# ICPC Team Reference

# University of Brasilia

| Contents  | 4.9 Suffix Automaton   |
|---|--|
| 1.1 Merge Sort Tree       2         1.2 Wavelet Tree       2         1.3 Order Set       2         1.4 Hash table       3         1.5 Convex Hull Trick Simple       3         1.6 Convex Hull Trick       3         1.7 Convex Hull Trick       3  | 2     5     Geometry     15       2     5.1     2D basics     15       2     5.2     Nearest Points     17       5.3     Convex Hull     18       3     5.4     Check point inside polygon     18       5.5     Check point inside polygon without lower/upper hull     18       3     5.6     Minkowski sum     18  |
| 1.9       Sparse Table       4         1.10       Treap       4         1.11       ColorUpdate       5         1.12       Heavy Light Decomposition       5         1.13       Iterative Segtree       5         1.14       LiChao's Segtree       5  | 4       4       6 Miscellaneous       18         4       6.1 LIS       18         5       6.2 DSU rollback       19         5       6.3 Buildings       19         5       6.4 Rand       20         5       6.5 Klondike       20         6       6.6 Hilbert Order       20         6.7 Modular Factorial       21   |
| 2.1       Extended Euclidean Algorithm       6         2.2       Chinese Remainder Theorem       6         2.3       Preffix inverse       6         2.4       Pollard Rho       6         2.5       Miller Rabin       7         2.6       Totiente       7         2.7       Mobius Function       7         2.8       Mulmod TOP       7         2.9       Matrix Determinant       7         2.10       Simplex Method       8         2.11       FFT       8 | 6       6.8       Enumeration all submasks of a bitmask       21         6       6.9       Slope Trick       21         6       6.10       Fast IO       21         6       6.11       Knapsack Bounded with Cost       21         6       6.12       LCA <o(nlgn), o(1)="">       21         7       6.13       Burnside's Lemma       22         7       6.14       Wilson's Theorem       22         6.15       Fibonacci       22         7       6.16       Kirchhoff's Theorem       22         6.16.1       Multigraphs       22         6.16.2       Directed multigraphs       22         8       6.17       Matroid       22         6.17.1       Matroid intersection algorithm       22</o(nlgn),> |
| 3.1 Dinic   | 10<br>11<br>11<br>11<br>12   |
| 4 Strings       13         4.1 Aho Corasick       13         4.2 Suffix Array       13         4.3 Z Algorithm       13         4.4 Prefix function/KMP       13         4.5 Min rotation       14         4.6 All palindrome       14         4.7 Manacher       14         4.8 Palindromic Tree       14  | 13<br>13<br>13<br>13<br>14<br>14<br>14   |

University of Brasilia Data Structures, 2

```
set ts=4 sw=4 sta nu rnu sc stl+=%F cindent
imap {<CR> {<CR>}<Esc>0
nmap <F2> 0V$%d
nmap <C-down> :m+1<CR>
nmap <C-up> :m-2<CR>
vmap < C-c > "+y
nmap <C-a> ggVG
syntax on
alias cmp='g++ -Wall -Wformat=2 -Wshadow -Wconversion -
  fsanitize=address -fsanitize=undefined -fno-sanitize-
  recover -std=c++14'
```

### **Data Structures**

## **Merge Sort Tree**

```
struct MergeTree{
   int n:
   vector<vector<int>> st;
   void build(int p, int L, int R, const int v[]){
       if(L == R){
          st[p].push_back(v[L]);
          return:
       }
      int mid = (L+R)/2;
      build(2*p, L, mid, v);
      build(2*p+1, mid+1, R, v);
       st[p].resize(R-L+1);
      merge(st[2*p].begin(), st[2*p].end(),
              st[2*p+1].begin(), st[2*p+1].end(),
              st[p].begin());
   }
   int query(int p, int L, int R, int i, int j, int x)
     const{
       if(L > j || R < i) return 0;</pre>
       if(L >= i && R <= j){</pre>
          int id = lower_bound(st[p].begin(), st[p].end
             (), x) - st[p].begin();
          return int(st[p].size()) - id;
      }
       int mid = (L+R)/2;
      return query(2*p, L, mid, i, j, x) +
          query(2*p+1, mid+1, R, i, j, x);
   }
public:
   MergeTree(int sz, const int v[]): n(sz), st(4*sz){
      build(1, 1, n, v);
   //number of elements >= x on segment [i, j]
   int query(int i, int j, int x) const{
       if(i > j) swap(i, j);
       return query(1, 1, n, i, j, x);
   }
};
Wavelet Tree
template<typename T>
class wavelet{
   T L, R;
```

```
vector<int> 1;
vector<T> sum; // <<</pre>
wavelet *lef, *rig;
```

```
int r(int i) const{ return i - l[i]; }
public:
   template<typename ITER>
   wavelet(ITER bg, ITER en){
       lef = rig = nullptr;
       L = *bg, R = *bg;
       for(auto it = bg; it != en; it++)
          L = min(L, *it), R = max(R, *it);
       if(L == R) return;
       T mid = L + (R - L)/2;
       1.reserve(std::distance(bg, en) + 1);
       sum.reserve(std::distance(bg, en) + 1);
       1.push_back(0), sum.push_back(0);
       for(auto it = bg; it != en; it++)
          l.push_back(l.back() + (*it <= mid)),</pre>
          sum.push_back(sum.back() + *it);
       auto tmp = stable_partition(bg, en, [mid](T x){
          return x <= mid;</pre>
       });
       if(bg != tmp) lef = new wavelet(bg, tmp);
       if(tmp != en) rig = new wavelet(tmp, en);
   }
    ~wavelet(){
       delete lef;
       delete rig;
   // 1 index, first is 1st
   T kth(int i, int j, int k) const{
       if(L >= R) return L;
       int c = l[j] - l[i-1];
       if(c >= k) return lef->kth(l[i-1]+1, l[j], k);
       else return rig->kth(r(i-1)+1, r(j), k - c);
   // # elements > x on [i, j]
   int cnt(int i, int j, T x) const{
       if(L > x) return j - i + 1;
       if(R <= x || L == R) return 0;
       int ans = 0;
       if(lef) ans += lef->cnt(l[i-1]+1, l[j], x);
       if(rig) ans += rig->cnt(r(i-1)+1, r(j), x);
       return ans;
   }
   // sum of elements <= k on [i, j]</pre>
   T sumk(int i, int j, T k){
       if(L == R) return R <= k ? L * (j - i + 1) : 0;
       if(R <= k) return sum[j] - sum[i-1];</pre>
       int ans = 0;
       if(lef) ans += lef->sumk(l[i-1]+1, l[j], k);
      if(rig) ans += rig->sumk(r(i-1)+1, r(j), k);
      return ans;
   // swap (i, i+1) just need to update "array" l[i]
};
Order Set
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
```

University of Brasilia Data Structures, 3

```
int L = 0, R = (int)cht.size()-2;
#include <ext/pb_ds/detail/standard_policies.hpp>
                                                                int bans = (int)cht.size()-1;
                                                                while(L <= R){</pre>
                                                                   int mid = (L+R)/2;
using namespace __gnu_pbds; // or pb_ds;
                                                                   if(cht[mid].eval(x) >= cht[mid+1].eval(x)) // <<<</pre>
template<typename T, typename B = null_type>
                                                                       I. = mid + 1:
using oset = tree<T, B, less<T>, rb_tree_tag,
                                                                   else bans = mid, R = mid - 1;
  tree_order_statistics_node_update>;
// find_by_order / order_of_key
                                                                return cht[bans].eval(x);
Hash table
                                                            Convex Hull Trick
#include <ext/pb_ds/assoc_container.hpp>
                                                            const ll is_query = -(1LL<<62);</pre>
using namespace __gnu_pbds;
                                                            struct Line{
struct custom_hash {
                                                               11 m. b:
   static uint64_t splitmix64(uint64_t x) {
                                                               mutable function<const Line*()> succ;
       // http://xorshift.di.unimi.it/splitmix64.c
                                                                bool operator<(const Line& rhs) const{</pre>
       x += 0x9e3779b97f4a7c15;
                                                                   if(rhs.b != is_query) return m < rhs.m;</pre>
       x = (x \hat{ } (x >> 30)) * 0xbf58476d1ce4e5b9;
                                                                   const Line* s = succ();
       x = (x ^(x >> 27)) * 0x94d049bb133111eb;
                                                                   if(!s) return 0;
       return x \hat{ } (x >> 31);
                                                                   11 x = rhs.m;
   }
                                                                   return b - s->b < (s->m - m) * x;
   size_t operator()(uint64_t x) const {
                                                            };
       static const uint64_t FIXED_RANDOM = chrono::
                                                            struct Cht : public multiset<Line>{ // maintain max
         steady_clock::now().time_since_epoch().count();
                                                                bool bad(iterator y){
       return splitmix64(x + FIXED_RANDOM);
                                                                   auto z = next(y);
   }
                                                                   if(y == begin()){
};
                                                                       if(z == end()) return 0;
                                                                       return y->m == z->m && y->b <= z->b;
gp_hash_table<long long, int, custom_hash> table;
unordered_map<long long, int, custom_hash> uhash;
                                                                   auto x = prev(y);
uhash.reserve(1 << 15);</pre>
                                                                   if(z == end()) return y->m == x->m && y->b <= x->
uhash.max_load_factor(0.25);
                                                                   return (long double) (x->b - y->b)*(z->m - y->m)
Convex Hull Trick Simple
                                                                     >= (long double)(y->b - z->b)*(y->m - x->m);
struct Line{
                                                                void insert_line(ll m, ll b){
   11 m, b;
                                                                   auto y = insert({ m, b });
   inline 11 eval(11 x) const{
                                                                   y->succ = [=]{return next(y) == end() ? 0 : &*
       return x * m + b;
                                                                     next(y); };
   }
                                                                   if(bad(y)){ erase(y); return; }
};
                                                                   while(next(y) != end() && bad(next(y))) erase(
                                                                     next(y));
// min => cht.back().m >= L.m
                                                                   while(y != begin() && bad(prev(y))) erase(prev(y)
// max => cht.back().m <= L.m
                                                                     );
void push_line(vector<Line> &cht, Line L){
 while((int)cht.size() >= 2){
                                                                11 eval(11 x){
   int sz = (int)cht.size();
                                                                   auto 1 = *lower_bound((Line) { x, is_query });
   if((long double)(L.b-cht[sz-1].b)*(cht[sz-2].m-L.m)
                                                                   return 1.m * x + 1.b;
  <= (long double)(L.b-cht[sz-2].b)*(cht[sz-1].m-L.m)){
     cht.pop_back();
                                                            };
   }
                                                            Convex Hull Trick
   else break;
 cht.push_back(L);
                                                             * Author: Simon Lindholm
                                                             * source: https://github.com/kth-competitive-
// x increasing; pos = 0 in first call
                                                               programming/kactl/blob/master/content/data-structures
11 linear_search(const vector<Line> &cht,ll x,int &pos){
                                                               /LineContainer.h
                                                             * License: CC0
   while(pos+1 < (int)cht.size()){</pre>
                                                             */
/*>>*/ if(cht[pos].eval(x) >= cht[pos+1].eval(x)) pos++;
       else break;
                                                            struct Line {
   }
   return cht[pos].eval(x);
                                                               mutable 11 m, b, p;
}
                                                                bool operator<(const Line& o) const { return m < o.m</pre>
                                                                  ; }
11 binary_search(const vector<Line> &cht, ll x){
                                                                bool operator<(11 x) const { return p < x; }</pre>
```

