

ICPC Team Reference

University of Brasilia

Contents

1 Data Structures	2	5 Geometry	14
1.1 Merge Sort Tree	2	5.1 2D basics	14
1.2 Wavelet Tree	2	5.2 Nearest Points	16
1.3 Order Set	2	5.3 Convex Hull	16
1.4 Hash table	3	5.4 Check point inside polygon	16
1.5 Convex Hull Trick	3	5.5 Check point inside polygon without lower/up- per hull	16
1.6 Min queue	3	5.6 Minkowski sum	17
1.7 Sparse Table	3	6 Miscellaneous	17
1.8 Treap	3	6.1 LIS	17
1.9 ColorUpdate	4	6.2 DSU rollback	17
1.10 Heavy Light Decomposition	4	6.3 Buildings	18
1.11 Iterative Segtree	4	6.4 Rand	18
1.12 LiChao's Segtree	5	6.5 Klondike	18
1.13 Palindromic tree	5	6.6 Hilbert Order	19
2 Math	5	6.7 Modular Factorial	19
2.1 Extended Euclidean Algorithm	5	6.8 Enumeration all submasks of a bitmask	19
2.2 Chinese Remainder Theorem	5	6.9 Slope Trick	19
2.3 Prefix inverse	6	6.10 Fast IO	19
2.4 Pollard Rho	6	6.11 Knapsack Bounded with Cost	20
2.5 Miller Rabin	6	6.12 LCA $<O(\text{nlgn}), O(1)>$	20
2.6 Totiente	6	6.13 Burnside's Lemma	20
2.7 Mobius Function	6	6.14 Wilson's Theorem	20
2.8 Mulmod TOP	6	6.15 Fibonacci	20
2.9 Matrix Determinant	6	6.16 Kirchhoff's Theorem	20
2.10 FFT	7	6.16.1 Multigraphs	20
2.11 NTT	7	6.16.2 Directed multigraphs	20
3 Graphs	7	6.17 Edge coloring	20
3.1 Dinic	7	6.18 Notes	21
3.2 Min Cost Max Flow	8		
3.3 Small to Large	9		
3.4 Junior e Falta de Ideias	9		
3.5 Kosaraju	9		
3.6 Tarjan	10		
3.7 Max Clique	10		
3.8 Dominator Tree	10		
3.9 Min Cost Matching	11		
4 Strings	11		
4.1 Aho Corasick	11		
4.2 Suffix Array	11		
4.3 Z Algorithm	12		
4.4 Prefix function/KMP	12		
4.5 Min rotation	12		
4.6 All palindrome	12		
4.7 Palindromic Tree	12		
4.8 Suffix Automaton	13		
4.9 Suffix Tree	13		

```
set ts=4 sw=4 sta nu rnu sc stl+=%F cindent
imap {<CR> {<CR>}<Esc>0
nmap <F2> 0V$%d
nmap <C-down> :m+1<CR>
nmap <C-up> :m-2<CR>
vmap <C-c> "+y
nmap <C-a> ggVG
syntax on
alias cmp='g++ -Wall -Wformat=2 -Wshadow -Wconversion -
fsanitize=address -fsanitize=undefined -fno-sanitize-
recover -std=c++14'
```

Data Structures

Merge Sort Tree

```
struct MergeTree{
    int n;
    vector<vector<int>> st;

    void build(int p, int L, int R, const int v[]){
        if(L == R){
            st[p].push_back(v[L]);
            return;
        }
        int mid = (L+R)/2;
        build(2*p, L, mid, v);
        build(2*p+1, mid+1, R, v);
        st[p].resize(R-L+1);
        merge(st[2*p].begin(), st[2*p].end(),
              st[2*p+1].begin(), st[2*p+1].end(),
              st[p].begin());
    }

    int query(int p, int L, int R, int i, int j, int x)
    const{
        if(L > j || R < i) return 0;
        if(L >= i && R <= j){
            int id = lower_bound(st[p].begin(), st[p].end()
                                (), x) - st[p].begin();
            return int(st[p].size()) - id;
        }
        int mid = (L+R)/2;
        return query(2*p, L, mid, i, j, x) +
               query(2*p+1, mid+1, R, i, j, x);
    }
};

public:
    MergeTree(int sz, const int v[]): n(sz), st(4*sz){
        build(1, 1, n, v);
    }

    //number of elements >= x on segment [i, j]
    int query(int i, int j, int x) const{
        if(i > j) swap(i, j);
        return query(1, 1, n, i, j, x);
    }
};
```

