

ICPC Team Reference

University of Brasilia

Contents

1 Data Structures	2	4 Strings	13
1.1 Merge Sort Tree	2	4.1 Aho Corasick	13
1.2 Wavelet Tree	2	4.2 Suffix Array	14
1.3 Order Set	2	4.3 Z Algorithm	14
1.4 Hash table	3	4.4 Prefix function/KMP	14
1.5 Convex Hull Trick Simple	3	4.5 Min rotation	15
1.6 Convex Hull Trick	3	4.6 All palindrome	15
1.7 Min queue	3	4.7 Suffix Automaton	15
1.8 Sparse Table	4	4.8 Suffix Tree	15
1.9 Treap	4	5 Geometry	15
1.10 ColorUpdate	4	5.1 2D basics	15
1.11 Heavy Light Decomposition	5	5.2 Circle line intersection	17
1.12 Iterative Segtree	5	5.3 Circle Circle intersection	18
1.13 LiChao's Segtree	5	5.4 Tangents of two circles	18
1.14 Palindromic tree	5	5.5 Convex Hull	18
2 Math	6	5.6 Check point inside polygon	18
2.1 Extended Euclidean Algorithm	6	5.7 Check point inside polygon without lower/up- per hull	18
2.2 Chinese Remainder Theorem	6	5.8 Minkowski sum	18
2.3 Prefix inverse	6	5.9 Geo Notes	19
2.4 Pollard Rho	6	5.9.1 Center of mass	19
2.5 Miller Rabin	7	5.9.2 Pick's Theorem	19
2.6 Totiente	7	6 Miscellaneous	19
2.7 Primitive root	7	6.1 LIS	19
2.8 Mobius Function	7	6.2 DSU rollback	19
2.9 Mulmod TOP	7	6.3 Buildings	19
2.10 Matrix Determinant	7	6.4 Rand	20
2.11 Simplex Method	8	6.5 Klondike	20
2.12 FFT	8	6.6 Hilbert Order	20
2.13 NTT	9	6.7 Modular Factorial	20
2.14 Gauss	9	6.8 Enumeration all submasks of a bitmask	21
2.15 Gauss Xor	9	6.9 Slope Trick	21
2.16 Simpson	9	6.10 Fast IO	21
3 Graphs	10	6.11 Knapsack Bounded with Cost	21
3.1 Dinic	10	6.12 LCA $<O(n \lg n), O(1)>$	21
3.2 Push relabel	10	6.13 Buffered reader	22
3.3 Min Cost Max Flow	11	6.14 Burnside's Lemma	22
3.4 Small to Large	11	6.15 Wilson's Theorem	22
3.5 Centroid Decomposition	11	6.16 Fibonacci	22
3.6 Kosaraju	12	6.17 Kirchhoff's Theorem	22
3.7 Tarjan	12	6.17.1 Multigraphs	22
3.8 Max Clique	12	6.17.2 Directed multigraphs	22
3.9 Dominator Tree	13	6.18 Matroid	22
3.10 Min Cost Matching	13	6.18.1 Matroid intersection	22
		6.18.2 Matroid Union	23
		6.19 Edge coloring	23
		6.20 Notes	23

```

set ts=4 sw=4 sta nu rnu sc stl+=%F cindent
set bg=dark ruler clipboard=unnamed,unnamedplus
  timeoutlen=100
imap {<CR> {<CR>}<Esc>O
nmap <F2> 0V$%d
nmap <C-down> :m+1<CR>
nmap <C-up> :m-2<CR>
vmap <C-c> "+y
nmap <C-a> ggVG
syntax on
alias cmp='g++ -Wall -Wformat=2 -Wshadow -Wconversion -
fsanitize=address -fsanitize=undefined -fno-sanitize-
recover -std=c++14'

```

Data Structures

Merge Sort Tree

```

struct MergeTree{
    int n;
    vector<vector<int>> st;

    void build(int p, int L, int R, const int v[]){
        if(L == R){
            st[p].push_back(v[L]);
            return;
        }
        int mid = (L+R)/2;
        build(2*p, L, mid, v);
        build(2*p+1, mid+1, R, v);
        st[p].resize(R-L+1);
        merge(st[2*p].begin(), st[2*p].end(),
              st[2*p+1].begin(), st[2*p+1].end(),
              st[p].begin());
    }

    int query(int p, int L, int R, int i, int j, int x)
    const{
        if(L > j || R < i) return 0;
        if(L >= i && R <= j){
            int id = lower_bound(st[p].begin(), st[p].end(),
                                  x) - st[p].begin();
            return int(st[p].size()) - id;
        }
        int mid = (L+R)/2;
        return query(2*p, L, mid, i, j, x) +
               query(2*p+1, mid+1, R, i, j, x);
    }

public:
    MergeTree(int sz, const int v[]): n(sz), st(4*sz){
        build(1, 1, n, v);
    }

    //number of elements >= x on segment [i, j]
    int query(int i, int j, int x) const{
        if(i > j) swap(i, j);
        return query(1, 1, n, i, j, x);
    }
};

```

Wavelet Tree

```

template<typename T>
class wavelet{
    T L, R;
    vector<int> l;
    vector<T> sum; // <<

```

```

    wavelet *lef, *rig;

    int r(int i) const{ return i - l[i]; }

public:
    template<typename ITER>
    wavelet(ITER bg, ITER en){
        lef = rig = nullptr;
        L = *bg, R = *bg;

        for(auto it = bg; it != en; it++){
            L = min(L, *it), R = max(R, *it);
            if(L == R) return;
        }

        T mid = L + (R - L)/2;
        l.reserve(std::distance(bg, en) + 1);
        sum.reserve(std::distance(bg, en) + 1);
        l.push_back(0), sum.push_back(0);
        for(auto it = bg; it != en; it++){
            l.push_back(l.back() + (*it <= mid)),
            sum.push_back(sum.back() + *it);
        }

        auto tmp = stable_partition(bg, en, [mid](T x){
            return x <= mid;
        });

        if(bg != tmp) lef = new wavelet(bg, tmp);
        if(tmp != en) rig = new wavelet(tmp, en);
    }

    ~wavelet(){
        delete lef;
        delete rig;
    }

    // 1 index, first is 1st
    T kth(int i, int j, int k) const{
        if(L >= R) return L;
        int c = l[j] - l[i-1];
        if(c >= k) return lef->kth(l[i-1]+1, l[j], k);
        else return rig->kth(r(i-1)+1, r(j), k - c);
    }

    // # elements > x on [i, j]
    int cnt(int i, int j, T x) const{
        if(L > x) return j - i + 1;
        if(R <= x || L == R) return 0;
        int ans = 0;
        if(lef) ans += lef->cnt(l[i-1]+1, l[j], x);
        if(rig) ans += rig->cnt(r(i-1)+1, r(j), x);
        return ans;
    }

    // sum of elements <= k on [i, j]
    T sumk(int i, int j, T k){
        if(L == R) return R <= k ? L * (j - i + 1) : 0;
        if(R <= k) return sum[j] - sum[i-1];
        int ans = 0;
        if(lef) ans += lef->sumk(l[i-1]+1, l[j], k);
        if(rig) ans += rig->sumk(r(i-1)+1, r(j), k);
        return ans;
    }

    // swap (i, i+1) just need to update "array" l[i]
};

```

Order Set

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

#include <ext/pb_ds/detail/standard_policies.hpp>

using namespace __gnu_pbds; // or pb_ds;
```

```
template<typename T, typename B = null_type>
using oset = tree<T, B, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
// find_by_order / order_of_key
```

Hash table

```
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
```

```
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        // http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }

    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM = chrono::
            steady_clock::now().time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};
```

```
gp_hash_table<long long, int, custom_hash> table;
unordered_map<long long, int, custom_hash> uhash;
uhash.reserve(1 << 15);
uhash.max_load_factor(0.25);
```

Convex Hull Trick Simple

```
struct Line{
    ll m, b;
    inline ll eval(ll x) const{
        return x * m + b;
    }
};

// min => cht.back().m >= L.m
// max => cht.back().m <= L.m
void push_line(vector<Line> &cht, Line L){
    while((int)cht.size() >= 2){
        int sz = (int)cht.size();
        if((long double)(L.b-cht[sz-1].b)*(cht[sz-2].m-L.m)
            <= (long double)(L.b-cht[sz-2].b)*(cht[sz-1].m-L.m)){
            cht.pop_back();
        }
        else break;
    }
    cht.push_back(L);
}

// x increasing; pos = 0 in first call
ll linear_search(const vector<Line> &cht, ll x, int &pos){
    while(pos+1 < (int)cht.size()){
        /*>>*/ if(cht[pos].eval(x) >= cht[pos+1].eval(x)) pos++;
        else break;
    }
    return cht[pos].eval(x);
}
```

```
ll binary_search(const vector<Line> &cht, ll x){
    int L = 0, R = (int)cht.size()-2;
    int bans = (int)cht.size()-1;
    while(L <= R){
        int mid = (L+R)/2;
        if(cht[mid].eval(x) >= cht[mid+1].eval(x)) // <<<
            L = mid + 1;
        else bans = mid, R = mid - 1;
    }
    return cht[bans].eval(x);
}
```

Convex Hull Trick

```
/**
 * Author: Simon Lindholm
 * source: https://github.com/kth-competitive-
    programming/kactl/blob/master/content/data-structures
    /LineContainer.h
 * License: CC0
 */
```

```
struct Line {
    mutable ll m, b, p;
    bool operator<(const Line& o) const { return m < o.m
        ; }
    bool operator<(ll x) const { return p < x; }
};
```

```
struct LineContainer : multiset<Line, less<>> { // C++14
    only
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    const ll inf = LLONG_MAX;
    ll div(ll a, ll b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = inf; return false; }
        if (x->m == y->m) x->p = x->b > y->b ? inf : -inf
            ;
        else x->p = div(y->b - x->b, x->m - y->m);
        return x->p >= y->p;
    }
    void add(ll m, ll b) {
        auto z = insert({m, b, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y =
            erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        assert(!empty());
        auto l = *lower_bound(x);
        return l.m * x + l.b;
    }
};
```

Min queue

```
template<typename T>
class minQ{
    deque<tuple<T, int, int> > p;
    T delta;
    int sz;
public:
    minQ() : delta(0), sz(0) {}
    inline int size() const{ return sz; }
    inline void add(T x){ delta += x; }
```

```

inline void push(T x, int id){
    x -= delta, sz++;
    int t = 1;
    while(p.size() > 0 && get<0>(p.back()) >= x)
        t += get<1>(p.back()), p.pop_back();
    p.emplace_back(x, t, id);
}
inline void pop(){
    get<1>(p.front())--, sz--;
    if(!get<1>(p.front())) p.pop_front();
}
T getmin() const{ return get<0>(p.front())+delta; }
int getid() const{ return get<2>(p.front()); }
};

```

Sparse Table

```

const int N = 100005;

int v[N], n;
int dn[N][20];
int fn(int i, int j){
    if(j == 0) return v[i];
    if(~dn[i][j]) return dn[i][j];
    return dn[i][j] = min(fn(i, j-1), fn(i + (1 << (j-1)
        ), j-1));
}

int lg(int x){ return 31 - __builtin_clz(x); }

```

```

int getmn(int l, int r){ // [l, r]
    int lz = lg(r - l + 1);
    return min(fn(l, lz), fn(r - (1 << lz) + 1, lz));
}

```

Treap

