# 100% É Pouco, Pagode Importa D+

# University of Brasilia

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University of Brasilia **Data Structures** 

```
set ai ts=4 sw=4 sta nu rnu sc stl+=%F autoindent
syntax on
alias cmp='g++ -Wall -Wshadow -Wconversion -fsanitize=
  address -std=c++11'
```

#### **Data Structures**

### **Merge Sort Tree**

```
struct MergeTree{
   int n;
   vector<vector<int>>> st;
   void build(int p, int L, int R, const int v[]){
      if(L == R){
          st[p].push_back(v[L]);
          return:
       int mid = (L+R)/2;
      build(2*p, L, mid, v);
      build(2*p+1, mid+1, R, v);
       st[p].resize(R-L+1);
       merge(st[2*p].begin(), st[2*p].end(),
              st[2*p+1].begin(), st[2*p+1].end(),
              st[p].begin());
   }
   int query(int p, int L, int R, int i, int j, int x)
     const.{
      if(L > j \mid \mid R < i) return 0;
      if(L >= i && R <= j){
          int id = lower_bound(st[p].begin(), st[p].end
            (), x) - st[p].begin();
          return int(st[p].size()) - id;
      int mid = (L+R)/2;
      return query(2*p, L, mid, i, j, x) +
          query(2*p+1, mid+1, R, i, j, x);
   }
public:
   MergeTree(int sz, const int v[]): n(sz), st(4*sz){
       build(1, 1, n, v);
   //number of elements >= x on segment [i, j]
   int query(int i, int j, int x) const{
       if(i > j) swap(i, j);
      return query(1, 1, n, i, j, x);
   }
ζ.
Ordered Set
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
using namespace __gnu_pbds; // or pb_ds;
template<typename T, typename B = null_type>
using oset = tree<T, B, less<T>, rb_tree_tag,
  tree_order_statistics_node_update>;
```

#### Convex Hull Trick

// find\_by\_order / order\_of\_key

```
const ll is_query = -(1LL<<62);</pre>
struct Line{
   11 m. b:
   mutable function<const Line*()> succ;
   bool operator<(const Line& rhs) const{</pre>
       if(rhs.b != is_query) return m < rhs.m;</pre>
       const Line* s = succ();
       if(!s) return 0;
       11 x = rhs.m;
       return b - s->b < (s->m - m) * x;
};
struct Cht : public multiset<Line>{ // maintain max
   bool bad(iterator y){
       auto z = next(y);
       if(y == begin()){}
          if(z == end()) return 0;
          return y->m == z->m \&\& y->b <= z->b;
       auto x = prev(y);
       if(z == end()) return y->m == x->m && y->b <= x->
       return (x->b - y->b)*(z->m - y->m) >= (y->b - z->
         b)*(y->m - x->m);
   void insert_line(ll m, ll b){
       auto y = insert({ m, b });
       y->succ = [=]{ return next(y) == end() ? 0 : &* }
         next(y); };
       if(bad(y)){ erase(y); return; }
       while(next(y) != end() && bad(next(y))) erase(
         next(v)):
       while(y != begin() && bad(prev(y))) erase(prev(y)
         );
   ll eval(ll x){
       auto 1 = *lower_bound((Line) { x, is_query });
      return 1.m * x + 1.b;
   }
};
Min queue
template<typename T>
class minQ{
   deque<tuple<T, int, int> > p;
   T delta;
   int sz;
public:
   minQ() : delta(0), sz(0) {}
   inline int size() const{ return sz; }
   inline void add(T x){ delta += x; }
   inline void push(T x, int id){
      x -= delta, sz++;
       int t = 1;
       while(p.size() > 0 && get<0>(p.back()) >= x)
          t += get<1>(p.back()), p.pop_back();
      p.emplace_back(x, t, id);
   inline void pop(){
       get<1>(p.front())--, sz--;
       if(!get<1>(p.front())) p.pop_front();
   T getmin() const{ return get<0>(p.front())+delta; }
   int getid() const{ return get<2>(p.front()); }
};
```

