

# ICPC Team Reference

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```
set ts=4 sw=4 sta nu rnu sc stl+=%F cindent
imap {<CR> {<CR>}<Esc>0
nmap <F2> 0V$%d
nmap <C-down> :m+1<CR>
nmap <C-up> :m-2<CR>
vmap <C-c> "+y
nmap <C-a> ggVG
syntax on
alias cmp='g++ -Wall -Wformat=2 -Wshadow -Wconversion -
fsanitize=address -fsanitize=undefined -fno-sanitize-
recover -std=c++14'
```

## Data Structures

### Merge Sort Tree

```
struct MergeTree{
    int n;
    vector<vector<int>> st;

    void build(int p, int L, int R, const int v[]){
        if(L == R){
            st[p].push_back(v[L]);
            return;
        }
        int mid = (L+R)/2;
        build(2*p, L, mid, v);
        build(2*p+1, mid+1, R, v);
        st[p].resize(R-L+1);
        merge(st[2*p].begin(), st[2*p].end(),
              st[2*p+1].begin(), st[2*p+1].end(),
              st[p].begin());
    }

    int query(int p, int L, int R, int i, int j, int x)
    const{
        if(L > j || R < i) return 0;
        if(L >= i && R <= j){
            int id = lower_bound(st[p].begin(), st[p].end(),
                                  x) - st[p].begin();
            return int(st[p].size()) - id;
        }
        int mid = (L+R)/2;
        return query(2*p, L, mid, i, j, x) +
               query(2*p+1, mid+1, R, i, j, x);
    }
};

public:
    MergeTree(int sz, const int v[]): n(sz), st(4*sz){
        build(1, 1, n, v);
    }

    //number of elements >= x on segment [i, j]
    int query(int i, int j, int x) const{
        if(i > j) swap(i, j);
        return query(1, 1, n, i, j, x);
    }
};
```

### Wavelet Tree

```
template<typename T>
class wavelet{
    T L, R;
    vector<int> l;
    vector<T> sum; // <<
    wavelet *lef, *rig;
```

```
int r(int i) const{ return i - l[i]; }

public:
    template<typename ITER>
    wavelet(ITER bg, ITER en){
        lef = rig = nullptr;
        L = *bg, R = *bg;

        for(auto it = bg; it != en; it++){
            L = min(L, *it), R = max(R, *it);
            if(L == R) return;

            T mid = L + (R - L)/2;
            l.reserve(std::distance(bg, en) + 1);
            sum.reserve(std::distance(bg, en) + 1);
            l.push_back(0), sum.push_back(0);
            for(auto it = bg; it != en; it++){
                l.push_back(l.back() + (*it <= mid)),
                sum.push_back(sum.back() + *it);

                auto tmp = stable_partition(bg, en, [mid](T x){
                    return x <= mid;
                });

                if(bg != tmp) lef = new wavelet(bg, tmp);
                if(tmp != en) rig = new wavelet(tmp, en);
            }

            ~wavelet(){
                delete lef;
                delete rig;
            }

            // 1 index, first is 1st
            T kth(int i, int j, int k) const{
                if(L >= R) return L;
                int c = l[j] - l[i-1];
                if(c >= k) return lef->kth(l[i-1]+1, l[j], k);
                else return rig->kth(r(i-1)+1, r(j), k - c);
            }

            // # elements > x on [i, j]
            int cnt(int i, int j, T x) const{
                if(L > x) return j - i + 1;
                if(R <= x || L == R) return 0;
                int ans = 0;
                if(lef) ans += lef->cnt(l[i-1]+1, l[j], x);
                if(rig) ans += rig->cnt(r(i-1)+1, r(j), x);
                return ans;
            }

            // sum of elements <= k on [i, j]
            T sumk(int i, int j, T k){
                if(L == R) return R <= k ? L * (j - i + 1) : 0;
                if(R <= k) return sum[j] - sum[i-1];
                int ans = 0;
                if(lef) ans += lef->sumk(l[i-1]+1, l[j], k);
                if(rig) ans += rig->sumk(r(i-1)+1, r(j), k);
                return ans;
            }

            // swap (i, i+1) just need to update "array" l[i]
};
```

### Order Set

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
```

```
#include <ext/pb_ds/detail/standard_policies.hpp>

using namespace __gnu_pbds; // or pb_ds;

template<typename T, typename B = null_type>
using oset = tree<T, B, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
// find_by_order / order_of_key
```

## Hash table

```
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;

struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        // http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }

    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM = chrono::
            steady_clock::now().time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};
```

```
gp_hash_table<long long, int, custom_hash> table;
unordered_map<long long, int, custom_hash> uhash;
uhash.reserve(1 << 15);
uhash.max_load_factor(0.25);
```

## Convex Hull Trick

```
const ll is_query = -(1LL<<62);
struct Line{
    ll m, b;
    mutable function<const Line*> succ;
    bool operator<(const Line& rhs) const{
        if(rhs.b != is_query) return m < rhs.m;
        const Line* s = succ();
        if(!s) return 0;
        ll x = rhs.m;
        return b - s->b < (s->m - m) * x;
    }
};

struct Cht : public multiset<Line>{ // maintain max
    bool bad(iterator y){
        auto z = next(y);
        if(y == begin()){
            if(z == end()) return 0;
            return y->m == z->m && y->b <= z->b;
        }
        auto x = prev(y);
        if(z == end()) return y->m == x->m && y->b <= x->b;
        return (x->b - y->b)*(z->m - y->m) >= (y->b - z->b)*
            (y->m - x->m);
    }
    void insert_line(ll m, ll b){
        auto y = insert({ m, b });
        y->succ = [=]{ return next(y) == end() ? 0 : &*
            next(y); };
        if(bad(y)){ erase(y); return; }
        while(next(y) != end() && bad(next(y))) erase(
```

```
        next(y));
        while(y != begin() && bad(prev(y))) erase(prev(y)
        );
    }
    ll eval(ll x){
        auto l = *lower_bound((Line) { x, is_query });
        return l.m * x + l.b;
    }
};
```

## Min queue

```
template<typename T>
class minQ{
    deque<tuple<T, int, int> > p;
    T delta;
    int sz;
public:
    minQ() : delta(0), sz(0) {}
    inline int size() const{ return sz; }
    inline void add(T x){ delta += x; }
    inline void push(T x, int id){
        x -= delta, sz++;
        int t = 1;
        while(p.size() > 0 && get<0>(p.back()) >= x)
            t += get<1>(p.back()), p.pop_back();
        p.emplace_back(x, t, id);
    }
    inline void pop(){
        get<1>(p.front())--, sz--;
        if(!get<1>(p.front())) p.pop_front();
    }
    T getmin() const{ return get<0>(p.front())+delta; }
    int getid() const{ return get<2>(p.front()); }
};
```

## Sparse Table

```
const int N = 100005;

int v[N], n;
int dn[N][20];
int fn(int i, int j){
    if(j == 0) return v[i];
    if(~dn[i][j]) return dn[i][j];
    return dn[i][j] = min(fn(i, j-1), fn(i + (1 << (j-1)
        ), j-1));
}

int lg(int x){ return 31 - __builtin_clz(x); }

int getmn(int l, int r){ // [l, r]
    int lz = lg(r - l + 1);
    return min(fn(l, lz), fn(r - (1 << lz) + 1, lz));
}
```

## Treap

```
// source: https://github.com/victorsenam/caderno/blob/
    master/code/treap.cpp
//const int N = ; typedef int num;
num X[N]; int en = 1, Y[N], sz[N], L[N], R[N];
void calc (int u) { // update node given children info
    sz[u] = sz[L[u]] + 1 + sz[R[u]];
    // code here, no recursion
}
void unlaze (int u) {
    if(!u) return;
    // code here, no recursion
}
```

```

void split_val(int u, num x, int &l, int &r) { // l gets
    <= x, r gets > x
    unlaze(u); if(!u) return (void) (l = r = 0);
    if(X[u] <= x) { split_val(R[u], x, l, r); R[u] = l;
        l = u; }
    else { split_val(L[u], x, l, r); L[u] = r; r = u; }
    calc(u);
}
void split_sz(int u, int s, int &l, int &r) { // l gets
    first s, r gets remaining
    unlaze(u); if(!u) return (void) (l = r = 0);
    if(sz[L[u]] < s) { split_sz(R[u], s - sz[L[u]] - 1,
        l, r); R[u] = l; l = u; }
    else { split_sz(L[u], s, l, r); L[u] = r; r = u; }
    calc(u);
}
int merge(int l, int r) { // els on l <= els on r
    unlaze(l); unlaze(r); if(!l || !r) return l + r; int
        u;
    if(Y[l] > Y[r]) { R[l] = merge(R[l], r); u = l; }
    else { L[r] = merge(l, L[r]); u = r; }
    calc(u); return u;
}
void init(int n=N-1) { // XXX call before using other
    funcs
    for(int i = en = 1; i <= n; i++) { Y[i] = i; sz[i] =
        1; L[i] = R[i] = 0; }
    random_shuffle(Y + 1, Y + n + 1);
}

```

## ColorUpdate

// source: <https://github.com/tfg50/Competitive-Programming/tree/master/Biblioteca/Data%20Structures>

```

#include <set>
#include <vector>

template <class Info = int>
class ColorUpdate {
public:
    struct Range {
        Range(int l = 0) { this->l = l; }
        Range(int l, int r, Info v) {
            this->l = l;
            this->r = r;
            this->v = v;
        }
        int l, r;
        Info v;

        bool operator < (const Range &b) const { return l
            < b.l; }
    };

    std::vector<Range> upd(int l, int r, Info v) {
        std::vector<Range> ans;
        if(l >= r) return ans;
        auto it = ranges.lower_bound(l);
        if(it != ranges.begin()) {
            it--;
            if(it->r > l) {
                auto cur = *it;
                ranges.erase(it);
                ranges.insert(Range(cur.l, l, cur.v));
                ranges.insert(Range(l, cur.r, cur.v));
            }
        }
    }
}

```

```

it = ranges.lower_bound(r);
if(it != ranges.begin()) {
    it--;
    if(it->r > r) {
        auto cur = *it;
        ranges.erase(it);
        ranges.insert(Range(cur.l, r, cur.v));
        ranges.insert(Range(r, cur.r, cur.v));
    }
}
for(it = ranges.lower_bound(l); it != ranges.end
    () && it->l < r; it++) {
    ans.push_back(*it);
}
ranges.erase(ranges.lower_bound(l), ranges.
    lower_bound(r));
ranges.insert(Range(l, r, v));
return ans;
}
private:
    std::set<Range> ranges;
};