```

// source: https://github.com/victorsenam/caderno/blob/
// master/code/treap.cpp
//const int N = ; typedef int num;
num X[N]; int en = 1, Y[N], sz[N], L[N], R[N];
void calc (int u) { // update node given children info
    if(!u) return;
    sz[u] = sz[L[u]] + 1 + sz[R[u]];
    // code here, no recursion
}
void unlaze (int u) {
    if(!u) return;
    // code here, no recursion
}
void split_val(int u, num x, int &l, int &r) { // l gets
    <= x, r gets > x
    unlaze(u); if(!u) return (l = r = 0);
    if(X[u] <= x) { split_val(R[u], x, l, r); R[u] = l;
        l = u; }
    else { split_val(L[u], x, l, r); L[u] = r; r = u; }
    calc(u);
}
void split_sz(int u, int s, int &l, int &r) { // l gets
    first s, r gets remaining
    unlaze(u); if(!u) return (l = r = 0);
    if(sz[L[u]] < s) { split_sz(R[u], s - sz[L[u]] - 1,
        l, r); R[u] = l; l = u; }
    else { split_sz(L[u], s, l, r); L[u] = r; r = u; }
    calc(u);
}
int merge(int l, int r) { // els on l <= els on r
    unlaze(l); unlaze(r); if(!l || !r) return l + r; int
    u;

```

```

    if(Y[l] > Y[r]) { R[l] = merge(R[l], r); u = l; }
    else { L[r] = merge(l, L[r]); u = r; }
    calc(u); return u;
}
void init(int n=N-1) { // XXX call before using other
    funcs
    for(int i = en = 1; i <= n; i++) { Y[i] = i; sz[i] =
        1; L[i] = R[i] = 0; }
    random_shuffle(Y + 1, Y + n + 1);
}
void insert(int &u, int it){
    unlaze(u);
    if(!u) u = it;
    else if(Y[it] > Y[u]) split_val(u, X[it], L[it], R[
        it]), u = it;
    else insert(X[it] < X[u] ? L[u] : R[u], it);
    calc(u);
}
void erase(int &u, num key){
    unlaze(u);
    if(!u) return;
    if(X[u] == key) u = merge(L[u], R[u]);
    else erase(key < X[u] ? L[u] : R[u], key);
    calc(u);
}
int create_node(num key){
    X[en] = key;
    sz[en] = 1;
    L[en] = R[en] = 0;
    return en++;
}
int query(int u, int l, int r){//0 index
    unlaze(u);
    if(u! or r < 0 or l >= sz[u]) return
        identity_element;
    if(l <= 0 and r >= sz[u] - 1) return subt_data[u];
    int ans = query(L[u], l, r);
    if(l <= sz[ L[u] ] and sz[ L[u] ] <= r)
        ans = max(ans, st[u]);
    ans = max(ans, query(R[u], l-sz[L[u]]-1, r-sz[L[u]
        ]-1));
    return ans;
}

```

ColorUpdate

```

// source: https://github.com/tfg50/Competitive-
// Programming/tree/master/Biblioteca/Data%20Structures

#include <set>
#include <vector>

template <class Info = int>
class ColorUpdate {
public:
    struct Range {
        Range(int l = 0) { this->l = l; }
        Range(int l, int r, Info v) {
            this->l = l;
            this->r = r;
            this->v = v;
        }
        int l, r;
        Info v;

        bool operator < (const Range &b) const { return l
            < b.l; }
    };
};

```

```

std::vector<Range> upd(int l, int r, Info v) {
    std::vector<Range> ans;
    if(l >= r) return ans;
    auto it = ranges.lower_bound(l);
    if(it != ranges.begin()) {
        it--;
        if(it->r > l) {
            auto cur = *it;
            ranges.erase(it);
            ranges.insert(Range(cur.l, l, cur.v));
            ranges.insert(Range(l, cur.r, cur.v));
        }
    }
    it = ranges.lower_bound(r);
    if(it != ranges.begin()) {
        it--;
        if(it->r > r) {
            auto cur = *it;
            ranges.erase(it);
            ranges.insert(Range(cur.l, r, cur.v));
            ranges.insert(Range(r, cur.r, cur.v));
        }
    }
    for(it = ranges.lower_bound(l); it != ranges.end()
        () && it->l < r; it++) {
        ans.push_back(*it);
    }
    ranges.erase(ranges.lower_bound(l), ranges.
        lower_bound(r));
    ranges.insert(Range(l, r, v));
    return ans;
}

private:
    std::set<Range> ranges;
};

```

Heavy Light Decomposition

```

void dfs_sz(int u){
    sz[u] = 1;

    for(auto &v : g[u]) if(v == p[u]){
        swap(v, g[u].back());
        g[u].pop_back();
        break;
    }

    for(auto &v : g[u]){
        p[v] = u;
        dfs_sz(v);
        sz[u] += sz[v];
        if(sz[v] > sz[ g[u][0] ])
            swap(v, g[u][0]);
    }
}

// nxt[u] = start of path with u
// set nxt[root] = root beforehand
void dfs_hld(int u){
    in[u] = t++;
    rin[in[u]] = u;
    for(auto v : g[u]){
        nxt[v] = (v == g[u][0] ? nxt[u] : v);
        dfs_hld(v);
    }
    out[u] = t;
}

```

```

// subtree of u => [ in[u], out[u] )
// path from nxt[u] to u => [ in[ nxt[u] ], in[u] ]

```

Iterative Segtree

```

T query(int l, int r, int &pos){ // [l, r]
    T rl, rr;
    for(l += n, r += n+1; l < r; l >>= 1, r >>= 1){
        if(l & 1) rl = merge(rl, st[l++]);
        if(r & 1) rr = merge(st[--r], rr);
    }
    return merge(rl, rr);
}

// initially save v[i] in st[n+i] for all i in [0, n)
void build(){
    for(int p = n-1; p > 0; p--){
        st[p] = merge(st[2*p], st[2*p+1]);
    }
}

void update(int p, T val){
    st[p += n] = val;
    while(p >>= 1) st[p] = merge(st[2*p], st[2*p+1]);
}

```

LiChao's Segtree

```

void add_line(line nw, int v = 1, int l = 0, int r =
    maxn) { // [l, r)
    int m = (l + r) / 2;
    bool lef = nw.eval(l) < st[v].eval(l);
    bool mid = nw.eval(m) < st[v].eval(m);
    if(mid) swap(st[v], nw);
    if(r - l == 1) {
        return;
    } else if(!lef != mid) {
        add_line(nw, 2 * v, l, m);
    } else {
        add_line(nw, 2 * v + 1, m, r);
    }
}

int get(int x, int v = 1, int l = 0, int r = maxn) {
    int m = (l + r) / 2;
    if(r - l == 1) {
        return st[v].eval(x);
    } else if(x < m) {
        return min(st[v].eval(x), get(x, 2*v, l, m));
    } else {
        return min(st[v].eval(x), get(x, 2*v+1, m, r));
    }
}

```

Palindromic tree

```

#include <bits/stdc++.h>

using namespace std;

const int maxn = 3e5 + 1, sigma = 26;
int len[maxn], link[maxn], to[maxn][sigma];
int slink[maxn], diff[maxn], series_ans[maxn];
int sz, last, n;
char s[maxn];

void init()
{
    s[n++] = -1;
    link[0] = 1;
}

```

```

    len[1] = -1;
    sz = 2;
}

int get_link(int v)
{
    while(s[n - len[v] - 2] != s[n - 1]) v = link[v];
    return v;
}

void add_letter(char c)
{
    s[n++] = c - 'a';
    last = get_link(last);
    if(!to[last][c])
    {
        len[sz] = len[last] + 2;
        link[sz] = to[get_link(link[last])][c];
        diff[sz] = len[sz] - len[link[sz]];
        if(diff[sz] == diff[link[sz]])
            slink[sz] = slink[link[sz]];
        else
            slink[sz] = link[sz];
        to[last][c] = sz++;
    }
    last = to[last][c];
}

int main()
{
    ios::sync_with_stdio(0);
    cin.tie(0);
    init();
    string s;
    cin >> s;
    int n = s.size();
    int ans[n + 1];
    memset(ans, 63, sizeof(ans));
    ans[0] = 0;
    for(int i = 1; i <= n; i++)
    {
        add_letter(s[i - 1]);
        for(int v = last; len[v] > 0; v = slink[v])
        {
            series_ans[v] = ans[i - (len[slink[v]] + diff[v])];
            if(diff[v] == diff[link[v]])
                series_ans[v] = min(series_ans[v], series_ans[link[v]]);
            ans[i] = min(ans[i], series_ans[v] + 1);
        }
        cout << ans[i] << "\n";
    }
    return 0;
}

```

Math

Extended Euclidean Algorithm

```

// a*x + b*y = gcd(a, b), <gcd, x, y>
tuple<int, int, int> gcd(int a, int b) {
    if(b == 0) return make_tuple(a, 1, 0);
    int q, w, e;
    tie(q, w, e) = gcd(b, a % b);
    return make_tuple(q, e, w - e * (a / b));
}

```

Chinese Remainder Theorem

```

// x = vet[i].first (mod vet[i].second)
ll crt(vector<pair<ll, ll>> vet){

    ll ans = vet[0].first, lcm = vet[0].second;
    ll a, b, g, x, y;

    for(int i = 1; i < (int)vet.size(); i++){
        tie(a, b) = vet[i];
        tie(g, x, y) = gcd(lcm, b);
        ans = ans + x * (a - ans) / g % (b / g) * lcm;
        lcm = lcm * b / g;
        ans = (ans % lcm + lcm) % lcm;
    }

    return ans;
}

```

Prefix inverse

```

inv[1] = 1;
for(int i = 2; i < p; i++)
    inv[i] = (p - (p/i) * inv[p/i] % p) % p;

```

Pollard Rho

```

ll rho(ll n){
    if(n % 2 == 0) return 2;

    ll d, c, x, y;
    do{
        c = llrand() % n, x = llrand() % n, y = x;
        do{
            x = add(mul(x, x, n), c, n);
            y = add(mul(y, y, n), c, n);
            y = add(mul(y, y, n), c, n);
            d = __gcd(abs(x - y), n);
        }while(d == 1);
    }while(d == n);
    return d;
}

ll pollard_rho(ll n){
    ll x, c, y, d, k;
    int i;
    do{
        i = 1;
        x = llrand() % n, c = llrand() % n;
        y = x, k = 4;
        do{
            if(++i == k) y = x, k *= 2;
            x = add(mul(x, x, n), c, n);
            d = __gcd(abs(x - y), n);
        }while(d == 1);
    }while(d == n);
    return d;
}

void factorize(ll val, map<ll, int> &fac){
    if(rabin(val)) fac[val]++;
    else{
        ll d = pollard_rho(val);
        factorize(d, fac);
        factorize(val / d, fac);
    }
}

map<ll, int> factor(ll val){

```

```

map<ll, int> fac;
if(val > 1) factorize(val, fac);
return fac;
}

```

Miller Rabin

```

bool rabin(ll n){
    if(n <= 1) return 0;
    if(n <= 3) return 1;
    ll s = 0, d = n - 1;
    while(d % 2 == 0) d /= 2, s++;
    for(int k = 0; k < 64; k++){
        ll a = (llrand() % (n - 3)) + 2;
        ll x = fexp(a, d, n);
        if(x != 1 && x != n-1){
            for(int r = 1; r < s; r++){
                x = mul(x, x, n);
                if(x == 1) return 0;
                if(x == n-1) break;
            }
            if(x != n-1) return 0;
        }
    }
    return 1;
}

```

Totiente

```

ll totiente(ll n){
    ll ans = n;
    for(ll i = 2; i*i <= n; i++){
        if(n % i == 0){
            ans = ans / i * (i - 1);
            while(n % i == 0) n /= i;
        }
    }

    if(n > 1) ans = ans / n * (n - 1);
    return ans;
}

```

Primitive root

// a primitive root modulo n is any number g such that any c coprime to n is congruent to a power of g modulo n.