University of Brasilia Data Structures, 4

```
int lg(int x){ return 31 - __builtin_clz(x); }
struct LineContainer : multiset<Line, less<>>> { // CPP14
                                                            int getmn(int 1, int r) { // [1, r]
                                                                int 1z = 1g(r - 1 + 1);
  only
   // (for doubles, use inf = 1/.0, div(a,b) = a/b)
                                                                return min(fn(1, lz), fn(r - (1 << lz) + 1, lz));
   const ll inf = LLONG_MAX;
                                                            }
   11 div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }</pre>
                                                            Treap
   bool isect(iterator x, iterator y) {
                                                            // source: https://github.com/victorsenam/caderno/blob/
       if (y == end()) { x->p = inf; return false; }
                                                              master/code/treap.cpp
       if (x->m == y->m) x->p = x->b > y->b? inf : -inf
                                                            //const int N = ; typedef int num;
                                                            num X[N]; int en = 1, Y[N], sz[N], L[N], R[N];
       else x->p = div(y->b - x->b, x->m - y->m);
                                                            void calc (int u) { // update node given children info
      return x->p >= y->p;
                                                                if(!u) return;
                                                                sz[u] = sz[L[u]] + 1 + sz[R[u]];
   void add(ll m, ll b) {
      auto z = insert(\{m, b, 0\}), y = z++, x = y;
                                                                // code here, no recursion
      while (isect(y, z)) z = erase(z);
                                                            void unlaze (int u) {
       if (x != begin() \&\& isect(--x, y)) isect(x, y =
                                                                if(!u) return;
         erase(y));
                                                                // code here, no recursion
       while ((y = x) != begin() \&\& (--x)->p >= y->p)
                                                            }
          isect(x, erase(y));
                                                            void split_val(int u, num x, int &l, int &r) { // l gets
                                                                <= x, r gets > x
   11 query(ll x) {
                                                                unlaze(u); if(!u) return (void) (l = r = 0);
      assert(!empty());
                                                                if(X[u] \le x) \{ split_val(R[u], x, 1, r); R[u] = 1;
       auto 1 = *lower_bound(x);
      return 1.m * x + 1.b;
                                                                else { split_val(L[u], x, 1, r); L[u] = r; r = u; }
   }
};
                                                                calc(u):
Min queue
                                                            void split_sz(int u, int s, int &l, int &r) { // l gets
                                                              first s, r gets remaining
template<typename T>
                                                                unlaze(u); if(!u) return (void) (l = r = 0);
class minQ{
                                                                if(sz[L[u]] < s)  { split_sz(R[u], s - sz[L[u]] - 1,
   deque<tuple<T, int, int> > p;
                                                                  1, r); R[u] = 1; 1 = u; }
   T delta;
                                                                else { split_sz(L[u], s, l, r); L[u] = r; r = u; }
   int sz;
                                                                calc(u);
public:
   minQ() : delta(0), sz(0) {}
                                                            int merge(int 1, int r) { // els on 1 <= els on r</pre>
   inline int size() const{ return sz; }
                                                                unlaze(1); unlaze(r); if(!1 || !r) return 1 + r; int
   inline void add(T x){ delta += x; }
                                                                   u;
   inline void push(T x, int id){
                                                                if(Y[1] > Y[r]) { R[1] = merge(R[1], r); u = 1; }
      x \rightarrow delta, sz++;
                                                                else { L[r] = merge(l, L[r]); u = r; }
      int t = 1;
                                                                calc(u); return u;
       while(p.size() > 0 && get<0>(p.back()) >= x)
          t += get<1>(p.back()), p.pop_back();
                                                            void init(int n=N-1) { // XXX call before using other
      p.emplace_back(x, t, id);
   }
                                                                for(int i = en = 1; i \le n; i++) { Y[i] = i; sz[i] =
   inline void pop(){
                                                                   1; L[i] = R[i] = 0; }
      get<1>(p.front())--, sz--;
                                                                random_shuffle(Y + 1, Y + n + 1);
       if(!get<1>(p.front())) p.pop_front();
                                                            void insert(int &u, int it){
   T getmin() const{ return get<0>(p.front())+delta; }
                                                                unlaze(u):
   int getid() const{ return get<2>(p.front()); }
                                                                if(!u) u = it:
};
                                                                else if(Y[it] > Y[u]) split_val(u, X[it], L[it], R[
Sparse Table
                                                                  it]), u = it;
                                                                else insert(X[it] < X[u] ? L[u] : R[u], it);</pre>
const int N = 100005;
                                                            }
int v[N], n;
                                                            void erase(int &u, num key){
int dn[N][20];
                                                                unlaze(u):
                                                                if(!u) return;
int fn(int i, int j){
   if(j == 0) return v[i];
                                                                if(X[u] == key) u = merge(L[u], R[u]);
   if(~dn[i][j]) return dn[i][j];
                                                                else erase(key < X[u] ? L[u] : R[u], key);</pre>
   return dn[i][j] = min(fn(i, j-1), fn(i + (1 << (j-1)))
                                                                calc(u);
     ), j-1));
                                                            }
}
                                                            int create_node(num key){
                                                                X[en] = key;
```

University of Brasilia Data Structures, 5

```
sz[en] = 1;
   L[en] = R[en] = 0;
                                                                   ranges.erase(ranges.lower_bound(1), ranges.
                                                                     lower_bound(r));
   return en++;
                                                                   ranges.insert(Range(1, r, v));
int query(int u, int 1, int r){//0 index
                                                                   return ans;
   unlaze(u);
                                                               }
   if(u! or r < 0 or l >= sz[u]) return
                                                            private:
     identity_element;
                                                               std::set<Range> ranges;
   if(1 \le 0 \text{ and } r \ge sz[u] - 1) \text{ return } subt\_data[u];
   int ans = query(L[u], 1, r);
                                                            Heavy Light Decomposition
   if(1 \le sz[L[u]] and sz[L[u]] \le r)
       ans = max(ans, st[u]);
                                                            void dfs_sz(int u){
   ans = max(ans, query(R[u], l-sz[L[u]]-1, r-sz[L[u]])
                                                               sz[u] = 1;
     ]]-1));
   return ans;
                                                               for(auto &v : g[u]) if(v == p[u]){
                                                                   swap(v, g[u].back());
                                                                   g[u].pop_back();
ColorUpdate
                                                                   break;
                                                               }
// source: https://github.com/tfg50/Competitive-
  Programming/tree/master/Biblioteca/Data%20Structures
                                                               for(auto &v : g[u]){
                                                                   p[v] = u;
#include <set>
                                                                   dfs_sz(v);
#include <vector>
                                                                   sz[u] += sz[v];
                                                                   if(sz[v] > sz[g[u][0]])
template <class Info = int>
                                                                       swap(v, g[u][0]);
class ColorUpdate {
                                                               }
public:
                                                            }
   struct Range {
      Range(int l = 0) { this->l = 1; }
                                                            // nxt[u] = start of path with u
      Range(int 1, int r, Info v) {
                                                            // set nxt[root] = root beforehand
          this \rightarrow 1 = 1;
                                                            void dfs_hld(int u){
          this -> r = r;
                                                               in[u] = t++;
          this->v = v;
                                                               rin[in[u]] = u;
                                                               for(auto v : g[u]){
       int 1, r;
                                                                   nxt[v] = (v == g[u][0] ? nxt[u] : v);
      Info v;
                                                                   dfs_hld(v);
      bool operator < (const Range &b) const { return 1</pre>
                                                               out[u] = t;
          < b.1; }
                                                            }
   }:
                                                            // subtree of u => [ in[u], out[u] )
   std::vector<Range> upd(int 1, int r, Info v) {
                                                            // path from nxt[u] to u => [ in[ nxt[u] ], in[u] ]
       std::vector<Range> ans;
                                                            Iterative Segtree
       if(1 >= r) return ans;
       auto it = ranges.lower_bound(1);
                                                            T query(int 1, int r, int &pos){ // [1, r]
       if(it != ranges.begin()) {
                                                               T rl, rr;
          it--;
                                                               for(1 += n, r += n+1; 1 < r; 1 >>= 1, r >>= 1){
          if(it->r>1) {
                                                                   if(1 & 1) rl = merge(rl, st[l++]);
              auto cur = *it;
                                                                   if(r & 1) rr = merge(st[--r], rr);
              ranges.erase(it);
                                                               }
              ranges.insert(Range(cur.1, 1, cur.v));
                                                               return merge(rl, rr);
              ranges.insert(Range(1, cur.r, cur.v));
                                                            }
                                                            // initially save v[i] in st[n+i] for all i in [0, n)
      it = ranges.lower_bound(r);
                                                            void build(){
       if(it != ranges.begin()) {
                                                               for(int p = n-1; p > 0; p--)
          it--:
                                                                   st[p] = merge(st[2*p], st[2*p+1]);
          if(it->r > r) {
                                                            }
              auto cur = *it;
              ranges.erase(it);
                                                            void update(int p, T val){
              ranges.insert(Range(cur.1, r, cur.v));
                                                               st[p += n] = val;
              ranges.insert(Range(r, cur.r, cur.v));
                                                               while(p \gg 1) st[p] = merge(st[2*p], st[2*p+1]);
       for(it = ranges.lower_bound(l); it != ranges.end
                                                            LiChao's Segtree
         () && it->l < r; it++) {
          ans.push_back(*it);
                                                            void add_line(line nw, int v = 1, int l = 0, int r =
```

University of Brasilia Math, 6

```
maxn) { // [1, r)
   int m = (1 + r) / 2;
   bool lef = nw.eval(1) < st[v].eval(1);</pre>
   bool mid = nw.eval(m) < st[v].eval(m);</pre>
   if(mid) swap(st[v], nw);
   if(r - 1 == 1) {
       return;
   } else if(lef != mid) {
       add_line(nw, 2 * v, 1, m);
   } else {
       add_line(nw, 2 * v + 1, m, r);
}
int get(int x, int v = 1, int l = 0, int r = maxn) {
   int m = (1 + r) / 2;
   if(r - 1 == 1) {
       return st[v].eval(x);
   } else if(x < m) {</pre>
       return min(st[v].eval(x), get(x, 2*v, 1, m));
       return min(st[v].eval(x), get(x, 2*v+1, m, r));
Palindromic tree
#include <bits/stdc++.h>
```

```
using namespace std;
const int maxn = 3e5 + 1, sigma = 26;
int len[maxn], link[maxn], to[maxn][sigma];
int slink[maxn], diff[maxn], series_ans[maxn];
int sz, last, n;
char s[maxn];
void init()
{
   s[n++] = -1;
   link[0] = 1;
   len[1] = -1;
   sz = 2;
}
int get_link(int v)
   while(s[n - len[v] - 2] != s[n - 1]) v = link[v];
   return v;
void add_letter(char c)
   s[n++] = c -= 'a';
   last = get_link(last);
   if(!to[last][c])
   {
      len[sz] = len[last] + 2;
       link[sz] = to[get_link(link[last])][c];
       diff[sz] = len[sz] - len[link[sz]];
       if(diff[sz] == diff[link[sz]])
          slink[sz] = slink[link[sz]];
       else
          slink[sz] = link[sz];
       to[last][c] = sz++;
   last = to[last][c];
}
```

```
int main()
{
   ios::sync_with_stdio(0);
   cin.tie(0);
   init();
   string s;
   cin >> s;
   int n = s.size();
   int ans[n + 1];
   memset(ans, 63, sizeof(ans));
   ans[0] = 0;
   for(int i = 1; i <= n; i++)
       add_letter(s[i - 1]);
       for(int v = last; len[v] > 0; v = slink[v])
          series_ans[v] = ans[i - (len[slink[v]] + diff
             [v])]:
          if(diff[v] == diff[link[v]])
              series_ans[v] = min(series_ans[v],
                series_ans[link[v]]);
          ans[i] = min(ans[i], series_ans[v] + 1);
       }
       cout << ans[i] << "\n";
   }
   return 0;
```