```

## Heavy Light Decomposition

```

void dfs_sz(int u){
    sz[u] = 1;

    for(auto &v : g[u]) if(v == p[u]){
        swap(v, g[u].back());
        g[u].pop_back();
        break;
    }

    for(auto &v : g[u]){
        p[v] = u;
        dfs_sz(v);
        sz[u] += sz[v];
        if(sz[v] > sz[g[u][0]])
            swap(v, g[u][0]);
    }
}

// nxt[u] = start of path with u
// set nxt[root] beforehand
void dfs_hld(int u){
    in[u] = t++;
    rin[in[u]] = u;
    for(auto v : g[u]){
        nxt[v] = (v == g[u][0] ? nxt[u] : v);
        dfs_hld(v);
    }
    out[u] = t;
}

// subtree of u => [ in[u], out[u] )
// path from nxt[u] to u => [ in[ nxt[u] ], in[u] ]

```

## Iterative Segtree

```

T query(int l, int r, int &pos){ // [l, r]
    T rl, rr;
    for(l += n, r += n+1; l < r; l >>= 1, r >>= 1){
        if(l & 1) rl = merge(rl, st[l++]);
        if(r & 1) rr = merge(st[--r], rr);
    }
    return merge(rl, rr);
}

```

```
// initially save v[i] in st[n+i] for all i
void build(){
    for(int p = n-1; p > 0; p--){
        st[p] = merge(st[2*p], st[2*p+1]);
    }

    void update(int p, T val){
        st[p += n] = val;
        while(p >= 1) st[p] = merge(st[2*p], st[2*p+1]);
    }
}
```

## LiChao's Segtree

```
void add_line(line nw, int v = 1, int l = 0, int r =
    maxn) { // [l, r)
    int m = (l + r) / 2;
    bool lef = nw.eval(l) < st[v].eval(l);
    bool mid = nw.eval(m) < st[v].eval(m);
    if(mid) swap(st[v], nw);
    if(r - l == 1) {
        return;
    } else if(lef != mid) {
        add_line(nw, 2 * v, l, m);
    } else {
        add_line(nw, 2 * v + 1, m, r);
    }
}

int get(int x, int v = 1, int l = 0, int r = maxn) {
    int m = (l + r) / 2;
    if(r - l == 1) {
        return st[v].eval(x);
    } else if(x < m) {
        return min(st[v].eval(x), get(x, 2*v, l, m));
    } else {
        return min(st[v].eval(x), get(x, 2*v+1, m, r));
    }
}
```

## Palindromic tree

```
#include <bits/stdc++.h>

using namespace std;

const int maxn = 3e5 + 1, sigma = 26;
int len[maxn], link[maxn], to[maxn][sigma];
int slink[maxn], diff[maxn], series_ans[maxn];
int sz, last, n;
char s[maxn];

void init()
{
    s[n++] = -1;
    link[0] = 1;
    len[1] = -1;
    sz = 2;
}

int get_link(int v)
{
    while(s[n - len[v] - 2] != s[n - 1]) v = link[v];
    return v;
}

void add_letter(char c)
{
    s[n++] = c - 'a';
    last = get_link(last);
```

```
if(!to[last][c])
{
    len[sz] = len[last] + 2;
    link[sz] = to[get_link(link[last])][c];
    diff[sz] = len[sz] - len[link[sz]];
    if(diff[sz] == diff[link[sz]])
        slink[sz] = slink[link[sz]];
    else
        slink[sz] = link[sz];
    to[last][c] = sz++;
}
last = to[last][c];
}

int main()
{
    ios::sync_with_stdio(0);
    cin.tie(0);
    init();
    string s;
    cin >> s;
    int n = s.size();
    int ans[n + 1];
    memset(ans, 63, sizeof(ans));
    ans[0] = 0;
    for(int i = 1; i <= n; i++)
    {
        add_letter(s[i - 1]);
        for(int v = last; len[v] > 0; v = slink[v])
        {
            series_ans[v] = ans[i - (len[slink[v]] + diff
                [v])];
            if(diff[v] == diff[link[v]])
                series_ans[v] = min(series_ans[v],
                    series_ans[link[v]]);
            ans[i] = min(ans[i], series_ans[v] + 1);
        }
        cout << ans[i] << "\n";
    }
    return 0;
}
```

## Math

### Euclides Extendido

```
// a*x + b*y = gcd(a, b), <gcd, x, y>
tuple<int, int, int> euclidesExt(int a, int b) {
    if(b == 0) return make_tuple(a, 1, 0);
    int q, w, e;
    tie(q, w, e) = euclidesExt(b, a % b);
    return make_tuple(q, e, w - e * (a / b));
}
```

### Prefix inverse

```
inv[1] = 1;
for(int i = 2; i < p; i++)
    inv[i] = (p - (p/i) * inv[p%i] % p) % p;
```

### Pollard Rho

```
ll rho(ll n){
    if(n % 2 == 0) return 2;

    ll d, c, x, y;
    do{
        c = llrand() % n, x = llrand() % n, y = x;
        do{
```

```

        x = add(mul(x, x, n), c, n);
        y = add(mul(y, y, n), c, n);
        y = add(mul(y, y, n), c, n);
        d = __gcd(abs(x - y), n);
    }while(d == 1);
}while(d == n);
return d;
}

ll pollard_rho(ll n){
    ll x, c, y, d, k;
    int i;
    do{
        i = 1;
        x = llrand() % n, c = llrand() % n;
        y = x, k = 4;
        do{
            if(++i == k) y = x, k *= 2;
            x = add(mul(x, x, n), c, n);
            d = __gcd(abs(x - y), n);
        }while(d == 1);
    }while(d == n);
    return d;
}

void factorize(ll val, map<ll, int> &fac){
    if(rabin(val)) fac[ val ]++;
    else{
        ll d = pollard_rho(val);
        factorize(d, fac);
        factorize(val / d, fac);
    }
}

map<ll, int> factor(ll val){
    map<ll, int> fac;
    if(val > 1) factorize(val, fac);
    return fac;
}

```

## Miller Rabin

```

bool rabin(ll n){
    if(n <= 1) return 0;
    if(n <= 3) return 1;
    ll s = 0, d = n - 1;
    while(d % 2 == 0) d /= 2, s++;
    for(int k = 0; k < 64; k++){
        ll a = (llrand() % (n - 3)) + 2;
        ll x = fexp(a, d, n);
        if(x != 1 && x != n-1){
            for(int r = 1; r < s; r++){
                x = mul(x, x, n);
                if(x == 1) return 0;
                if(x == n-1) break;
            }
            if(x != n-1) return 0;
        }
    }
    return 1;
}

```

## Totiente

```

ll totiente(ll n){
    ll ans = n;
    for(ll i = 2; i*i <= n; i++){
        if(n % i == 0){
            ans = ans / i * (i - 1);

```

```

        while(n % i == 0) n /= i;
    }
}

if(n > 1) ans = ans / n * (n - 1);
return ans;
}

```

## Mobius Function

```

memset(mu, 0, sizeof mu);
mu[1] = 1;
for(int i = 1; i < N; i++){
    for(int j = i + i; j < N; j += i)
        mu[j] -= mu[i];
}
// g(n) = sum{f(d)} => f(n) = sum{mu(d)*g(n/d)}

```

## Mulmod TOP

```

constexpr uint64_t mod = (1ull<<61) - 1;
uint64_t modmul(uint64_t a, uint64_t b){
    uint64_t l1 = (uint32_t)a, h1 = a>>32, l2 = (
        uint32_t)b, h2 = b>>32;
    uint64_t l = l1*l2, m = l1*h2 + l2*h1, h = h1*h2;
    uint64_t ret = (l&mod) + (l>>61) + (h << 3) + (m >>
        29) + (m << 35 >> 3) + 1;
    ret = (ret & mod) + (ret>>61);
    ret = (ret & mod) + (ret>>61);
    return ret-1;
}

```

## Determinant

```

int n;
long double a[n][n];

long double gauss(){
    long double det = 1;
    for(int i = 0; i < n; i++){
        int q = i;
        for(int j = i+1; j < n; j++){
            if(abs(a[j][i]) > abs(a[q][i]))
                q = j;
        }
        if(abs(a[q][i]) < EPS){
            det = 0;
            break;
        }
        if(i != q){
            for(int w = 0; w < n; w++){
                swap(a[i][w], a[q][w]);
                det = -det;
            }
        }
        det *= a[i][i];
        for(int j = i+1; j < n; j++) a[i][j] /= a[i][i];

        for(int j = 0; j < n; j++) if(j != i){
            if(abs(a[j][i]) > EPS)
                for(int k = i+1; k < n; k++)
                    a[j][k] -= a[i][k] * a[j][i];
        }
    }

    return det;
}

```

## FFT

```

struct base{
    double r, i;
    base(double _r = 0, double _i = 0) : r(_r), i(_i) {}
}

```

```

base operator*(base &o) const{
    return {r*o.r - i*o.i, r*o.i + o.r*i};
}
double real() const{ return r; }
void operator*=(const base &o){
    (*this) = {r*o.r-i*o.i, r*o.i+o.r*i};
}
void operator+=(const base &o){r += o.r, i += o.i; }
void operator/=(const double &o){ r /= o, i /= o; }
void operator-=(const base &o){r -= o.r, i -= o.i; }
base operator+(const base &o){return {r+o.r,i+o.i};}
base operator-(const base &o){return {r-o.r,i-o.i};}
};

double PI = acos(-1);

void fft(vector<base> &a, bool inv){
    int n = (int)a.size();

    for(int i = 1, j = 0; i < n; i++){
        int bit = n >> 1;
        for(; j >= bit; bit >>= 1) j -= bit;
        j += bit;
        if(i < j) swap(a[i], a[j]);
    }

    for(int sz = 2; sz <= n; sz <= 1) {
        double ang = 2*PI/sz * (inv ? -1 : 1);
        base wlen(cos(ang), sin(ang));
        for(int i = 0; i < n; i += sz){
            base w(1);
            for(int j = 0; j < sz/2; j++){
                base u = a[i+j], v = a[i+j+sz/2] * w;
                a[i+j] = u + v;
                a[i+j+sz/2] = u - v;
                w *= wlen;
            }
        }
    }
    if(inv) for(int i = 0; i < n; i++) a[i] /= 1.0 * n;
}

void multiply(const vector<int> &a, const vector<int> &b, vector<int> &res){
    vector<base> fa(a.begin(), a.end());
    vector<base> fb(b.begin(), b.end());
    size_t n = 1;
    while(n < a.size()) n <= 1;
    while(n < b.size()) n <= 1;
    n <= 1;
    fa.resize(n), fb.resize(n);

    fft(fa, false), fft(fb, false);
    for(size_t i = 0; i < n; i++)
        fa[i] *= fb[i];
    fft(fa, true);

    res.resize(n);
    for(size_t i = 0; i < n; ++i)
        res[i] = int(fa[i].real() + 0.5);
}