```

bool exists_root(ll n){
    if(n == 1 || n == 2 || n == 4) return true;
    if(n % 2 == 0) n /= 2;
    if(n % 2 == 0) return false;

    // test if n is a power of only one prime
    for(ll i = 3; i * i <= n; i += 2) if(n % i == 0){
        while(n % i == 0) n /= i;
        return n == 1;
    }
    return true;
}

ll primitive_root(ll n){
    if(n == 1 || n == 2 || n == 4) return n - 1;
    if(not exists_root(n)) return -1;
    ll x = phi(n);
    auto pr = factorize(x);
    auto check = [x, n, pr](ll m){
        for(ll p : pr) if(fexp(m, x / p, n) == 1)
            return false;
        return true;
    };
}

```

```

};
for(ll m = 2; ; m++) if(__gcd(m, n) == 1)
    if(check(m)) return m;
}

```

// Let's denote $R(n)$ as the set of primitive roots modulo n , p is prime
 $g \in R(p) \Rightarrow (g^{p-1} \equiv 1 \pmod{p} \wedge g \not\equiv 1 \pmod{p})$
 $g \in R(p^2) \Rightarrow (g^{p-1} \not\equiv 1 \pmod{p^2})$
 $g \in R(2^k) \Rightarrow (g \equiv 5 \pmod{8})$

Mobius Function

```

memset(mu, 0, sizeof mu);
mu[1] = 1;
for(int i = 1; i < N; i++){
    for(int j = i + i; j < N; j += i)
        mu[j] -= mu[i];
}
// g(n) = sum{f(d)} => f(n) = sum{mu(d)*g(n/d)}

```

Mulmod TOP

```

constexpr uint64_t mod = (1ull<<61) - 1;
uint64_t modmul(uint64_t a, uint64_t b){
    uint64_t l1 = (uint32_t)a, h1 = a>>32, l2 = (
        uint32_t)b, h2 = b>>32;
    uint64_t l = l1*l2, m = l1*h2 + l2*h1, h = h1*h2;
    uint64_t ret = (l&mod) + (l>>61) + (h<<3) + (m>>
        29) + (m<<35>>3) + 1;
    ret = (ret & mod) + (ret>>61);
    ret = (ret & mod) + (ret>>61);
    return ret-1;
}

```

Matrix Determinant

```

int n;
long double a[n][n];

long double gauss(){
    long double det = 1;
    for(int i = 0; i < n; i++){
        int q = i;
        for(int j = i+1; j < n; j++){
            if(abs(a[j][i]) > abs(a[q][i]))
                q = j;
        }
        if(abs(a[q][i]) < EPS){
            det = 0;
            break;
        }
        if(i != q){
            for(int w = 0; w < n; w++)
                swap(a[i][w], a[q][w]);
            det = -det;
        }
        det *= a[i][i];
        for(int j = i+1; j < n; j++) a[i][j] /= a[i][i];

        for(int j = 0; j < n; j++) if(j != i){
            if(abs(a[j][i]) > EPS)
                for(int k = i+1; k < n; k++)
                    a[j][k] -= a[i][k] * a[j][i];
        }
    }

    return det;
}

```

Simplex Method

```
typedef long double dbl;
const dbl eps = 1e-6;
const int N = , M = ;

mt19937 rng(chrono::steady_clock::now().time_since_epoch()
().count());

struct simplex {
    int X[N], Y[M];
    dbl A[M][N], b[M], c[N];
    dbl ans;
    int n, m;
    dbl sol[N];

    void pivot(int x, int y){
        swap(X[y], Y[x]);
        b[x] /= A[x][y];
        for(int i = 0; i < n; i++)
            if(i != y)
                A[x][i] /= A[x][y];
        A[x][y] = 1. / A[x][y];
        for(int i = 0; i < m; i++)
            if(i != x && abs(A[i][y]) > eps) {
                b[i] -= A[i][y] * b[x];
                for(int j = 0; j < n; j++) if(j != y)
                    A[i][j] -= A[i][y] * A[x][j];
                A[i][y] = -A[i][y] * A[x][y];
            }
        ans += c[y] * b[x];
        for(int i = 0; i < n; i++)
            if(i != y)
                c[i] -= c[y] * A[x][i];
        c[y] = -c[y] * A[x][y];
    }

    // maximiza sum(x[i] * c[i])
    // sujeito a
    // sum(a[i][j] * x[j]) <= b[i] para 0 <= i < m (Ax
    <= b)
    // x[i] >= 0 para 0 <= i < n (x >= 0)
    // (n variaveis, m restricoes)
    // guarda a resposta em ans e retorna o valor otimo
    dbl solve(int _n, int _m) {
        this->n = _n; this->m = _m;

        for(int i = 1; i < m; i++){
            int id = uniform_int_distribution<int>(0, i)(
                rng);
            swap(b[i], b[id]);
            for(int j = 0; j < n; j++)
                swap(A[i][j], A[id][j]);
        }

        ans = 0.;
        for(int i = 0; i < n; i++) X[i] = i;
        for(int i = 0; i < m; i++) Y[i] = i + n;
        while(true) {
            int x = min_element(b, b + m) - b;
            if(b[x] >= -eps)
                break;
            int y = find_if(A[x], A[x] + n, [](dbl d) {
                return d < -eps; }) - A[x];
            if(y == n) throw 1; // no solution
            pivot(x, y);
        }
        while(true) {
            int y = max_element(c, c + n) - c;
```

```
            if(c[y] <= eps) break;
            int x = -1;
            dbl mn = 1. / 0.;
            for(int i = 0; i < m; i++){
                if(A[i][y] > eps && b[i] / A[i][y] < mn)
                    mn = b[i] / A[i][y], x = i;
            }
            if(x == -1) throw 2; // unbounded
            pivot(x, y);
        }
        memset(sol, 0, sizeof(dbl) * n);
        for(int i = 0; i < m; i++)
            if(Y[i] < n)
                sol[Y[i]] = b[i];
        return ans;
    }
};
```

FFT

```
struct base{
    double r, i;
    base(double _r = 0, double _i = 0) : r(_r), i(_i) {}
    base operator*(const base &o) const{
        return {r*o.r - i*o.i, r*o.i + o.r*i};
    }
    double real() const{ return r; }
    void operator*=(const base &o){
        (*this) = {r*o.r - i*o.i, r*o.i + o.r*i};
    }
    void operator+=(const base &o){r += o.r, i += o.i; }
    void operator/=(const double &o){ r /= o, i /= o; }
    void operator-=(const base &o){r -= o.r, i -= o.i; }
    base operator+(const base &o){return {r+o.r,i+o.i};}
    base operator-(const base &o){return {r-o.r,i-o.i};}
};
```

```
double PI = acos(-1);
```

```
void fft(vector<base> &a, bool inv){
    int n = (int)a.size();

    for(int i = 1, j = 0; i < n; i++){
        int bit = n >> 1;
        for(; j >= bit; bit >>= 1) j -= bit;
        j += bit;
        if(i < j) swap(a[i], a[j]);
    }

    for(int sz = 2; sz <= n; sz <= 1) {
        double ang = 2*PI/sz * (inv ? -1 : 1);
        base wlen(cos(ang), sin(ang));
        for(int i = 0; i < n; i += sz){
            base w(1);
            for(int j = 0; j < sz/2; j++){
                base u = a[i+j], v = a[i+j+sz/2] * w;
                a[i+j] = u + v;
                a[i+j+sz/2] = u - v;
                w *= wlen;
            }
        }
        if(inv) for(int i = 0; i < n; i++) a[i] /= 1.0 * n;
    }

    void multiply(const vector<int> &a, const vector<int> &b
, vector<int> &res){
        vector<base> fa(a.begin(), a.end());
        vector<base> fb(b.begin(), b.end());
```



```

size_t n = 1;
while(n < a.size()) n <= 1;
while(n < b.size()) n <= 1;
n <= 1;
fa.resize(n), fb.resize(n);

fft(fa, false), fft(fb, false);
for(size_t i = 0; i < n; i++)
    fa[i] *= fb[i];
fft(fa, true);

res.resize(n);
for(size_t i = 0; i < n; ++i)
    res[i] = int(fa[i].real() + 0.5);
}

```

NTT

```

const int mod = 7340033;
const int root = 5;
const int root_1 = 4404020;
const int root_pw = 1<<20;

void fft (vector<int> & a, bool invert) {
    int n = (int) a.size();

    for (int i=1, j=0; i<n; ++i) {
        int bit = n >> 1;
        for (; j>=bit; bit>>=1)
            j -= bit;
        j += bit;
        if (i < j)
            swap (a[i], a[j]);
    }

    for (int len=2; len<=n; len<=1) {
        int wlen = invert ? root_1 : root;
        for (int i=len; i<root_pw; i<=1)
            wlen = int (wlen * 1ll * wlen % mod);
        for (int i=0; i<n; i+=len) {
            int w = 1;
            for (int j=0; j<len/2; ++j) {
                int u = a[i+j], v = int (a[i+j+len/2] * 1
                    ll * w % mod);
                a[i+j] = u+v < mod ? u+v : u+v-mod;
                a[i+j+len/2] = u-v >= 0 ? u-v : u-v+mod;
                w = int (w * 1ll * wlen % mod);
            }
        }
    }
    if (invert) {
        int nrev = reverse (n, mod);
        for (int i=0; i<n; ++i)
            a[i] = int (a[i] * 1ll * nrev % mod);
    }
}

```

Gauss

// Solves systems of linear equations.
 // To use, build a matrix of coefficients and call run(
 mat, R, C). If the i-th variable is free, row[i] will
 be -1, otherwise it's value will be ans[i].