Wavelet Tree

```
template<typename T>
class wavelet{
    T L, R;
    vector<int> l;
    vector<T> sum; // <<
    wavelet *lef, *rig;
```

```
    int r(int i) const{ return i - l[i]; }

public:
    template<typename ITER>
    wavelet(ITER bg, ITER en){
        lef = rig = nullptr;
        L = *bg, R = *bg;

        for(auto it = bg; it != en; it++){
            L = min(L, *it), R = max(R, *it);
            if(L == R) return;

            T mid = L + (R - L)/2;
            l.reserve(std::distance(bg, en) + 1);
            sum.reserve(std::distance(bg, en) + 1);
            l.push_back(0), sum.push_back(0);
            for(auto it = bg; it != en; it++){
                l.push_back(l.back() + (*it <= mid)),
                sum.push_back(sum.back() + *it);
            }

            auto tmp = stable_partition(bg, en, [mid](T x){
                return x <= mid;
            });

            if(bg != tmp) lef = new wavelet(bg, tmp);
            if(tmp != en) rig = new wavelet(tmp, en);
        }

        ~wavelet(){
            delete lef;
            delete rig;
        }

        // 1 index, first is 1st
        T kth(int i, int j, int k) const{
            if(L >= R) return L;
            int c = l[j] - l[i-1];
            if(c >= k) return lef->kth(l[i-1]+1, l[j], k);
            else return rig->kth(r(i-1)+1, r(j), k - c);
        }

        // # elements > x on [i, j]
        int cnt(int i, int j, T x) const{
            if(L > x) return j - i + 1;
            if(R <= x || L == R) return 0;
            int ans = 0;
            if(lef) ans += lef->cnt(l[i-1]+1, l[j], x);
            if(rig) ans += rig->cnt(r(i-1)+1, r(j), x);
            return ans;
        }

        // sum of elements <= k on [i, j]
        T sumk(int i, int j, T k){
            if(L == R) return R <= k ? L * (j - i + 1) : 0;
            if(R <= k) return sum[j] - sum[i-1];
            int ans = 0;
            if(lef) ans += lef->sumk(l[i-1]+1, l[j], k);
            if(rig) ans += rig->sumk(r(i-1)+1, r(j), k);
            return ans;
        }

        // swap (i, i+1) just need to update "array" l[i]
    };
```

Order Set

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
```

```
#include <ext/pb_ds/detail/standard_policies.hpp>
```

```
using namespace __gnu_pbds; // or pb_ds;
```

```
template<typename T, typename B = null_type>
using oset = tree<T, B, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
// find_by_order / order_of_key
```

Hash table

```
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
```

```
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        // http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }

    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM = chrono::
            steady_clock::now().time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};
```

```
gp_hash_table<long long, int, custom_hash> table;
unordered_map<long long, int, custom_hash> uhash;
uhash.reserve(1 << 15);
uhash.max_load_factor(0.25);
```

Convex Hull Trick

```
const ll is_query = -(1LL<<62);
struct Line{
    ll m, b;
    mutable function<const Line*> succ;
    bool operator<(const Line& rhs) const{
        if(rhs.b != is_query) return m < rhs.m;
        const Line* s = succ();
        if(!s) return 0;
        ll x = rhs.m;
        return b - s->b < (s->m - m) * x; // OVERFLOW?
    }
};

struct Cht : public multiset<Line>{ // maintain max
    bool bad(iterator y){
        auto z = next(y);
        if(y == begin()){
            if(z == end()) return 0;
            return y->m == z->m && y->b <= z->b;
        }
        auto x = prev(y);
        if(z == end()) return y->m == x->m && y->b <= x->b;
        return (x->b - y->b)*(z->m - y->m) >= (y->b - z->b)*
            (y->m - x->m); // BE CAREFUL WITH OVERFLOW
    }
    void insert_line(ll m, ll b){
        auto y = insert({ m, b });
        y->succ = [=]{ return next(y) == end() ? 0 : &*
            next(y); };
        if(bad(y)){ erase(y); return; }
        while(next(y) != end() && bad(next(y))) erase(
```

```
        next(y));
        while(y != begin() && bad(prev(y))) erase(prev(y)
        );
    }
    ll eval(ll x){
        auto l = *lower_bound((Line) { x, is_query });
        return l.m * x + l.b;
    }
};
```

Min queue

```
template<typename T>
class minQ{
    deque<tuple<T, int, int> > p;
    T delta;
    int sz;
public:
    minQ() : delta(0), sz(0) {}
    inline int size() const{ return sz; }
    inline void add(T x){ delta += x; }
    inline void push(T x, int id){
        x -= delta, sz++;
        int t = 1;
        while(p.size() > 0 && get<0>(p.back()) >= x)
            t += get<1>(p.back()), p.pop_back();
        p.emplace_back(x, t, id);
    }
    inline void pop(){
        get<1>(p.front())--, sz--;
        if(!get<1>(p.front())) p.pop_front();
    }
    T getmin() const{ return get<0>(p.front())+delta; }
    int getid() const{ return get<2>(p.front()); }
};
```

Sparse Table

```
const int N = 100005;

int v[N], n;
int dn[N][20];
int fn(int i, int j){
    if(j == 0) return v[i];
    if(!dn[i][j]) return dn[i][j];
    return dn[i][j] = min(fn(i, j-1), fn(i + (1 << (j-1)
        ), j-1));
}

int lg(int x){ return 31 - __builtin_clz(x); }

int getmn(int l, int r){ // [l, r]
    int lz = lg(r - l + 1);
    return min(fn(l, lz), fn(r - (1 << lz) + 1, lz));
}
```

Treap

```
// source: https://github.com/victorsenam/caderno/blob/
    master/code/treap.cpp
//const int N = ; typedef int num;
num X[N]; int en = 1, Y[N], sz[N], L[N], R[N];
void calc(int u) { // update node given children info
    sz[u] = sz[L[u]] + 1 + sz[R[u]];
    // code here, no recursion
}
void unlaze(int u) {
    if(!u) return;
    // code here, no recursion
}
```