```

## NTT

```

const int mod = 7340033;
const int root = 5;
const int root_1 = 4404020;
const int root_pw = 1<<20;

```

```

void fft (vector<int> &a, bool invert) {
    int n = (int) a.size();

    for (int i=1, j=0; i<n; ++i) {
        int bit = n >> 1;
        for (; j>=bit; bit>>=1)
            j -= bit;
        j += bit;
        if (i < j)
            swap (a[i], a[j]);
    }

    for (int len=2; len<=n; len<=1) {
        int wlen = invert ? root_1 : root;
        for (int i=len; i<root_pw; i<=1)
            wlen = int (wlen * 1ll * wlen % mod);
        for (int i=0; i<n; i+=len) {
            int w = 1;
            for (int j=0; j<len/2; ++j) {
                int u = a[i+j], v = int (a[i+j+len/2] * 1
                    ll * w % mod);
                a[i+j] = u+v < mod ? u+v : u+v-mod;
                a[i+j+len/2] = u-v >= 0 ? u-v : u-v+mod;
                w = int (w * 1ll * wlen % mod);
            }
        }
    }
    if (invert) {
        int nrev = reverse (n, mod);
        for (int i=0; i<n; ++i)
            a[i] = int (a[i] * 1ll * nrev % mod);
    }
}

```

## Graphs

### Dinic

```

const int N = 1000005;
const int E = 20000006;
vector<int> g[N];

int ne;
struct Edge{
    int from, to;
    ll flow, cap;
} edge[E];

int lvl[N], vis[N], pass, start = N-2, target = N-1;
int qu[N], qt, px[N];

ll run(int s, int sink, ll minE){
    if(s == sink) return minE;

    ll ans = 0;

    for(; px[s] < (int)g[s].size(); px[s]++){
        int e = g[s][ px[s] ];
        auto &v = edge[e], &rev = edge[e^1];
        if(lvl[v.to] != lvl[s]+1 || v.flow >= v.cap)
            continue; // v.cap - v.flow < lim
        ll tmp = run(v.to, sink, min(minE, v.cap-v.flow));
        v.flow += tmp, rev.flow -= tmp;
        ans += tmp, minE -= tmp;
        if(minE == 0) break;
    }
    return ans;
}

```

```

}

bool bfs(int source, int sink){
    qt = 0;
    qu[qt++] = source;
    lvl[source] = 1;
    vis[source] = ++pass;

    for(int i = 0; i < qt; i++){
        int u = qu[i];
        px[u] = 0;
        if(u == sink) return true;

        for(int e : g[u]){
            auto v = edge[e];
            if(v.flow >= v.cap || vis[v.to] == pass)
                continue; // v.cap - v.flow < lim
            vis[v.to] = pass;
            lvl[v.to] = lvl[u]+1;
            qu[qt++] = v.to;
        }
    }
    return false;
}

ll flow(int source = start, int sink = target){
    ll ans = 0;
    //for(lim = (1LL << 62); lim >= 1; lim /= 2)
    while(bfs(source, sink))
        ans += run(source, sink, oo);
    return ans;
}

void addEdge(int u, int v, ll c = 1, ll rc = 0){
    edge[ne] = {u, v, 0, c};
    g[u].push_back(ne++);
    edge[ne] = {v, u, 0, rc};
    g[v].push_back(ne++);
}

void reset_flow(){
    for(int i = 0; i < ne; i++)
        edge[i].flow = 0;
}

Min Cost Max Flow

const ll oo = 1e18;
const int N = 505;
const int E = 30006;

vector<int> g[N];

int ne;

struct Edge{
    int from, to;
    ll cap, cost;
} edge[E];

int lvl[N], vis[N], pass, source, target, p[N], px[N];

ll d[N];

ll back(int s, ll minE){
    if(s == source) return minE;

    int e = p[s];

```

```

    ll f = back(edge[e].from, min(minE, edge[e].cap));
    edge[e].cap -= f;
    edge[e^1].cap += f;
    return f;
}

int dijkstra(){
    for(i, N) d[i] = oo;

    priority_queue<pair<ll, int> > q;

    d[source] = 0;

    q.emplace(0, source);

    while(!q.empty()){
        ll dis = -q.top().ff;
        int u = q.top().ss; q.pop();

        if(dis > d[u]) continue;

        for(int e : g[u]){
            auto v = edge[e];
            if(v.cap <= 0) continue;
            if(d[u] + v.cost < d[v.to]){
                d[v.to] = d[u] + v.cost;
                p[v.to] = e;
                q.emplace(-d[v.to], v.to);
            }
        }
    }
    return d[target] != oo;
}

pair<ll, ll> mincost(){
    ll ans = 0, mf = 0;
    while(dijkstra()){
        ll f = back(target, oo);
        mf += f;
        ans += f * d[target];
    }
    return {mf, ans};
}

void addEdge(int u, int v, ll c, ll cost){
    edge[ne] = {u, v, c, cost};
    g[u].pb(ne++);
}

Small to Large

void cnt_sz(int u, int p = -1){
    sz[u] = 1;

    for(int v : g[u]) if(v != p)
        cnt_sz(v, u), sz[u] += sz[v];
}

void add(int u, int p, int big = -1){
    // Update info about this vx in global answer

    for(int v : g[u]) if(v != p && v != big)
        add(v, u);
}

void dfs(int u, int p, int keep){

```



```

int big = -1, mmx = -1;

for(int v : g[u]) if(v != p && sz[v] > mmx)
    mmx = sz[v], big = v;

for(int v : g[u]) if(v != p && v != big)
    dfs(v, u, 0);

if(big != -1) dfs(big, u, 1);

add(u, p, big);

for(auto x : q[u]){
    // answer all queries for this vx
}

if(!keep){
    // Remove data from this subtree
}
}

```

## Junior e Falta de Ideias

```

#include <bits/stdc++.h>

#define ff first
#define ss second
#define mp make_pair

using namespace std;

typedef long long ll;

vector<pair<int,int>> G[500005];
int subtree[500005], treesize, k;
bool vis[500005];
ll dist[500005], ans;

int dfs(int v, int p){
    subtree[v] = 1;
    for(pair<int,int> x : G[v])
        if(x.ff != p && !vis[x.ff]) subtree[v] += dfs(x.ff, v);
    return subtree[v];
}

int centroid(int v, int p){
    for(pair<int,int> x : G[v]){
        if(x.ff == p || vis[x.ff]) continue;
        if(subtree[x.ff]*2 > treesize) return centroid(x.ff, v);
    }
    return v;
}

void procurar_ans(int v, int p, int d_atual, ll custo){
    ans = min(ans, dist[k-d_atual] + custo);
    if(d_atual == k) return;
    for(pair<int,int> x : G[v]){
        if(!vis[x.ff] && x.ff != p)
            procurar_ans(x.ff, v, d_atual+1, custo+x.ss);
    }
}

void atualiza_distancia(int v, int p, int d_atual, ll custo){
    dist[d_atual] = min(dist[d_atual], custo);
    if(d_atual == k) return;
}

```

```

for(pair<int,int> x : G[v]){
    if(!vis[x.ff] && x.ff != p)
        atualiza_distancia(x.ff, v, d_atual+1, custo+x.ss);
}

void decomp(int v, int p){
    treesize = dfs(v, v);
    // if(treesize < k) return;
    int cent = centroid(v, v);
    vis[cent] = 1;

    for(int i = 1; i <= treesize; i++)
        dist[i] = 1e18;

    for(pair<int,int> x : G[cent]){
        if(!vis[x.ff]){
            procurar_ans(x.ff, cent, 1, x.ss);
            atualiza_distancia(x.ff, cent, 1, x.ss);
        }
    }

    for(pair<int,int> x : G[cent]){
        if(!vis[x.ff])
            decomp(x.ff, cent);
    }
}

int main(){
    int n, i, a, b;

    scanf("%d%d", &n, &k);
    for(i = 2; i <= n; i++){
        scanf("%d%d", &a, &b);
        G[i].push_back(mp(a, b));
        G[a].push_back(mp(i, b));
    }
    ans = 1e18;
    decomp(1, -1);

    printf("%lld\n", ans == 1e18 ? -1 : ans);

    return 0;
}

```

## Kosaraju

```

vector<int> g[N], gt[N], S;

int vis[N], cor[N], tempo = 1;

void dfs(int u){
    vis[u] = 1;
    for(int v : g[u]) if(!vis[v]) dfs(v);
    S.push_back(u);
}

int e;

void dfst(int u){
    cor[u] = e;
    for(int v : gt[u]) if(!cor[v]) dfst(v);
}

int main(){

    for(int i = 1; i <= n; i++) if(!vis[i]) dfs(i);

    e = 0;
}

```

```

reverse(S.begin(), S.end());
for(int u : S) if(!cor[u])
    e++, dfst(u);

return 0;
}

```

## Tarjan

```

int cnt = 0, root;
void dfs(int u, int p = -1){
    low[u] = num[u] = ++t;
    for(int v : g[u]){
        if(!num[v]){
            dfs(v, u);
            if(v == root) cnt++;
            if(low[v] >= num[u]) u PONTO DE ARTICULACAO;
            if(low[v] > num[u]) ARESTA u->v PONTE;
            low[u] = min(low[u], low[v]);
        }
        else if(v != p) low[u] = min(low[u], num[v]);
    }
}

```

root PONTO DE ARTICULACAO <=> cnt > 1

```

void tarjanSCC(int u){
    low[u] = num[u] = cnt++;
    vis[u] = 1;
    S.push_back(u);
    for(int v : g[u]){
        if(!num[v]) tarjanSCC(v);
        if(vis[v]) low[u] = min(low[u], low[v]);
    }
    if(low[u] == num[u]){
        ssc[u] = ++ssc_cnt; int v;
        do{
            v = S.back(); S.pop_back(); vis[v] = 0;
            ssc[v] = ssc_cnt;
        }while(u != v);
    }
}

```

## Max Clique

long long adj[N], dp[N];

```

for(int i = 0; i < n; i++){
    for(int j = 0; j < n; j++){
        int x;
        scanf("%d",&x);
        if(x || i == j)
            adj[i] |= 1LL << j;
    }
}

int resto = n - n/2;
int C = n/2;
for(int i = 1; i < (1 << resto); i++){
    int x = i;
    for(int j = 0; j < resto; j++){
        if(i & (1 << j))
            x &= adj[j + C] >> C;
        if(x == i){
            dp[i] = __builtin_popcount(i);
        }
    }
}

for(int i = 1; i < (1 << resto); i++)