### Math

# **Extended Euclidean Algorithm**

```
// a*x + b*y = gcd(a, b), < gcd, x, y>
tuple<int, int, int> gcd(int a, int b) {
   if(b == 0) return make_tuple(a, 1, 0);
   int q, w, e;
   tie(q, w, e) = gcd(b, a % b);
   return make_tuple(q, e, w - e * (a / b));
```

#### Chinese Remainder Theorem

```
// x = vet[i].first (mod vet[i].second)
11 crt(vector<pair<ll, ll>> vet){
   11 ans = vet[0].first, lcm = vet[0].second;
   ll a, b, g, x, y;
   for(int i = 1; i < (int)vet.size(); i++){</pre>
       tie(a, b) = vet[i];
       tie(g, x, y) = gcd(lcm, b);
       ans = ans + x * (a - ans) / g % (b / g) * lcm;
       lcm = lcm * b / g;
       ans = (ans \% lcm + lcm) \% lcm;
   return ans;
```

#### **Preffix inverse**

```
inv[1] = 1;
for(int i = 2; i < p; i++)</pre>
    inv[i] = (p - (p/i) * inv[p%i] % p) % p;
```

#### Pollard Rho

```
11 rho(ll n){
   if(n % 2 == 0) return 2;
```

University of Brasilia Math, 7

```
11 totiente(ll n){
   11 d, c, x, y;
                                                                11 \text{ ans} = n;
                                                                for(11 i = 2; i*i <= n; i++){</pre>
   do{
       c = 11rand() % n, x = 11rand() % n, y = x;
                                                                   if(n \% i == 0){
                                                                       ans = ans / i * (i - 1);
       do{
                                                                       while(n % i == 0) n /= i;
          x = add(mul(x, x, n), c, n);
          y = add(mul(y, y, n), c, n);
                                                                   }
          y = add(mul(y, y, n), c, n);
                                                                }
          d = \_gcd(abs(x - y), n);
       }while(d == 1);
                                                                if(n > 1) ans = ans / n * (n - 1);
   }while(d == n);
                                                                return ans;
   return d;
                                                            Mobius Function
ll pollard_rho(ll n){
                                                            memset(mu, 0, sizeof mu);
   ll x, c, y, d, k;
                                                            mu[1] = 1;
   int i;
                                                            for(int i = 1; i < N; i++)</pre>
   do{
                                                                for(int j = i + i; j < N; j += i)
      i = 1;
                                                                   mu[j] -= mu[i];
      x = 11rand() % n, c = 11rand() % n;
                                                            // g(n) = sum{f(d)} => f(n) = sum{mu(d)*g(n/d)}
      y = x, k = 4;
                                                            Mulmod TOP
       do{
          if(++i == k) y = x, k *= 2;
                                                            constexpr uint64_t mod = (1ull<<61) - 1;</pre>
          x = add(mul(x, x, n), c, n);
                                                            uint64_t modmul(uint64_t a, uint64_t b){
          d = \_gcd(abs(x - y), n);
                                                                uint64_t 11 = (uint32_t)a, h1 = a>>32, 12 = (
       }while(d == 1);
                                                                  uint32_t)b, h2 = b>>32;
   }while(d == n);
                                                                uint64_t 1 = 11*12, m = 11*h2 + 12*h1, h = h1*h2;
   return d;
                                                                uint64_t ret = (1\&mod) + (1>>61) + (h << 3) + (m >>
}
                                                                  29) + (m \ll 35 \gg 3) + 1;
                                                                ret = (ret & mod) + (ret>>61);
void factorize(ll val, map<ll, int> &fac){
                                                                ret = (ret & mod) + (ret>>61);
   if(rabin(val)) fac[ val ]++;
                                                                return ret-1;
   else{
       11 d = pollard_rho(val);
                                                            Matrix Determinant
       factorize(d, fac);
       factorize(val / d, fac);
                                                            int n;
   }
                                                            long double a[n][n];
}
map<ll, int> factor(ll val){
                                                            long double gauss(){
                                                                long double det = 1;
   map<ll, int> fac;
                                                                for(int i = 0; i < n; i++){</pre>
   if(val > 1) factorize(val, fac);
                                                                   int q = i;
   return fac;
                                                                   for(int j = i+1; j < n; j++){
                                                                       if(abs(a[j][i]) > abs(a[q][i]))
Miller Rabin
                                                                          q = j;
bool rabin(ll n){
                                                                   if(abs(a[q][i]) < EPS){
   if(n <= 1) return 0;</pre>
                                                                       det = 0;
   if(n <= 3) return 1;
                                                                       break;
   11 s = 0, d = n - 1;
   while(d % 2 == 0) d /= 2, s++;
                                                                   if(i != q){
   for(int k = 0; k < 64; k++){
                                                                       for(int w = 0; w < n; w++)
      11 a = (11rand() \% (n - 3)) + 2;
                                                                          swap(a[i][w], a[q][w]);
      11 x = fexp(a, d, n);
                                                                       det = -det;
       if(x != 1 \&\& x != n-1){
          for(int r = 1; r < s; r++){
                                                                   det *= a[i][i];
              x = mul(x, x, n);
                                                                   for(int j = i+1; j < n; j++) a[i][j] /= a[i][i];</pre>
              if(x == 1) return 0;
              if(x == n-1) break;
                                                                    for(int j = 0; j < n; j++) if(j != i){
                                                                       if(abs(a[j][i]) > EPS)
          if(x != n-1) return 0;
                                                                          for(int k = i+1; k < n; k++)
      }
                                                                              a[j][k] -= a[i][k] * a[j][i];
   }
                                                                   }
   return 1;
                                                                }
Totiente
                                                                return det;
```

University of Brasilia Math, 8

# Simplex Method

```
typedef long double dbl;
const dbl eps = 1e-6;
const int N = , M = ;
mt19937 rng(chrono::steady_clock::now().time_since_epoch
  ().count()):
struct simplex {
   int X[N], Y[M];
   dbl A[M][N], b[M], c[N];
   dbl ans;
   int n, m;
   dbl sol[N];
   void pivot(int x, int y){
       swap(X[y], Y[x]);
      b[x] /= A[x][y];
       for(int i = 0; i < n; i++)</pre>
          if(i != y)
              A[x][i] /= A[x][y];
      A[x][y] = 1. / A[x][y];
       for(int i = 0; i < m; i++)</pre>
          if(i != x && abs(A[i][y]) > eps) {
              b[i] -= A[i][y] * b[x];
              for(int j = 0; j < n; j++) if(j != y)
                 A[i][j] -= A[i][y] * A[x][j];
              A[i][y] = -A[i][y] * A[x][y];
          }
       ans += c[y] * b[x];
       for(int i = 0; i < n; i++)</pre>
          if(i != y)
              c[i] -= c[y] * A[x][i];
      c[y] = -c[y] * A[x][y];
   }
   // maximiza sum(x[i] * c[i])
   // sujeito a
   // sum(a[i][j] * x[j]) <= b[i] para 0 <= i < m (Ax)
   // x[i] >= 0 para 0 <= i < n (x >= 0)
   // (n variaveis, m restricoes)
   // guarda a resposta em ans e retorna o valor otimo
   dbl solve(int _n, int _m) {
       this->n = _n; this->m = _m;
       for(int i = 1; i < m; i++){
          int id = uniform_int_distribution<int>(0, i)(
            rng);
          swap(b[i], b[id]);
          for(int j = 0; j < n; j++)
              swap(A[i][j], A[id][j]);
      }
       ans = 0.;
       for(int i = 0; i < n; i++) X[i] = i;
       for(int i = 0; i < m; i++) Y[i] = i + n;
          int x = min_element(b, b + m) - b;
          if(b[x] >= -eps)
             break:
          int y = find_if(A[x], A[x] + n, [](dbl d) {
            return d < -eps; }) - A[x];</pre>
          if(y == n) throw 1; // no solution
          pivot(x, y);
      while(true) {
          int y = max_element(c, c + n) - c;
```

```
if(c[y] <= eps) break;</pre>
          int x = -1;
          dbl mn = 1. / 0.;
          for(int i = 0; i < m; i++)
              if(A[i][y] > eps && b[i] / A[i][y] < mn)</pre>
                 mn = b[i] / A[i][y], x = i;
          if(x == -1) throw 2; // unbounded
          pivot(x, y);
       memset(sol, 0, sizeof(dbl) * n);
       for(int i = 0; i < m; i++)
          if(Y[i] < n)
              sol[Y[i]] = b[i];
       return ans;
};
FFT
struct base{
   double r, i;
   base(double _r = 0, double _i = 0) : r(_r), i(_i) {}
   base operator*(base &o) const{
       return {r*o.r - i*o.i, r*o.i + o.r*i};
   double real() const{ return r; }
   void operator*=(const base &o){
       (*this) = \{r*o.r-i*o.i, r*o.i+o.r*i\};
   }
   void operator+=(const base &o){r += o.r, i += o.i; }
   void operator/=(const double &o){ r /= o, i /= o; }
   void operator==(const base &o){r == o.r, i == o.i; }
   base operator+(const base &o){return {r+o.r,i+o.i};}
   base operator-(const base &o){return {r-o.r,i-o.i};}
};
double PI = acos(-1);
void fft(vector<base> &a, bool inv){
   int n = (int)a.size();
   for(int i = 1, j = 0; i < n; i++){
       int bit = n \gg 1;
       for(; j >= bit; bit >>= 1) j -= bit;
       j += bit;
       if(i < j) swap(a[i], a[j]);
   for(int sz = 2; sz <= n; sz <<= 1) {
       double ang = 2*PI/sz * (inv ? -1 : 1);
       base wlen(cos(ang), sin(ang));
       for(int i = 0; i < n; i += sz){
          base w(1);
          for(int j = 0; j < sz/2; j++){
              base u = a[i+j], v = a[i+j+sz/2] * w;
              a[i+j] = u + v;
              a[i+j+sz/2] = u - v;
              w *= wlen;
          }
       }
   if(inv) for(int i = 0; i < n; i++) a[i] /= 1.0 * n;
}
void multiply(const vector<int> &a, const vector<int> &b
  , vector<int> &res){
   vector<base> fa(a.begin(), a.end());
   vector<base> fb(b.begin(), b.end());
```