```

void split_val(int u, num x, int &l, int &r) { // l gets
    <= x, r gets > x
    unlaze(u); if(!u) return (void) (l = r = 0);
    if(X[u] <= x) { split_val(R[u], x, l, r); R[u] = l;
        l = u; }
    else { split_val(L[u], x, l, r); L[u] = r; r = u; }
    calc(u);
}
void split_sz(int u, int s, int &l, int &r) { // l gets
    first s, r gets remaining
    unlaze(u); if(!u) return (void) (l = r = 0);
    if(sz[L[u]] < s) { split_sz(R[u], s - sz[L[u]] - 1,
        l, r); R[u] = l; l = u; }
    else { split_sz(L[u], s, l, r); L[u] = r; r = u; }
    calc(u);
}
int merge(int l, int r) { // els on l <= els on r
    unlaze(l); unlaze(r); if(!l || !r) return l + r; int
        u;
    if(Y[l] > Y[r]) { R[l] = merge(R[l], r); u = l; }
    else { L[r] = merge(l, L[r]); u = r; }
    calc(u); return u;
}
void init(int n=N-1) { // XXX call before using other
    funcs
    for(int i = en = 1; i <= n; i++) { Y[i] = i; sz[i] =
        1; L[i] = R[i] = 0; }
    random_shuffle(Y + 1, Y + n + 1);
}

```

ColorUpdate

// source: <https://github.com/tfg50/Competitive-Programming/tree/master/Biblioteca/Data%20Structures>

```

#include <set>
#include <vector>

template <class Info = int>
class ColorUpdate {
public:
    struct Range {
        Range(int l = 0) { this->l = l; }
        Range(int l, int r, Info v) {
            this->l = l;
            this->r = r;
            this->v = v;
        }
        int l, r;
        Info v;

        bool operator < (const Range &b) const { return l
            < b.l; }
    };

    std::vector<Range> upd(int l, int r, Info v) {
        std::vector<Range> ans;
        if(l >= r) return ans;
        auto it = ranges.lower_bound(l);
        if(it != ranges.begin()) {
            it--;
            if(it->r > l) {
                auto cur = *it;
                ranges.erase(it);
                ranges.insert(Range(cur.l, l, cur.v));
                ranges.insert(Range(l, cur.r, cur.v));
            }
        }
    }
}

```

```

it = ranges.lower_bound(r);
if(it != ranges.begin()) {
    it--;
    if(it->r > r) {
        auto cur = *it;
        ranges.erase(it);
        ranges.insert(Range(cur.l, r, cur.v));
        ranges.insert(Range(r, cur.r, cur.v));
    }
}
for(it = ranges.lower_bound(l); it != ranges.end
    () && it->l < r; it++) {
    ans.push_back(*it);
}
ranges.erase(ranges.lower_bound(l), ranges.
    lower_bound(r));
ranges.insert(Range(l, r, v));
return ans;
}
private:
    std::set<Range> ranges;
};

```

Heavy Light Decomposition

```

void dfs_sz(int u){
    sz[u] = 1;

    for(auto &v : g[u]) if(v == p[u]){
        swap(v, g[u].back());
        g[u].pop_back();
        break;
    }

    for(auto &v : g[u]){
        p[v] = u;
        dfs_sz(v);
        sz[u] += sz[v];
        if(sz[v] > sz[ g[u][0] ])
            swap(v, g[u][0]);
    }
}

// nxt[u] = start of path with u
// set nxt[root] = root beforehand
void dfs_hld(int u){
    in[u] = t++;
    rin[in[u]] = u;
    for(auto v : g[u]){
        nxt[v] = (v == g[u][0] ? nxt[u] : v);
        dfs_hld(v);
    }
    out[u] = t;
}

// subtree of u => [ in[u], out[u] )
// path from nxt[u] to u => [ in[ nxt[u] ], in[u] ]

```

Iterative Segtree

```

T query(int l, int r, int &pos){ // [l, r]
    T rl, rr;
    for(l += n, r += n+1; l < r; l >>= 1, r >>= 1){
        if(l & 1) rl = merge(rl, st[l++]);
        if(r & 1) rr = merge(st[--r], rr);
    }
    return merge(rl, rr);
}

```

```
// initially save v[i] in st[n+i] for all i in [0, n)
void build(){
    for(int p = n-1; p > 0; p--){
        st[p] = merge(st[2*p], st[2*p+1]);
    }

    void update(int p, T val){
        st[p += n] = val;
        while(p >= 1) st[p] = merge(st[2*p], st[2*p+1]);
    }
}
```

LiChao's Segtree