```

```

for(int j = 0; j < resto; j++){
    if(i & (1 << j))
        dp[i] = max(dp[i], dp[i ^ (1 << j)]);
}

```

```

int maxCliq = 0;
for(int i = 0; i < (1 << C); i++){
    int x = i, y = (1 << resto) - 1;
    for(int j = 0; j < C; j++){
        if(i & (1 << j))
            x &= adj[j] & ((1 << C) - 1), y &= adj[j] >> C;
        if(x != i) continue;
        maxCliq = max(maxCliq, __builtin_popcount(i) + dp[y]);
    }
}

```

## Dominator Tree

```

vector<int> g[N], gt[N], T[N];
vector<int> S;
int dsu[N], label[N];
int sdom[N], idom[N], dfs_time, id[N];

```

```

vector<int> bucket[N];
vector<int> down[N];

```

```

void prep(int u){
    S.push_back(u);
    id[u] = ++dfs_time;
    label[u] = sdom[u] = dsu[u] = u;

    for(int v : g[u]){
        if(!id[v])
            prep(v), down[u].push_back(v);
        gt[v].push_back(u);
    }
}

```

```

int fnd(int u, int flag = 0){
    if(u == dsu[u]) return u;
    int v = fnd(dsu[u], 1), b = label[ dsu[u] ];
    if(id[ sdom[b] ] < id[ sdom[ label[u] ] ])
        label[u] = b;
    dsu[u] = v;
    return flag ? v : label[u];
}

```

```

void build_dominator_tree(int root, int sz){
    // memset(id, 0, sizeof(int) * (sz + 1));
    // for(int i = 0; i <= sz; i++) T[i].clear();
    prep(root);
}

```

reverse(S.begin(), S.end());

```

int w;
for(int u : S){
    for(int v : gt[u]){
        w = fnd(v);
        if(id[ sdom[w] ] < id[ sdom[u] ])
            sdom[u] = sdom[w];
    }
    gt[u].clear();

    if(u != root) bucket[ sdom[u] ].push_back(u);

    for(int v : bucket[u]){
        w = fnd(v);
    }
}

```

```

        if(sdom[w] == sdom[v]) idom[v] = sdom[v];
        else idom[v] = w;
    }
    bucket[u].clear();

    for(int v : down[u]) dsu[v] = u;
    down[u].clear();
}

reverse(S.begin(), S.end());

for(int u : S) if(u != root){
    if(idom[u] != sdom[u]) idom[u] = idom[ idom[u] ];
    T[ idom[u] ].push_back(u);
}

S.clear();
}

```

## Min Cost Matching

```

// Min cost matching
// O(n^2 * m)
// n == nro de linhas
// m == nro de colunas
// n <= m | flow == n
// a[i][j] = custo pra conectar i a j
vector<int> u(n + 1), v(m + 1), p(m + 1), way(m + 1);
for(int i = 1; i <= n; ++i){
    p[0] = i;
    int j0 = 0;
    vector<int> minv(m + 1, oo);
    vector<char> used(m + 1, false);
    do{
        used[j0] = true;
        int i0 = p[j0], delta = oo, j1;
        for(int j = 1; j <= m; ++j)
            if(!used[j]){
                int cur = a[i0][j] - u[i0] - v[j];
                if(cur < minv[j])
                    minv[j] = cur, way[j] = j0;
                if(minv[j] < delta)
                    delta = minv[j], j1 = j;
            }
        for(int j = 0; j <= m; ++j)
            if(used[j])
                u[p[j]] += delta, v[j] -= delta;
            else
                minv[j] -= delta;
        j0 = j1;
    }while(p[j0] != 0);

    do{
        int j1 = way[j0];
        p[j0] = p[j1];
        j0 = j1;
    }while(j0);
}

// match[i] = coluna escolhida para linha i
vector<int> match(n + 1);
for(int j = 1; j <= m; ++j)
    match[p[j]] = j;

int cost = -v[0];

```

## Strings

### Aho Corasick

```

map<char, int> to[N];
int ne = 1, term[N], fail[N];

void add_string(char *str){
    int p = 0;

    for(int i = 0; str[i]; i++){
        if(!to[p][ str[i] ]) to[p][ str[i] ] = ne++;
        p = to[p][ str[i] ];
    }
    term[p] = 1;
}

int go(int s, char c){
    while(s && !to[s].count(c)) s = fail[s];
    if(to[s].count(c)) return to[s][c];
    return s;
}

void init(){
    queue<int> q;
    q.push(0);

    int u, v; char c;
    while(!q.empty()){
        u = q.front(); q.pop();

        for(auto w : to[u]){
            tie(c, v) = w;
            q.push(v);
            if(u){
                fail[v] = go(fail[u], c);
                term[v] |= term[ fail[v] ];
            }
        }
    }
}

// Suffix Array

char s[N];
int n, sa[N], tsa[N], lcp[N], r[N], nr[N], c[N];

void sort(int k, int mx){
    mx += 2;
    memset(c, 0, sizeof(int) * mx);
    for(int i = 0; i < n; i++) c[i + k < n ? r[i+k]+2 : 1]++;
    partial_sum(c, c+mx, c);
    int t;
    for(int i = 0; i < n; i++)
        t = sa[i]+k < n ? r[ sa[i]+k ]+1 : 0,
        tsa[ c[t]++ ] = sa[i];
    memcpy(sa, tsa, sizeof(int) * n);
}

void build_sa(){
    for(int i = 0; i < n; i++) sa[i] = i, r[i] = s[i];

    int t = 300, a, b;
    for(int sz = 1; sz < n; sz *= 2){

```

```

    sort(sz, t), sort(0, t);
    t = nr[ sa[0] ] = 0;
    for(int i = 1; i < n; i++){
        a = sa[i]+sz < n ? r[ sa[i]+sz ] : -1;
        b = sa[i-1]+sz < n ? r[ sa[i-1]+sz ] : -1;
        nr[ sa[i] ] = r[ sa[i] ] == r[ sa[i-1] ] && a
            == b ? t : ++t;
    }
    if(t == n-1) break;
    memcpy(r, nr, sizeof(int) * n);
}

void build_lcp(){ // lcp[i] = lcp(s[:i], s[:i+1])
    int k = 0;
    for(int i = 0; i < n; i++) r[ sa[i] ] = i;

    for(int i = 0; i < n; i++){
        if(r[i] == n-1) k = 0;
        else{
            int j = sa[r[i]+1];
            while(i+k < n && j+k < n && s[i+k] == s[j+k])
                k++;
        }
        lcp[r[i]] = k;
        if(k) k--;
    }
}

int comp_lcp(int i, int j){
    if(i == j) return n - i;
    if(r[i] > r[j]) swap(i, j);
    return min(lcp[k] for k in [r[i], r[j]-1]);
}

```

## Z Algorithm

```

vector<int> z_algo(const string &s) {
    int n = s.size(), L = 0, R = 0;
    vector<int> z(n, 0);
    for(int i = 1; i < n; i++){
        if(i <= R) z[i] = min(z[i-L], R - i + 1);
        while(z[i]+i < n && s[ z[i]+i ] == s[ z[i] ])
            z[i]++;
        if(i+z[i]-1 > R) L = i, R = i + z[i] - 1;
    }
    return z;
}

```

## Prefix function/KMP

```

vector<int> prefix_function(const string &s){
    int n = s.size();
    vector<int> b(n+1);
    b[0] = -1;
    int i = 0, j = -1;
    while(i < n){
        while(j >= 0 && s[i] != s[j]) j = b[j];
        b[++i] = ++j;
    }
    return b;
}

void kmp(const string &t, const string &p){
    vector<int> b = prefix_function(p);
    int n = t.size(), m = p.size();
    int j = 0;
    for(int i = 0; i < n; i++){
        while(j >= 0 && t[i] != p[j]) j = b[j];

```

```

        j++;
        if(j == m){
            //patern of p found on t
            j = b[j];
        }
    }
}

```

## Min rotation

```

int min_rotation(int *s, int N) {
    REP(i, N) s[N+i] = s[i];

    int a = 0;
    REP(b, N) REP(i, N) {
        if (a+i == b || s[a+i] < s[b+i]) { b += max(0, i-1);
            break; }
        if (s[a+i] > s[b+i]) { a = b; break; }
    }
    return a;
}

```

## All palindrome

```

void manacher(char *s, int N, int *rad) {
    static char t[2*MAX];
    int m = 2*N - 1;

    REP(i, m) t[i] = -1;
    REP(i, N) t[2*i] = s[i];

    int x = 0;
    FOR(i, 1, m) {
        int &r = rad[i] = 0;
        if (i <= x+rad[x]) r = min(rad[x+x-i], x+rad[x]-i);
        while (i-r-1 >= 0 && i+r+1 < m && t[i-r-1] == t[i+r+1]) ++r;
        if (i+r >= x+rad[x]) x = i;
    }

    REP(i, m) if (i-rad[i] == 0 || i+rad[i] == m-1) ++rad[i];
    REP(i, m) rad[i] /= 2;
}

```

## Palindromic Tree

```

const int MAXN = 105000;

struct node {
    int next[26];
    int len;
    int sufflink;
    int num;
};

int len;
char s[MAXN];
node tree[MAXN];
int num; // node 1 - root with len -1, node 2 - root
           with len 0
int suff; // max suffix palindrome
long long ans;

bool addLetter(int pos) {
    int cur = suff, curlen = 0;
    int let = s[pos] - 'a';

    while(true){
        curlen = tree[cur].len;

```

```

    if (pos-1 - curlen >= 0 && s[pos-1 - curlen] == s
        [pos])
        break;
    cur = tree[cur].sufflink;
}
if (tree[cur].next[let]) {
    suff = tree[cur].next[let];
    return false;
}

num++;
suff = num;
tree[num].len = tree[cur].len + 2;
tree[cur].next[let] = num;

if (tree[num].len == 1){
    tree[num].sufflink = 2;
    tree[num].num = 1;
    return true;
}

while (true){
    cur = tree[cur].sufflink;
    curlen = tree[cur].len;
    if(pos-1 - curlen >= 0 && s[pos-1 - curlen] == s[
        pos]){
        tree[num].sufflink = tree[cur].next[let];
        break;
    }
}

tree[num].num = 1 + tree[tree[num].sufflink].num;

return true;
}

void initTree() {
    num = 2; suff = 2;
    tree[1].len = -1; tree[1].sufflink = 1;
    tree[2].len = 0; tree[2].sufflink = 1;
}

int main() {

    initTree();

    for (int i = 0; i < len; i++) {
        addLetter(i);
    }

    return 0;
}