University of Brasilia Graphs, 9

```
size_t n = 1;
                                                              } edge[E];
   while(n < a.size()) n <<= 1;</pre>
                                                              int lvl[N], vis[N], pass, start = N-2, target = N-1;
   while(n < b.size()) n <<= 1;</pre>
                                                              int qu[N], qt, px[N];
   n <<= 1:
   fa.resize(n), fb.resize(n);
                                                              11 run(int s, int sink, 11 minE){
   fft(fa, false), fft(fb, false);
                                                                 if(s == sink) return minE;
   for(size_t i = 0; i < n; i++)</pre>
       fa[i] *= fb[i];
                                                                 11 ans = 0;
   fft(fa, true);
                                                                 for(; px[s] < (int)g[s].size(); px[s]++){</pre>
   res.resize (n);
                                                                     int e = g[s][ px[s] ];
   for(size_t i = 0; i < n; ++i)</pre>
                                                                     auto &v = edge[e], &rev = edge[e^1];
                                                                     if(lvl[v.to] != lvl[s]+1 || v.flow >= v.cap)
       res[i] = int(fa[i].real() + 0.5);
                                                                         continue; // v.cap - v.flow < lim</pre>
                                                                     11 tmp = run(v.to, sink,min(minE, v.cap-v.flow));
NTT
                                                                     v.flow += tmp, rev.flow -= tmp;
                                                                     ans += tmp, minE -= tmp;
const int mod = 7340033;
                                                                     if(minE == 0) break;
const int root = 5;
                                                                 }
const int root_1 = 4404020;
                                                                 return ans;
const int root_pw = 1<<20;</pre>
                                                              }
void fft (vector<int> & a, bool invert) {
                                                              bool bfs(int source, int sink){
   int n = (int) a.size();
                                                                 at = 0;
                                                                 qu[qt++] = source;
   for (int i=1, j=0; i<n; ++i) {
                                                                 lvl[source] = 1;
       int bit = n \gg 1;
                                                                 vis[source] = ++pass;
       for (; j>=bit; bit>>=1)
          j -= bit;
                                                                 for(int i = 0; i < qt; i++){</pre>
       j += bit;
                                                                     int u = qu[i];
       if (i < j)
                                                                     px[u] = 0;
          swap (a[i], a[j]);
                                                                     if(u == sink) return true;
                                                                     for(int e : g[u]){
   for (int len=2; len<=n; len<<=1) {</pre>
                                                                         auto v = edge[e];
       int wlen = invert ? root_1 : root;
                                                                         if(v.flow >= v.cap || vis[v.to] == pass)
       for (int i=len; i<root_pw; i<<=1)</pre>
                                                                            continue; // v.cap - v.flow < lim</pre>
          wlen = int (wlen * 111 * wlen % mod);
                                                                         vis[v.to] = pass;
       for (int i=0; i<n; i+=len) {</pre>
                                                                         lvl[v.to] = lvl[u]+1;
          int w = 1;
                                                                         qu[qt++] = v.to;
           for (int j=0; j<len/2; ++j) {</pre>
                                                                     }
              int u = a[i+j], v = int (a[i+j+len/2] * 1
                                                                 }
                ll * w % mod);
                                                                 return false;
              a[i+j] = u+v < mod ? u+v : u+v-mod;
              a[i+j+len/2] = u-v >= 0 ? u-v : u-v+mod;
              w = int (w * 111 * wlen % mod);
                                                              11 flow(int source = start, int sink = target){
          }
                                                                 11 \text{ ans} = 0;
       }
                                                                 //for(lim = (1LL << 62); lim >= 1; lim /= 2)
   }
                                                                 while(bfs(source, sink))
   if (invert) {
                                                                     ans += run(source, sink, oo);
       int nrev = reverse (n, mod);
                                                                 return ans;
       for (int i=0; i<n; ++i)</pre>
           a[i] = int (a[i] * 111 * nrev % mod);
   }
                                                              void addEdge(int u, int v, ll c = 1, ll rc = 0){
}
                                                                 edge[ne] = \{u, v, 0, c\};
Graphs
                                                                 g[u].push_back(ne++);
                                                                 edge[ne] = \{v, u, 0, rc\};
Dinic
                                                                 g[v].push_back(ne++);
                                                              }
const int N = 100005;
const int E = 2000006;
                                                              void reset_flow(){
vector<int> g[N];
                                                                 for(int i = 0; i < ne; i++)</pre>
                                                                     edge[i].flow = 0;
int ne;
                                                              }
struct Edge{
                                                              Min Cost Max Flow
   int from, to;
```

11 flow, cap;

University of Brasilia Graphs, 10

```
const 11 oo = 1e18;
const int N = 505;
const int E = 30006;
vector<int> g[N];
int ne:
struct Edge{
   int from, to;
   11 cap, cost;
} edge[E];
int lvl[N], vis[N], pass, source, target, p[N], px[N];
11 d[N];
11 back(int s, ll minE){
   if(s == source) return minE;
   int e = p[s];
   11 f = back(edge[e].from, min(minE, edge[e].cap));
   edge[e].cap -= f;
   edge[e^1].cap += f;
   return f:
}
int dijkstra(){
   forn(i, N) d[i] = oo;
   priority_queue<pair<ll, int> > q;
   d[source] = 0;
   q.emplace(0, source);
   while(!q.empty()){
       11 dis = -q.top().ff;
       int u = q.top().ss; q.pop();
       if(dis > d[u]) continue;
       for(int e : g[u]){
          auto v = edge[e];
          if(v.cap <= 0) continue;</pre>
          if(d[u] + v.cost < d[v.to]){
              d[v.to] = d[u] + v.cost;
              p[v.to] = e;
              q.emplace(-d[v.to], v.to);
   return d[target] != oo;
pair<11, 11> mincost(){
   ll ans = 0, mf = 0;
   while(dijkstra()){
       11 f = back(target, oo);
       mf += f;
       ans += f * d[target];
   return {mf, ans};
}
void addEdge(int u, int v, 11 c, 11 cost){
```

```
edge[ne] = \{u, v, c, cost\};
   g[u].pb(ne++);
}
Small to Large
void cnt_sz(int u, int p = -1){
   sz[u] = 1;
   for(int v : g[u]) if(v != p)
       cnt_sz(v, u), sz[u] += sz[v];
void add(int u, int p, int big = -1){
   // Update info about this vx in global answer
   for(int v : g[u]) if(v != p && v != big)
       add(v, u);
}
void dfs(int u, int p, int keep){
   int big = -1, mmx = -1;
   for(int v : g[u]) if(v != p \&\& sz[v] > mmx)
       mmx = sz[v], big = v;
   for(int v : g[u]) if(v != p && v != big)
       dfs(v, u, 0);
   if(big != -1) dfs(big, u, 1);
   add(u, p, big);
   for(auto x : q[u]){
       // answer all queries for this vx
   if(!keep){
       // Remove data from this subtree
Junior e Falta de Ideias
#include <bits/stdc++.h>
#define ff first
#define ss second
#define mp make_pair
using namespace std;
typedef long long 11;
vector<pair<int,int>> G[500005];
int subtree[500005], treesize, k;
bool vis[500005];
ll dist[500005], ans;
int dfs(int v, int p){
   subtree[v] = 1;
   for(pair<int,int> x : G[v])
       if(x.ff != p \&\& !vis[x.ff]) subtree[v] += dfs(x.
         ff,v);
   return subtree[v];
}
int centroid(int v, int p){
```

University of Brasilia Graphs, 11

```
for(pair<int,int> x : G[v]){
       if(x.ff == p || vis[x.ff]) continue;
       if(subtree[x.ff]*2 > treesize) return centroid(x.
   }
   return v;
}
void procurar_ans(int v, int p, int d_atual, ll custo){
   ans = min(ans, dist[k-d_atual] + custo);
   if(d_atual == k) return;
   for(pair<int,int> x : G[v]){
       if(!vis[x.ff] && x.ff != p)
          procurar_ans(x.ff,v,d_atual+1,custo+x.ss);
   }
}
void atualiza_distancia(int v, int p, int d_atual, 11
   dist[d_atual] = min(dist[d_atual], custo);
   if(d_atual == k) return;
   for(pair<int,int> x : G[v]){
       if(!vis[x.ff] && x.ff != p)
          atualiza_distancia(x.ff,v,d_atual+1,custo+x.
   }
}
void decomp(int v, int p){
   treesize = dfs(v,v);
   // if(treesize < k) return;</pre>
   int cent = centroid(v,v);
   vis[cent] = 1;
   for(int i = 1; i <= treesize; i++)</pre>
       dist[i] = 1e18;
   for(pair<int,int> x : G[cent]){
       if(!vis[x.ff]){
          procurar_ans(x.ff,cent,1,x.ss);
          atualiza_distancia(x.ff,cent,1,x.ss);
   }
   for(pair<int,int> x : G[cent]){
       if(!vis[x.ff])
          decomp(x.ff, cent);
   }
}
int main(){
   int n,i,a,b;
   scanf("%d%d", &n,&k);
   for(i = 2; i \le n; i++){
       scanf("%d%d", &a,&b);
       G[i].push_back(mp(a,b));
       G[a].push_back(mp(i,b));
   }
   ans = 1e18:
   decomp(1,-1);
   printf("%11d\n", ans == 1e18 ? -1 : ans);
   return 0;
}
```

## Kosaraju

```
vector<int> g[N], gt[N], S;
int vis[N], cor[N], tempo = 1;
void dfs(int u){
   vis[u] = 1;
   for(int v : g[u]) if(!vis[v]) dfs(v);
   S.push_back(u);
}
int e;
void dfst(int u){
   cor[u] = e;
   for(int v : gt[u]) if(!cor[v]) dfst(v);
}
int main(){
   for(int i = 1; i <= n; i++) if(!vis[i]) dfs(i);</pre>
   e = 0;
   reverse(S.begin(), S.end());
   for(int u : S) if(!cor[u])
       e++, dfst(u);
   return 0;
}
Tarjan
int cnt = 0, root;
void dfs(int u, int p = -1){
   low[u] = num[u] = ++t;
   for(int v : g[u]){
       if(!num[v]){
          dfs(v, u);
              if(u == root) cnt++;
          if(low[v] >= num[u]) u PONTO DE ARTICULAÇÃO;
          if(low[v] > num[u]) ARESTA u->v PONTE;
          low[u] = min(low[u], low[v]);
       else if(v != p) low[u] = min(low[u], num[v]);
   }
}
root PONTO DE ARTICULAÇÃO <=> cnt > 1
void tarjanSCC(int u){
   low[u] = num[u] = ++cnt;
   vis[u] = 1;
   S.push_back(u);
   for(int v : g[u]){
       if(!num[v]) tarjanSCC(v);
       if(vis[v]) low[u] = min(low[u], low[v]);
   if(low[u] == num[u]){
       ssc[u] = ++ssc_cnt; int v;
       do{
          v = S.back(); S.pop_back(); vis[v] = 0;
          ssc[v] = ssc_cnt;
       }while(u != v);
   }
Max Clique
long long adj[N], dp[N];
```

University of Brasilia Graphs, 12

```
for(int i = 0; i < n; i++){
   for(int j = 0; j < n; j++){
       int x;
       scanf("%d",&x);
       if(x \mid | i == j)
           adj[i] |= 1LL << j;
   }
}
int resto = n - n/2;
int C = n/2;
for(int i = 1; i < (1 << resto); i++){</pre>
   int x = i;
   for(int j = 0; j < resto; j++)
       if(i & (1 << j))</pre>
           x \&= adj[j + C] >> C;
   if(x == i){
       dp[i] = __builtin_popcount(i);
   }
}
for(int i = 1; i < (1 << resto); i++)</pre>
   for(int j = 0; j < resto; j++)
       if(i \& (1 << j))
           dp[i] = max(dp[i], dp[i ^ (1 << j)]);
int maxCliq = 0;
for(int i = 0; i < (1 << C); i++){</pre>
   int x = i, y = (1 << resto) - 1;
   for(int j = 0; j < C; j++)
       if(i & (1 << j))
           x \&= adj[j] \& ((1 << C) - 1), y \&= adj[j] >>
   if(x != i) continue;
   maxCliq = max(maxCliq, __builtin_popcount(i) + dp[y
     1);
```