```

namespace Gauss {
    const int MAXC = 1001;
    int row[MAXC];
    double ans[MAXC];
}

```

```

void run(double mat[][MAXC], int R, int C) {
    REP(i, C) row[i] = -1;

    int r = 0;
    REP(c, C) {
        int k = r;
        FOR(i, r, R) if(fabs(mat[i][c]) > fabs(mat[k][c]))
            k = i;
        if(fabs(mat[k][c]) < eps) continue;

        REP(j, C+1) swap(mat[r][j], mat[k][j]);
        REP(i, R) if (i != r) {
            double w = mat[i][c] / mat[r][c];
            REP(j, C+1) mat[i][j] -= mat[r][j] * w;
        }
        row[c] = r++;
    }

    REP(i, C) {
        int r = row[i];
        ans[i] = r == -1 ? 0 : mat[r][C] / mat[r][i];
    }
}

```

Gauss Xor

```

const ll MAX = 1e9;
const ll LOG_MAX = 64 - __builtin_clzll((ll)MAX);

struct gauss{
    vector<ll> vet;
    gauss(){}
    gauss(ll val){
        if(val) vet.push_back(val);
    }
    void add(ll val){
        int sig = LOG_MAX;
        for(int i = 0; i < (int)vet.size(); i++){
            while(!(vet[i] & (1LL << sig))) sig--;
            if(val & (1LL << sig)) val ^= vet[i];
        }
        if(!val) return;
        sig = LOG_MAX;
        while(!(val & (1 << sig))) sig--;
        for(auto &x : vet) if(x & (1LL << sig)) x ^= val;
        vet.push_back(val);
        for(int i = (int)vet.size() - 2; i >= 0 && vet[i]
            < vet[i + 1]; i--){
            swap(vet[i], vet[i + 1]);
        }
    }
};

```

Simpson

```

inline double simpson(double fl,double fr,double fmid,
    double l,double r) { return (fl+fr+4.0*fmid)*(r-l)
    /6.0; }
double rsimpson(double slr,double fl,double fr,double
    fmid,double l,double r)
{
    double mid = (l+r)*0.5;
    double fml = f((l+mid)*0.5);
    double fmr = f((mid+r)*0.5);
    double slm = simpson(fl,fmid,fml,l,mid);
    double smr = simpson(fmid,fr,fmr,mid,r);
    if(fabs(slr-slm-smr) < eps) return slm+smr;
    return rsimpson(slm,fl,fmid,fml,l,mid)+rsimpson(smr,

```

```

    fmid, fr, fmr, mid, r);
}
double integrate(double l, double r) {
    double mid = (l+r)*0.5;
    double fl = f(l);
    double fr = f(r);
    double fmid = f(mid);
    return rsimpson(simpson(fl, fr, fmid, l, r), fl, fr, fmid, l,
        , r);
}

```

Graphs

Dinic

```

const int N = 1000005;
const int E = 20000006;
vector<int> g[N];

int ne;
struct Edge{
    int from, to;
    ll flow, cap;
} edge[E];

int lvl[N], vis[N], pass, start = N-2, target = N-1;
int qu[N], qt, px[N];

ll run(int s, int sink, ll minE){
    if(s == sink) return minE;

    ll ans = 0;

    for(; px[s] < (int)g[s].size(); px[s]++){
        int e = g[s][px[s]];
        auto &v = edge[e], &rev = edge[e^1];
        if(lvl[v.to] != lvl[s]+1 || v.flow >= v.cap)
            continue; // v.cap - v.flow < lim
        ll tmp = run(v.to, sink, min(minE, v.cap-v.flow));
        v.flow += tmp, rev.flow -= tmp;
        ans += tmp, minE -= tmp;
        if(minE == 0) break;
    }
    return ans;
}

bool bfs(int source, int sink){
    qt = 0;
    qu[qt++] = source;
    lvl[source] = 1;
    vis[source] = ++pass;

    for(int i = 0; i < qt; i++){
        int u = qu[i];
        px[u] = 0;
        if(u == sink) return true;

        for(int e : g[u]){
            auto v = edge[e];
            if(v.flow >= v.cap || vis[v.to] == pass)
                continue; // v.cap - v.flow < lim
            vis[v.to] = pass;
            lvl[v.to] = lvl[u]+1;
            qu[qt++] = v.to;
        }
    }
    return false;
}

```

```

ll flow(int source = start, int sink = target){
    ll ans = 0;
    //for(lim = (1LL << 62); lim >= 1; lim /= 2)
    while(bfs(source, sink))
        ans += run(source, sink, oo);
    return ans;
}

```

```

void addEdge(int u, int v, ll c = 1, ll rc = 0){
    edge[ne] = {u, v, 0, c};
    g[u].push_back(ne++);
    edge[ne] = {v, u, 0, rc};
    g[v].push_back(ne++);
}

```

```

void reset_flow(){
    for(int i = 0; i < ne; i++)
        edge[i].flow = 0;
}

```

Push relabel

```

// Push relabel in  $O(V^2 E^{0.5})$  with gap heuristic
// It's quite fast
template<typename flow_t = long long>
struct PushRelabel {
    struct Edge { int to, rev; flow_t f, c; };
    vector<vector<Edge>> g;
    vector<flow_t> ec;
    vector<Edge*> cur;
    vector<vector<int>> hs;
    vector<int> H;
    PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(n) {}
    void add_edge(int s, int t, flow_t cap, flow_t rcap = 0) {
        if (s == t) return;
        Edge a = {t, (int)g[t].size(), 0, cap};
        Edge b = {s, (int)g[s].size(), 0, rcap};
        g[s].push_back(a);
        g[t].push_back(b);
    }
    void add_flow(Edge& e, flow_t f) {
        Edge &back = g[e.to][e.rev];
        if (!ec[e.to] && f)
            hs[H[e.to]].push_back(e.to);
        e.f += f, ec[e.to] += f;
        back.f -= f, ec[back.to] -= f;
    }
    flow_t max_flow(int s, int t) {
        int v = g.size();
        H[s] = v; ec[t] = 1;
        vector<int> co(2 * v);
        co[0] = v-1;
        for(int i = 0; i < v; ++i) cur[i] = g[i].data();
        for(auto &e : g[s]) add_flow(e, e.c);

        if(hs[0].size())
            for (int hi = 0; hi >= 0; ) {
                int u = hs[hi].back();
                hs[hi].pop_back();
                while (ec[u] > 0) // discharge u
                    if (cur[u] == g[u].data() + g[u].size()) {
                        H[u] = 1e9;
                        for(auto &e:g[u])
                            if (e.c - e.f && H[u] > H[e.to]+1)
                                H[u] = H[e.to]+1, cur[u] = &e;
                    }
            }
    }
}

```

```

        if (++co[H[u]],!--co[hi] && hi < v)
            for(int i = 0; i < v; ++i)
                if (hi < H[i] && H[i] < v){
                    --co[H[i]];
                    H[i] = v + 1;
                }
            hi = H[u];
        } else if (cur[u]->c - cur[u]->f && H[u]
        == H[cur[u]->to]+1)
            add_flow(*cur[u], min(ec[u], cur[u]->c
            - cur[u]->f));
        else ++cur[u];
        while (hi >= 0 && hs[hi].empty()) --hi;
    }
    return -ec[s];
}
};

```

Min Cost Max Flow

```

const ll oo = 1e18;
const int N = 222, E = 2 * 1000006;

vector<int> g[N];
int ne;
struct Edge{
    int from, to;
    ll cap, cost;
} edge[E];

int start = N-1, target = N-2, p[N];
int inqueue[N];
ll d[N];

bool spfa(int source, int sink){
    for(int i = 0; i < N; i++) d[i] = oo, inqueue[i] =
        0;

    d[source] = 0;
    queue<int> q;
    q.push(source);
    inqueue[source] = 1;

    while(!q.empty()){
        int u = q.front(); q.pop();
        inqueue[u] = 0;

        for(int e : g[u]){
            auto v = edge[e];
            if(v.cap > 0 and d[u] + v.cost < d[v.to]){
                d[v.to] = d[u] + v.cost;
                p[v.to] = e;
                if(!inqueue[v.to]){
                    q.push(v.to);
                    inqueue[v.to] = 1;
                }
            }
        }
    }
    return d[sink] != oo;
}

// <max flow, min cost>
pair<ll, ll> mincost(int source = start, int sink =
target){
    ll ans = 0, mf = 0;
    while(spfa(source, sink)){
        ll f = oo;

```

```

        for(int u = sink; u != source; u = edge[ p[u] ].
        from)
            f = min(f, edge[ p[u] ].cap);

        for(int u = sink; u != source; u = edge[ p[u] ].
        from){
            edge[ p[u] ].cap -= f;
            edge[ p[u] ^ 1 ].cap += f;
        }

        mf += f;
        ans += f * d[sink];
    }
    return {mf, ans};
}

```

```

void addEdge(int u, int v, ll c, ll cost){
    edge[ne] = {u, v, c, cost};
    g[u].push_back(ne++);
    edge[ne] = {v, u, 0, -cost};
    g[v].push_back(ne++);
}

```

Small to Large

```

void cnt_sz(int u, int p = -1){
    sz[u] = 1;

    for(int v : g[u]) if(v != p)
        cnt_sz(v, u), sz[u] += sz[v];
}

void add(int u, int p, int big = -1){
    // Update info about this vx in global answer

    for(int v : g[u]) if(v != p && v != big)
        add(v, u);
}

void dfs(int u, int p, int keep){

    int big = -1, mmx = -1;

    for(int v : g[u]) if(v != p && sz[v] > mmx)
        mmx = sz[v], big = v;

    for(int v : g[u]) if(v != p && v != big)
        dfs(v, u, 0);

    if(big != -1) dfs(big, u, 1);

    add(u, p, big);

    for(auto x : q[u]){
        // answer all queries for this vx
    }

    if(!keep){
        // Remove data from this subtree
    }
}

```

Centroid Decomposition

```

vector<pair<int,int>> G[500005];
int subtree[500005], k;
bool erased[500005];
ll dist[500005], ans;

```

```

int calc_sz(int v, int p){
    subtree[v] = 1;
    for(auto x : G[v]) if(!erased[x.ff] && x.ff != p){
        subtree[v] += calc_sz(x.ff, v);
    }
    return subtree[v];
}

int centroid(int v, int p, int treesize){
    for(auto x : G[v]) if(!erased[x.ff] && x.ff != p){
        if(subtree[x.ff] * 2 > treesize)
            return centroid(x.ff, v, treesize);
    }
    return v;
}

void procurar_ans(int v, int p, int d_atual, ll custo){
    ans = min(ans, dist[k - d_atual] + custo);
    if(d_atual == k) return;
    for(auto x : G[v]) if(!erased[x.ff] && x.ff != p)
        procurar_ans(x.ff, v, d_atual+1, custo+x.ss);
}

void atualiza_dist(int v, int p, int d_atual, ll custo){
    dist[d_atual] = min(dist[d_atual], custo);
    if(d_atual == k) return;
    for(auto x : G[v]) if(!erased[x.ff] && x.ff != p)
        atualiza_dist(x.ff, v, d_atual+1, custo+x.ss);
}

void decomp(int v, int p){
    int treesize = calc_sz(v, v);
    if(treesize < k) return;
    int cent = centroid(v, v, treesize);
    erased[cent] = 1;

    for(int i = 1; i <= treesize; i++) dist[i] = 1e18;

    for(pair<int, int> x : G[cent]) if(!erased[x.ff]){
        procurar_ans(x.ff, cent, 1, x.ss);
        atualiza_dist(x.ff, cent, 1, x.ss);
    }

    for(pair<int, int> x : G[cent]) if(!erased[x.ff])
        decomp(x.ff, cent);
}

```

Kosaraju

```

vector<int> g[N], gt[N], S;
int vis[N], cor[N];

void dfs(int u){
    vis[u] = 1;
    for(int v : g[u]) if(!vis[v]) dfs(v);
    S.push_back(u);
}

void dfst(int u, int e){
    cor[u] = e;
    for(int v : gt[u]) if(!cor[v]) dfst(v, e);
}

void kosaraju(){
    for(int i = 1; i <= n; i++) if(!vis[i]) dfs(i);
    for(int i = 1; i <= n; i++) for(int j : g[i])
        gt[j].push_back(i);
    int e = 0;
    reverse(S.begin(), S.end());
}

```

```

for(int u : S) if(!cor[u])
    dfst(u, ++e);
}

```

Tarjan

```

int cnt = 0, root;
void dfs(int u, int p = -1){
    low[u] = num[u] = ++t;
    for(int v : g[u]){
        if(!num[v]){
            dfs(v, u);
            if(u == root) cnt++;
            if(low[v] >= num[u]) u PONTO DE ARTICULACAO;
            if(low[v] > num[u]) ARESTA u->v PONTE;
            low[u] = min(low[u], low[v]);
        }
        else if(v != p) low[u] = min(low[u], num[v]);
    }
}

root PONTO DE ARTICULACAO <=> cnt > 1

```

```

void tarjanSCC(int u){
    low[u] = num[u] = ++cnt;
    vis[u] = 1;
    S.push_back(u);
    for(int v : g[u]){
        if(!num[v]) tarjanSCC(v);
        if(vis[v]) low[u] = min(low[u], low[v]);
    }
    if(low[u] == num[u]){
        ssc[u] = ++ssc_cnt; int v;
        do{
            v = S.back(); S.pop_back(); vis[v] = 0;
            ssc[v] = ssc_cnt;
        }while(u != v);
    }
}

```

Max Clique