```

## Suffix Automaton

```

map<char, int> to[2*N];
int link[2*N], len[2*N], last = 0, sz = 1;

void add_letter(char c){
    int p = last;
    last = sz++;
    len[last] = len[p] + 1;
    for(; !to[p][c]; p = link[p]) to[p][c] = last;
    if(to[p][c] == last){
        link[last] = 0;
        return;
    }
    int u = to[p][c];

```

```

    if(len[u] == len[p]+1){
        link[last] = u;
        return;
    }
    int c1 = sz++;
    to[c1] = to[u];
    link[c1] = link[u];
    len[c1] = len[p]+1;
    link[last] = link[u] = c1;
    for(; to[p][c] == u; p = link[p]) to[p][c] = c1;
}

```

## Suffix Tree

```

namespace sf {
    // const int NS = ; const int N = * 2;
    int cn, cd, ns, en = 1, lst;
    string S[NS]; int si = -1;
    vector<int> sufn[N]; // sufn[si][i] no do sufixo S[si][i]
    ...
    struct node {
        int l, r, si, p, suf;
        map<char, int> adj;
        node() : l(0), r(-1), suf(0), p(0) {}
        node(int L, int R, int S, int P) : l(L), r(R), si(S)
            , p(P) {}
        inline int len() { return r - l + 1; }
        inline int operator[](int i) { return S[si][l + i]; }
    }
    inline int& operator()(char c) { return adj[c]; }
} t[N];
inline int new_node(int L, int R, int S, int P) { t[en]
    = node(L, R, S, P); return en++; }
void add_string(string s) {
    s += '$'; S[++si] = s; sufn[si].resize(s.size() + 1)
        ; cn = cd = 0;
    int i = 0; const int n = s.size();
    for(int j = 0; j < n; j++)
        for(; i <= j; i++) {
            if(cd == t[cn].len() && t[cn][s[j]]) { cn = t
                [cn][s[j]]; cd = 0; }
            if(cd < t[cn].len() && t[cn][cd] == s[j]) {
                cd++;
                if(j < s.size() - 1) break;
            }
            else {
                if(i) t[lst].suf = cn;
                for(; i <= j; i++) { sufn[si][i] = cn;
                    cn = t[cn].suf; }
            }
        }
    else if(cd == t[cn].len()) {
        sufn[si][i] = en;
        if(i) t[lst].suf = en; lst = en;
        t[cn][s[j]] = new_node(j, n - 1, si, cn);
        cn = t[cn].suf; cd = t[cn].len();
    }
    else {
        int mid = new_node(t[cn].l, t[cn].l + cd -
            1, t[cn].si, t[cn].p);
        t[t[cn].p][t[cn][0]] = mid;
        if(ns) t[ns].suf = mid;
        if(i) t[lst].suf = en; lst = en;
        sufn[si][i] = en;
        t[mid][s[j]] = new_node(j, n - 1, si, mid)
            ;
        t[mid][t[cn][cd]] = cn;
        t[cn].p = mid; t[cn].l += cd; cn = t[mid].
            p;
        int g = cn? j - cd : i + 1; cn = t[cn].suf
            ;
    }
}

```

```

        while(g < j && g + t[t[cn](S[si][g])].len()
              <= j) {
            cn = t[cn](S[si][g]); g += t[cn].len();
        }
        if(g == j) { ns = 0; t[mid].suf = cn; cd =
            t[cn].len(); }
        else { ns = mid; cn = t[cn](S[si][g]); cd
            = j - g; }
    }
}
};

```

## Geometry

### 2D basics

```

typedef double cod;
double eps = 1e-7;
bool eq(cod a, cod b){ return abs(a - b) <= eps; }

```

```

struct vec{
    cod x, y; int id;
    vec(cod a = 0, cod b = 0) : x(a), y(b) {}
    vec operator+(const vec &o) const{
        return {x + o.x, y + o.y};
    }
    vec operator-(const vec &o) const{
        return {x - o.x, y - o.y};
    }
    vec operator*(cod t) const{
        return {x * t, y * t};
    }
    vec operator/(cod t) const{
        return {x / t, y / t};
    }
    cod operator*(const vec &o) const{ // cos
        return x * o.x + y * o.y;
    }
    cod operator^(const vec &o) const{ // sin
        return x * o.y - y * o.x;
    }
    bool operator==(const vec &o) const{
        return eq(x, o.x) && eq(y, o.y);
    }
    bool operator<(const vec &o) const{
        if(!eq(x, o.x)) return x < o.x;
        return y < o.y;
    }
    cod cross(const vec &a, const vec &b) const{
        return (a-(*this)) ^ (b-(*this));
    }
    int ccw(const vec &a, const vec &b) const{
        cod tmp = cross(a, b);
        return (tmp > eps) - (tmp < -eps);
    }
    cod dot(const vec &a, const vec &b) const{
        return (a-(*this)) * (b-(*this));
    }
    cod len() const{
        return sqrt(x * x + y * y); // <
    }
    double angle(const vec &a, const vec &b) const{
        return atan2(cross(a, b), dot(a, b));
    }
    double tan(const vec &a, const vec &b) const{
        return cross(a, b) / dot(a, b);
    }
}

```

```

vec unit() const{
    return operator/(len());
}
int quad() const{
    if(x > 0 && y >=0) return 0;
    if(x <=0 && y > 0) return 1;
    if(x < 0 && y <=0) return 2;
    return 3;
}
bool comp(const vec &a, const vec &b) const{
    return (a - *this).comp(b - *this);
}
bool comp(vec b){
    if(quad() != b.quad()) return quad() < b.quad();
    if(!eq(operator^(b), 0)) return operator^(b) > 0;
    return (*this) * (*this) < b * b;
}
template<class T>
void sort_by_angle(T first, T last) const{
    std::sort(first, last, [=](const vec &a, const
        vec &b){
        return comp(a, b);
    });
}
vec rot90() const{ return {-y, x}; }
vec rot(double a) const{
    return {cos(a)*x - sin(a)*y, sin(a)*x + cos(a)*y};
}
};

struct line{
    cod a, b, c; vec n;
    line(vec q, vec w){ // q.cross(w, (x, y)) = 0
        a = -(w.y-q.y);
        b = w.x-q.x;
        c = -(a * q.x + b * q.y);
        n = {a, b};
    }
    cod dist(const vec &o) const{
        return abs(eval(o)) / n.len();
    }
    bool contains(const vec &o) const{
        return eq(a * o.x + b * o.y + c, 0);
    }
    cod dist(const line &o) const{
        if(!parallel(o)) return 0;
        if(!eq(o.a * b, o.b * a)) return 0;
        if(!eq(a, 0))
            return abs(c - o.c * a / o.a) / n.len();
        if(!eq(b, 0))
            return abs(c - o.c * b / o.b) / n.len();
        return abs(c - o.c);
    }
    bool parallel(const line &o) const{
        return eq(n ^ o.n, 0);
    }
    bool operator==(const line &o) const{
        if(!eq(a*o.b, b*o.a)) return false;
        if(!eq(a*o.c, c*o.a)) return false;
        if(!eq(c*o.b, b*o.c)) return false;
        return true;
    }
    bool intersect(const line &o) const{
        return !parallel(o) || *this == o;
    }
    vec inter(const line &o) const{
        if(parallel(o)){

```

```

        if(*this == o){ }
        else{ /* dont intersect */ }
    }

    auto tmp = n ^ o.n;
    return {(o.c*b -c*o.b)/tmp, (o.a*c -a*o.c)/tmp};
}
vec at_x(cod x) const{
    return {x, (-c-a*x)/b};
}
vec at_y(cod y) const{
    return {(-c-b*y)/a, y};
}
cod eval(const vec &o) const{
    return a * o.x + b * o.y + c;
}
};

struct segment{
    vec p, q;
    segment(vec a = vec(), vec b = vec()): p(a), q(b) {}
    bool onstrip(const vec &o) const{ // onstrip strip
        return p.dot(o, q) >= -eps && q.dot(o, p) >= -eps;
    }
    cod len() const{
        return (p-q).len();
    }
    cod dist(const vec &o) const{
        if(onstrip(o)) return line(p, q).dist(o);
        return min((o-q).len(), (o-p).len());
    }
    bool contains(const vec &o) const{
        return eq(p.cross(q, o), 0) && onstrip(o);
    }
    bool intersect(const segment &o) const{
        if(contains(o.p)) return true;
        if(contains(o.q)) return true;
        if(o.contains(p)) return true;
        if(o.contains(q)) return true;
        return p.ccw(q, o.p) * p.ccw(q, o.q) == -1
            && o.p.ccw(o.q, q) * o.p.ccw(o.q, p) == -1;
    }
    bool intersect(const line &o) const{
        return o.eval(p) * o.eval(q) <= 0;
    }
    cod dist(const segment &o) const{
        if(line(p, q).parallel(line(o.p, o.q))){
            if(onstrip(o.p) || onstrip(o.q)
                || o.onstrip(p) || o.onstrip(q))
                return line(p, q).dist(line(o.p, o.q));
            else if(intersect(o)) return 0;
            return min(min(dist(o.p), dist(o.q)),
                min(o.dist(p), o.dist(q)));
        }
    }
    cod dist(const line &o) const{
        if(line(p, q).parallel(o))
            return line(p, q).dist(o);
        else if(intersect(o)) return 0;
        return min(o.dist(p), o.dist(q));
    }
};

struct hray{
    vec p, q;
    hray(vec a = vec(), vec b = vec()): p(a), q(b){}