# **Dominator Tree**

```
vector<int> g[N], gt[N], T[N];
vector<int> S;
int dsu[N], label[N];
int sdom[N], idom[N], dfs_time, id[N];
vector<int> bucket[N];
vector<int> down[N];
void prep(int u){
   S.push_back(u);
   id[u] = ++dfs_time;
   label[u] = sdom[u] = dsu[u] = u;
   for(int v : g[u]){
       if(!id[v])
          prep(v), down[u].push_back(v);
       gt[v].push_back(u);
   }
}
int fnd(int u, int flag = 0){
   if(u == dsu[u]) return u;
   int v = fnd(dsu[u], 1), b = label[ dsu[u] ];
   if(id[ sdom[b] ] < id[ sdom[ label[u] ] ])</pre>
       label[u] = b;
   dsu[u] = v;
   return flag ? v : label[u];
}
```

```
void build_dominator_tree(int root, int sz){
   // memset(id, 0, sizeof(int) * (sz + 1));
   // for(int i = 0; i <= sz; i++) T[i].clear();
   prep(root);
   reverse(S.begin(), S.end());
   int w;
   for(int u : S){
       for(int v : gt[u]){
          w = fnd(v);
          if(id[ sdom[w] ] < id[ sdom[u] ])</pre>
              sdom[u] = sdom[w];
       gt[u].clear();
       if(u != root) bucket[ sdom[u] ].push_back(u);
       for(int v : bucket[u]){
          w = fnd(v);
          if(sdom[w] == sdom[v]) idom[v] = sdom[v];
          else idom[v] = w;
       bucket[u].clear();
       for(int v : down[u]) dsu[v] = u;
       down[u].clear();
   }
   reverse(S.begin(), S.end());
   for(int u : S) if(u != root){
       if(idom[u] != sdom[u]) idom[u] = idom[ idom[u] ];
       T[ idom[u] ].push_back(u);
   S.clear();
Min Cost Matching
// Min cost matching
// O(n^2 * m)
// n == nro de linhas
```

```
// m == nro de colunas
// n <= m | flow == n
// a[i][j] = custo pra conectar i a j
vector<int> u(n + 1), v(m + 1), p(m + 1), way(m + 1);
for(int i = 1; i \le n; ++i){
   p[0] = i;
   int j0 = 0;
   vector<int> minv(m + 1 , oo);
   vector<char> used(m + 1 , false);
       used[j0] = true;
       int i0 = p[j0] , delta = oo, j1;
       for(int j = 1; j \le m; ++j)
           if(! used[j]){
              int cur = a[i0][j] - u[i0] - v[j];
              if(cur < minv[j])</pre>
                  minv[j] = cur, way[j] = j0;
              if(minv[j] < delta)</pre>
                  delta = minv[j] , j1 = j;
           }
       for(int j = 0; j \le m; ++j)
           if(used[j])
              u[p[j]] \leftarrow delta, v[j] \leftarrow delta;
              minv[j] -= delta;
       j0 = j1;
```

University of Brasilia Strings, 13

```
}while(p[j0] != 0);
                                                                1]++;
                                                              partial_sum(c, c+mx, c);
   do{
                                                              int t;
      int j1 = way[j0];
                                                              for(int i = 0; i < n; i++)
      p[j0] = p[j1];
                                                                  t = sa[i]+k < n ? r[ sa[i]+k ]+1 : 0,
      j0 = j1;
                                                                  tsa[c[t]++] = sa[i];
                                                              memcpy(sa, tsa, sizeof(int) * n);
   }while(j0);
}
                                                           }
// match[i] = coluna escolhida para linha i
                                                           void build_sa(){
vector<int> match(n + 1);
                                                              for(int i = 0; i < n; i++) sa[i] = i, r[i] = s[i];
for(int j = 1; j \le m; ++j)
   match[p[j]] = j;
                                                              int t = 300, a, b;
int cost = -v[0];
                                                              for(int sz = 1; sz < n; sz *= 2){
                                                                  sort(sz, t), sort(0, t);
Strings
                                                                  t = nr[ sa[0] ] = 0;
                                                                  for(int i = 1; i < n; i++){
Aho Corasick
                                                                     a = sa[i]+sz < n ? r[ sa[i]+sz ] : -1;
                                                                     b = sa[i-1]+sz < n ? r[ sa[i-1]+sz ] : -1;
map<char, int> to[N];
                                                                     nr[sa[i]] = r[sa[i]] == r[sa[i-1]] && a
int ne = 1, term[N], fail[N];
                                                                         == b ? t : ++t;
                                                                  }
void add_string(char *str){
                                                                  if(t == n-1) break;
   int p = 0;
                                                                  memcpy(r, nr, sizeof(int) * n);
                                                              }
   for(int i = 0; str[i]; i++){
                                                           }
      if(!to[p][ str[i] ]) to[p][ str[i] ] = ne++;
      p = to[p][ str[i] ];
                                                           void build_lcp(){ // lcp[i] = lcp(s[:i], s[:i+1])
   }
                                                              int k = 0;
   term[p] = 1;
                                                              for(int i = 0; i < n; i++) r[ sa[i] ] = i;
}
                                                              for(int i = 0; i < n; i++){
int go(int s, char c){
                                                                  if(r[i] == n-1) k = 0;
   while(s && !to[s].count(c)) s = fail[s];
                                                                  else{
   if(to[s].count(c)) return to[s][c];
                                                                     int j = sa[r[i]+1];
   return s;
                                                                     while(i+k < n && j+k < n && s[i+k] == s[j+k])
                                                                        k++:
                                                                  }
void init(){
                                                                  lcp[r[i]] = k;
                                                                  if(k) k--;
   queue<int> q;
                                                              }
   q.push(0);
                                                           }
   int u, v; char c;
                                                           int comp_lcp(int i, int j){
   while(!q.empty()){
                                                              if(i == j) return n - i;
      u = q.front(); q.pop();
                                                              if(r[i] > r[j]) swap(i, j);
                                                              return min(lcp[k] for k in [r[i], r[j]-1]);
       for(auto w : to[u]){
          tie(c, v) = w;
                                                           Z Algorithm
          q.push(v);
          if(u){}
              fail[v] = go(fail[u], c);
                                                           vector<int> z_algo(const string &s) {
                                                              int n = s.size(), L = 0, R = 0;
             term[v] |= term[ fail[v] ];
                                                              vector < int > z(n, 0);
          }
                                                              for(int i = 1; i < n; i++){
      }
                                                                  if(i \le R) z[i] = min(z[i-L], R - i + 1);
   }
                                                                  while(z[i]+i < n \&\& s[z[i]+i] == s[z[i]])
                                                                     z[i]++;
Suffix Array
                                                                  if(i+z[i]-1 > R) L = i, R = i + z[i] - 1;
                                                              }
                                                              return z;
char s[N];
int n, sa[N], tsa[N], lcp[N], r[N], nr[N], c[N];
                                                           Prefix function/KMP
void sort(int k, int mx){
   mx += 2;
                                                           vector<int> preffix_function(const string &s){
   memset(c, 0, sizeof(int) * mx);
                                                              int n = s.size();
   for(int i = 0; i < n; i++) c[i + k < n ? r[i+k]+2 :
                                                              vector<int> b(n+1);
```

University of Brasilia Strings, 14

```
b[0] = -1;
                                                               vector<int> vet[2];
   int i = 0, j = -1;
                                                               for(int add = 0; add < 2; add++){
   while(i < n){
       while(j \ge 0 \& s[i] != s[j]) j = b[j];
                                                                   vet[add].resize(n);
                                                                   for(int i = 0, l = 0, r = -1; i < n; i++){
      b[++i] = ++j;
                                                                      int k = i > r? add : min(vet[add][1 + r - i
   return b;
                                                                         + !add], r - i + !add);
                                                                      while(i - k - !add >= 0 \&\& i + k < n \&\& s[i -
}
                                                                         k - !add] == s[i + k]) k++;
void kmp(const string &t, const string &p){
                                                                      vet[add][i] = k--;
   vector<int> b = preffix_function(p);
                                                                      if(i + k > r){
   int n = t.size(), m = p.size();
                                                                          r = i + k;
   int j = 0;
                                                                          l = i - k - !add;
   for(int i = 0; i < n; i++){
                                                                      }
      while(j \ge 0 \& t[i] != p[j]) j = b[j];
                                                                  }
                                                               }
       i++:
      if(j == m){
          //patern of p found on t
                                                               return make_pair(vet[0], vet[1]);
          i = b[i];
                                                            Palindromic Tree
   }
                                                            const int MAXN = 105000;
Min rotation
                                                            struct node {
int min_rotation(int *s, int N) {
                                                               int next[26];
 REP(i, N) s[N+i] = s[i];
                                                               int len;
                                                               int sufflink;
 int a = 0;
                                                               int num:
 REP(b, N) REP(i, N) {
                                                            };
   if (a+i == b \mid \mid s[a+i] < s[b+i]) { b += max(0, i-1);}
      break; }
                                                            int len:
   if (s[a+i] > s[b+i]) \{ a = b; break; \}
                                                            char s[MAXN];
                                                            node tree[MAXN];
 return a;
                                                            int num; // node 1 - root with len -1, node 2 - root
                                                              with len 0
All palindrome
                                                            int suff; // max suffix palindrome
                                                            long long ans;
void manacher(char *s, int N, int *rad) {
 static char t[2*MAX];
                                                            bool addLetter(int pos) {
 int m = 2*N - 1;
                                                               int cur = suff, curlen = 0;
                                                               int let = s[pos] - 'a';
 REP(i, m) t[i] = -1;
 REP(i, N) t[2*i] = s[i];
                                                               while(true){
                                                                   curlen = tree[cur].len;
 int x = 0;
                                                                   if (pos-1 - curlen >= 0 \&\& s[pos-1 - curlen] == s
 FOR(i, 1, m) {
                                                                     [pos])
   int &r = rad[i] = 0;
                                                                      break:
   if (i <= x+rad[x]) r = min(rad[x+x-i], x+rad[x]-i);</pre>
                                                                   cur = tree[cur].sufflink;
   while (i-r-1 >= 0 \&\& i+r+1 < m \&\& t[i-r-1] == t[i+r]
     +1]) ++r;
                                                               if (tree[cur].next[let]) {
   if (i+r >= x+rad[x]) x = i;
                                                                   suff = tree[cur].next[let];
 }
                                                                   return false;
                                                               }
 REP(i, m) if (i-rad[i] == 0 || i+rad[i] == m-1) ++rad[
                                                               num++;
 REP(i, m) rad[i] /= 2;
                                                               suff = num;
                                                               tree[num].len = tree[cur].len + 2;
Manacher
                                                               tree[cur].next[let] = num;
// odd[i] = largest palindrome with center in character
                                                               if (tree[num].len == 1){
                                                                   tree[num].sufflink = 2;
 i(0-based)
// even[i] = largest palindrome with center between
                                                                   tree[num].num = 1;
  chars i-1 and i(0-based)
                                                                   return true;
// pair<even. odd>
                                                               }
pair<vector<int>, vector<int>> build(const string &s){
                                                               while (true){
   int n = (int)s.size();
                                                                   cur = tree[cur].sufflink;
```