```
void add_line(line nw, int v = 1, int l = 0, int r =
    maxn) { // [l, r)
    int m = (l + r) / 2;
    bool lef = nw.eval(l) < st[v].eval(l);
    bool mid = nw.eval(m) < st[v].eval(m);
    if(mid) swap(st[v], nw);
    if(r - l == 1) {
        return;
    } else if(lef != mid) {
        add_line(nw, 2 * v, l, m);
    } else {
        add_line(nw, 2 * v + 1, m, r);
    }
}

int get(int x, int v = 1, int l = 0, int r = maxn) {
    int m = (l + r) / 2;
    if(r - l == 1) {
        return st[v].eval(x);
    } else if(x < m) {
        return min(st[v].eval(x), get(x, 2*v, l, m));
    } else {
        return min(st[v].eval(x), get(x, 2*v+1, m, r));
    }
}
```

Palindromic tree

```
#include <bits/stdc++.h>

using namespace std;

const int maxn = 3e5 + 1, sigma = 26;
int len[maxn], link[maxn], to[maxn][sigma];
int slink[maxn], diff[maxn], series_ans[maxn];
int sz, last, n;
char s[maxn];

void init()
{
    s[n++] = -1;
    link[0] = 1;
    len[1] = -1;
    sz = 2;
}

int get_link(int v)
{
    while(s[n - len[v] - 2] != s[n - 1]) v = link[v];
    return v;
}

void add_letter(char c)
{
    s[n++] = c - 'a';
    last = get_link(last);
```

```
if(!to[last][c])
{
    len[sz] = len[last] + 2;
    link[sz] = to[get_link(link[last])][c];
    diff[sz] = len[sz] - len[link[sz]];
    if(diff[sz] == diff[link[sz]])
        slink[sz] = slink[link[sz]];
    else
        slink[sz] = link[sz];
    to[last][c] = sz++;
}
last = to[last][c];
}

int main()
{
    ios::sync_with_stdio(0);
    cin.tie(0);
    init();
    string s;
    cin >> s;
    int n = s.size();
    int ans[n + 1];
    memset(ans, 63, sizeof(ans));
    ans[0] = 0;
    for(int i = 1; i <= n; i++)
    {
        add_letter(s[i - 1]);
        for(int v = last; len[v] > 0; v = slink[v])
        {
            series_ans[v] = ans[i - (len[slink[v]] + diff[v])];
            if(diff[v] == diff[link[v]])
                series_ans[v] = min(series_ans[v],
                    series_ans[link[v]]);
            ans[i] = min(ans[i], series_ans[v] + 1);
        }
        cout << ans[i] << "\n";
    }
    return 0;
}
```

Math

Extended Euclidean Algorithm

```
// a*x + b*y = gcd(a, b), <gcd, x, y>
tuple<int, int, int> gcd(int a, int b) {
    if(b == 0) return make_tuple(a, 1, 0);
    int q, w, e;
    tie(q, w, e) = gcd(b, a % b);
    return make_tuple(q, e, w - e * (a / b));
}
```

Chinese Remainder Theorem

```
// x = vet[i].first (mod vet[i].second)
ll crt(vector<pair<ll, ll>> vet){

    ll ans = vet[0].first, lcm = vet[0].second;
    ll a, b, g, x, y;

    for(int i = 1; i < (int)vet.size(); i++){
        tie(a, b) = vet[i];
        tie(g, x, y) = gcd(lcm, b);
        ans = ans + x * (a - ans) / g % (b / g) * lcm;
        lcm = lcm * b / g;
        ans = (ans % lcm + lcm) % lcm;
    }
}
```

```
    return ans;
}
```

Prefix inverse

```
inv[1] = 1;
for(int i = 2; i < p; i++)
    inv[i] = (p - (p/i) * inv[p%i] % p) % p;
```

Pollard Rho

```
ll rho(ll n){
    if(n % 2 == 0) return 2;

    ll d, c, x, y;
    do{
        c = llrand() % n, x = llrand() % n, y = x;
        do{
            x = add(mul(x, x, n), c, n);
            y = add(mul(y, y, n), c, n);
            y = add(mul(y, y, n), c, n);
            d = __gcd(abs(x - y), n);
        }while(d == 1);
    }while(d == n);
    return d;
}
```

```
ll pollard_rho(ll n){
    ll x, c, y, d, k;
    int i;
    do{
        i = 1;
        x = llrand() % n, c = llrand() % n;
        y = x, k = 4;
        do{
            if(++i == k) y = x, k *= 2;
            x = add(mul(x, x, n), c, n);
            d = __gcd(abs(x - y), n);
        }while(d == 1);
    }while(d == n);
    return d;
}
```

```
void factorize(ll val, map<ll, int> &fac){
    if(rabin(val)) fac[ val ]++;
    else{
        ll d = pollard_rho(val);
        factorize(d, fac);
        factorize(val / d, fac);
    }
}
```

```
map<ll, int> factor(ll val){
    map<ll, int> fac;
    if(val > 1) factorize(val, fac);
    return fac;
}
```

Miller Rabin

```
bool rabin(ll n){
    if(n <= 1) return 0;
    if(n <= 3) return 1;
    ll s = 0, d = n - 1;
    while(d % 2 == 0) d /= 2, s++;
    for(int k = 0; k < 64; k++){
        ll a = (llrand() % (n - 3)) + 2;
        ll x = fexp(a, d, n);
        if(x != 1 && x != n-1){
```

```
            for(int r = 1; r < s; r++){
                x = mul(x, x, n);
                if(x == 1) return 0;
                if(x == n-1) break;
            }
            if(x != n-1) return 0;
        }
    }
    return 1;
}
```

Totiente