Sparse Table

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```
const int N = 100005;
                                                               vector<base> fa(a.begin(), a.end());
                                                               vector<base> fb(b.begin(), b.end());
int v[N], n;
                                                               size t n = 1:
int dn[N][20];
                                                               while(n < a.size()) n <<= 1;
int fn(int i, int j){
                                                               while(n < b.size()) n <<= 1;
   if(j == 0) return v[i];
                                                               n <<= 1:
   \textbf{if}(\texttt{``dn[i][j]') return dn[i][j];}\\
                                                               fa.resize(n), fb.resize(n);
   return dn[i][j] = min(fn(i, j-1), fn(i + (1 << (j-1)))
     ), j-1));
                                                               fft(fa, false), fft(fb, false);
                                                               for(size_t i = 0; i < n; i++)</pre>
                                                                   fa[i] *= fb[i];
int lg(int x){ return 31 - __builtin_clz(x); }
                                                               fft(fa, true);
int getmn(int 1, int r){ // [1, r]
                                                               res.resize (n);
   int 1z = 1g(r - 1 + 1);
                                                               for(size_t i = 0; i < n; ++i)</pre>
   return min(fn(1, lz), fn(r - (1 << lz) + 1, lz));
                                                                  res[i] = int(fa[i].real() + 0.5);
                                                           NTT
Paradigms
                                                           const int mod = 7340033;
FFT
                                                           const int root = 5;
                                                           const int root_1 = 4404020;
// typedef complex<double> base;
                                                           const int root_pw = 1<<20;</pre>
struct base{
   double r, i;
                                                           void fft (vector<int> & a, bool invert) {
   base(double _r = 0, double _i = 0) : r(_r), i(_i) {}
                                                               int n = (int) a.size();
   base operator*(base &o) const{
      return {r*o.r - i*o.i, r*o.i + o.r*i};
                                                               for (int i=1, j=0; i<n; ++i) {
                                                                   int bit = n \gg 1;
   double real() const{ return r; }
                                                                   for (; j>=bit; bit>>=1)
   void operator*=(base &o){ r*o.r-i*o.i,r*o.i+o.r*i; }
                                                                      j -= bit;
   void operator+=(base &o){ r += o.r, i += o.i; }
                                                                   j += bit;
   void operator/=(double &o){ r /= o, i /= o; }
                                                                   if (i < j)
   void operator==(base &o){ r == o.r, i == o.i; }
                                                                      swap (a[i], a[j]);
   base operator+(base &o){ return {r+o.r, i+o.i}; }
                                                               }
   base operator-(base &o){ return {r-o.r, i-o.i}; }
                                                               int wlen = invert ? root_1 : root;
double PI = acos(-1);
                                                                   for (int i=len; i<root_pw; i<<=1)</pre>
                                                                      wlen = int (wlen * 111 * wlen % mod);
void fft(vector<base> &a, bool inv){
                                                                   for (int i=0; i<n; i+=len) {
   int n = (int)a.size();
                                                                      int w = 1;
                                                                      for (int j=0; j<len/2; ++j) {</pre>
   for(int i = 1, j = 0; i < n; i++){
                                                                          int u = a[i+j], v = int (a[i+j+len/2] * 1
       int bit = n \gg 1;
                                                                            ll * w % mod);
       for(; j >= bit; bit >>= 1) j -= bit;
                                                                          a[i+j] = u+v < mod ? u+v : u+v-mod;
       j += bit;
                                                                          a[i+j+len/2] = u-v >= 0 ? u-v : u-v+mod;
       if(i < j) swap(a[i], a[j]);
                                                                          w = int (w * 111 * wlen % mod);
                                                                      }
                                                                  }
   for(int sz = 2; sz <= n; sz <<= 1) {
                                                               }
       double ang = 2*PI/sz * (inv ? -1 : 1);
                                                               if (invert) {
       base wlen(cos(ang), sin(ang));
                                                                   int nrev = reverse (n, mod);
       for(int i = 0; i < n; i += sz){</pre>
                                                                   for (int i=0; i<n; ++i)
          base w(1);
                                                                      a[i] = int (a[i] * 111 * nrev % mod);
          for(int j = 0; j < sz/2; j++){
              base u = a[i+j], v = a[i+j+sz/2] * w;
                                                           }
              a[i+j] = u + v;
                                                           Math
             a[i+j+sz/2] = u - v;
             w *= wlen;
                                                           Euclides Extendido
          }
      }
                                                            // a*x + b*y = gcd(a, b), < gcd, x, y>
   if(inv) for(int i = 0; i < n; i++) a[i] /= n;
}
                                                               int q, w, e;
void multiply(const vector<int> &a, const vector<int> &b
  , vector<int> &res){
```

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```
Preffix inverse
inv[1] = 1;
for(int i = 2; i < p; i++)
   inv[i] = (p - (p/i) * inv[p%i] % p) % p;
Pollard Rho
ll rho(ll n){
   if(n \% 2 == 0) return 2;
   11 d, c, x, y;
       c = 11rand() % n, x = 11rand() % n, y = x;
       do{
          x = add(mul(x, x, n), c, n);
          y = add(mul(y, y, n), c, n);
          y = add(mul(y, y, n), c, n);
          d = \_gcd(abs(x - y), n);
       }while(d == 1);
   }while(d == n);
   return d;
11 pollard_rho(ll n){
   ll x, c, y, d, k;
   int i;
   do{
       i = 1;
       x = 11rand() % n, c = 11rand() % n;
       y = x, k = 4;
       do{
          if(++i == k) y = x, k *= 2;
          x = add(mul(x, x, n), c, n);
          d = \_gcd(abs(x - y), n);
       }while(d == 1);
   \}while(d == n);
   return d;
void factorize(ll val, map<ll, int> &fac){
   if(rabin(val)) fac[ val ]++;
   else{
       11 d = pollard_rho(val);
       factorize(d, fac);
       factorize(val / d, fac);
   }
}
map<ll, int> factor(ll val){
   map<ll, int> fac;
   if(val > 1) factorize(val, fac);
   return fac;
}
Miller Rabin
bool rabin(ll n){
   if(n \ll 1) return 0;
   if(n <= 3) return 1;
   11 s = 0, d = n - 1;
   while(d % 2 == 0) d /= 2, s++;
   for(int k = 0; k < 64; k++){
       11 a = (11rand() \% (n - 3)) + 2;
       11 x = fexp(a, d, n);
       if(x != 1 \&\& x != n-1){
          for(int r = 1; r < s; r++){
              x = mul(x, x, n);
```

```
if(x == 1) return 0;
              if(x == n-1) break;
          }
          if(x != n-1) return 0;
       }
   return 1;
Totiente
ll totiente(ll n){
   11 \text{ ans} = n;
   for(ll i = 2; i*i <= n; i++){
       if(n \% i == 0){
           ans = ans / i * (i - 1);
           while(n \% i == 0) n /= i;
       }
   if(n > 1) ans = ans / n * (n - 1);
   return ans:
Mulmod TOP
constexpr uint64_t mod = (1ull<<61) - 1;</pre>
uint64_t modmul(uint64_t a, uint64_t b){
   uint64_t 11 = (uint32_t)a, h1 = a>>32, 12 = (
     uint32_t)b, h2 = b >> 32;
   uint64_t l = 11*12, m = 11*h2 + 12*h1, h = h1*h2;
   uint64_t ret = (1&mod) + (1>>61) + (h << 3) + (m >>
     29) + (m << 35 >> 3) + 1;
   ret = (ret & mod) + (ret>>61);
   ret = (ret & mod) + (ret>>61);
   return ret-1;
Determinant
const double EPS = 1E-9;
int n:
vector < vector<double> > a (n, vector<double> (n));
double det = 1;
for (int i=0; i<n; ++i) {</pre>
   int k = i;
   for (int j=i+1; j<n; ++j)
       if (abs (a[j][i]) > abs (a[k][i]))
          k = j;
   if (abs (a[k][i]) < EPS) {</pre>
       det = 0;
       break;
   }
   swap (a[i], a[k]);
   if (i != k)
       det = -det;
   det *= a[i][i];
   for (int j=i+1; j< n; ++j)
       a[i][j] /= a[i][i];
   for (int j=0; j<n; ++j)
       if (j != i && abs (a[j][i]) > EPS)
           for (int k=i+1; k<n; ++k)
              a[j][k] = a[i][k] * a[j][i];
}
cout << det;</pre>
```