University of Brasilia Geometry, 15

```
curlen = tree[cur].len;
                                                               node(int L, int R, int S, int P) : 1(L), r(R), si(S)
       if(pos-1 - curlen) = 0 & s[pos-1 - curlen] == s[
                                                                  , p(P) \{ \}
                                                               inline int len() { return r - l + 1; }
         }([sog
          tree[num].sufflink = tree[cur].next[let];
                                                               inline int operator[](int i) { return S[si][l + i];
          break;
                                                                 }
                                                               inline int& operator()(char c) { return adj[c]; }
      }
                                                            } t[N];
   }
                                                            inline int new_node(int L, int R, int S, int P) { t[en]
   tree[num].num = 1 + tree[tree[num].sufflink].num;
                                                              = node(L, R, S, P); return en++; }
                                                            void add_string(string s) {
   return true;
                                                               s += '; S[++si] = s; sufn[si].resize(s.size() + 1)
}
                                                                  ; cn = cd = 0;
                                                               int i = 0; const int n = s.size();
                                                               for(int j = 0; j < n; j++)
void initTree() {
   num = 2; suff = 2;
                                                                   for(; i <= j; i++) {</pre>
   tree[1].len = -1; tree[1].sufflink = 1;
                                                                      if(cd == t[cn].len() \&\& t[cn](s[j])) { cn = t}
   tree[2].len = 0; tree[2].sufflink = 1;
                                                                         [cn](s[j]); cd = 0; }
                                                                       if(cd < t[cn].len() \& t[cn][cd] == s[j]) {
                                                                          cd++:
int main() {
                                                                          if(j < s.size() - 1) break;</pre>
                                                                          else {
   initTree();
                                                                             if(i) t[lst].suf = cn;
                                                                              for(; i <= j; i++) { sufn[si][i] = cn;</pre>
   for (int i = 0; i < len; i++) {</pre>
                                                                                cn = t[cn].suf; }
                                                                          }
      addLetter(i);
                                                                       } else if(cd == t[cn].len()) {
                                                                          sufn[si][i] = en;
                                                                          if(i) t[lst].suf = en; lst = en;
   return 0:
}
                                                                          t[cn](s[j]) = new_node(j, n - 1, si, cn);
                                                                          cn = t[cn].suf; cd = t[cn].len();
Suffix Automaton
                                                                      } else {
                                                                          int mid = new_node(t[cn].1, t[cn].1 + cd -
map<char, int> to[2*N];
                                                                             1, t[cn].si, t[cn].p);
int link[2*N], len[2*N], last = 0, sz = 1;
                                                                          t[t[cn].p](t[cn][0]) = mid;
                                                                          if(ns) t[ns].suf = mid;
void add_letter(char c){
                                                                          if(i) t[lst].suf = en; lst = en;
   int p = last;
                                                                          sufn[si][i] = en;
   last = sz++;
                                                                          t[mid](s[j]) = new_node(j, n - 1, si, mid)
   len[last] = len[p] + 1;
   for(; !to[p][c]; p = link[p]) to[p][c] = last;
                                                                          t[mid](t[cn][cd]) = cn;
   if(to[p][c] == last){
                                                                          t[cn].p = mid; t[cn].l += cd; cn = t[mid].
      link[last] = 0;
       return;
                                                                          int g = cn? j - cd : i + 1; cn = t[cn].suf
   int u = to[p][c];
                                                                          while (g < j \& g + t[t[cn](S[si][g])].len
   if(len[u] == len[p]+1){
                                                                            () <= j) {
      link[last] = u;
                                                                             cn = t[cn](S[si][g]); g += t[cn].len();
      return;
   }
                                                                          if(g == j) \{ ns = 0; t[mid].suf = cn; cd = 
   int c1 = sz++;
                                                                             t[cn].len(); }
   to[c1] = to[u];
                                                                          else { ns = mid; cn = t[cn](S[si][g]); cd
   link[c1] = link[u];
                                                                            = j - g; }
   len[c1] = len[p]+1;
                                                                      }
   link[last] = link[u] = c1;
                                                                   }
   for(; to[p][c] == u; p = link[p]) to[p][c] = c1;
                                                            };
Suffix Tree
                                                            Geometry
namespace sf {
                                                            2D basics
// const int NS = ; const int N = * 2;
int cn, cd, ns, en = 1, lst;
                                                            typedef double cod;
string S[NS]; int si = -1;
vector<int> sufn[N]; // sufn[si][i] no do sufixo S[si][i
                                                            double eps = 1e-7;
                                                            bool eq(cod a, cod b){ return abs(a - b) <= eps; }</pre>
  . . . ]
struct node {
   int 1, r, si, p, suf;
                                                            struct vec{
   map<char, int> adj;
                                                               cod x, y; int id;
   node() : 1(0), r(-1), suf(0), p(0) {}
                                                               vec(cod a = 0, cod b = 0) : x(a), y(b) {}
```

University of Brasilia Geometry, 16

```
vec operator+(const vec &o) const{
   return \{x + o.x, y + o.y\};
}
vec operator-(const vec &o) const{
   return \{x - o.x, y - o.y\};
}
vec operator*(cod t) const{
   return {x * t, y * t};
vec operator/(cod t) const{
   return {x / t, y / t};
cod operator*(const vec &o) const{ // cos
   return x * o.x + y * o.y;
}
cod operator^(const vec &o) const{ // sin
   return x * o.y - y * o.x;
bool operator==(const vec &o) const{
   return eq(x, o.x) && eq(y, o.y);
bool operator<(const vec &o) const{</pre>
   if(!eq(x, o.x)) return x < o.x;
   return y < o.y;</pre>
}
cod cross(const vec &a, const vec &b) const{
   return (a-(*this)) ^ (b-(*this));
int ccw(const vec &a, const vec &b) const{
   cod tmp = cross(a, b);
   return (tmp > eps) - (tmp < -eps);</pre>
cod dot(const vec &a, const vec &b) const{
   return (a-(*this)) * (b-(*this));
}
cod len() const{
   return sqrt(x * x + y * y); // <</pre>
double angle(const vec &a, const vec &b) const{
   return atan2(cross(a, b), dot(a, b));
double tan(const vec &a, const vec &b) const{
   return cross(a, b) / dot(a, b);
}
vec unit() const{
   return operator/(len());
}
int quad() const{
   if(x > 0 \&\& y >=0) return 0;
   if(x \le 0 \&\& y > 0) return 1;
   if(x < 0 \&\& y <=0) return 2;
   return 3;
bool comp(const vec &a, const vec &b) const{
   return (a - *this).comp(b - *this);
bool comp(vec b){
   if(quad() != b.quad()) return quad() < b.quad();</pre>
   if(!eq(operator^(b), 0)) return operator^(b) > 0;
   return (*this) * (*this) < b * b;</pre>
template<class T>
void sort_by_angle(T first, T last) const{
   std::sort(first, last, [=](const vec &a, const
     vec &b){
       return comp(a, b);
   });
```

```
vec rot90() const{ return {-y, x}; }
   vec rot(double a) const{
       return \{\cos(a)*x - \sin(a)*y, \sin(a)*x + \cos(a)*y\};
};
struct line{
   cod a, b, c; vec n;
   line(vec q, vec w){ // q.cross(w, (x, y)) = 0
       a = -(w.y-q.y);
      b = w.x-q.x;
      c = -(a * q.x + b * q.y);
      n = \{a, b\};
   cod dist(const vec &o) const{
       return abs(eval(o)) / n.len();
   bool contains(const vec &o) const{
       return eq(a * o.x + b * o.y + c, 0);
   cod dist(const line &o) const{
       if(!parallel(o)) return 0;
       if(!eq(o.a * b, o.b * a)) return 0;
       if(!eq(a, 0))
          return abs(c - o.c * a / o.a) / n.len();
       if(!eq(b, 0))
          return abs(c - o.c * b / o.b) / n.len();
       return abs(c - o.c);
   bool parallel(const line &o) const{
       return eq(n ^ o.n, 0);
   bool operator==(const line &o) const{
       if(!eq(a*o.b, b*o.a)) return false;
       if(!eq(a*o.c, c*o.a)) return false;
       if(!eq(c*o.b, b*o.c)) return false;
       return true;
   }
   bool intersect(const line &o) const{
      return !parallel(o) || *this == o;
   vec inter(const line &o) const{
       if(parallel(o)){
          if(*this == o){ }
          else{ /* dont intersect */ }
       }
       auto tmp = n ^ o.n;
       return {(o.c*b -c*o.b)/tmp, (o.a*c -a*o.c)/tmp};
   vec at_x(cod x) const{
       return \{x, (-c-a*x)/b\};
   vec at_y(cod y) const{
       return \{(-c-b*y)/a, y\};
   cod eval(const vec &o) const{
       return a * o.x + b * o.y + c;
};
struct segment{
   segment(vec a = vec(), vec b = vec()): p(a), q(b) {}
   bool onstrip(const vec &o) const{ // onstrip strip
       return p.dot(o, q) >= -eps && q.dot(o, p) >= -eps
```