```
ll totiente(ll n){
    ll ans = n;
    for(ll i = 2; i*i <= n; i++){
        if(n % i == 0){
            ans = ans / i * (i - 1);
            while(n % i == 0) n /= i;
        }
    }

    if(n > 1) ans = ans / n * (n - 1);
    return ans;
}
```

Mobius Function

```
memset(mu, 0, sizeof mu);
mu[1] = 1;
for(int i = 1; i < N; i++){
    for(int j = i + i; j < N; j += i)
        mu[j] -= mu[i];
}
// g(n) = sum{f(d)} => f(n) = sum{mu(d)*g(n/d)}
```

Mulmod TOP

```
constexpr uint64_t mod = (1ull<<61) - 1;
uint64_t modmul(uint64_t a, uint64_t b){
    uint64_t l1 = (uint32_t)a, h1 = a>>32, l2 = (
        uint32_t)b, h2 = b>>32;
    uint64_t l = l1*l2, m = l1*h2 + l2*h1, h = h1*h2;
    uint64_t ret = (l&mod) + (l>>61) + (h << 3) + (m >>
        29) + (m << 35 >> 3) + 1;
    ret = (ret & mod) + (ret>>61);
    ret = (ret & mod) + (ret>>61);
    return ret-1;
}
```

Matrix Determinant

```
int n;
long double a[n][n];

long double gauss(){
    long double det = 1;
    for(int i = 0; i < n; i++){
        int q = i;
        for(int j = i+1; j < n; j++){
            if(abs(a[j][i]) > abs(a[q][i]))
                q = j;
        }
        if(abs(a[q][i]) < EPS){
            det = 0;
            break;
        }
        if(i != q){
            for(int w = 0; w < n; w++){
                swap(a[i][w], a[q][w]);
                det = -det;
            }
        }
    }
}
```

```

    det *= a[i][i];
    for(int j = i+1; j < n; j++) a[i][j] /= a[i][i];

    for(int j = 0; j < n; j++) if(j != i){
        if(abs(a[j][i]) > EPS)
            for(int k = i+1; k < n; k++)
                a[j][k] -= a[i][k] * a[j][i];
    }
}

return det;
}

```

FFT

```

struct base{
    double r, i;
    base(double _r = 0, double _i = 0) : r(_r), i(_i) {}
    base operator*(const base &o) const{
        return {r*o.r - i*o.i, r*o.i + o.r*i};
    }
    double real() const{ return r; }
    void operator*=(const base &o){
        (*this) = {r*o.r - i*o.i, r*o.i + o.r*i};
    }
    void operator+=(const base &o){r += o.r, i += o.i; }
    void operator/=(const double &o){ r /= o, i /= o; }
    void operator-=(const base &o){r -= o.r, i -= o.i; }
    base operator+(const base &o){return {r+o.r,i+o.i};}
    base operator-(const base &o){return {r-o.r,i-o.i};}
};

```

```
double PI = acos(-1);
```

```

void fft(vector<base> &a, bool inv){
    int n = (int)a.size();

    for(int i = 1, j = 0; i < n; i++){
        int bit = n >> 1;
        for(; j >= bit; bit >>= 1) j -= bit;
        j += bit;
        if(i < j) swap(a[i], a[j]);
    }

    for(int sz = 2; sz <= n; sz <= 1) {
        double ang = 2*PI/sz * (inv ? -1 : 1);
        base wlen(cos(ang), sin(ang));
        for(int i = 0; i < n; i += sz){
            base w(1);
            for(int j = 0; j < sz/2; j++){
                base u = a[i+j], v = a[i+j+sz/2] * w;
                a[i+j] = u + v;
                a[i+j+sz/2] = u - v;
                w *= wlen;
            }
        }
    }
    if(inv) for(int i = 0; i < n; i++) a[i] /= 1.0 * n;
}

```

```

void multiply(const vector<int> &a, const vector<int> &b
, vector<int> &res){
    vector<base> fa(a.begin(), a.end());
    vector<base> fb(b.begin(), b.end());
    size_t n = 1;
    while(n < a.size()) n <= 1;
    while(n < b.size()) n <= 1;
    n <= 1;
}

```

```

fa.resize(n), fb.resize(n);

fft(fa, false), fft(fb, false);
for(size_t i = 0; i < n; i++)
    fa[i] *= fb[i];
fft(fa, true);

res.resize(n);
for(size_t i = 0; i < n; ++i)
    res[i] = int(fa[i].real() + 0.5);
}

```

NTT

```

const int mod = 7340033;
const int root = 5;
const int root_1 = 4404020;
const int root_pw = 1<<20;

void fft(vector<int> &a, bool invert) {
    int n = (int) a.size();

    for (int i=1, j=0; i<n; ++i) {
        int bit = n >> 1;
        for (; j>=bit; bit>>=1)
            j -= bit;
        j += bit;
        if (i < j)
            swap (a[i], a[j]);
    }

    for (int len=2; len<=n; len<=1) {
        int wlen = invert ? root_1 : root;
        for (int i=len; i<root_pw; i<=1)
            wlen = int (wlen * 111 * wlen % mod);
        for (int i=0; i<n; i+=len) {
            int w = 1;
            for (int j=0; j<len/2; ++j) {
                int u = a[i+j], v = int (a[i+j+len/2] * 1
                    11 * w % mod);
                a[i+j] = u+v < mod ? u+v : u+v-mod;
                a[i+j+len/2] = u-v >= 0 ? u-v : u-v+mod;
                w = int (w * 111 * wlen % mod);
            }
        }
    }
    if (invert) {
        int nrev = reverse (n, mod);
        for (int i=0; i<n; ++i)
            a[i] = int (a[i] * 111 * nrev % mod);
    }
}

```

Graphs

Dinic