```

long long adj[N], dp[N];

for(int i = 0; i < n; i++){
    for(int j = 0; j < n; j++){
        int x;
        scanf("%d", &x);
        if(x || i == j)
            adj[i] |= 1LL << j;
    }
}

int resto = n - n/2;
int C = n/2;
for(int i = 1; i < (1 << resto); i++){
    int x = i;
    for(int j = 0; j < resto; j++){
        if(i & (1 << j))
            x &= adj[j + C] >> C;
    }
    if(x == i){
        dp[i] = __builtin_popcount(i);
    }
}

for(int i = 1; i < (1 << resto); i++)
    for(int j = 0; j < resto; j++)
        if(i & (1 << j))
            dp[i] = max(dp[i], dp[i ^ (1 << j)]);
}

```

```

int maxCliq = 0;
for(int i = 0; i < (1 << C); i++){
    int x = i, y = (1 << resto) - 1;
    for(int j = 0; j < C; j++){
        if(i & (1 << j))
            x &= adj[j] & ((1 << C) - 1), y &= adj[j] >> C;
        if(x != i) continue;
        maxCliq = max(maxCliq, __builtin_popcount(i) + dp[y]);
    }
}

```

Dominator Tree

```

vector<int> g[N], gt[N], T[N];
vector<int> S;
int dsu[N], label[N];
int sdom[N], idom[N], dfs_time, id[N];

vector<int> bucket[N];
vector<int> down[N];

void prep(int u){
    S.push_back(u);
    id[u] = ++dfs_time;
    label[u] = sdom[u] = dsu[u] = u;

    for(int v : g[u]){
        if(!id[v])
            prep(v), down[u].push_back(v);
        gt[v].push_back(u);
    }
}

int fnd(int u, int flag = 0){
    if(u == dsu[u]) return u;
    int v = fnd(dsu[u], 1), b = label[ dsu[u] ];
    if(id[ sdom[b] ] < id[ sdom[ label[u] ] ])
        label[u] = b;
    dsu[u] = v;
    return flag ? v : label[u];
}

void build_dominator_tree(int root, int sz){
    // memset(id, 0, sizeof(int) * (sz + 1));
    // for(int i = 0; i <= sz; i++) T[i].clear();
    prep(root);
    reverse(S.begin(), S.end());

    int w;
    for(int u : S){
        for(int v : gt[u]){
            w = fnd(v);
            if(id[ sdom[w] ] < id[ sdom[u] ])
                sdom[u] = sdom[w];
        }
        gt[u].clear();

        if(u != root) bucket[ sdom[u] ].push_back(u);

        for(int v : bucket[u]){
            w = fnd(v);
            if(sdom[w] == sdom[v]) idom[v] = sdom[v];
            else idom[v] = w;
        }
        bucket[u].clear();
    }
}

```

```

        for(int v : down[u]) dsu[v] = u;
        down[u].clear();
    }

    reverse(S.begin(), S.end());
    for(int u : S) if(u != root){
        if(idom[u] != sdom[u]) idom[u] = idom[ idom[u] ];
        T[ idom[u] ].push_back(u);
    }
    S.clear();
}

```

Min Cost Matching

```

// Min cost matching
// O(n^2 * m)
// n == nro de linhas
// m == nro de colunas
// n <= m | flow == n
// a[i][j] = custo pra conectar i a j
vector<int> u(n + 1), v(m + 1), p(m + 1), way(m + 1);
for(int i = 1; i <= n; ++i){
    p[0] = i;
    int j0 = 0;
    vector<int> minv(m + 1, oo);
    vector<char> used(m + 1, false);
    do{
        used[j0] = true;
        int i0 = p[j0], delta = oo, j1;
        for(int j = 1; j <= m; ++j)
            if(!used[j]){
                int cur = a[i0][j] - u[i0] - v[j];
                if(cur < minv[j])
                    minv[j] = cur, way[j] = j0;
                if(minv[j] < delta)
                    delta = minv[j], j1 = j;
            }
        for(int j = 0; j <= m; ++j)
            if(used[j])
                u[p[j]] += delta, v[j] -= delta;
            else
                minv[j] -= delta;
        j0 = j1;
    }while(p[j0] != 0);

    do{
        int j1 = way[j0];
        p[j0] = p[j1];
        j0 = j1;
    }while(j0);

    // match[i] = coluna escolhida para linha i
    vector<int> match(n + 1);
    for(int j = 1; j <= m; ++j)
        match[p[j]] = j;
}

```

```
int cost = -v[0];
```

Strings

Aho Corasick

```

map<char, int> to[N];
int ne = 1, term[N], fail[N];

void add_string(char *str){
    int p = 0;

```

```

    for(int i = 0; str[i]; i++){
        if(!to[p][ str[i] ]) to[p][ str[i] ] = ne++;
        p = to[p][ str[i] ];
    }
    term[p] = 1;
}

int go(int s, char c){
    while(s && !to[s].count(c)) s = fail[s];
    if(to[s].count(c)) return to[s][c];
    return s;
}

void init(){
    queue<int> q;
    q.push(0);

    int u, v; char c;
    while(!q.empty()){
        u = q.front(); q.pop();

        for(auto w : to[u]){
            tie(c, v) = w;
            q.push(v);
            if(u){
                fail[v] = go(fail[u], c);
                term[v] |= term[ fail[v] ];
            }
        }
    }
}

```

Suffix Array

```

char s[N];
int n, sa[N], tsa[N], lcp[N], r[N], nr[N], c[N];

void sort(int k, int mx){
    mx += 2;
    memset(c, 0, sizeof(int) * mx);
    for(int i = 0; i < n; i++) c[i + k < n ? r[i+k]+2 : 1]++;
    partial_sum(c, c+mx, c);
    int t;
    for(int i = 0; i < n; i++){
        t = sa[i]+k < n ? r[ sa[i]+k ]+1 : 0;
        tsa[ c[t]++ ] = sa[i];
    }
    memcpy(sa, tsa, sizeof(int) * n);
}

void build_sa(){
    for(int i = 0; i < n; i++) sa[i] = i, r[i] = s[i];

    int t = 300, a, b;
    for(int sz = 1; sz < n; sz *= 2){
        sort(sz, t), sort(0, t);
        t = nr[ sa[0] ] = 0;
        for(int i = 1; i < n; i++){
            a = sa[i]+sz < n ? r[ sa[i]+sz ] : -1;
            b = sa[i-1]+sz < n ? r[ sa[i-1]+sz ] : -1;
            nr[ sa[i] ] = r[ sa[i] ] == r[ sa[i-1] ] && a == b ? t : ++t;
        }
        if(t == n-1) break;
        memcpy(r, nr, sizeof(int) * n);
    }
}

```

```

    }
}

void build_lcp(){ // lcp[i] = lcp(s[:i], s[:i+1])
    int k = 0;
    for(int i = 0; i < n; i++) r[ sa[i] ] = i;

    for(int i = 0; i < n; i++){
        if(r[i] == n-1) k = 0;
        else{
            int j = sa[r[i]+1];
            while(i+k < n && j+k < n && s[i+k] == s[j+k])
                k++;
            lcp[r[i]] = k;
            if(k) k--;
        }
    }

    int comp_lcp(int i, int j){
        if(i == j) return n - i;
        if(r[i] > r[j]) swap(i, j);
        return min(lcp[k] for k in [r[i], r[j]-1]);
    }
}

```

Z Algorithm

```

vector<int> z_algo(const string &s) {
    int n = s.size(), L = 0, R = 0;
    vector<int> z(n, 0);
    for(int i = 1; i < n; i++){
        if(i <= R) z[i] = min(z[i-L], R - i + 1);
        while(z[i]+i < n && s[ z[i]+i ] == s[ z[i] ])
            z[i]++;
        if(i+z[i]-1 > R) L = i, R = i + z[i] - 1;
    }
    return z;
}

```

Prefix function/KMP

```

vector<int> prefix_function(const string &s){
    int n = s.size();
    vector<int> b(n+1);
    b[0] = -1;
    int i = 0, j = -1;
    while(i < n){
        while(j >= 0 && s[i] != s[j]) j = b[j];
        b[++i] = ++j;
    }
    return b;
}

void kmp(const string &t, const string &p){
    vector<int> b = prefix_function(p);
    int n = t.size(), m = p.size();
    int j = 0;
    for(int i = 0; i < n; i++){
        while(j >= 0 && t[i] != p[j]) j = b[j];
        j++;
        if(j == m){
            //patern of p found on t
            j = b[j];
        }
    }
}

Min rotation

int min_rotation(int *s, int N) {

```

```

REP(i, N) s[N+i] = s[i];

int a = 0;
REP(b, N) REP(i, N) {
    if (a+i == b || s[a+i] < s[b+i]) { b += max(0, i-1);
        break; }
    if (s[a+i] > s[b+i]) { a = b; break; }
}
return a;
}

```

All palindrome

```

void manacher(char *s, int N, int *rad) {
    static char t[2*MAX];
    int m = 2*N - 1;

    REP(i, m) t[i] = -1;
    REP(i, N) t[2*i] = s[i];

    int x = 0;
    FOR(i, 1, m) {
        int &r = rad[i] = 0;
        if (i <= x+rad[x]) r = min(rad[x+x-i], x+rad[x]-i);
        while (i-r-1 >= 0 && i+r+1 < m && t[i-r-1] == t[i+r+1]) ++r;
        if (i+r >= x+rad[x]) x = i;
    }

    REP(i, m) if (i-rad[i] == 0 || i+rad[i] == m-1) ++rad[i];
    REP(i, m) rad[i] /= 2;
}

```

Suffix Automaton

```

map<char, int> to[2*N];
int link[2*N], len[2*N], last = 0, sz = 1;

void add_letter(char c){
    int p = last;
    last = sz++;
    len[last] = len[p] + 1;
    for(; !to[p][c]; p = link[p]) to[p][c] = last;
    if(to[p][c] == last){
        link[last] = 0;
        return;
    }
    int u = to[p][c];
    if(len[u] == len[p]+1){
        link[last] = u;
        return;
    }
    int c1 = sz++;
    to[c1] = to[u];
    link[c1] = link[u];
    len[c1] = len[p]+1;
    link[last] = link[u] = c1;
    for(; to[p][c] == u; p = link[p]) to[p][c] = c1;
}

```

Suffix Tree

```

namespace sf {
    // const int NS = ; const int N = * 2;
    int cn, cd, ns, en = 1, lst;
    string S[NS]; int si = -1;
    vector<int> sufn[N]; // sufn[si][i] no do sufixo S[si][i]
    ...
    struct node {

```