University of Brasilia Geometry, 17

```
}
   cod len() const{
      return (p-q).len();
   cod dist(const vec &o) const{
      if(onstrip(o)) return line(p, q).dist(o);
      return min((o-q).len(), (o-p).len());
   bool contains(const vec &o) const{
      return eq(p.cross(q, o), 0) && onstrip(o);
   bool intersect(const segment &o) const{
      if(contains(o.p)) return true;
      if(contains(o.q)) return true;
      if(o.contains(q)) return true;
      if(o.contains(p)) return true;
      return p.ccw(q, o.p) * p.ccw(q, o.q) == -1
      && o.p.ccw(o.q, q) * o.p.ccw(o.q, p) == -1;
   bool intersect(const line &o) const{
      return o.eval(p) * o.eval(q) <= 0;</pre>
   cod dist(const segment &o) const{
      if(line(p, q).parallel(line(o.p, o.q))){
          if(onstrip(o.p) || onstrip(o.q)
          || o.onstrip(p) || o.onstrip(q))
             return line(p, q).dist(line(o.p, o.q));
      else if(intersect(o)) return 0;
      return min(min(dist(o.p), dist(o.q)),
                min(o.dist(p), o.dist(q)));
   cod dist(const line &o) const{
      if(line(p, q).parallel(o))
          return line(p, q).dist(o);
      else if(intersect(o)) return 0;
      return min(o.dist(p), o.dist(q));
   }
};
struct hray{
   vec p, q;
   hray(vec a = vec(), vec b = vec()): p(a), q(b){}
   bool onstrip(const vec &o) const{ // onstrip strip
      return p.dot(q, o) >= -eps;
   }
   cod dist(const vec &o) const{
      if(onstrip(o)) return line(p, q).dist(o);
      return (o-p).len();
   bool intersect(const segment &o) const{
       if(!o.intersect(line(p,q))) return false;
       if(line(o.p, o.q).parallel(line(p,q)))
          return contains(o.p) || contains(o.q);
      return contains(line(p,q).inter(line(o.p,o.q)));
   bool contains(const vec &o) const{
      return eq(line(p, q).eval(o), 0) && onstrip(o);
   cod dist(const segment &o) const{
      if(line(p, q).parallel(line(o.p, o.q))){
          if(onstrip(o.p) || onstrip(o.q))
             return line(p, q).dist(line(o.p, o.q));
          return o.dist(p);
      else if(intersect(o)) return 0;
```

```
return min(min(dist(o.p), dist(o.q)),
                 o.dist(p));
   bool intersect(const hray &o) const{
       if(!line(p, q).parallel(line(o.p, o.q)))
          return false;
       auto pt = line(p, q).inter(line(o.p, o.q));
       return contains(pt) && o.contains(pt); // <<</pre>
   bool intersect(const line &o) const{
       if(line(p, q).parallel(o)) return line(p, q)== o;
       if(o.contains(p) || o.contains(q)) return true;
       return (o.eval(p) >= -eps)^(o.eval(p)<o.eval(q));</pre>
       return contains(o.inter(line(p, q)));
   cod dist(const line &o) const{
       if(line(p,q).parallel(o))
          return line(p,q).dist(o);
       else if(intersect(o)) return 0;
       return o.dist(p);
   cod dist(const hray &o) const{
       if(line(p, q).parallel(line(o.p, o.q))){
          if(onstrip(o.p) || o.onstrip(p))
              return line(p,q).dist(line(o.p, o.q));
          return (p-o.p).len();
       else if(intersect(o)) return 0;
       return min(dist(o.p), o.dist(p));
   }
};
double heron(cod a, cod b, cod c){
   cod s = (a + b + c) / 2;
   return sqrt(s * (s - a) * (s - b) * (s - c));
Nearest Points
struct pt {
   int x, y, id;
};
inline bool cmp_x (const pt & a, const pt & b) {
   return a.x < b.x | | a.x == b.x && a.y < b.y;
inline bool cmp_y (const pt & a, const pt & b) {
   return a.y < b.y;</pre>
}
pt a[MAXN];
double mindist;
int ansa, ansb;
inline void upd_ans (const pt & a, const pt & b) {
   double dist = sqrt((a.x-b.x)*(a.x-b.x) + (a.y-b.y)
     *(a.y-b.y) + .0);
   if (dist < mindist)</pre>
       mindist = dist, ansa = a.id, ansb = b.id;
void rec (int 1, int r) {
   if (r - 1 \le 3) {
       for (int i=1; i<=r; ++i)</pre>
          for (int j=i+1; j<=r; ++j)</pre>
              upd_ans (a[i], a[j]);
```

```
sort (a+1, a+r+1, &cmp_y);
                                                               return prev(it)->cross(*it, p) >= 0;
                                                           }
       return;
   }
                                                           // lowerhull, upperhull and point, borders included
   int m = (1 + r) >> 1;
                                                           bool inside_poly(const vector<vec> &lo, const vector<vec</pre>
   int midx = a[m].x;
                                                             > &hi, vec p){
   rec (1, m), rec (m+1, r);
                                                               return below(hi, p) && above(lo, p);
   static pt t[MAXN];
   merge (a+1, a+m+1, a+m+1, a+r+1, t, &cmp_y);
                                                           Check point inside polygon without lower/upper
   copy (t, t+r-l+1, a+l);
                                                                 hull
   int tsz = 0;
                                                           // borders included
   for (int i=1; i<=r; ++i)</pre>
                                                            // must not have 3 colinear consecutive points
       if (abs (a[i].x - midx) < mindist) {</pre>
                                                           bool inside_poly(const vector<vec> &v, vec p){
          for (int j=tsz-1; j>=0 && a[i].y - t[j].y <</pre>
                                                               if(v[0].ccw(v[1], p) < 0) return false;</pre>
            mindist; --j)
                                                               if(v[0].ccw(v.back(), p) > 0) return 0;
              upd_ans (a[i], t[j]);
                                                               if(v[0].ccw(v.back(), p) == 0)
          t[tsz++] = a[i];
                                                                  return v[0].dot(p, v.back()) >= 0
                                                                      && v.back().dot(p, v[0]) >= 0;
}
                                                               int L = 1, R = (int)v.size() - 1, ans = 1;
sort (a, a+n, &cmp_x);
mindist = 1E20;
                                                               while(L <= R){</pre>
rec (0, n-1);
                                                                  int mid = (L+R)/2;
Convex Hull
                                                                  if(v[0].ccw(v[mid], p) >= 0) ans = mid, L = mid
vector<vec> monotone_chain_ch(vector<vec> P){
                                                                  else R = mid-1;
   sort(P.begin(), P.end());
                                                               }
   vector<vec> L, U;
                                                               return v[ans].ccw(v[(ans+1)%v.size()], p) >= 0;
   for(auto p : P){
       while(L.size() >= 2 && L[L.size() - 2].cross(L.
                                                           Minkowski sum
         back(), p) < 0)
          L.pop_back();
                                                           vector<vec> mk(const vector<vec>&a,const vector<vec>&b){
                                                               int i = 0, j = 0;
       L.push_back(p);
                                                               for(int k = 0; k < (int)a.size(); k++)if(a[k] < a[i])
   }
                                                                  i = k;
                                                               for(int k = 0; k < (int)b.size(); k++)if(b[k] < b[j])
   reverse(P.begin(), P.end());
                                                                  j = k;
   for(auto p : P){
       while(U.size() >= 2 && U[U.size() - 2].cross(U.
                                                               vector<vec> c:
         back(), p) < 0)
                                                               c.reserve(a.size() + b.size());
          U.pop_back();
                                                               for(int k = 0; k < int(a.size()+b.size()); k++){
                                                                  vec pt{a[i] + b[j]};
       U.push_back(p);
                                                                  if((int)c.size() >= 2
   }
                                                                   && c[c.size()-2].ccw(c.back(), pt) == 0)
                                                                      c.pop_back();
   L.pop_back(), U.pop_back();
                                                                  c.push_back(pt);
                                                                  int q = i+1, w = j+1;
   L.reserve(L.size() + U.size());
                                                                  if(q == int(a.size())) q = 0;
   L.insert(L.end(), U.begin(), U.end());
                                                                  if(w == int(b.size())) w = 0;
                                                                  if(c.back().ccw(a[i]+b[w], a[q]+b[j]) < 0) i = q;
   return L;
                                                                  else j = w;
Check point inside polygon
                                                               c.shrink_to_fit();
bool below(const vector<vec> &vet, vec p){
                                                               return c;
   auto it = lower_bound(vet.begin(), vet.end(), p);
   if(it == vet.end()) return false;
                                                           Miscellaneous
   if(it == vet.begin()) return *it == p;
   return prev(it)->cross(*it, p) <= 0;</pre>
                                                           LIS
}
bool above(const vector<vec> &vet, vec p){
                                                           multiset<int> S;
   auto it = lower_bound(vet.begin(), vet.end(), p);
                                                           for(int i = 0; i < n; i++){
   if(it == vet.end()) return false;
                                                               auto it = S.upper_bound(a[i]); // low for inc
   if(it == vet.begin()) return *it == p;
                                                               if(it != S.end()) S.erase(it);
```

```
S.insert(a[i]);
}
ans = S.size();
DSU rollback
#include <bits/stdc++.h>
using namespace std;
struct DSU{
   vector<int> sz, p, change;
   vector<tuple<int, int, int>> modifications;
   vector<size_t> saves;
   bool bipartite;
   DSU(int n): sz(n+1, 1), p(n+1), change(n+1),
     bipartite(true){
      iota(p.begin(), p.end(), 0);
   }
   void add_edge(int u, int v){
       if(!bipartite) return;
       int must_change = get_colour(u) == get_colour(v);
       int a = rep(u), b = rep(v);
       if(sz[a] < sz[b]) swap(a, b);
       if(a != b){
          p[b] = a;
          modifications.emplace_back(b, change[b],
            bipartite);
          change[b] ^= must_change;
          sz[a] += sz[b];
      }
      else if(must_change){
          modifications.emplace_back(0, change[0],
            bipartite);
          bipartite = false;
       }
   }
   int rep(int u){
      return p[u] == u ? u : rep(p[u]);
   }
   int get_colour(int u){
       if(p[u] == u) return change[u];
       return change[u] ^ get_colour(p[u]);
   void reset(){
      modifications.clear();
       saves.clear();
      iota(p.begin(), p.end(), 0);
       fill(sz.begin(), sz.end(), 1);
       fill(change.begin(), change.end(), 0);
      bipartite = true;
   }
   void rollback(){
       int u = get<0>(modifications.back());
       tie(ignore, change[u], bipartite) = modifications
         .back();
       sz[p[u]] -= sz[u];
      p[u] = u;
      modifications.pop_back();
   }
   void reload(){
```

```
while(modifications.size() > saves.back())
          rollback();
       saves.pop_back();
   }
   void save(){
       saves.push_back(modifications.size());
};
const int N = 100005;
const int B = 318;
int n, m, q;
int x[N], y[N], l[N], r[N], ans[N];
vector<int> qu[N];
int brute(int lef, int rig, DSU &s){
   s.save();
   for(int i = lef; i <= rig; i++)</pre>
       s.add_edge(x[i], y[i]);
   int ret = s.bipartite;
   s.reload();
   return ret;
}
int main(){
   scanf("%d %d %d", &n, &m, &q);
   for(int i = 1; i <= m; i++)</pre>
       scanf("%d %d", x+i, y+i);
   DSU s(n);
   for(int i = 0; i < q; i++){
       scanf("%d %d", l+i, r+i);
       if(r[i] - 1[i] \le B + 10)
           ans[i] = brute(l[i], r[i], s);
       else qu[l[i] / B].push_back(i);
   }
   for(int i = 0; i <= m / B; i++){</pre>
       sort(qu[i].begin(), qu[i].end(),[](int a, int b){
          return r[a] < r[b];</pre>
       });
       s.reset();
       int R = (i+1)*B-1;
       for(int id : qu[i]){
          while(R < r[id]) ++R, s.add_edge(x[R], y[R]);
           for(int k = 1[id]; k < (i+1)*B; k++)
              s.add_edge(x[k], y[k]);
           ans[id] = s.bipartite;
           s.reload();
       }
   }
   for(int i = 0; i < q; i++)
       printf("%s\n",ans[i] ? "Possible":"Impossible");
}
Buildings
// count the number of circular arrays
// of size m, with elements on range
```

```
// [1, c**(x*x)]
                                                            iota(permutation.begin(), permutation.end(), 0);
#include<bits/stdc++.h>
using namespace std;
                                                            for(int i = 1; i < N; i++){
                                                               swap(permutation[i], permutation[
#define debug(x) cerr << fixed << #x << " = " << x <<
                                                                 uniform_int_distribution<int>(0, i)(rng)]);
  endl:
                                                            }
                                                            Klondike
const int MOD = 1e9 + 7, MAX = 1e5 + 5;
int dp[MAX];
                                                            // minimum number of moves to make
                                                            // all elements equal
inline int add(int a, int b) {
                                                            // move: change a segment of equal value
 return a + b >= MOD ? a + b - MOD : a;
                                                            // elements to any value
inline int sub(int a, int b) {
                                                            int v[305];
 return a - b < 0? a - b + MOD: a;
                                                            int dp[305][305]:
                                                            int rec[305][305];
inline int mult(int a, int b) {
 return (1LL * a * b) % MOD;
                                                            int f(int 1, int r){
                                                             if(r == 1) return 1;
int f_exp(int x, int exp) {
                                                             if(r < 1) return 0;</pre>
 if(exp == 0) return 1;
                                                             if(dp[l][r] != -1) return dp[l][r];
 else if(exp & 1) return mult(x, f_exp(x, exp - 1));
                                                             int ans = f(1+1, r) + 1;
 return f_exp(mult(x, x), exp / 2);
                                                             for(int i = l+1; i <= r; i++)
                                                               if(v[i] == v[l])
inline int inv(int x) {
                                                                 ans = min(ans, f(1, i - 1) + f(i+1, r));
 return f_exp(x, MOD - 2);
                                                             return dp[l][r] = ans;
int main() {
 ios::sync_with_stdio(false);
 cin.tie(NULL); cout.tie(NULL);
                                                            int main() {
                                                             int n, m;
 int n, m, c;
                                                             memset(dp, -1, sizeof dp);
 cin >> n >> m >> c;
                                                             scanf("%d %d",&n , &m);
                                                             for(int i = 0; i < n; i++){
 int x = f_{exp}(c, n * n);
                                                               scanf("%d",v+i);
 int ans = f_exp(x, m);
                                                               if(i \& v[i] == v[i-1]){
 for(int i = 1; i <= m; i++) {</pre>
                                                                 i--:
   if(m % i == 0) {
                                                                 n--;
     int y = f_exp(x, i);
                                                               }
     for(int j = 1; j < i; j++) {
                                                             }
       if(i % j == 0)
                                                             printf("%d\n",f(0, n-1) - 1);
        y = sub(y, mult(j, dp[j]));
                                                             // printf("%d\n",rec[0][n-1] );
                                                             // printf("%d\n",rec[1][n-1] );
     dp[i] = mult(y, inv(i));
                                                             // printf("%d\n",rec[2][n-3] );
     ans = sub(ans, mult(i - 1, dp[i]));
                                                            Hilbert Order
                                                            // maybe use B = n / sqrt(q)
 cout << ans << '\n';</pre>
                                                            inline int64_t hilbertOrder(int x, int y, int pow = 21,
                                                              int rotate = 0) {
 return 0;
                                                               if(pow == 0) return 0;
                                                               int hpow = 1 \ll (pow-1);
Rand
                                                               int seg = (x < hpow) ? (
                                                                   (y < hpow) ? 0 : 3
#include <random>
                                                               ):(
#include <chrono>
                                                                   (y < hpow) ? 1 : 2
                                                               );
cout << RAND_MAX << endl;</pre>
                                                               seg = (seg + rotate) & 3;
mt19937 rng(chrono::steady_clock::now().time_since_epoch
                                                               const int rotateDelta[4] = \{3, 0, 0, 1\};
                                                               int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
  ().count());
vector<int> permutation(N);
                                                               int nrot = (rotate + rotateDelta[seg]) & 3;
                                                               int64_t subSquareSize = int64_t(1) << (2*pow - 2);</pre>
iota(permutation.begin(), permutation.end(), 0);
                                                               int64_t ans = seg * subSquareSize;
                                                               int64_t add = hilbertOrder(nx, ny, pow-1, nrot);
                                                               ans += (seg == 1 || seg == 2) ? add : (subSquareSize
shuffle(permutation.begin(), permutation.end(), rng);
                                                                   - add - 1);
```

```
return ans;
                                                           #define gc(x) getchar_unlocked(x)
}
                                                           inline void scan_int(int &x){
Modular Factorial
                                                               register int c = gc();
                                                               x = 0;
// Compute (1*2*...*(p-1)*1*(p+1)*(p+2)*..*n) % p
                                                               int neg = 0;
// in O(p*lg(n))
                                                               for(; ((c < '0' || c > '9') && c != '-'); c = gc());
                                                               if(c == '-'){
int factmod(int n, int p){
                                                                  neg = 1;
   int ans = 1;
                                                                  c = gc();
   while (n > 1) {
       for(int i = 2; i <= n % p; i++)
                                                               for(; c >= '0' && c <= '9'; c = gc())
          ans = (ans * i) % p;
                                                                  x = (x << 1) + (x << 3) + c - '0';
      n /= p;
                                                               if(neg) x = -x;
      if(n \% 2) ans = p - ans;
                                                           }
   }
   return ans % p;
                                                           inline void print_int(int n){
}
                                                               int rev = 0, count = 0, neg;
                                                               if(n == 0){
int fac_pow(int n, int p){
                                                                  pc('0');
   int ans = 0;
                                                                  return;
   while(n) n \neq p, ans += n;
   return ans;
                                                               if(n < 0) n = -n, neg = 1;
                                                               while(n % 10 == 0) count++, n /= 10;
                                                               for(rev = 0; n != 0; n /= 10)
int C(int n, int k, int p){
                                                                  rev = (rev << 3) + (rev << 1) + n % 10;
   if(fac_pow(n, p) > fac_pow(n-k, p) + fac_pow(k, p))
                                                               if(neg) pc('-');
                                                               while(rev != 0) pc(rev % 10 + '0'), rev /= 10;
   int tmp = factmod(k, p) * factmod(n-k, p);
                                                               while(count--) pc('0');
   return (f_exp(tmp, p-2, p) * factmod(n, p)) % p;
                                                               pc('\n');
                                                           }
Enumeration all submasks of a bitmask
                                                           inline void print_string(char *str){
// loop through all submask of a given bitmask
                                                               while(*str) pc(*str++);
// it does not include mask 0
                                                               pc('\n');
for(int sub = mask; sub; sub = (sub-1)&mask){
                                                           Knapsack Bounded with Cost
Slope Trick
                                                           // menor custo para conseguir peso ate M usando N tipos
                                                             diferentes de elementos, sendo que o i-esimo elemento
///By wogja125, contest: Codeforces Round #371 (Div. 1),
                                                             pode ser usado b[i] vezes, tem peso w[i] e custo c[i]
  problem: (C) Sonya and Problem Wihtout a Legend,
                                                           // O(N * M)
  Accepted, #
                                                           int b[N], w[N], c[N];
#include <stdio.h>
                                                           MinQueue Q[M]
#include <queue>
                                                           int d[M] //d[i] = custo minimo para conseguir peso i
int main() {
                                                           for(int i = 0; i \le M; i++) d[i] = i ? oo : 0;
   int n, t;
                                                           for(int i = 0; i < N; i++){</pre>
   long long ans = 0;
                                                               for(int j = 0; j < w[i]; j++)</pre>
   std::priority_queue<int> Q;
                                                                  Q[j].clear();
   scanf("%d%d", &n, &t);
                                                               for(int j = 0; j <= M; j++){</pre>
   Q.push(t);
                                                                  q = Q[j \% w[i]];
   for(int i = 1; i < n; i++) {</pre>
                                                                  if(q.size() >= q) q.pop();
       scanf("%d", &t); t -= i;
                                                                  q.add(c[i]);
      Q.push(t);
                                                                  q.push(d[j]);
       if(Q.top() > t) {
                                                                  d[j] = q.getmin();
          ans += Q.top() - t;
          Q.pop();
          Q.push(t);
                                                           LCA < O(nlgn), O(1) >
   printf("%11d", ans);
                                                           int start[N], dfs_time;
   return 0;
                                                           int tour[2*N], id[2*N];
Fast IO
                                                           void dfs(int u){
                                                               start[u] = dfs_time;
#define pc(x) putchar_unlocked(x)
                                                               id[dfs_time] = u;
```

```
tour[dfs_time++] = start[u];
  for(int v : g[u]){
     dfs(v);
     id[dfs_time] = u;
     tour[dfs_time++] = start[u];
  }
}
int LCA(int u, int v){
  if(start[u] > start[v]) swap(u, v);
  return id[min(tour[k]for k in [start[u],start[v]])];
}
```