University of Brasilia Graphs

### Graphs

#### Dinic

```
const int N = 100005;
const int E = 2000006;
vector<int> g[N];
int ne;
struct Edge{
   int from, to:
   11 flow, cap;
} edge[E];
int lvl[N], vis[N], pass, start = N-2, target = N-1;
int qu[N], qt, px[N];
11 run(int s, int sink, ll minE){
   if(s == sink) return minE;
   11 ans = 0:
   for(; px[s] < (int)g[s].size(); px[s]++){</pre>
       int e = g[s][ px[s] ];
       auto &v = edge[e], &rev = edge[e^1];
       if(lvl[v.to] != lvl[s]+1 || v.flow >= v.cap)
         continue;
       11 tmp = run(v.to, sink,min(minE, v.cap-v.flow));
       v.flow += tmp, rev.flow -= tmp;
       ans += tmp, minE -= tmp;
       if(minE == 0) break;
   }
   return ans;
}
bool bfs(int source, int sink){
   qt = 0;
   qu[qt++] = source;
   lvl[source] = 1;
   vis[source] = ++pass;
   for(int i = 0; i < qt; i++){
       int u = qu[i];
       px[u] = 0;
       if(u == sink) return true;
       for(int e : g[u]){
          auto v = edge[e];
          if(v.flow >= v.cap || vis[v.to] == pass)
            continue:
          vis[v.to] = pass;
          lvl[v.to] = lvl[u]+1;
          qu[qt++] = v.to;
   return false;
11 flow(int source = start, int sink = target){
   11 \text{ ans} = 0;
   while(bfs(source, sink))
       ans += run(source, sink, oo);
   return ans;
}
void addEdge(int u, int v, ll c = 1, ll rc = 0){
   edge[ne] = \{u, v, 0, c\};
```

```
g[u].push_back(ne++);
   edge[ne] = \{v, u, 0, rc\};
   g[v].push_back(ne++);
void reset_flow(){
   for(int i = 0; i < ne; i++)
       edge[i].flow = 0;
Min Cost Max Flow
const 11 oo = 1e18;
const int N = 505;
const int E = 30006;
vector<int> g[N];
int ne;
struct Edge{
   int from, to;
   11 cap, cost;
} edge[E];
int lvl[N], vis[N], pass, source, target, p[N], px[N];
11 d[N];
11 back(int s, ll minE){
   if(s == source) return minE;
   int e = p[s];
   11 f = back(edge[e].from, min(minE, edge[e].cap));
   edge[e].cap -= f;
   edge[e^1].cap += f;
   return f;
}
int dijkstra(){
   forn(i, N) d[i] = oo;
   priority_queue<pair<ll, int> > q;
   d[source] = 0;
   q.emplace(0, source);
   while(!q.empty()){
       11 dis = -q.top().ff;
       int u = q.top().ss; q.pop();
       if(dis > d[u]) continue;
       for(int e : g[u]){
          auto v = edge[e];
          if(v.cap <= 0) continue;</pre>
          if(d[u] + v.cost < d[v.to]){
              d[v.to] = d[u] + v.cost;
              p[v.to] = e;
              q.emplace(-d[v.to], v.to);
          }
      }
   }
   return d[target] != oo;
}
```

University of Brasilia Graphs

```
pair<11, 11> mincost(){
   11 ans = 0, mf = 0;
   while(dijkstra()){
       11 f = back(target, oo);
       mf += f;
       ans += f * d[target];
   return {mf, ans};
}
void addEdge(int u, int v, 11 c, 11 cost){
   edge[ne] = \{u, v, c, cost\};
   g[u].pb(ne++);
Small to Large
void cnt_sz(int u, int p = -1){
                                                             }
   sz[u] = 1;
   for(int v : g[u]) if(v != p)
       cnt_sz(v, u), sz[u] += sz[v];
}
void add(int u, int p, int big = -1){
   // Update info about this vx in global answer
                                                             }
   for(int v : g[u]) if(v != p && v != big)
       add(v, u);
void dfs(int u, int p, int keep){
   int big = -1, mmx = -1;
   for(int v : g[u]) if(v != p \&\& sz[v] > mmx)
       mmx = sz[v], big = v;
                                                             }
   for(int v : g[u]) if(v != p && v != big)
       dfs(v, u, 0);
   if(big != -1) dfs(big, u, 1);
   add(u, p, big);
   \quad \text{for}(\text{auto } x \,:\, q[u])\{
       // answer all queries for this vx
   if(!keep){
       // Remove data from this subtree
Iunior e Falta de Ideias
#include <bits/stdc++.h>
#define ff first
#define ss second
#define mp make_pair
                                                             }
using namespace std;
typedef long long 11;
vector<pair<int,int>> G[500005];
int subtree[500005], treesize, k;
bool vis[500005];
```

```
ll dist[500005], ans;
int dfs(int v, int p){
   subtree[v] = 1;
   for(pair<int,int> x : G[v])
       if(x.ff != p \&\& !vis[x.ff]) subtree[v] += dfs(x.
         ff.v):
   return subtree[v];
int centroid(int v, int p){
   for(pair<int,int> x : G[v]){
       if(x.ff == p || vis[x.ff]) continue;
       if(subtree[x.ff]*2 > treesize) return centroid(x.
   }
   return v;
void procurar_ans(int v, int p, int d_atual, ll custo){
   ans = min(ans, dist[k-d_atual] + custo);
   if(d_atual == k) return;
   for(pair<int,int> x : G[v]){
       if(!vis[x.ff] && x.ff != p)
          procurar_ans(x.ff,v,d_atual+1,custo+x.ss);
   }
void atualiza_distancia(int v, int p, int d_atual, 11
  custo){
   dist[d_atual] = min(dist[d_atual], custo);
   if(d_atual == k) return;
   for(pair<int,int> x : G[v]){
       if(!vis[x.ff] && x.ff != p)
          atualiza_distancia(x.ff,v,d_atual+1,custo+x.
            ss);
   }
void decomp(int v, int p){
   treesize = dfs(v,v);
   // if(treesize < k) return;</pre>
   int cent = centroid(v,v);
   vis[cent] = 1;
   for(int i = 1; i <= treesize; i++)
       dist[i] = 1e18;
   for(pair<int,int> x : G[cent]){
       if(!vis[x.ff]){
          procurar_ans(x.ff,cent,1,x.ss);
          atualiza_distancia(x.ff,cent,1,x.ss);
   for(pair<int,int> x : G[cent]){
       if(!vis[x.ff])
          decomp(x.ff, cent);
int main(){
   int n,i,a,b;
   scanf("%d%d", &n,&k);
   for(i = 2; i \le n; i++){
       scanf("%d%d", &a,&b);
```

