant we recursively go into $[l, m]$ and $[m+1, r]$ subsegments. Finally when we come into segment of length 1.

$$a > b \implies a \bmod b < \frac{a}{2}$$