```

const int N = 100005;
const int E = 2000006;
vector<int> g[N];

int ne;
struct Edge{
    int from, to;
    ll flow, cap;
} edge[E];

int lvl[N], vis[N], pass, start = N-2, target = N-1;
int qu[N], qt, px[N];

```

```

ll run(int s, int sink, ll minE){
    if(s == sink) return minE;

    ll ans = 0;

    for(; px[s] < (int)g[s].size(); px[s]++){
        int e = g[s][ px[s] ];
        auto &v = edge[e], &rev = edge[e^1];
        if(lvl[v.to] != lvl[s]+1 || v.flow >= v.cap)
            continue; // v.cap - v.flow < lim
        ll tmp = run(v.to, sink, min(minE, v.cap-v.flow));
        v.flow += tmp, rev.flow -= tmp;
        ans += tmp, minE -= tmp;
        if(minE == 0) break;
    }
    return ans;
}

bool bfs(int source, int sink){
    qt = 0;
    qu[qt++] = source;
    lvl[source] = 1;
    vis[source] = ++pass;

    for(int i = 0; i < qt; i++){
        int u = qu[i];
        px[u] = 0;
        if(u == sink) return true;

        for(int e : g[u]){
            auto v = edge[e];
            if(v.flow >= v.cap || vis[v.to] == pass)
                continue; // v.cap - v.flow < lim
            vis[v.to] = pass;
            lvl[v.to] = lvl[u]+1;
            qu[qt++] = v.to;
        }
    }
    return false;
}

ll flow(int source = start, int sink = target){
    ll ans = 0;
    //for(lim = (1LL << 62); lim >= 1; lim /= 2)
    while(bfs(source, sink))
        ans += run(source, sink, oo);
    return ans;
}

void addEdge(int u, int v, ll c = 1, ll rc = 0){
    edge[ne] = {u, v, 0, c};
    g[u].push_back(ne++);
    edge[ne] = {v, u, 0, rc};
    g[v].push_back(ne++);
}

void reset_flow(){
    for(int i = 0; i < ne; i++)
        edge[i].flow = 0;
}

```

Min Cost Max Flow

```

const ll oo = 1e18;
const int N = 505;
const int E = 30006;

```

```

vector<int> g[N];

int ne;

struct Edge{
    int from, to;
    ll cap, cost;
} edge[E];

int lvl[N], vis[N], pass, source, target, p[N], px[N];

ll d[N];

ll back(int s, ll minE){
    if(s == source) return minE;

    int e = p[s];

    ll f = back(edge[e].from, min(minE, edge[e].cap));
    edge[e].cap -= f;
    edge[e^1].cap += f;
    return f;
}

int dijkstra(){
    for(i, N) d[i] = oo;

    priority_queue<pair<ll, int> > q;

    d[source] = 0;

    q.emplace(0, source);

    while(!q.empty()){
        ll dis = -q.top().ff;
        int u = q.top().ss; q.pop();

        if(dis > d[u]) continue;

        for(int e : g[u]){
            auto v = edge[e];
            if(v.cap <= 0) continue;
            if(d[u] + v.cost < d[v.to]){
                d[v.to] = d[u] + v.cost;
                p[v.to] = e;
                q.emplace(-d[v.to], v.to);
            }
        }
    }
    return d[target] != oo;
}

pair<ll, ll> mincost(){
    ll ans = 0, mf = 0;
    while(dijkstra()){
        ll f = back(target, oo);
        mf += f;
        ans += f * d[target];
    }
    return {mf, ans};
}

void addEdge(int u, int v, ll c, ll cost){
    edge[ne] = {u, v, c, cost};
    g[u].pb(ne++);
}

```


Small to Large

```

void cnt_sz(int u, int p = -1){
    sz[u] = 1;

    for(int v : g[u]) if(v != p)
        cnt_sz(v, u), sz[u] += sz[v];
}

void add(int u, int p, int big = -1){
    // Update info about this vx in global answer

    for(int v : g[u]) if(v != p && v != big)
        add(v, u);
}

void dfs(int u, int p, int keep){

    int big = -1, mmx = -1;

    for(int v : g[u]) if(v != p && sz[v] > mmx)
        mmx = sz[v], big = v;

    for(int v : g[u]) if(v != p && v != big)
        dfs(v, u, 0);

    if(big != -1) dfs(big, u, 1);

    add(u, p, big);

    for(auto x : q[u]){
        // answer all queries for this vx
    }

    if(!keep){
        // Remove data from this subtree
    }
}

Junior e Falta de Ideias

#include <bits/stdc++.h>

#define ff first
#define ss second
#define mp make_pair

using namespace std;

typedef long long ll;

vector<pair<int,int>> G[500005];
int subtree[500005], treesize, k;
bool vis[500005];
ll dist[500005], ans;

int dfs(int v, int p){
    subtree[v] = 1;
    for(pair<int,int> x : G[v])
        if(x.ff != p && !vis[x.ff]) subtree[v] += dfs(x.
ff,v);
    return subtree[v];
}

int centroid(int v, int p){
    for(pair<int,int> x : G[v]){
        if(x.ff == p || vis[x.ff]) continue;
        if(subtree[x.ff]*2 > treesize) return centroid(x.

```