University of Brasilia Strings

```
G[i].push_back(mp(a,b));
                                                            }
       G[a].push_back(mp(i,b));
   }
                                                            Max Clique
   ans = 1e18:
   decomp(1,-1);
                                                            long long adj[N], dp[N];
   printf("%lld\n", ans == 1e18 ? -1 : ans);
                                                            for(int i = 0; i < n; i++){
                                                                for(int j = 0; j < n; j++){
   return 0;
                                                                   int x;
                                                                   scanf("%d",&x);
Kosaraju
                                                                   if(x \mid | i == j)
                                                                       adj[i] |= 1LL << j;
vector<int> g[N], gt[N], S;
                                                                }
                                                            }
int vis[N], cor[N], tempo = 1;
                                                            int resto = n - n/2;
                                                            int C = n/2;
void dfs(int u){
                                                            for(int i = 1; i < (1 << resto); i++){</pre>
   vis[u] = 1;
                                                                int x = i;
   for(int v : g[u]) if(!vis[v]) dfs(v);
                                                                for(int j = 0; j < resto; j++)
   S.push_back(u);
                                                                   if(i & (1 << j))
                                                                       x \&= adj[j + C] >> C;
int e;
                                                                if(x == i){
void dfst(int u){
                                                                   dp[i] = __builtin_popcount(i);
   cor[u] = e;
   for(int v : gt[u]) if(!cor[v]) dfst(v);
                                                            }
                                                            for(int i = 1; i < (1 << resto); i++)</pre>
int main(){
                                                                for(int j = 0; j < resto; j++)
                                                                   if(i & (1 << j))
   for(int i = 1; i <= n; i++) if(!vis[i]) dfs(i);</pre>
                                                                       dp[i] = max(dp[i], dp[i ^ (1 << j)]);
   e = 0;
                                                            int maxCliq = 0;
   reverse(S.begin(), S.end());
                                                            for(int i = 0; i < (1 << C); i++){
   for(int u : S) if(!cor[u])
                                                                int x = i, y = (1 << resto) - 1;
       e++, dfst(u);
                                                                for(int j = 0; j < C; j++)
                                                                   if(i & (1 << j))
   return 0;
                                                                       x \&= adj[j] \& ((1 << C) - 1), y \&= adj[j] >>
                                                                         С;
Tarjan
                                                                if(x != i) continue;
                                                                maxCliq = max(maxCliq, __builtin_popcount(i) + dp[y
void dfs(int u, int p = -1){
                                                                  ]);
   low[u] = num[u] = ++t;
   for(int v : g[u]){
                                                            Strings
       if(!num[v]){
          dfs(v, u);
                                                            Aho Corasick
          if(low[v] >= num[u]) u PONTO DE ARTICULAÇÃO;
          if(low[v] > num[u]) ARESTA u->v PONTE;
                                                            void init_aho(){
          low[u] = min(low[u], low[v]);
                                                                queue<int> q;
       else if(v != p) low[u] = min(low[u], num[v]);
   }
                                                                q.push(0);
}
                                                                while(!q.empty()){
void tarjanSCC(int u){
                                                                   int t = q.front(); q.pop();
   low[u] = num[u] = cnt++;
   vis[u] = 1;
                                                                   for(int i = 0; i < 52; i++) if(trie[t][i]){</pre>
   S.push_back(u);
                                                                       int x = trie[t][i];
   for(int v : g[u]){
                                                                       Q.push(x);
       if(!num[v]) tarjanSCC(v);
                                                                       if(t){
       if(vis[v]) low[u] = min(low[u], low[v]);
                                                                          fn[x] = fn[t];
   if(low[u] == num[u]){
                                                                          while(fn[x] \& trie[fn[x]][i] == 0) fn[x]
       ssc[u] = ++ssc_cnt; int v;
       do{
                                                                            = fn[fn[x]];
          v = S.back(); S.pop_back(); vis[v] = 0;
                                                                          if(trie[fn[x]][i]) fn[x] = trie[fn[x]][i];
          ssc[v] = ssc_cnt;
                                                                       }
       }while(u != v);
                                                                   }
```