```

```

    bool onstrip(const vec &o) const{ // onstrip strip
        return p.dot(q, o) >= -eps;
    }
    cod dist(const vec &o) const{
        if(onstrip(o)) return line(p, q).dist(o);
        return (o-p).len();
    }
    bool intersect(const segment &o) const{
        if(!o.intersect(line(p,q))) return false;
        if(line(o.p, o.q).parallel(line(p,q)))
            return contains(o.p) || contains(o.q);
        return contains(line(p,q).inter(line(o.p,o.q)));
    }
    bool contains(const vec &o) const{
        return eq(line(p, q).eval(o), 0) && onstrip(o);
    }
    cod dist(const segment &o) const{
        if(line(p, q).parallel(line(o.p, o.q))){
            if(onstrip(o.p) || onstrip(o.q))
                return line(p, q).dist(line(o.p, o.q));
            return o.dist(p);
        }
        else if(intersect(o)) return 0;
        return min(min(dist(o.p), dist(o.q)),
            o.dist(p));
    }
    bool intersect(const hray &o) const{
        if(!line(p, q).parallel(line(o.p, o.q)))
            return false;
        auto pt = line(p, q).inter(line(o.p, o.q));
        return contains(pt) && o.contains(pt); // <<
    }
    bool intersect(const line &o) const{
        if(line(p, q).parallel(o)) return line(p, q) == o;
        if(o.contains(p) || o.contains(q)) return true;
        return (o.eval(p) >= -eps)^(o.eval(p)<o.eval(q));
        return contains(o.inter(line(p, q)));
    }
    cod dist(const line &o) const{
        if(line(p,q).parallel(o))
            return line(p,q).dist(o);
        else if(intersect(o)) return 0;
        return o.dist(p);
    }
    cod dist(const hray &o) const{
        if(line(p, q).parallel(line(o.p, o.q))){
            if(onstrip(o.p) || o.onstrip(p))
                return line(p,q).dist(line(o.p, o.q));
            return (p-o.p).len();
        }
        else if(intersect(o)) return 0;
        return min(dist(o.p), o.dist(p));
    }
};

double heron(cod a, cod b, cod c){
    cod s = (a + b + c) / 2;
    return sqrt(s * (s - a) * (s - b) * (s - c));
}

Nearest Points

struct pt {
    int x, y, id;
};

inline bool cmp_x (const pt &a, const pt &b) {
    return a.x < b.x || a.x == b.x && a.y < b.y;

```

```

}

inline bool cmp_y (const pt & a, const pt & b) {
    return a.y < b.y;
}

pt a[MAXN];

double mindist;
int ansa, ansb;

inline void upd_ans (const pt & a, const pt & b) {
    double dist = sqrt ((a.x-b.x)*(a.x-b.x) + (a.y-b.y)
        *(a.y-b.y) + .0);
    if (dist < mindist)
        mindist = dist, ansa = a.id, ansb = b.id;
}

void rec (int l, int r) {
    if (r - l <= 3) {
        for (int i=l; i<=r; ++i)
            for (int j=i+1; j<=r; ++j)
                upd_ans (a[i], a[j]);
        sort (a+l, a+r+1, &cmp_y);
        return;
    }

    int m = (l + r) >> 1;
    int midx = a[m].x;
    rec (l, m), rec (m+1, r);
    static pt t[MAXN];
    merge (a+l, a+m+1, a+m+1, a+r+1, t, &cmp_y);
    copy (t, t+r-l+1, a+l);

    int tsz = 0;
    for (int i=l; i<=r; ++i)
        if (abs (a[i].x - midx) < mindist) {
            for (int j=tsz-1; j>=0 && a[i].y - t[j].y <
                mindist; --j)
                upd_ans (a[i], t[j]);
            t[tsz++] = a[i];
        }
}

sort (a, a+n, &cmp_x);
mindist = 1E20;
rec (0, n-1);

```

## Convex Hull

```

vector<vec> monotone_chain_ch(vector<vec> P){
    sort(P.begin(), P.end());

    vector<vec> L, U;
    for(auto p : P){
        while(L.size() >= 2 && L[L.size() - 2].cross(L.
            back(), p) < 0)
            L.pop_back();

        L.push_back(p);
    }

    reverse(P.begin(), P.end());
    for(auto p : P){
        while(U.size() >= 2 && U[U.size() - 2].cross(U.
            back(), p) < 0)
            U.pop_back();
    }
}

```

```

        U.push_back(p);
    }

    L.pop_back(), U.pop_back();

    L.reserve(L.size() + U.size());
    L.insert(L.end(), U.begin(), U.end());

    return L;
}

```

## Check point inside polygon

```

bool below(const vector<vec> &vet, vec p){
    auto it = lower_bound(vet.begin(), vet.end(), p);
    if(it == vet.end()) return false;
    if(it == vet.begin()) return *it == p;
    return prev(it)->cross(*it, p) <= 0;
}

bool above(const vector<vec> &vet, vec p){
    auto it = lower_bound(vet.begin(), vet.end(), p);
    if(it == vet.end()) return false;
    if(it == vet.begin()) return *it == p;
    return prev(it)->cross(*it, p) >= 0;
}

// lowerhull, upperhull and point, borders included
bool inside_poly(const vector<vec> &lo, const vector<vec>
    &hi, vec p){
    return below(hi, p) && above(lo, p);
}

```

## Check point inside polygon without lower/upper hull

```

// borders included
// must not have 3 colinear consecutive points
bool inside_poly(const vector<vec> &v, vec p){
    if(v[0].ccw(v[1], p) < 0) return false;
    if(v[0].ccw(v.back(), p) > 0) return 0;
    if(v[0].ccw(v.back(), p) == 0)
        return v[0].dot(p, v.back()) >= 0
            && v.back().dot(p, v[0]) >= 0;

    int L = 1, R = (int)v.size() - 1, ans = 1;

    while(L <= R){
        int mid = (L+R)/2;
        if(v[0].ccw(v[mid], p) >= 0) ans = mid, L = mid
            +1;
        else R = mid-1;
    }

    return v[ans].ccw(v[(ans+1)%v.size()], p) >= 0;
}

```

## Minkowski sum

```

vector<vec> mk(const vector<vec> &a, const vector<vec> &b){
    int i = 0, j = 0;
    for(int k = 0; k < (int)a.size(); k++){if(a[k] < a[i])
        i = k;
    for(int k = 0; k < (int)b.size(); k++){if(b[k] < b[j])
        j = k;

    vector<vec> c;
    c.reserve(a.size() + b.size());
    for(int k = 0; k < int(a.size()+b.size()); k++){

```



```

    vec pt{a[i] + b[j]};
    if((int)c.size() >= 2
        && c[c.size()-2].ccw(c.back(), pt) == 0)
        c.pop_back();
    c.push_back(pt);
    int q = i+1, w = j+1;
    if(q == int(a.size())) q = 0;
    if(w == int(b.size())) w = 0;
    if(c.back().ccw(a[i]+b[w], a[q]+b[j]) < 0) i = q;
    else j = w;
}
c.shrink_to_fit();

return c;
}

```

## Miscellaneous

### LIS

```

multiset<int> S;
for(int i = 0; i < n; i++){
    auto it = S.upper_bound(a[i]); // low for inc
    if(it != S.end()) S.erase(it);
    S.insert(a[i]);
}
ans = S.size();

```

### DSU rollback

```

#include <bits/stdc++.h>

using namespace std;

struct DSU{
    vector<int> sz, p, change;
    vector<tuple<int, int, int>> modifications;
    vector<size_t> saves;
    bool bipartite;

    DSU(int n): sz(n+1, 1), p(n+1), change(n+1),
        bipartite(true){
        iota(p.begin(), p.end(), 0);
    }

    void add_edge(int u, int v){
        if(!bipartite) return;
        int must_change = get_colour(u) == get_colour(v);
        int a = rep(u), b = rep(v);
        if(sz[a] < sz[b]) swap(a, b);
        if(a != b){
            p[b] = a;
            modifications.emplace_back(b, change[b],
                bipartite);
            change[b] ^= must_change;
            sz[a] += sz[b];
        }
        else if(must_change){
            modifications.emplace_back(0, change[0],
                bipartite);
            bipartite = false;
        }
    }

    int rep(int u){
        return p[u] == u ? u : rep(p[u]);
    }

    int get_colour(int u){

```

```

        if(p[u] == u) return change[u];
        return change[u] ^ get_colour(p[u]);
    }

    void reset(){
        modifications.clear();
        saves.clear();
        iota(p.begin(), p.end(), 0);
        fill(sz.begin(), sz.end(), 1);
        fill(change.begin(), change.end(), 0);
        bipartite = true;
    }

    void rollback(){
        int u = get<0>(modifications.back());
        tie(ignore, change[u], bipartite) = modifications
            .back();
        sz[ p[u] ] -= sz[u];
        p[u] = u;
        modifications.pop_back();
    }

    void reload(){
        while(modifications.size() > saves.back())
            rollback();
        saves.pop_back();
    }

    void save(){
        saves.push_back(modifications.size());
    }
};

const int N = 100005;
const int B = 318;

int n, m, q;
int x[N], y[N], l[N], r[N], ans[N];

vector<int> qu[N];

int brute(int lef, int rig, DSU &s){
    s.save();
    for(int i = lef; i <= rig; i++)
        s.add_edge(x[i], y[i]);
    int ret = s.bipartite;
    s.reload();
    return ret;
}

int main(){
    scanf("%d %d %d", &n, &m, &q);

    for(int i = 1; i <= m; i++)
        scanf("%d %d", x+i, y+i);

    DSU s(n);
    for(int i = 0; i < q; i++){
        scanf("%d %d", l+i, r+i);
        if(r[i] - l[i] <= B + 10)
            ans[i] = brute(l[i], r[i], s);
        else qu[l[i] / B].push_back(i);
    }

    for(int i = 0; i <= m / B; i++){
        sort(qu[i].begin(), qu[i].end(), [](int a, int b){

```

```

        return r[a] < r[b];
    });
    s.reset();

    int R = (i+1)*B-1;

    for(int id : qu[i]){
        while(R < r[id]) ++R, s.add_edge(x[R], y[R]);
        s.save();
        for(int k = l[id]; k < (i+1)*B; k++)
            s.add_edge(x[k], y[k]);
        ans[id] = s.bipartite;
        s.reload();
    }
}

for(int i = 0; i < q; i++)
    printf("%s\n", ans[i] ? "Possible" : "Impossible");
}

```

## Buildings

```

// count the number of circular arrays
// of size m, with elements on range
// [1, c**(x*x)]

```

```

#include<bits/stdc++.h>
using namespace std;

```

```

#define debug(x) cerr << fixed << #x << " = " << x << endl;
#define ll long long

```

```

const int MOD = 1e9 + 7;
const int MAX = 1e5 + 5;
int dp[MAX];

```

```

inline int add(int a, int b) {
    a += b;
    if(a >= MOD) {
        a -= MOD;
    }
    return a;
}

```

```

inline int sub(int a, int b) {
    a -= b;
    if(0 > a) {
        a += MOD;
    }
    return a;
}

```

```

inline int mult(int a, int b) {
    return (1LL * a * b) % MOD;
}

```

```

int f_exp(int x, int exp) {
    if(exp == 0) {
        return 1;
    }
    else if(exp & 1) {
        return mult(x, f_exp(x, exp - 1));
    }
    return f_exp(mult(x, x), exp / 2);
}