#### Burnside's Lemma

Let  $(G, \oplus)$  be a finite group that acts on a set X. It should hold that  $e_g * x = x$  and  $g_1 * (g_2 * x) = (g_1 \oplus g_2) * x$ ,  $\forall x \in X, g_1, g_2 \in G$ . For each  $g \in G$  let  $X^g = \{x \in X \mid g * x = x\}$ . The number of orbits its given by:

 $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$ 

#### Wilson's Theorem

 $(n-1)! = -1 \mod n \iff n \text{ is prime}$ 

#### Fibonacci

- $F_{n-1}F_{n+1} F_n^2 = (-1)^n$
- $F_{n+k} = F_k F_{n+1} + F_{k-1} F_n$
- $GCD(F_n, F_m) = F_{GCD(n,m)}$
- $F_n = \frac{(\frac{1+\sqrt{5}}{2})^n (\frac{1-\sqrt{5}}{2})^n}{\sqrt{5}}$

# Kirchhoff's Theorem

Laplacian matrix is L = D - A, where D is a diagonal matrix with vertex degrees on the diagonals and A is adjacency matrix.

The number of spanning trees is any cofactor of L. i-th cofactor is determinant of the matrix gotten by removing i-th row and column of L.

### Multigraphs

In D[i][i] all loops are excluded. A[i][j] = number of edges from i to j.

#### **Directed multigraphs**

D[i][i] = indegree of i minus the number of loops at i. A[i][j] = number of edges from i to j.

The number of oriented spanning trees rooted at a vertex i is the determinant of the matrix gotten by removing the ith row and column of L.

#### **Matroid**

Let *X* set of objects,  $I \subseteq 2^X$  set of independents sets such that:

- 1.  $\emptyset \in I$
- 2.  $A \in I, B \subseteq A \implies B \in I$
- 3. Exchange axiom,  $A \in I, B \in I, |B| > |A| \implies \exists x \in B \setminus A : A \cup \{x\} \in I$

4.  $A \subseteq X$  and I and I' are maximal independent subsets of A then |I| = |I'|

Then (X, I) is a matroid. The combinatorial optimization problem associated with it is: Given a weight  $w(e) \ge 0 \ \forall e \in X$ , find an independet subset that has the largest possible total weight.

#### Matroid intersection algorithm

```
Data: Two matroids (X, I_a) and (X, I_b)
Result: Set I with maximum size, I \in I_a and I \in I_b
while True do
    for e_i \in X \setminus I do
        if I + e_i \in I_a and I + e_i \in I_b then
         I = I + e_i
        end
    end
    A = \emptyset, Q = \emptyset, T = \emptyset
                                       ▶ Q is a queue;
    label[x \in X] = MARK1;
    for e_i \in X \setminus I do
        if I + e_i \in I_a then
            Q.push(e_i), label[e_i] := MARK2
        else
             for x \in I \mid I \setminus \{x\} \cup \{e_i\} \in I_a do
             A := A \cup \{(x, e_i)\}
             end
        end
        if I + e_i \in I_b then
         T := T \cup \{e_i\}
        else
             for x \in I \mid I \setminus \{x\} \cup \{e_i\} \in I_b do
             A := A \cup \{(e_i, x)\}
             end
        end
    end
    if T = \emptyset then
     break
    end
    found := False;
    while Q \neq \emptyset and found = False do
        e := Q.front(), Q.pop();
        for x \mid (e, x) \in A and label[x] = MARK1 do
             label[x] = e;
             Q.push(x);
             if x \in T then
                 found := True;
                 I := I \oplus path(x);
                 break;
             end
        end
    end
    if found = False then
     break
    end
end
```

Where path(e) = [e] if label[e] = MARK2, path(label[e]) + [e] otherwise.

#### **Matroid Union**

Given k matroids over the same set of objects  $(X, I_1)$ ,  $(X, I_2)$ , ...,  $(X, I_k)$  find  $A_1 \in I_1$ ,  $A_2 \in I_2$ , ...,  $A_k \in I_k$  such that  $i \neq j, A_i \cap A_j = \emptyset$  and  $|\bigcup_{i=1}^k A_i|$  is maximum. Matroid union can be reduced to matroid intersection as follows.

Let  $X' = X \times \{1, 2, ..., k\}$ , ie, k copies of each element of X with different colors. M1 = (X', Q) where  $B \in Q \iff \forall 1 \le i \le k$ ,  $\{x \mid (x, i) \in B\} \in I_i$ , ie, for each color, B is independent. M2 = (X', W) where  $B \in W \iff i \ne j \implies \neg((x, i) \in B \land (x, j) \in B)$ , ie, each element is picked by at most one color.

Intersection of *M*1 and *M*2 is the answer for the combinatorial problem of matroid union.

# **Edge coloring**

```
Data: A graph G 

Result: A proper coloring c of the edges of G 

Let U := E(G); while U \neq \emptyset do 

Let (u,v) be any edge in U; 

Let F[1:k] be a maximal fan of u starting at F[1]=v; 

Let c be a color that is free on u and d be a color 

that is free on F[k]; 

Invert the cd_u path; 

Let w \in V(G) be such that w \in F, F' = [F[1]...w] is a 

fan and d is free on w; 

Rotate F' and set c(u,w)=d; 

U := U - (u,v); end
```

#### **Notes**

When we repeat something and each time we have probability p to succeed then the expected number or tries is  $\frac{1}{p}$ , till we succeed.

### Small to large

**Trick in statement** If k sets are given you should note that the amount of different set sizes is  $O(\sqrt{s})$  where s is total size of those sets. And no more than  $\sqrt{s}$  sets have size greater than  $\sqrt{s}$ . For example, a path to the root in Aho-Corasick through suffix links will have at most  $O(\sqrt{s})$  vertices.

**gcd on subsegment**, we have at most  $log(a_i)$  different values in  $\{gcd(a_j, a_{j+1}, ..., a_i) \text{ for } j < i\}$ .

From static set to expandable. To insert, create a new set with the new element. While there are two sets with same size, merge them. There will be at most log(n) disjoints sets.

**Matrix exponentiation optimization**. Save binary power of  $A_{nxn}$  and answer q queries  $b = A^m x$  in  $O((n^3 + qn^2)log(m))$ .

**Ternary search on integers into binary search**, comparing f(mid) and f(mid+1), binary search on derivative

**Dynamic offline set** For each element we will wind segment of time [a, b] such that element is present in the set during this whole segment. Now we can come up with recursive procedure which handles [l, r] time segment considering that all elements such that  $[l, r] \subset [a, b]$  are already included into the set. Now, keeping this invariant we recursively go into [l, m] and [m + 1, r] subsegments. Finally when we come into segment of length 1.

 $a > b \implies a \mod b < \frac{a}{2}$