University of Brasilia Strings

```
}
}
                                                           Prefix function/KMP
Suffix Array
                                                           vector<int> preffix_function(const string &s){
                                                               int n = s.size();
                                                               vector<int> b(n+1);
char s[N];
                                                               b[0] = -1;
int n, sa[N], tsa[N], lcp[N], r[N], nr[N], c[N];
                                                               int i = 0, j = -1;
                                                               while(i < n){
void sort(int k, int mx){
                                                                  while(j \ge 0 \& s[i] != s[j]) j = b[j];
   mx++;
                                                                  b[++i] = ++j;
   memset(c, 0, sizeof(int) * mx);
   for(int i = 0; i < n; i++) c[i + k < n ? r[i+k]+1 :
                                                               return b;
     1]++;
                                                           }
   partial_sum(c, c+mx, c);
   int t:
                                                           void kmp(const string &t, const string &p){
   for(int i = 0; i < n; i++)
                                                               vector<int> b = preffix_function(p);
       t = sa[i]+k < n ? r[ sa[i]+k ] : 0,
                                                               int n = t.size(), m = p.size();
       tsa[c[t]++] = sa[i];
                                                               int j = 0;
   memcpy(sa, tsa, sizeof(int) * n);
                                                               for(int i = 0; i < n; i++){
}
                                                                  while(j \ge 0 \& t[i] != p[j]) j = b[j];
                                                                  j++;
void build_sa(){
                                                                  if(j == m){
                                                                      //patern of p found on t
   for(int i = 0; i < n; i++) sa[i] = i, r[i] = s[i];
                                                                      j = b[j];
                                                                  }
   int t = 300, a, b;
                                                               }
   for(int sz = 1; sz < n; sz *= 2){
       sort(sz, t), sort(0, t);
       t = nr[sa[0]] = 0;
                                                           Min rotation
       for(int i = 1; i < n; i++){
          a = sa[i]+sz < n ? r[ sa[i]+sz ] : -1;
                                                           int min_rotation(int *s, int N) {
                                                             REP(i, N) s[N+i] = s[i];
          b = sa[i-1]+sz < n ? r[ sa[i-1]+sz ] : -1;
          nr[ sa[i] ] = r[ sa[i] ] == r[ sa[i-1] ] && a
             == b ? t : ++t;
                                                             int a = 0;
                                                             REP(b, N) REP(i, N) {
       if(t == n-1) break;
                                                               if (a+i == b \mid \mid s[a+i] < s[b+i]) { b += max(0, i-1);}
       memcpy(r, nr, sizeof(int) * n);
                                                                  break; }
   }
                                                               if (s[a+i] > s[b+i]) \{ a = b; break; \}
}
                                                             return a;
void build_lcp(){ // lcp[i] = lcp(s[:i], s[:i+1])
   int k = 0;
                                                           All palindrome
   for(int i = 0; i < n; i++) r[ sa[i] ] = i;
                                                           void manacher(char *s, int N, int *rad) {
   for(int i = 0; i < n; i++){
                                                             static char t[2*MAX];
       if(r[i] == n-1) k = 0;
                                                             int m = 2*N - 1;
       else{
          int j = sa[r[i]+1];
                                                             REP(i, m) t[i] = -1;
          while(i+k < n \&\& j+k < n \&\& s[i+k] == s[j+k])
                                                             REP(i, N) t[2*i] = s[i];
                                                             int x = 0;
       lcp[r[i]] = k;
                                                             FOR(i, 1, m) {
       if(k) k--;
                                                               int &r = rad[i] = 0;
   }
                                                               if (i <= x+rad[x]) r = min(rad[x+x-i], x+rad[x]-i);</pre>
}
                                                               while (i-r-1 >= 0 \& i+r+1 < m \& t[i-r-1] == t[i+r]
                                                                 +1]) ++r;
Z Algorithm
                                                               if (i+r >= x+rad[x]) x = i;
vector<int> z_algo(const string &s) {
   int n = s.size(), L = 0, R = 0;
                                                             REP(i, m) if (i-rad[i] == 0 || i+rad[i] == m-1) ++rad[
   vector<int> z(n, 0);
   for(int i = 1; i < n; i++){
                                                             REP(i, m) rad[i] /= 2;
       if(i \le R) z[i] = min(z[i-L], R - i + 1);
       while(z[i]+i < n \&\& s[z[i]+i] == s[z[i]])
                                                           Palindromic Tree
          z[i]++:
       if(i+z[i]-1 > R) L = i, R = i + z[i] - 1;
                                                           const int MAXN = 105000;
   }
   return z;
```

University of Brasilia Geometry

```
struct node {
   int next[26];
   int len:
   int sufflink;
   int num;
};
int len;
char s[MAXN];
node tree[MAXN];
int num; // node 1 - root with len -1, node 2 - root
  with len 0
int suff; // max suffix palindrome
long long ans;
bool addLetter(int pos) {
   int cur = suff, curlen = 0;
   int let = s[pos] - 'a';
   while(true){
       curlen = tree[cur].len;
      if (pos-1 - curlen \geq 0 && s[pos-1 - curlen] == s
          break:
       cur = tree[cur].sufflink;
   if (tree[cur].next[let]) {
       suff = tree[cur].next[let];
       return false;
   }
   num++:
   suff = num;
   tree[num].len = tree[cur].len + 2;
   tree[cur].next[let] = num;
   if (tree[num].len == 1){
      tree[num].sufflink = 2;
       tree[num].num = 1;
      return true;
   }
   while (true){
      cur = tree[cur].sufflink;
      curlen = tree[cur].len;
      if(pos-1 - curlen) == 0 \&\& s[pos-1 - curlen] == s[
         posl){
          tree[num].sufflink = tree[cur].next[let];
          break;
       }
   }
   tree[num].num = 1 + tree[tree[num].sufflink].num;
   return true;
}
void initTree() {
   num = 2; suff = 2;
   tree[1].len = -1; tree[1].sufflink = 1;
   tree[2].len = 0; tree[2].sufflink = 1;
int main() {
   initTree();
```

```
for (int i = 0; i < len; i++) {</pre>
       addLetter(i);
   return 0:
Geometry
2D basics
typedef double coord;
double eps = 1e-7;
bool eq(coord a, coord b){ return abs(a - b) <= eps; }</pre>
struct vec{
   coord x, y; int id;
   vec(coord \ a = 0, \ coord \ b = 0) : x(a), y(b) \{\}
   vec operator+(const vec &o) const{
      return \{x + o.x, y + o.y\};
   vec operator-(const vec &o) const{
      return \{x - o.x, y - o.y\};
   vec operator*(coord t) const{
      return {x * t, y * t};
   vec operator/(coord t) const{
      return {x / t, y / t};
   coord operator*(const vec &o) const{ // cos
      return x * o.x + y * o.y;
   coord operator^(const vec &o) const{ // sin
      return x * o.y - y * o.x;
   bool operator==(const vec &o) const{
      return eq(x, o.x) & eq(y, o.y);
   bool operator<(const vec &o) const{</pre>
      if(!eq(x, o.x)) return x < o.x;
      return y < o.y;</pre>
   coord cross(const vec &a, const vec &b) const{
      return (a-(*this)) ^ (b-(*this));
   coord dot(const vec &a, const vec &b) const{
      return (a-(*this)) * (b-(*this));
   coord len() const{
      return sqrt(x * x + y * y); // <
   double angle(const vec &a, const vec &b) const{
      return atan2(cross(a, b), dot(a, b));
   double tan(const vec &a, const vec &b) const{
      return cross(a, b) / dot(a, b);
   vec unit() const{
      return operator/(len());
   int quad() const{
      if(x > 0 \& y >=0) return 0;
      if(x \le 0 \&\& y > 0) return 1;
```