```

```

inline int inv(int x) {

```

```

    return f_exp(x, MOD - 2);
}

```

```

int main()
{
    ios::sync_with_stdio(false);
    cin.tie(NULL); cout.tie(NULL);

    int n, m, c;
    cin >> n >> m >> c;

    int x = f_exp(c, n * n);
    int ans = f_exp(x, m);
    for(int i = 1; i <= m; i++) {
        if(m % i == 0) {
            int y = f_exp(x, i);
            for(int j = 1; j < i; j++) {
                if(i % j == 0) {
                    y = sub(y, mult(j, dp[j]));
                }
            }
            dp[i] = mult(y, inv(i));
            ans = sub(ans, mult(i - 1, dp[i]));
        }
    }
}

```

```

cout << ans << '\n';

```

```

    return 0;
}

```

## Rand

```

cout << RAND_MAX << endl;
mt19937 rng(chrono::steady_clock::now().time_since_epoch
().count());
vector<int> permutation(N);

```

```

iota(permutation.begin(), permutation.end(), 0);

```

```

shuffle(permutation.begin(), permutation.end(), rng);

```

```

iota(permutation.begin(), permutation.end(), 0);

```

```

for(int i = 1; i < N; i++){
    swap(permutation[i], permutation[
        uniform_int_distribution<int>(0, i)(rng)]);
}

```

## Klondike

```

// minimum number of moves to make
// all elements equal
// move: change a segment of equal value
// elements to any value

```

```

int v[305];
int dp[305][305];
int rec[305][305];

```

```

int f(int l, int r){
    if(r == l) return 1;
    if(r < l) return 0;
    if(dp[l][r] != -1) return dp[l][r];
    int ans = f(l+1, r) + 1;
    for(int i = l+1; i <= r; i++)
        if(v[i] == v[l])
            ans = min(ans, f(l, i - 1) + f(i+1, r));
}

```

```

    return dp[l][r] = ans;
}

int main() {
    int n, m;
    memset(dp, -1, sizeof dp);
    scanf("%d %d", &n, &m);
    for(int i = 0; i < n; i++){
        scanf("%d", &v[i]);
        if(i && v[i] == v[i-1]){
            i--;
            n--;
        }
    }
    printf("%d\n", f(0, n-1) - 1);
    // printf("%d\n", rec[0][n-1] );
    // printf("%d\n", rec[1][n-1] );
    // printf("%d\n", rec[2][n-3] );
}

```

## Hilbert Order

```

// maybe use B = n / sqrt(q)
inline int64_t hilbertOrder(int x, int y, int pow = 21,
    int rotate = 0) {
    if(pow == 0) return 0;
    int hpow = 1 << (pow-1);
    int seg = (x < hpow) ? (
        (y < hpow) ? 0 : 3
    ) : (
        (y < hpow) ? 1 : 2
    );
    seg = (seg + rotate) & 3;
    const int rotateDelta[4] = {3, 0, 0, 1};
    int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
    int nrot = (rotate + rotateDelta[seg]) & 3;
    int64_t subSquareSize = int64_t(1) << (2*pow - 2);
    int64_t ans = seg * subSquareSize;
    int64_t add = hilbertOrder(nx, ny, pow-1, nrot);
    ans += (seg == 1 || seg == 2) ? add : (subSquareSize
        - add - 1);
    return ans;
}

```

## Modular Factorial

```

// Compute (1*2*...*(p-1)*1*(p+1)*(p+2)*...*n) % p
// in O(p*lg(n))

int factmod(int n, int p){
    int ans = 1;
    while(n > 1){
        for(int i = 2; i <= n % p; i++){
            ans = (ans * i) % p;
            n /= p;
        }
        if(n % 2) ans = p - ans;
    }
    return ans % p;
}

int fac_pow(int n, int p){
    int ans = 0;
    while(n) n /= p, ans += n;
    return ans;
}

int C(int n, int k, int p){
    if(fac_pow(n, p) > fac_pow(n-k, p) + fac_pow(k, p))

```

```

        return 0;
    int tmp = factmod(k, p) * factmod(n-k, p);
    return (f_exp(tmp, p-2, p) * factmod(n, p)) % p;
}

```

## Enumeration all submasks of a bitmask

```

// loop through all submask of a given bitmask
// it does not include mask 0
for(int sub = mask; sub; sub = (sub-1)&mask){
}

```

## Slope Trick

///By woqjal25, contest: Codeforces Round #371 (Div. 1),  
problem: (C) Sonya and Problem Wihtout a Legend,  
Accepted, #

```

#include<stdio.h>
#include<queue>

int main()
{
    int n, t;
    long long ans = 0;
    std::priority_queue<int> Q;
    scanf("%d%d", &n, &t);
    Q.push(t);
    for(int i=1; i<n; i++){
        scanf("%d", &t); t-=i;
        Q.push(t);
        if(Q.top() > t)
        {
            ans += Q.top() - t;
            Q.pop();
            Q.push(t);
        }
    }
    printf("%lld", ans);
    return 0;
}

```

## Fast IO

```

#define pc(x) putchar_unlocked(x)
#define gc(x) getchar_unlocked(x)

inline void scan_int(int &x){
    register int c = gc();
    x = 0;
    int neg = 0;
    for(;; ((c < '0' || c > '9') && c != '-'); c = gc());
    if(c == '-'){
        neg = 1;
        c = gc();
    }
    for(;; c >= '0' && c <= '9'; c = gc()){
        x = (x << 1) + (x << 3) + c - '0';
    }
    if(neg) x = -x;
}

inline void print_int(int n){
    int rev = 0, count = 0, neg;
    if(n == 0){
        pc('0');
        return;
    }
    if(n < 0) n = -n, neg = 1;

```

```

while(n % 10 == 0) count++, n /= 10;
for(rev = 0; n != 0; n /= 10)
    rev = (rev << 3) + (rev << 1) + n % 10;
if(neg) pc('-');
while(rev != 0) pc(rev % 10 + '0'), rev /= 10;
while(count-->0) pc('0');
pc('\n');
}

```

```

inline void print_string(char *str){
    while(*str) pc(*str++);
    pc('\n');
}

```

## Big int

```

/*
#####
##### THE BIG INT
#####
*/
const int base = 1000000000;
const int base_digits = 9;
struct bigint {
    vector<int> a;
    int sign;
    /*<arpa>*/
    int size(){
        if(a.empty())return 0;
        int ans=(a.size()-1)*base_digits;
        int ca=a.back();
        while(ca)
            ans++,ca/=10;
        return ans;
    }
    bigint operator^(const bigint &v){
        bigint ans=1,a=*this,b=v;
        while(!b.isZero()){
            if(b%2)
                ans*=a;
            a*=a,b/=2;
        }
        return ans;
    }
    string to_string(){
        stringstream ss;
        ss << *this;
        string s;
        ss >> s;
        return s;
    }
    int sumof(){
        string s = to_string();
        int ans = 0;
        for(auto c : s) ans += c - '0';
        return ans;
    }
    /*</arpa>*/
    bigint() :
        sign(1) {
    }

    bigint(long long v) {
        *this = v;
    }

    bigint(const string &s) {
        read(s);
    }

```

```

}

void operator=(const bigint &v) {
    sign = v.sign;
    a = v.a;
}

void operator=(long long v) {
    sign = 1;
    a.clear();
    if (v < 0)
        sign = -1, v = -v;
    for (; v > 0; v = v / base)
        a.push_back(v % base);
}

bigint operator+(const bigint &v) const {
    if (sign == v.sign) {
        bigint res = v;

        for (int i = 0, carry = 0; i < (int) max(a.size(), v.a.size()) || carry; ++i) {
            if (i == (int) res.a.size())
                res.a.push_back(0);
            res.a[i] += carry + (i < (int) a.size() ? a[i] : 0);
            carry = res.a[i] >= base;
            if (carry)
                res.a[i] -= base;
        }
        return res;
    }
    return *this - (-v);
}

bigint operator-(const bigint &v) const {
    if (sign == v.sign) {
        if (abs() >= v.abs()) {
            bigint res = *this;
            for (int i = 0, carry = 0; i < (int) v.a.size() || carry; ++i) {
                res.a[i] -= carry + (i < (int) v.a.size() ? v.a[i] : 0);
                carry = res.a[i] < 0;
                if (carry)
                    res.a[i] += base;
            }
            res.trim();
            return res;
        }
        return -(v - *this);
    }
    return *this + (-v);
}

void operator*=(int v) {
    if (v < 0)
        sign = -sign, v = -v;
    for (int i = 0, carry = 0; i < (int) a.size() || carry; ++i) {
        if (i == (int) a.size())
            a.push_back(0);
        long long cur = a[i] * (long long) v + carry;
        carry = (int) (cur / base);
        a[i] = (int) (cur % base);
        //asm("divl %%ecx" : "=a"(carry), "=d"(a[i])
        : "A"(cur), "c"(base));
    }
}

```

```

    }
    trim();
}

bigint operator*(int v) const {
    bigint res = *this;
    res *= v;
    return res;
}

void operator*=(long long v) {
    if (v < 0)
        sign = -sign, v = -v;
    for (int i = 0, carry = 0; i < (int) a.size() ||
        carry; ++i) {
        if (i == (int) a.size())
            a.push_back(0);
        long long cur = a[i] * (long long) v + carry;
        carry = (int) (cur / base);
        a[i] = (int) (cur % base);
        //asm("divl %%ecx" : "=a"(carry), "=d"(a[i])
        : "A"(cur), "c"(base));
    }
    trim();
}

bigint operator*(long long v) const {
    bigint res = *this;
    res *= v;
    return res;
}

friend pair<bigint, bigint> divmod(const bigint &a1,
    const bigint &b1) {
    int norm = base / (b1.a.back() + 1);
    bigint a = a1.abs() * norm;
    bigint b = b1.abs() * norm;
    bigint q, r;
    q.a.resize(a.a.size());

    for (int i = a.a.size() - 1; i >= 0; i--) {
        r *= base;
        r += a.a[i];
        int s1 = r.a.size() <= b.a.size() ? 0 : r.a[b
            .a.size()];
        int s2 = r.a.size() <= b.a.size() - 1 ? 0 : r
            .a[b.a.size() - 1];
        int d = ((long long) base * s1 + s2) / b.a
            .back();
        r -= b * d;
        while (r < 0)
            r += b, --d;
        q.a[i] = d;
    }

    q.sign = a1.sign * b1.sign;
    r.sign = a1.sign;
    q.trim();
    r.trim();
    return make_pair(q, r / norm);
}

bigint operator/(const bigint &v) const {
    return divmod(*this, v).first;
}

bigint operator%(const bigint &v) const {