```

        ff,v);
    }
    return v;
}

void procurar_ans(int v, int p, int d_atual, ll custo){
    ans = min(ans, dist[k-d_atual] + custo);
    if(d_atual == k) return;
    for(pair<int,int> x : G[v]){
        if(!vis[x.ff] && x.ff != p)
            procurar_ans(x.ff,v,d_atual+1,custo+x.ss);
    }
}

void atualiza_distancia(int v, int p, int d_atual, ll
custo){
    dist[d_atual] = min(dist[d_atual], custo);
    if(d_atual == k) return;
    for(pair<int,int> x : G[v]){
        if(!vis[x.ff] && x.ff != p)
            atualiza_distancia(x.ff,v,d_atual+1,custo+x.
ss);
    }
}

void decomp(int v, int p){
    treesize = dfs(v,v);
    // if(treesize < k) return;
    int cent = centroid(v,v);
    vis[cent] = 1;

    for(int i = 1; i <= treesize; i++)
        dist[i] = 1e18;

    for(pair<int,int> x : G[cent]){
        if(!vis[x.ff]){
            procurar_ans(x.ff,cent,1,x.ss);
            atualiza_distancia(x.ff,cent,1,x.ss);
        }
    }

    for(pair<int,int> x : G[cent]){
        if(!vis[x.ff])
            decomp(x.ff, cent);
    }
}

int main(){
    int n,i,a,b;

    scanf("%d%d", &n,&k);
    for(i = 2; i <= n; i++){
        scanf("%d%d", &a,&b);
        G[i].push_back(mp(a,b));
        G[a].push_back(mp(i,b));
    }
    ans = 1e18;
    decomp(1,-1);

    printf("%lld\n", ans == 1e18 ? -1 : ans);

    return 0;
}

```

Kosaraju

```
vector<int> g[N], gt[N], S;
```

```

int vis[N], cor[N], tempo = 1;

void dfs(int u){
    vis[u] = 1;
    for(int v : g[u]) if(!vis[v]) dfs(v);
    S.push_back(u);
}

int e;
void dfst(int u){
    cor[u] = e;
    for(int v : gt[u]) if(!cor[v]) dfst(v);
}

int main(){

    for(int i = 1; i <= n; i++) if(!vis[i]) dfs(i);

    e = 0;
    reverse(S.begin(), S.end());
    for(int u : S) if(!cor[u])
        e++, dfst(u);

    return 0;
}

```

Tarjan

```

int cnt = 0, root;
void dfs(int u, int p = -1){
    low[u] = num[u] = ++t;
    for(int v : g[u]){
        if(!num[v]){
            dfs(v, u);
            if(v == root) cnt++;
            if(low[v] >= num[u]) u PONTO DE ARTICULACAO;
            if(low[v] > num[u]) ARESTA u->v PONTE;
            low[u] = min(low[u], low[v]);
        }
        else if(v != p) low[u] = min(low[u], num[v]);
    }
}

root PONTO DE ARTICULACAO <=> cnt > 1

```

```

void tarjanSCC(int u){
    low[u] = num[u] = cnt++;
    vis[u] = 1;
    S.push_back(u);
    for(int v : g[u]){
        if(!num[v]) tarjanSCC(v);
        if(vis[v]) low[u] = min(low[u], low[v]);
    }
    if(low[u] == num[u]){
        ssc[u] = ++ssc_cnt; int v;
        do{
            v = S.back(); S.pop_back(); vis[v] = 0;
            ssc[v] = ssc_cnt;
        }while(u != v);
    }
}

```

Max Clique

```

long long adj[N], dp[N];

for(int i = 0; i < n; i++){
    for(int j = 0; j < n; j++){
        int x;
        scanf("%d", &x);
    }
}

```

```

        if(x || i == j)
            adj[i] |= 1LL << j;
    }
}

int resto = n - n/2;
int C = n/2;
for(int i = 1; i < (1 << resto); i++){
    int x = i;
    for(int j = 0; j < resto; j++){
        if(i & (1 << j))
            x &= adj[j + C] >> C;
    }
    if(x == i){
        dp[i] = __builtin_popcount(i);
    }
}

for(int i = 1; i < (1 << resto); i++){
    for(int j = 0; j < resto; j++){
        if(i & (1 << j))
            dp[i] = max(dp[i], dp[i ^ (1 << j)]);
    }
}

int maxCliq = 0;
for(int i = 0; i < (1 << C); i++){
    int x = i, y = (1 << resto) - 1;
    for(int j = 0; j < C; j++){
        if(i & (1 << j))
            x &= adj[j] & ((1 << C) - 1), y &= adj[j] >> C;
    }
    if(x != i) continue;
    maxCliq = max(maxCliq, __builtin_popcount(i) + dp[y]);
}

```

Dominator Tree