if(x < 0 & y <=0) return 2;

bool comp(const vec &a, const vec &b) const{

return 3;

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```
return (a - *this).comp(b - *this);
   }
   bool comp(vec b){
      if(quad() != b.quad()) return quad() < b.quad();</pre>
      if(!eq(operator^(b), 0)) return operator^(b) > 0;
       return (*this) * (*this) < b * b;
   template<class T>
   void sort_by_angle(T first, T last) const{
       std::sort(first, last, [=](const vec &a, const
          return comp(a, b);
   }
   vec rot90() const{ return {-y, x}; }
   vec rot(double a) const{
      return \{\cos(a)*x - \sin(a)*y, \sin(a)*x + \cos(a)*y\}
};
struct line{
   coord a, b, c; vec n;
   line(vec q, vec w){ // q.cross(w, (x, y)) = 0
      a = -(w.y-q.y);
      b = w.x-q.x;
      c = -(a * q.x + b * q.y);
      n = \{a, b\};
   coord dist(const vec &o) const{
      return abs(eval(o)) / n.len();
   bool contains(const vec &o) const{
      return eq(a * o.x + b * o.y + c, 0);
   coord dist(const line &o) const{
      if(!parallel(o)) return 0;
       if(!eq(o.a * b, o.b * a)) return 0;
       if(!eq(a, 0))
          return abs(c - o.c * a / o.a) / n.len();
       if(!eq(b, 0))
          return abs(c - o.c * b / o.b) / n.len();
      return abs(c - o.c);
   bool parallel(const line &o) const{
      return eq(n ^ o.n, 0);
   bool operator==(const line &o) const{
      if(!eq(a*o.b, b*o.a)) return false;
       if(!eq(a*o.c, c*o.a)) return false;
       if(!eq(c*o.b, b*o.c)) return false;
       return true;
   bool intersect(const line &o) const{
      return !parallel(o) || *this == o;
   vec inter(const line &o) const{
      if(parallel(o)){
          if(*this == o){ }
          else{ /* dont intersect */ }
       auto tmp = n \circ o.n;
      return {(o.c*b - c*o.b)/tmp, (o.a*c - a*o.c)/tmp
         }:
   vec at_x(coord x) const{
```