```

        int l, r, si, p, suf;
        map<char, int> adj;
        node() : l(0), r(-1), suf(0), p(0) {}
        node(int L, int R, int S, int P) : l(L), r(R), si(S), p(P) {}
        inline int len() { return r - l + 1; }
        inline int operator[](int i) { return S[si][l + i]; }
        inline int& operator()(char c) { return adj[c]; }
    } t[N];
    inline int new_node(int L, int R, int S, int P) { t[en] = node(L, R, S, P); return en++; }
    void add_string(string s) {
        s += '$'; S[++si] = s; sufn[si].resize(s.size() + 1);
        ; cn = cd = 0;
        int i = 0; const int n = s.size();
        for(int j = 0; j < n; j++)
            for(; i <= j; i++) {
                if(cd == t[cn].len() && t[cn][s[j]]) { cn = t[cn][s[j]]; cd = 0; }
                if(cd < t[cn].len() && t[cn][cd] == s[j]) { cd++;
                    if(j < s.size() - 1) break;
                } else {
                    if(i) t[lst].suf = cn;
                    for(; i <= j; i++) { sufn[si][i] = cn;
                        cn = t[cn].suf; }
                }
            } else if(cd == t[cn].len()) {
                sufn[si][i] = en;
                if(i) t[lst].suf = en; lst = en;
                t[cn][s[j]] = new_node(j, n - 1, si, cn);
                cn = t[cn].suf; cd = t[cn].len();
            } else {
                int mid = new_node(t[cn].l, t[cn].l + cd - 1, t[cn].si, t[cn].p);
                t[t[cn].p][t[cn][0]] = mid;
                if(ns) t[ns].suf = mid;
                if(i) t[lst].suf = en; lst = en;
                sufn[si][i] = en;
                t[mid][s[j]] = new_node(j, n - 1, si, mid);
                ;
                t[mid][t[cn][cd]] = cn;
                t[cn].p = mid; t[cn].l += cd; cn = t[mid].p;
                int g = cn? j - cd : i + 1; cn = t[cn].suf;
                ;
                while(g < j && g + t[t[cn].S[si][g]].len() <= j) {
                    cn = t[cn][S[si][g]]; g += t[cn].len();
                }
                if(g == j) { ns = 0; t[mid].suf = cn; cd = t[cn].len(); }
                else { ns = mid; cn = t[cn][S[si][g]]; cd = j - g; }
            }
        }
    }
}

```

Geometry

2D basics

```

typedef double cod;
double eps = 1e-7;
bool eq(cod a, cod b){ return abs(a - b) <= eps; }

```

```

struct vec{
    cod x, y; int id;
    vec(cod a = 0, cod b = 0) : x(a), y(b) {}
    vec operator+(const vec &o) const{
        return {x + o.x, y + o.y};
    }
    vec operator-(const vec &o) const{
        return {x - o.x, y - o.y};
    }
    vec operator*(cod t) const{
        return {x * t, y * t};
    }
    vec operator/(cod t) const{
        return {x / t, y / t};
    }
    cod operator*(const vec &o) const{ // cos
        return x * o.x + y * o.y;
    }
    cod operator^(const vec &o) const{ // sin
        return x * o.y - y * o.x;
    }
    bool operator==(const vec &o) const{
        return eq(x, o.x) && eq(y, o.y);
    }
    bool operator<(const vec &o) const{
        if(!eq(x, o.x)) return x < o.x;
        return y < o.y;
    }
    cod cross(const vec &a, const vec &b) const{
        return (a-(*this)) ^ (b-(*this));
    }
    int ccw(const vec &a, const vec &b) const{
        cod tmp = cross(a, b);
        return (tmp > eps) - (tmp < -eps);
    }
    cod dot(const vec &a, const vec &b) const{
        return (a-(*this)) * (b-(*this));
    }
    cod len() const{
        return sqrt(x * x + y * y); // <
    }
    double angle(const vec &a, const vec &b) const{
        return atan2(cross(a, b), dot(a, b));
    }
    double tan(const vec &a, const vec &b) const{
        return cross(a, b) / dot(a, b);
    }
    vec unit() const{
        return operator/(len());
    }
    int quad() const{
        if(x > 0 && y >=0) return 0;
        if(x <=0 && y > 0) return 1;
        if(x < 0 && y <=0) return 2;
        return 3;
    }
    bool comp(const vec &a, const vec &b) const{
        return (a - *this).comp(b - *this);
    }
    bool comp(vec b){
        if(quad() != b.quad()) return quad() < b.quad();
        if(!eq(operator^(b), 0)) return operator^(b) > 0;
        return (*this) * (*this) < b * b;
    }
    template<class T>
    void sort_by_angle(T first, T last) const{
        std::sort(first, last, [=](const vec &a, const

```

```

        vec &b){
            return comp(a, b);
        }
    };

    vec rot90() const{ return {-y, x}; }
    vec rot(double a) const{
        return {cos(a)*x -sin(a)*y, sin(a)*x +cos(a)*y};
    }
    vec proj(const vec &b) const{ // proj of *this onto
        b
        cod k = operator*(b) / (b * b);
        return b * k;
    }
    // proj of (*this) onto the plane orthogonal to b
    vec rejection(vec b) const{
        return (*this) - proj(b);
    }
};

struct line{
    cod a, b, c; vec n;
    line(vec q, vec w){ // q.cross(w, (x, y)) = 0
        a = -(w.y-q.y);
        b = w.x-q.x;
        c = -(a * q.x + b * q.y);
        n = {a, b};
    }
    cod dist(const vec &o) const{
        return abs(eval(o)) / n.len();
    }
    bool contains(const vec &o) const{
        return eq(a * o.x + b * o.y + c, 0);
    }
    cod dist(const line &o) const{
        if(!parallel(o)) return 0;
        if(!eq(o.a * b, o.b * a)) return 0;
        if(!eq(a, 0))
            return abs(c - o.c * a / o.a) / n.len();
        if(!eq(b, 0))
            return abs(c - o.c * b / o.b) / n.len();
        return abs(c - o.c);
    }
    bool parallel(const line &o) const{
        return eq(n ^ o.n, 0);
    }
    bool operator==(const line &o) const{
        if(!eq(a*o.b, b*o.a)) return false;
        if(!eq(a*o.c, c*o.a)) return false;
        if(!eq(c*o.b, b*o.c)) return false;
        return true;
    }
    bool intersect(const line &o) const{
        return !parallel(o) || *this == o;
    }
    vec inter(const line &o) const{
        if(parallel(o)){
            if(*this == o){ }
            else{ /* dont intersect */ }
        }

        auto tmp = n ^ o.n;
        return {(o.c*b -c*o.b)/tmp, (o.a*c -a*o.c)/tmp};
    }
    vec at_x(cod x) const{
        return {x, (-c-a*x)/b};
    }
    vec at_y(cod y) const{

```



```

        return {(-c-b*y)/a, y};
    }
    cod eval(const vec &o) const{
        return a * o.x + b * o.y + c;
    }
};

struct segment{
    vec p, q;
    segment(vec a = vec(), vec b = vec()): p(a), q(b) {}
    bool onstrip(const vec &o) const{ // onstrip strip
        return p.dot(o, q) >= -eps && q.dot(o, p) >= -eps;
    }
    cod len() const{
        return (p-q).len();
    }
    cod dist(const vec &o) const{
        if(onstrip(o)) return line(p, q).dist(o);
        return min((o-q).len(), (o-p).len());
    }
    bool contains(const vec &o) const{
        return eq(p.cross(q, o), 0) && onstrip(o);
    }
    bool intersect(const segment &o) const{
        if(contains(o.p)) return true;
        if(contains(o.q)) return true;
        if(o.contains(q)) return true;
        if(o.contains(p)) return true;
        return p.ccw(q, o.p) * p.ccw(q, o.q) == -1
            && o.p.ccw(o.q, q) * o.p.ccw(o.q, p) == -1;
    }
    bool intersect(const line &o) const{
        return o.eval(p) * o.eval(q) <= 0;
    }
    cod dist(const segment &o) const{
        if(line(p, q).parallel(line(o.p, o.q))){
            if(onstrip(o.p) || onstrip(o.q)
                || o.onstrip(p) || o.onstrip(q))
                return line(p, q).dist(line(o.p, o.q));
        }
        else if(intersect(o)) return 0;
        return min(min(dist(o.p), dist(o.q)),
            min(o.dist(p), o.dist(q)));
    }
    cod dist(const line &o) const{
        if(line(p, q).parallel(o))
            return line(p, q).dist(o);
        else if(intersect(o)) return 0;
        return min(o.dist(p), o.dist(q));
    }
};

struct hray{
    vec p, q;
    hray(vec a = vec(), vec b = vec()): p(a), q(b){}
    bool onstrip(const vec &o) const{ // onstrip strip
        return p.dot(q, o) >= -eps;
    }
    cod dist(const vec &o) const{
        if(onstrip(o)) return line(p, q).dist(o);
        return (o-p).len();
    }
    bool intersect(const segment &o) const{
        if(!o.intersect(line(p,q))) return false;
        if(line(o.p, o.q).parallel(line(p,q)))
            return contains(o.p) || contains(o.q);
    }
};

```

```

        return contains(line(p,q).inter(line(o.p,o.q)));
    }
    bool contains(const vec &o) const{
        return eq(line(p, q).eval(o), 0) && onstrip(o);
    }
    cod dist(const segment &o) const{
        if(line(p, q).parallel(line(o.p, o.q))){
            if(onstrip(o.p) || onstrip(o.q))
                return line(p, q).dist(line(o.p, o.q));
            return o.dist(p);
        }
        else if(intersect(o)) return 0;
        return min(min(dist(o.p), dist(o.q)),
            o.dist(p));
    }
    bool intersect(const hray &o) const{
        if(!line(p, q).parallel(line(o.p, o.q)))
            return false;
        auto pt = line(p, q).inter(line(o.p, o.q));
        return contains(pt) && o.contains(pt); // <<
    }
    bool intersect(const line &o) const{
        if(line(p, q).parallel(o)) return line(p, q) == o;
        if(o.contains(p) || o.contains(q)) return true;
        return (o.eval(p) >= -eps)^(o.eval(p)<o.eval(q));
        return contains(o.inter(line(p, q)));
    }
    cod dist(const line &o) const{
        if(line(p,q).parallel(o))
            return line(p,q).dist(o);
        else if(intersect(o)) return 0;
        return o.dist(p);
    }
    cod dist(const hray &o) const{
        if(line(p, q).parallel(line(o.p, o.q))){
            if(onstrip(o.p) || o.onstrip(p))
                return line(p,q).dist(line(o.p, o.q));
            return (p-o.p).len();
        }
        else if(intersect(o)) return 0;
        return min(dist(o.p), o.dist(p));
    }
};

```

```

double heron(cod a, cod b, cod c){
    cod s = (a + b + c) / 2;
    return sqrt(s * (s - a) * (s - b) * (s - c));
}

```

Circle line intersection

```

// intersection of line a * x + b * y + c = 0
// and circle centered at the origin with radius r
double r, a, b, c; // given as input
double x0 = -a*c/(a*a+b*b), y0 = -b*c/(a*a+b*b);
if(c*c > r*r*(a*a+b*b)+EPS)
    puts("no points");
else if(abs(c*c - r*r*(a*a+b*b)) < EPS){
    puts("1 point");
    cout << x0 << ' ' << y0 << '\n';
}
else {
    double d = r*r - c*c/(a*a+b*b);
    double mult = sqrt(d / (a*a+b*b));
    double ax, ay, bx, by;
    ax = x0 + b * mult;
    bx = x0 - b * mult;
    ay = y0 - a * mult;
    by = y0 + a * mult;
}

```