```

```

        return divmod(*this, v).second;
    }

void operator/=(int v) {
    if (v < 0)
        sign = -sign, v = -v;
    for (int i = (int) a.size() - 1, rem = 0; i >= 0;
        --i) {
        long long cur = a[i] + rem * (long long) base
            ;
        a[i] = (int) (cur / v);
        rem = (int) (cur % v);
    }
    trim();
}

bigint operator/(int v) const {
    bigint res = *this;
    res /= v;
    return res;
}

int operator%(int v) const {
    if (v < 0)
        v = -v;
    int m = 0;
    for (int i = a.size() - 1; i >= 0; --i)
        m = (a[i] + m * (long long) base) % v;
    return m * sign;
}

void operator+=(const bigint &v) {
    *this = *this + v;
}

void operator-=(const bigint &v) {
    *this = *this - v;
}

void operator*=(const bigint &v) {
    *this = *this * v;
}

void operator/=(const bigint &v) {
    *this = *this / v;
}

bool operator<(const bigint &v) const {
    if (sign != v.sign)
        return sign < v.sign;
    if (a.size() != v.a.size())
        return a.size() * sign < v.a.size() * v.sign;
    for (int i = a.size() - 1; i >= 0; i--)
        if (a[i] != v.a[i])
            return a[i] * sign < v.a[i] * sign;
    return false;
}

bool operator>(const bigint &v) const {
    return v < *this;
}

bool operator<=(const bigint &v) const {
    return !(v < *this);
}

bool operator>=(const bigint &v) const {
    return !(*this < v);
}

bool operator==(const bigint &v) const {
    return !(*this < v) && !(v < *this);
}

```

```

bool operator!=(const bigint &v) const {
    return *this < v || v < *this;
}

void trim() {
    while (!a.empty() && !a.back())
        a.pop_back();
    if (a.empty())
        sign = 1;
}

bool isZero() const {
    return a.empty() || (a.size() == 1 && !a[0]);
}

bigint operator-() const {
    bigint res = *this;
    res.sign = -sign;
    return res;
}

bigint abs() const {
    bigint res = *this;
    res.sign *= res.sign;
    return res;
}

long long longValue() const {
    long long res = 0;
    for (int i = a.size() - 1; i >= 0; i--)
        res = res * base + a[i];
    return res * sign;
}

friend bigint gcd(const bigint &a, const bigint &b)
{
    return b.isZero() ? a : gcd(b, a % b);
}

friend bigint lcm(const bigint &a, const bigint &b)
{
    return a / gcd(a, b) * b;
}

void read(const string &s) {
    sign = 1;
    a.clear();
    int pos = 0;
    while (pos < (int) s.size() && (s[pos] == '-' ||
        s[pos] == '+')) {
        if (s[pos] == '-')
            sign = -sign;
        ++pos;
    }
    for (int i = s.size() - 1; i >= pos; i -=
        base_digits) {
        int x = 0;
        for (int j = max(pos, i - base_digits + 1); j
            <= i; j++)
            x = x * 10 + s[j] - '0';
        a.push_back(x);
    }
    trim();
}

friend istream& operator>>(istream &stream, bigint &
v) {
    string s;

```

```

    stream >> s;
    v.read(s);
    return stream;
}

friend ostream& operator<<(ostream &stream, const
bigint &v) {
    if (v.sign == -1)
        stream << '-';
    stream << (v.a.empty() ? 0 : v.a.back());
    for (int i = (int) v.a.size() - 2; i >= 0; --i)
        stream << setw(base_digits) << setfill('0')
            << v.a[i];
    return stream;
}

static vector<int> convert_base(const vector<int> &a
, int old_digits, int new_digits) {
    vector<long long> p(max(old_digits, new_digits) +
        1);
    p[0] = 1;
    for (int i = 1; i < (int) p.size(); i++)
        p[i] = p[i - 1] * 10;
    vector<int> res;
    long long cur = 0;
    int cur_digits = 0;
    for (int i = 0; i < (int) a.size(); i++) {
        cur += a[i] * p[cur_digits];
        cur_digits += old_digits;
        while (cur_digits >= new_digits) {
            res.push_back((int)(cur % p[new_digits]));
            cur /= p[new_digits];
            cur_digits -= new_digits;
        }
    }
    res.push_back((int) cur);
    while (!res.empty() && !res.back())
        res.pop_back();
    return res;
}

typedef vector<long long> vll;

static vll karatsubaMultiply(const vll &a, const vll
&b) {
    int n = a.size();
    vll res(n + n);
    if (n <= 32) {
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
                res[i + j] += a[i] * b[j];
        return res;
    }

    int k = n >> 1;
    vll a1(a.begin(), a.begin() + k);
    vll a2(a.begin() + k, a.end());
    vll b1(b.begin(), b.begin() + k);
    vll b2(b.begin() + k, b.end());

    vll a1b1 = karatsubaMultiply(a1, b1);
    vll a2b2 = karatsubaMultiply(a2, b2);

    for (int i = 0; i < k; i++)
        a2[i] += a1[i];
    for (int i = 0; i < k; i++)
        b2[i] += b1[i];

```

```

vll r = karatsubaMultiply(a2, b2);
for (int i = 0; i < (int) a1b1.size(); i++)
    r[i] -= a1b1[i];
for (int i = 0; i < (int) a2b2.size(); i++)
    r[i] -= a2b2[i];

for (int i = 0; i < (int) r.size(); i++)
    res[i + k] += r[i];
for (int i = 0; i < (int) a1b1.size(); i++)
    res[i] += a1b1[i];
for (int i = 0; i < (int) a2b2.size(); i++)
    res[i + n] += a2b2[i];
return res;
}

bigint operator*(const bigint &v) const {
    vector<int> a6 = convert_base(this->a,
        base_digits, 6);
    vector<int> b6 = convert_base(v.a, base_digits,
        6);
    vll a(a6.begin(), a6.end());
    vll b(b6.begin(), b6.end());
    while (a.size() < b.size())
        a.push_back(0);
    while (b.size() < a.size())
        b.push_back(0);
    while (a.size() & (a.size() - 1))
        a.push_back(0), b.push_back(0);
    vll c = karatsubaMultiply(a, b);
    bigint res;
    res.sign = sign * v.sign;
    for (int i = 0, carry = 0; i < (int) c.size(); i++) {
        long long cur = c[i] + carry;
        res.a.push_back((int) (cur % 10000000));
        carry = (int) (cur / 10000000);
    }
    res.a = convert_base(res.a, 6, base_digits);
    res.trim();
    return res;
}
};
/*
##### THE BIG INT #####
#####
*/

```

## Knapsack Bounded with Cost

```

// menor custo para conseguir peso ate M usando N tipos
diferentes de elementos, sendo que o i-esimo elemento
pode ser usado b[i] vezes, tem peso w[i] e custo c[i]
// O(N * M)

```

```

int b[N], w[N], c[N];
MinQueue Q[M]
int d[M] //d[i] = custo minimo para conseguir peso i

for(int i = 0; i <= M; i++) d[i] = i ? oo : 0;
for(int i = 0; i < N; i++){
    for(int j = 0; j < w[i]; j++){
        Q[j].clear();
    }
    for(int j = 0; j <= M; j++){
        q = Q[j % w[i]];
        if(q.size() >= q) q.pop();
        q.add(c[i]);
        q.push(d[j]);
    }
}

```

```

        d[j] = q.getmin();
    }
}

LCA <O(nlgn), O(1)>

int start[N], dfs_time;
int tour[2*N], id[2*N];

void dfs(int u){
    start[u] = dfs_time;
    id[dfs_time] = u;
    tour[dfs_time++] = start[u];
    for(int v : g[u]){
        dfs(v);
        id[dfs_time] = u;
        tour[dfs_time++] = start[u];
    }
}

```

```

int LCA(int u, int v){
    if(start[u] > start[v]) swap(u, v);
    return id[min(tour[k] for k in [start[u], start[v]])];
}

```

## Burnside's Lemma

Let  $(G, \oplus)$  be a finite group that acts on a set  $X$ . It should hold that  $e_g * x = x$  and  $g_1 * (g_2 * x) = (g_1 \oplus g_2) * x, \forall x \in X, g_1, g_2 \in G$ . For each  $g \in G$  let  $X^g = \{x \in X \mid g * x = x\}$ . The number of orbits is given by:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

## Wilson's Theorem

$(n-1)! \equiv -1 \pmod n \iff n$  is prime

## Fibonacci

- $F_{n-1}F_{n+1} - F_n^2 = (-1)^n$
- $F_{n+k} = F_kF_{n+1} + F_{k-1}F_n$
- $GCD(F_n, F_m) = F_{GCD(n,m)}$
- $F_n = \frac{(\frac{1+\sqrt{5}}{2})^n - (\frac{1-\sqrt{5}}{2})^n}{\sqrt{5}}$

## Kirchhoff's Theorem

Laplacian matrix is  $L = D - A$ , where  $D$  is a diagonal matrix with vertex degrees on the diagonals and  $A$  is adjacency matrix.

The number of spanning trees is any cofactor of  $L$ .  $i$ -th cofactor is determinant of the matrix gotten by removing  $i$ -th row and column of  $L$ .

## Multigraphs

In  $D[i][i]$  all loops are excluded.  $A[i][j]$  = number of edges from  $i$  to  $j$ .

## Directed multigraphs

$D[i][i]$  = indegree of  $i$  minus the number of loops at  $i$ .  $A[i][j]$  = number of edges from  $i$  to  $j$ .

The number of oriented spanning trees rooted at a vertex  $i$  is the determinant of the matrix gotten by removing the  $i$ th row and column of  $L$ .

## Edge coloring

**Data:** A graph  $G$

**Result:** A proper coloring  $c$  of the edges of  $G$

Let  $U := E(G)$ ;

**while**  $U \neq \emptyset$  **do**

    Let  $(u,v)$  be any edge in  $U$ ;

    Let  $F[1:k]$  be a maximal fan of  $u$  starting at  $F[1]=v$ ;

    Let  $c$  be a color that is free on  $u$  and  $d$  be a color that is free on  $F[k]$ ;

    Invert the  $cd_u$  path;

    Let  $w \in V(G)$  be such that  $w \in F, F' = [F[1]...w]$  is a fan and  $d$  is free on  $w$ ;

    Rotate  $F'$  and set  $c(u,w)=d$ ;

$U := U - (u,v)$ ;

**end**

## Notes

When we repeat something and each time we have probability  $p$  to succeed then the expected number of tries is  $\frac{1}{p}$ , till we succeed.