```
return \{x, (-c-a*x)/b\};
   }
   vec at_y(coord y) const{
      return \{(-c-b*y)/a, y\};
   coord eval(const vec &o) const{
       return a * o.x + b * o.y + c;
};
struct segment{
   vec p, q;
   segment(vec a = vec(), vec b = vec()): p(a), q(b) {}
   bool onstrip(const vec &o) const{ // onstrip strip
       return p.dot(o, q) >= -eps && q.dot(o, p) >= -eps
   }
   coord len() const{
      return (p-q).len();
   coord dist(const vec &o) const{
       if(onstrip(o)) return line(p, q).dist(o);
       return min((o-q).len(), (o-p).len());
   bool contains(const vec &o) const{
       return eq(p.cross(q, o), 0) && onstrip(o);
   bool intersect(const segment &o) const{
       auto d1 = p.cross(q, o.p);
       if(eq(d1, 0) && contains(o.p)) return true;
       auto d2 = p.cross(q, o.q);
       if(eq(d2, 0) && contains(o.q)) return true;
       auto d3 = o.p.cross(o.q, q);
       if(eq(d3, 0) && o.contains(q)) return true;
       auto d4 = o.p.cross(o.q, p);
       if(eq(d4, 0) && o.contains(p)) return true;
       return d1 * d2 < 0 && d3 * d4 < 0;
   bool intersect(const line &o) const{
      return o.eval(p) * o.eval(q) <= 0;</pre>
   coord dist(const segment &o) const{
       if(line(p, q).parallel(line(o.p, o.q))){
          if(onstrip(o.p) || onstrip(o.q)
          || o.onstrip(p) || o.onstrip(q))
              return line(p, q).dist(line(o.p, o.q));
       else if(intersect(o)) return 0;
       return min(min(dist(o.p), dist(o.q)),
                min(o.dist(p), o.dist(q)));
   coord dist(const line &o) const{
       if(line(p, q).parallel(o))
          return line(p, q).dist(o);
       else if(intersect(o)) return 0;
       return min(o.dist(p), o.dist(q));
};
struct hray{
   vec p. a:
   hray(vec a = vec(), vec b = vec()): p(a), q(b){}
   bool onstrip(const vec &o) const{ // onstrip strip
       return p.dot(q, o) >= -eps;
   }
   coord dist(const vec &o) const{
       if(onstrip(o)) return line(p, q).dist(o);
```

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```
return (o-p).len();
                                                            pt a[MAXN];
   }
   bool intersect(const segment &o) const{
      if(!o.intersect(line(p,q))) return false;
                                                            double mindist;
       if(line(o.p, o.q).parallel(line(p,q)))
                                                            int ansa, ansb;
          return contains(o.p) || contains(o.q);
       return contains(line(p,q).inter(line(o.p,o.q)));
                                                            inline void upd_ans (const pt & a, const pt & b) {
                                                               double dist = sqrt ((a.x-b.x)*(a.x-b.x) + (a.y-b.y)
   bool contains(const vec &o) const{
                                                                  *(a.y-b.y) + .0);
                                                               if (dist < mindist)</pre>
      return eq(line(p, q).eval(o), 0) && onstrip(o);
                                                                   mindist = dist, ansa = a.id, ansb = b.id;
   coord dist(const segment &o) const{
       if(line(p, q).parallel(line(o.p, o.q))){
                                                            void rec (int 1, int r) {
          if(onstrip(o.p) || onstrip(o.q))
                                                               if (r - 1 <= 3) {
              return line(p, q).dist(line(o.p, o.q));
          return o.dist(p);
                                                                   for (int i=1; i<=r; ++i)
                                                                      for (int j=i+1; j<=r; ++j)
      else if(intersect(o)) return 0;
                                                                          upd_ans (a[i], a[j]);
      return min(min(dist(o.p), dist(o.q)),
                                                                   sort (a+1, a+r+1, &cmp_y);
                o.dist(p));
                                                                   return:
   bool intersect(const hray &o) const{
      if(!line(p, q).parallel(line(o.p, o.q)))
                                                               int m = (1 + r) >> 1;
          return false;
                                                               int midx = a[m].x;
       auto pt = line(p, q).inter(line(o.p, o.q));
                                                               rec (1, m), rec (m+1, r);
      return contains(pt) && o.contains(pt); // <<</pre>
                                                               static pt t[MAXN];
                                                               merge (a+1, a+m+1, a+m+1, a+r+1, t, &cmp_y);
   bool intersect(const line &o) const{
                                                               copy (t, t+r-l+1, a+l);
      if(line(p, q).parallel(o)) return line(p, q)== o;
       if(o.contains(p) || o.contains(q)) return true;
                                                               int tsz = 0;
      return (o.eval(p) >= -eps)^(o.eval(p)<o.eval(q));</pre>
                                                               for (int i=1; i<=r; ++i)
       return contains(o.inter(line(p, q)));
                                                                   if (abs (a[i].x - midx) < mindist) {</pre>
                                                                      for (int j=tsz-1; j>=0 && a[i].y - t[j].y <</pre>
   coord dist(const line &o) const{
                                                                        mindist; --j)
                                                                          upd_ans (a[i], t[j]);
      if(line(p,q).parallel(o))
          return line(p,q).dist(o);
                                                                      t[tsz++] = a[i];
       else if(intersect(o)) return 0;
                                                                   }
      return o.dist(p);
   }
   coord dist(const hray &o) const{
                                                            sort (a, a+n, \&cmp_x);
       if(line(p, q).parallel(line(o.p, o.q))){
                                                            mindist = 1E20;
          if(onstrip(o.p) || o.onstrip(p))
                                                            rec (0, n-1);
             return line(p,q).dist(line(o.p, o.q));
                                                            Convex Hull
          return (p-o.p).len();
                                                            vector<vec> monotone_chain_ch(vector<vec> P){
       else if(intersect(o)) return 0;
                                                               sort(P.begin(), P.end());
      return min(dist(o.p), o.dist(p));
   }
                                                               vector<vec> L, U;
                                                               for(auto p : P){
                                                                   while(L.size() >= 2 && L[L.size() - 2].cross(L.
double heron(coord a, coord b, coord c){
                                                                     back(), p) < 0)
   coord s = (a + b + c) / 2;
                                                                      L.pop_back();
   return sqrt(s * (s - a) * (s - b) * (s - c));
                                                                  L.push_back(p);
Nearest Points
struct pt {
                                                               reverse(P.begin(), P.end());
   int x, y, id;
                                                               for(auto p : P){
                                                                   while(U.size() >= 2 && U[U.size() - 2].cross(U.
                                                                     back(), p) < 0)
inline bool cmp_x (const pt & a, const pt & b) {
                                                                      U.pop_back();
   return a.x < b.x | | a.x == b.x && a.y < b.y;
                                                                   U.push_back(p);
                                                               }
inline bool cmp_y (const pt & a, const pt & b) {
   return a.y < b.y;</pre>
                                                               L.pop_back(), U.pop_back();
```

};