```

    by = y0 + a * mult;
    puts ("2 points");
    cout<<ax<< ' '<<ay<<'\n'<<bx<< ' '<<by<<'\n';
}

```

Circle Circle intersection

Assume that the first circle is centered at the origin and second at (x_2, y_2) . Find circle line intersection of first circle and line $Ax + By + C = 0$, where $A = -2x_2$, $B = -2y_2$, $C = x_2^2 + y_2^2 + r_1^2 - r_2^2$.

Be aware of corner case with two circles centered at the same point.

Tangents of two circles

```

// solve first for same circle(and infinitely many
// tangents)
// Find up to four tangents of two circles
void tangents(pt c, double r1, double r2, vector<line> &
ans){
    double r = r2 - r1;
    double z = c.x * c.x + c.y * c.y;
    double d = z - r * r;
    if(d < -EPS) return;
    d = sqrt(abs(d));
    line l;
    l.a = (c.x * r + c.y * d) / z;
    l.b = (c.y * r - c.x * d) / z;
    l.c = r1;
    ans.push_back (l);
}

```

```

vector<line> tangents(circle a, circle b){
    vector<line> ans;
    pt aux = a.center - b.center;
    for(int i = -1; i <= 1; i += 2)
        for(int j = -1; j <= 1; j += 2)
            tangents(aux, a.r * i, b.r * j, ans);
    for(size_t i = 0; i < ans.size(); ++i)
        ans[i].c -= ans[i].a * a.x + ans[i].b * a.y;
    return ans;
}

```

Convex Hull

```

vector<vec> monotone_chain_ch(vector<vec> P){
    sort(P.begin(), P.end());

    vector<vec> L, U;
    for(auto p : P){
        while(L.size() >= 2 && L[L.size() - 2].cross(L.
            back(), p) < 0)
            L.pop_back();

        L.push_back(p);
    }

    reverse(P.begin(), P.end());
    for(auto p : P){
        while(U.size() >= 2 && U[U.size() - 2].cross(U.
            back(), p) < 0)
            U.pop_back();

        U.push_back(p);
    }

    L.pop_back(), U.pop_back();
}

```

```

L.reserve(L.size() + U.size());
L.insert(L.end(), U.begin(), U.end());

```

```
return L;
```

Check point inside polygon

```

bool below(const vector<vec> &vet, vec p){
    auto it = lower_bound(vet.begin(), vet.end(), p);
    if(it == vet.end()) return false;
    if(it == vet.begin()) return *it == p;
    return prev(it)->cross(*it, p) <= 0;
}

```

```

bool above(const vector<vec> &vet, vec p){
    auto it = lower_bound(vet.begin(), vet.end(), p);
    if(it == vet.end()) return false;
    if(it == vet.begin()) return *it == p;
    return prev(it)->cross(*it, p) >= 0;
}

```

```

// lowerhull, upperhull and point, borders included
bool inside_poly(const vector<vec> &lo, const vector<vec>
    &hi, vec p){
    return below(hi, p) && above(lo, p);
}

```

Check point inside polygon without lower/upper hull

```

// borders included
// must not have 3 colinear consecutive points
bool inside_poly(const vector<vec> &v, vec p){
    if(v[0].ccw(v[1], p) < 0) return false;
    if(v[0].ccw(v.back(), p) > 0) return 0;
    if(v[0].ccw(v.back(), p) == 0)
        return v[0].dot(p, v.back()) >= 0
            && v.back().dot(p, v[0]) >= 0;

    int L = 1, R = (int)v.size() - 1, ans = 1;

    while(L <= R){
        int mid = (L+R)/2;
        if(v[0].ccw(v[mid], p) >= 0) ans = mid, L = mid
            +1;
        else R = mid-1;
    }

    return v[ans].ccw(v[(ans+1)%v.size()], p) >= 0;
}

```

Minkowski sum

```

vector<vec> mk(const vector<vec> &a, const vector<vec> &b){
    int i = 0, j = 0;
    for(int k = 0; k < (int)a.size(); k++){
        if(a[k] < a[i])
            i = k;
        for(int k = 0; k < (int)b.size(); k++){
            if(b[k] < b[j])
                j = k;
        }

        vector<vec> c;
        c.reserve(a.size() + b.size());
        for(int k = 0; k < (int)(a.size()+b.size()); k++){
            vec pt{a[i] + b[j]};
            if((int)c.size() >= 2
                && c[c.size()-2].ccw(c.back(), pt) == 0)
                c.pop_back();
        }
    }
}

```

```

    c.push_back(pt);
    int q = i+1, w = j+1;
    if(q == int(a.size())) q = 0;
    if(w == int(b.size())) w = 0;
    if(c.back().ccw(a[i]+b[w], a[q]+b[j]) < 0) i = q;
    else j = w;
}
c.shrink_to_fit();

return c;
}

```

Geo Notes

Center of mass

System of points(2D/3D): Mass weighted average of points.

Frame(2D/3D): Get middle point of each segment solve as previously.

Triangle: Average of vertices.

2D Polygon: Compute **signed** area and center of mass of triangle $((0,0), p_i, p_{i+1})$. Then solve as system of points.

Polyhedron surface: Solve each face as a 2D polygon (be aware of $(0,0)$) then replace each face with its center of mass and solve as system of points.

Tetrahedron(Triangular pyramid): As triangles, its the average of points.

Polyhedron: Can be done as 2D polygon, but with tetrahedralization instead of triangulation.

Pick's Theorem

Given a polygon without self-intersections and all its vertices on integer coordinates in some 2D grid. Let A be its area, I the number of points with interger coordinates stricly inside the polygon and B the number of points with interger coordinates in the border of the polygon. The following formula holds: $A = I + \frac{B}{2} - 1$.

Miscellaneous

LIS

```

multiset<int> S;
for(int i = 0; i < n; i++){
    auto it = S.upper_bound(a[i]); // low for inc
    if(it != S.end()) S.erase(it);
    S.insert(a[i]);
}
ans = S.size();

```

DSU rollback

```

struct DSU{
    vector<int> sz, p, change;
    vector<tuple<int, int, int>> modifications;
    vector<size_t> saves;
    bool bipartite;

    DSU(int n): sz(n+1, 1), p(n+1), change(n+1),
        bipartite(true){
        iota(p.begin(), p.end(), 0);
    }

    void add_edge(int u, int v){
        if(!bipartite) return;
        int must_change = get_colour(u) == get_colour(v);
        int a = rep(u), b = rep(v);

```

```

        if(sz[a] < sz[b]) swap(a, b);
        if(a != b){
            p[b] = a;
            modifications.emplace_back(b, change[b],
                bipartite);
            change[b] ^= must_change;
            sz[a] += sz[b];
        }
        else if(must_change){
            modifications.emplace_back(0, change[0],
                bipartite);
            bipartite = false;
        }
    }

    int rep(int u){
        return p[u] == u ? u : rep(p[u]);
    }

    int get_colour(int u){
        if(p[u] == u) return change[u];
        return change[u] ^ get_colour(p[u]);
    }

    void reset(){
        modifications.clear();
        saves.clear();
        iota(p.begin(), p.end(), 0);
        fill(sz.begin(), sz.end(), 1);
        fill(change.begin(), change.end(), 0);
        bipartite = true;
    }

    void rollback(){
        int u = get<0>(modifications.back());
        tie(ignore, change[u], bipartite) = modifications
            .back();
        sz[ p[u] ] -= sz[u];
        p[u] = u;
        modifications.pop_back();
    }

    void reload(){
        while(modifications.size() > saves.back())
            rollback();
        saves.pop_back();
    }

    void save(){
        saves.push_back(modifications.size());
    }
};

```

Buildings

```

// count the number of circular arrays
// of size m, with elements on range
// [1, c**(x*x)]
#include<bits/stdc++.h>
using namespace std;

#define debug(x) cerr << fixed << #x << " = " << x << endl;

const int MOD = 1e9 + 7, MAX = 1e5 + 5;
int dp[MAX];

inline int add(int a, int b) {

```

```

    return a + b >= MOD ? a + b - MOD : a;
}
inline int sub(int a, int b) {
    return a - b < 0 ? a - b + MOD : a;
}
inline int mult(int a, int b) {
    return (1LL * a * b) % MOD;
}
int f_exp(int x, int exp) {
    if(exp == 0) return 1;
    else if(exp & 1) return mult(x, f_exp(x, exp - 1));
    return f_exp(mult(x, x), exp / 2);
}
inline int inv(int x) {
    return f_exp(x, MOD - 2);
}

```

```

int main() {
    ios::sync_with_stdio(false);
    cin.tie(NULL); cout.tie(NULL);

    int n, m, c;
    cin >> n >> m >> c;

    int x = f_exp(c, n * n);
    int ans = f_exp(x, m);
    for(int i = 1; i <= m; i++) {
        if(m % i == 0) {
            int y = f_exp(x, i);
            for(int j = 1; j < i; j++) {
                if(i % j == 0)
                    y = sub(y, mult(j, dp[j]));
            }
            dp[i] = mult(y, inv(i));
            ans = sub(ans, mult(i - 1, dp[i]));
        }
    }
}

```

```
cout << ans << '\n';
```

```
return 0;
```

Rand

```

#include <random>
#include <chrono>

cout << RAND_MAX << endl;
mt19937 rng(chrono::steady_clock::now().time_since_epoch
().count());
vector<int> permutation(N);

iota(permutation.begin(), permutation.end(), 0);

shuffle(permutation.begin(), permutation.end(), rng);

iota(permutation.begin(), permutation.end(), 0);

for(int i = 1; i < N; i++){
    swap(permutation[i], permutation[
        uniform_int_distribution<int>(0, i)(rng)]);
}

```

Klondike

```

// minimum number of moves to make
// all elements equal
// move: change a segment of equal value

```

```
// elements to any value
```

```

int v[305];
int dp[305][305];
int rec[305][305];

int f(int l, int r){
    if(r == l) return 1;
    if(r < l) return 0;
    if(dp[l][r] != -1) return dp[l][r];
    int ans = f(l+1, r) + 1;
    for(int i = l+1; i <= r; i++){
        if(v[i] == v[l])
            ans = min(ans, f(l, i - 1) + f(i+1, r));
    }

    return dp[l][r] = ans;
}

```

```

int main() {
    int n, m;
    memset(dp, -1, sizeof dp);
    scanf("%d %d", &n, &m);
    for(int i = 0; i < n; i++){
        scanf("%d", &v[i]);
        if(i && v[i] == v[i-1]){
            i--;
            n--;
        }
    }
    printf("%d\n", f(0, n-1) - 1);
    // printf("%d\n", rec[0][n-1]);
    // printf("%d\n", rec[1][n-1]);
    // printf("%d\n", rec[2][n-3]);
}

```

Hilbert Order

```

// maybe use B = n / sqrt(q)
inline int64_t hilbertOrder(int x, int y, int pow = 21,
    int rotate = 0) {
    if(pow == 0) return 0;
    int hpow = 1 << (pow-1);
    int seg = (x < hpow) ? (
        (y < hpow) ? 0 : 3
    ) : (
        (y < hpow) ? 1 : 2
    );
    seg = (seg + rotate) & 3;
    const int rotateDelta[4] = {3, 0, 0, 1};
    int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
    int nrot = (rotate + rotateDelta[seg]) & 3;
    int64_t subSquareSize = int64_t(1) << (2*pow - 2);
    int64_t ans = seg * subSquareSize;
    int64_t add = hilbertOrder(nx, ny, pow-1, nrot);
    ans += (seg == 1 || seg == 2) ? add : (subSquareSize
        - add - 1);
    return ans;
}

```

Modular Factorial

```

// Compute (1*2*...*(p-1)*1*(p+1)*(p+2)*...*n) % p
// in O(p*log(n))

int factmod(int n, int p){
    int ans = 1;
    while(n > 1){
        for(int i = 2; i <= n % p; i++)

```

```

        ans = (ans * i) % p;
        n /= p;
        if(n % 2) ans = p - ans;
    }
    return ans % p;
}

int fac_pow(int n, int p){
    int ans = 0;
    while(n) n /= p, ans += n;
    return ans;
}

int C(int n, int k, int p){
    if(fac_pow(n, p) > fac_pow(n-k, p) + fac_pow(k, p))
        return 0;
    int tmp = factmod(k, p) * factmod(n-k, p);
    return (f_exp(tmp, p-2, p) * factmod(n, p)) % p;
}

```

Enumeration all submasks of a bitmask

```

// loop through all submask of a given bitmask
// it does not include mask 0
for(int sub = mask; sub; sub = (sub-1)&mask){

}

```

Slope Trick

///By woqja125, contest: Codeforces Round #371 (Div. 1),
 problem: (C) Sonya and Problem Wihtout a Legend,
 Accepted, #

```

#include <stdio.h>
#include <queue>

int main() {
    int n, t;
    long long ans = 0;
    std::priority_queue<int> Q;
    scanf("%d%d", &n, &t);
    Q.push(t);
    for(int i = 1; i < n; i++) {
        scanf("%d", &t); t -= i;
        Q.push(t);
        if(Q.top() > t) {
            ans += Q.top() - t;
            Q.pop();
            Q.push(t);
        }
    }
    printf("%lld", ans);
    return 0;
}

```

Fast IO

```

#define pc(x) putchar_unlocked(x)
#define gc(x) getchar_unlocked(x)

inline void scan_int(int &x){
    register int c = gc();
    x = 0;
    int neg = 0;
    for(; ((c < '0' || c > '9') && c != '-'); c = gc());
    if(c == '-'){
        neg = 1;
        c = gc();
    }
}

```

```

for(; c >= '0' && c <= '9'; c = gc())
    x = (x << 1) + (x << 3) + c - '0';
if(neg) x = -x;
}

inline void print_int(int n){
    int rev = 0, count = 0, neg;
    if(n == 0){
        pc('0');
        return;
    }
    if(n < 0) n = -n, neg = 1;
    while(n % 10 == 0) count++, n /= 10;
    for(rev = 0; n != 0; n /= 10)
        rev = (rev << 3) + (rev << 1) + n % 10;
    if(neg) pc('-');
    while(rev != 0) pc(rev % 10 + '0'), rev /= 10;
    while(count-->0) pc('0');
    pc('\n');
}

```

```

inline void print_string(char *str){
    while(*str) pc(*str++);
    pc('\n');
}

```

Knapsack Bounded with Cost

// menor custo para conseguir peso ate M usando N tipos
 diferentes de elementos, sendo que o i-esimo elemento
 pode ser usado b[i] vezes, tem peso w[i] e custo c[i]
 // O(N * M)

```

int b[N], w[N], c[N];
MinQueue Q[M]
int d[M] //d[i] = custo minimo para conseguir peso i

for(int i = 0; i <= M; i++) d[i] = i ? oo : 0;
for(int i = 0; i < N; i++){
    for(int j = 0; j < w[i]; j++){
        Q[j].clear();
        for(int j = 0; j <= M; j++){
            q = Q[j % w[i]];
            if(q.size() >= q) q.pop();
            q.add(c[i]);
            q.push(d[j]);
            d[j] = q.getmin();
        }
    }
}

```

LCA <O(nlgn), O(1)>

```

int start[N], dfs_time;
int tour[2*N], id[2*N];

void dfs(int u){
    start[u] = dfs_time;
    id[dfs_time] = u;
    tour[dfs_time++] = start[u];
    for(int v : g[u]){
        dfs(v);
        id[dfs_time] = u;
        tour[dfs_time++] = start[u];
    }
}

int LCA(int u, int v){
    if(start[u] > start[v]) swap(u, v);
    return id[min(tour[k] for k in [start[u], start[v]])];
}

```

```
}
```

Buffered reader

```
// source: https://github.com/ngthanhtTrung23/
// ACM_Notebook_new/blob/master/buffered_reader.h
int INP, AM, REACHEOF;
#define BUFSIZE (1<<12)
char BUF[BUFSIZE+1], *inp=BUF;
#define GETCHAR(INP) { \
    if(!*inp && !REACHEOF) { \
        memset(BUF, 0, sizeof BUF); \
        int inpzzz = fread(BUF, 1, BUFSIZE, stdin); \
        if (inpzzz != BUFSIZE) REACHEOF = true; \
        inp=BUF; \
    } \
    INP=*inp++; \
}
#define DIG(a) (((a)>='0')&&((a)<='9'))
#define GN(j) { \
    AM=0; \
    GETCHAR(INP); while(!DIG(INP) && INP!='-') GETCHAR( \
        INP); \
    if (INP=='-') {AM=1; GETCHAR(INP);} \
    j=INP-'0'; GETCHAR(INP); \
    while(DIG(INP)) {j=10*j+(INP-'0'); GETCHAR(INP);} \
    if (AM) j=-j; \
}
```

Burnside's Lemma

Let (G, \oplus) be a finite group that acts on a set X . It should hold that $e_g * x = x$ and $g_1 * (g_2 * x) = (g_1 \oplus g_2) * x, \forall x \in X, g_1, g_2 \in G$. For each $g \in G$ let $X^g = \{x \in X \mid g * x = x\}$. The number of orbits its given by:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

Wilson's Theorem

$(n-1)! = -1 \pmod n \iff n$ is prime

Fibonacci

- $F_{n-1}F_{n+1} - F_n^2 = (-1)^n$
- $F_{n+k} = F_kF_{n+1} + F_{k-1}F_n$
- $\text{GCD}(F_n, F_m) = F_{\text{GCD}(n, m)}$
- $F_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}$

Kirchhoff's Theorem

Laplacian matrix is $L = D - A$, where D is a diagonal matrix with vertex degrees on the diagonals and A is adjacency matrix.

The number of spanning trees is any cofactor of L . i -th cofactor is determinant of the matrix gotten by removing i -th row and column of L .

Multigraphs

In $D[i][i]$ all loops are excluded. $A[i][j]$ = number of edges from i to j .

Directed multigraphs

$D[i][i]$ = indegree of i minus the number of loops at i . $A[i][j]$ = number of edges from i to j .

The number of oriented spanning trees rooted at a vertex i is the determinant of the matrix gotten by removing the i th row and column of L .

Matroid

Let X set of objects, $I \subseteq 2^X$ set of independents sets such that:

- $\emptyset \in I$
- $A \in I, B \subseteq A \implies B \in I$
- Exchange axiom, $A \in I, B \in I, |B| > |A| \implies \exists x \in B \setminus A : A \cup \{x\} \in I$
- $A \subseteq X$ and I and I' are maximal independent subsets of A then $|I| = |I'|$

Then (X, I) is a matroid. The combinatorial optimization problem associated with it is: Given a weight $w(e) \geq 0 \forall e \in X$, find an independent subset that has the largest possible total weight.

Matroid intersection

```
// Input two matroids (X, I_a) and (X, I_b)
// output set I of maximum size, I \in I_a and I \in I_b
set<> I;
while(1){
    for(e_i : X \ I)
        if(I + e_i \in I_a and I + e_i \in I_b)
            I = I + e_i;
    set<> A, T; queue<> Q;
    for(x : X) label[x] = MARK1;
    for(e_i : X \ I){
        if(I + e_i \in I_a)
            Q.push(e_i), label[e_i] = MARK2;
        else{
            for(x such that I - x + e_i \in I_a)
                A[x].push(e_i);
        }
        if(I + e_i \in I_b)
            T = T + {e_i}
        else{
            for(x such that I - x + e_i \in I_b)
                A[e_i].push(x);
        }
    }
    if(T.empty()) break;
    bool found = false;
    while(!Q.empty() and !found){
        auto e = Q.front(); Q.pop();
        for(x : A[e]) if(label[x] == MARK1){
            label[x] = e;
            Q.push(x);
            if(x \in T){
                found = true;
                put = 1;
                while(label[x] != MARK2){
                    if(put) I = I + x;
                    else I = I - x;
                    put = 1 - put;
                }
            }
            I = I + x;
        }
    }
}
```

```

        break;
    }
}
}
if(!found) break;
}
return I;

```

Where $\text{path}(e) = [e]$ if $\text{label}[e] = \text{MARK2}$, $\text{path}(\text{label}[e]) + [e]$ otherwise.

Matroid Union

Given k matroids over the same set of objects $(X, I_1), (X, I_2), \dots, (X, I_k)$ find $A_1 \in I_1, A_2 \in I_2, \dots, A_k \in I_k$ such that $i \neq j, A_i \cap A_j = \emptyset$ and $|\bigcup_{i=1}^k A_i|$ is maximum. Matroid union can be reduced to matroid intersection as follows.

Let $X' = X \times \{1, 2, \dots, k\}$, ie, k copies of each element of X with different colors. $M1 = (X', Q)$ where $B \in Q \iff \forall 1 \leq i \leq k, \{x \mid (x, i) \in B\} \in I_i$, ie, for each color, B is independent. $M2 = (X', W)$ where $B \in W \iff i \neq j \implies \neg((x, i) \in B \wedge (x, j) \in B)$, ie, each element is picked by at most one color.

Intersection of $M1$ and $M2$ is the answer for the combinatorial problem of matroid union.

Edge coloring

Data: A graph G

Result: A proper coloring c of the edges of G

Let $U := E(G)$;

while $U \neq \emptyset$ **do**

 Let (u, v) be any edge in U ;

 Let $F[1:k]$ be a maximal fan of u starting at $F[1]=v$;

 Let c be a color that is free on u and d be a color that is free on $F[k]$;

 Invert the cd_u path;

 Let $w \in V(G)$ be such that $w \in F, F' = [F[1]...w]$ is a fan and d is free on w ;

 Rotate F' and set $c(u, w)=d$;

$U := U - (u, v)$;

end

Notes

When we repeat something and each time we have probability p to succeed then the expected number of tries is $\frac{1}{p}$, till we succeed.

Small to large

Trick in statement If k sets are given you should note that the amount of different set sizes is $O(\sqrt{s})$ where s is total size of those sets. And no more than \sqrt{s} sets have size greater than \sqrt{s} . For example, a path to the root in Aho-Corasick through suffix links will have at most $O(\sqrt{s})$ vertices.

gcd on subsegment, we have at most $\log(a_i)$ different values in $\{\text{gcd}(a_j, a_{j+1}, \dots, a_i) \text{ for } j < i\}$.

From static set to expandable. To insert, create a new set with the new element. While there are two sets with same size, merge them. There will be at most $\log(n)$ disjoint sets.

Matrix exponentiation optimization. Save binary power of $A_{n \times n}$ and answer q queries $b = A^m x$ in $O((n^3 + qn^2)\log(m))$.

Ternary search on integers into binary search, comparing $f(\text{mid})$ and $f(\text{mid}+1)$, binary search on derivative

Dynamic offline set For each element we will wind segment of time $[a, b]$ such that element is present in the set during this whole segment. Now we can come up with recursive procedure which handles $[l, r]$ time segment considering that all elements such that $[l, r] \subset [a, b]$ are already included into the set. Now, keeping this invariant we recursively go into $[l, m]$ and $[m+1, r]$ subsegments. Finally when we come into segment of length 1.

$$a > b \implies a \bmod b < \frac{a}{2}$$