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```
L.insert(L.end(), U.begin(), U.end());
   return L:
Check point inside polygon, borders included
bool below(const vector<vec> &vet, vec p){
   auto it = lower_bound(vet.begin(), vet.end(), p);
   if(eq(it->x, p.x)) return p.y <= it->y;
   return prev(it)->cross(*it, p) <= 0;</pre>
}
bool above(const vector<vec> &vet, vec p){
   auto it = lower_bound(vet.begin(), vet.end(), p);
   if(eq(it->x, p.x)) return p.y >= it->y;
   return prev(it)->cross(*it, p) >= 0;
}
// lowerhull, upperhull and point
bool inside_poly(const vector<vec> &lo, const vector<vec</pre>
  > &hi, vec p){
   if(p.x < lo[0].x || lo.back().x < p.x)
      return false:
   return below(hi, p) && above(lo, p);
```

L.reserve(L.size() + U.size());

### Triangulo

Baricentro (centroide ou centro de massa), que é o ponto de interseção entre as três medianas (segmentos de reta que unem um vértice ao ponto médio do lado oposto). O baricentro divide uma mediana na proporção de 2:1, isto é, ele está a um terço de distância do lado oposto.

As coordenadas do baricentro podem ser computadas diretamente a partir das coordenadas dos vértices: serão a média aritmética entre as mesmas.

O ortocentro de um triângulo é o ponto de encontro de suas três alturas. O ortocentro pode mesmo estar fora do triângulo (no caso de um obtusângulo). No caso de um triângulo retângulo, o ortocentro sempre coincide com o vértice oposto à hipotenusa.

O **incentro** de um triângulo é o ponto de encontro de suas bissetrizes (retas que dividem um ângulo interno na metade). Além de ser sempre um ponto interior do triângulo, o incentro é o centro do círculo inscrito no triângulo, isto é, o maior círculo que cabe dentro do triângulo e que toca todos os seus três lados (os lados são tangentes ao círculo inscrito).

O raio do círculo inscrito é dado pela razão entre o dobro da área e o perímetro. As coordenadas do centro O do círculo inscrito são obtidas pela média ponderada das coordenadas x e y pelos comprimentos dos lados opostos. As fórmulas

$$r = \frac{2A}{p}$$
,  $Ox = \frac{a*A_x + b*B_x + c*C_x}{p}$ ,  $Oy = \frac{a*A_y + b*B_y + c*C_y}{p}$ 

abaixo sintetizam estas afirmações.  $r = \frac{2A}{p}, Ox = \frac{a*A_x + b*B_x + c*C_x}{p}, Oy = \frac{a*A_y + b*B_y + c*C_y}{p}$  O circuncentro é o ponto de encontro entre as retas bisectoras perpendiculares (isto é, retas perpendiculares aos lados do triângulo que os interceptam nos pontos médios). O circuncentro é o centro do círculo circunscrito do triângulo, isto é, o círculo que passa pelos três vértices do triângulo.

O circuncentro, assim como o ortocentro, pode estar localizado do lado externo do triângulo. Um caso especial interessante é o do triângulo retângulo, onde o circuncentro se localiza no ponto médio da hipotenusa.

O raio do circuncentro é dado pela razão entre o produto

das medidas de seus lados e o quádruplo de sua área. As coordenadas do circuncentro podem ser determinadas pelas expressões abaixo, onde  $|A|^2 = Ax^2 + Ay^2$ .

$$r = \frac{a*b*c}{4A}, S_{x} = \frac{1}{2d} * \begin{vmatrix} |A|^{2} & A_{y} & 1 \\ |B|^{2} & B_{y} & 1 \\ |C|^{2} & C_{y} & 1 \end{vmatrix}, S_{y} = \frac{1}{2d} * \begin{vmatrix} A_{x} & |A|^{2} & 1 \\ B_{x} & |B|^{2} & 1 \\ C_{x} & |C|^{2} & 1 \end{vmatrix}$$

$$d = \begin{vmatrix} A_{x} & A_{y} & 1 \\ B_{x} & B_{y} & 1 \\ C_{x} & C_{y} & 1 \end{vmatrix}$$

$$|A|^{2} \text{ é dot product do vetor } A.$$

#### Miscellaneous

#### LIS

```
multiset<int> S:
for(int i = 0; i < n; i++){
   auto it = S.upper_bound(a[i]); // low for inc
   if(it != S.end()) S.erase(it);
   S.insert(a[i]);
ans = S.size();
DSU rollback
#include <bits/stdc++.h>
using namespace std;
struct DSU{
   vector<int> sz, p, change;
   vector<tuple<int, int, int>> modifications;
   vector<size_t> saves;
   bool bipartite;
   DSU(int n): sz(n+1, 1), p(n+1), change(n+1),
     bipartite(true){
       iota(p.begin(), p.end(), 0);
   void add_edge(int u, int v){
       if(!bipartite) return;
       int must_change = get_colour(u) == get_colour(v);
       int a = rep(u), b = rep(v);
       if(sz[a] < sz[b]) swap(a, b);
       if(a != b){
          p[b] = a;
          modifications.emplace_back(b, change[b],
            bipartite);
          change[b] ^= must_change;
          sz[a] += sz[b];
       else if(must_change){
          modifications.emplace_back(0, change[0],
            bipartite);
          bipartite = false;
       }
   }
   int rep(int u){
       return p[u] == u ? u : rep(p[u]);
   }
   int get_colour(int u){
       if(p[u] == u) return change[u];
       return change[u] ^ get_colour(p[u]);
```

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```
void reset(){
       modifications.clear();
       saves.clear();
       iota(p.begin(), p.end(), 0);
       fill(sz.begin(), sz.end(), 1);
       fill(change.begin(), change.end(), 0);
       bipartite = true;
   }
   void rollback(){
       int u = get<0>(modifications.back());
       tie(ignore, change[u], bipartite) = modifications
         .back();
       sz[ p[u] ] -= sz[u];
       p[u] = u;
       modifications.pop_back();
   }
   void reload(){
       while(modifications.size() > saves.back())
          rollback();
       saves.pop_back();
   }
   void save(){
       saves.push_back(modifications.size());
   }
};
const int N = 100005;
const int B = 318;
int n, m, q;
int x[N], y[N], 1[N], r[N], ans[N];
vector<int> qu[N];
int brute(int lef, int rig, DSU &s){
   s.save();
   for(int i = lef; i <= rig; i++)</pre>
       s.add_edge(x[i], y[i]);
```

```
int ret = s.bipartite;
   s.reload();
   return ret;
}
int main(){
   scanf("%d %d %d", &n, &m, &q);
   for(int i = 1; i <= m; i++)
       scanf("%d %d", x+i, y+i);
   DSU s(n);
   for(int i = 0; i < q; i++){
       scanf("%d %d", l+i, r+i);
       if(r[i] - 1[i] \le B + 10)
          ans[i] = brute(l[i], r[i], s);
       else qu[l[i] / B].push_back(i);
   }
   for(int i = 0; i \le m / B; i++){
       sort(qu[i].begin(), qu[i].end(),[](int a, int b){
          return r[a] < r[b];
       });
       s.reset();
       int R = (i+1)*B-1;
       for(int id : qu[i]){
          while(R < r[id]) ++R, s.add_edge(x[R], y[R]);
          s.save();
          for(int k = 1[id]; k < (i+1)*B; k++)
              s.add_edge(x[k], y[k]);
          ans[id] = s.bipartite;
          s.reload();
       }
   }
   for(int i = 0; i < q; i++)
      printf("%s\n",ans[i] ? "Possible":"Impossible");
```