```

vector<int> g[N], gt[N], T[N];
vector<int> S;
int dsu[N], label[N];
int sdom[N], idom[N], dfs_time, id[N];

vector<int> bucket[N];
vector<int> down[N];

void prep(int u){
    S.push_back(u);
    id[u] = ++dfs_time;
    label[u] = sdom[u] = dsu[u] = u;

    for(int v : g[u]){
        if(!id[v])
            prep(v), down[u].push_back(v);
        gt[v].push_back(u);
    }
}

int fnd(int u, int flag = 0){
    if(u == dsu[u]) return u;
    int v = fnd(dsu[u], 1), b = label[ dsu[u] ];
    if(id[ sdom[b] ] < id[ sdom[ label[u] ] ] )
        label[u] = b;
    dsu[u] = v;
    return flag ? v : label[u];
}

void build_dominator_tree(int root, int sz){
    // memset(id, 0, sizeof(int) * (sz + 1));
    // for(int i = 0; i <= sz; i++) T[i].clear();
}

```

```

prep(root);

reverse(S.begin(), S.end());

int w;
for(int u : S){
    for(int v : gt[u]){
        w = fnd(v);
        if(id[ sdom[w] ] < id[ sdom[u] ])
            sdom[u] = sdom[w];
    }
    gt[u].clear();

    if(u != root) bucket[ sdom[u] ].push_back(u);

    for(int v : bucket[u]){
        w = fnd(v);
        if(sdom[w] == sdom[v]) idom[v] = sdom[v];
        else idom[v] = w;
    }
    bucket[u].clear();

    for(int v : down[u]) dsu[v] = u;
    down[u].clear();
}

reverse(S.begin(), S.end());

for(int u : S) if(u != root){
    if(idom[u] != sdom[u]) idom[u] = idom[ idom[u] ];
    T[ idom[u] ].push_back(u);
}

S.clear();
}

```

Min Cost Matching

```

// Min cost matching
// O(n^2 * m)
// n == nro de linhas
// m == nro de colunas
// n <= m | flow == n
// a[i][j] = custo pra conectar i a j
vector<int> u(n + 1), v(m + 1), p(m + 1), way(m + 1);
for(int i = 1; i <= n; ++i){
    p[0] = i;
    int j0 = 0;
    vector<int> minv(m + 1, oo);
    vector<char> used(m + 1, false);
    do{
        used[j0] = true;
        int i0 = p[j0], delta = oo, j1;
        for(int j = 1; j <= m; ++j)
            if(!used[j]){
                int cur = a[i0][j] - u[i0] - v[j];
                if(cur < minv[j])
                    minv[j] = cur, way[j] = j0;
                if(minv[j] < delta)
                    delta = minv[j], j1 = j;
            }
        for(int j = 0; j <= m; ++j)
            if(used[j])
                u[p[j]] += delta, v[j] -= delta;
            else
                minv[j] -= delta;
        j0 = j1;
    }
}

```

```

}while(p[j0] != 0);

do{
    int j1 = way[j0];
    p[j0] = p[j1];
    j0 = j1;
}while(j0);
}

// match[i] = coluna escolhida para linha i
vector<int> match(n + 1);
for(int j = 1; j <= m; ++j)
    match[p[j]] = j;

```

```
int cost = -v[0];
```

Strings

Aho Corasick

```

map<char, int> to[N];
int ne = 1, term[N], fail[N];

void add_string(char *str){
    int p = 0;

    for(int i = 0; str[i]; i++){
        if(!to[p][ str[i] ]) to[p][ str[i] ] = ne++;
        p = to[p][ str[i] ];
    }
    term[p] = 1;
}

int go(int s, char c){
    while(s && !to[s].count(c)) s = fail[s];
    if(to[s].count(c)) return to[s][c];
    return s;
}

void init(){
    queue<int> q;
    q.push(0);

    int u, v; char c;
    while(!q.empty()){
        u = q.front(); q.pop();

        for(auto w : to[u]){
            tie(c, v) = w;
            q.push(v);
            if(u){
                fail[v] = go(fail[u], c);
                term[v] |= term[ fail[v] ];
            }
        }
    }
}

```

Suffix Array

```

char s[N];
int n, sa[N], tsa[N], lcp[N], r[N], nr[N], c[N];

void sort(int k, int mx){
    mx += 2;
    memset(c, 0, sizeof(int) * mx);
    for(int i = 0; i < n; i++) c[i + k < n ? r[i+k]+2 :

```

```

    l]++;
    partial_sum(c, c+mx, c);
    int t;
    for(int i = 0; i < n; i++){
        t = sa[i]+k < n ? r[ sa[i]+k ]+1 : 0;
        tsa[ c[t]++ ] = sa[i];
        memcpy(sa, tsa, sizeof(int) * n);
    }

    void build_sa(){

        for(int i = 0; i < n; i++) sa[i] = i, r[i] = s[i];

        int t = 300, a, b;
        for(int sz = 1; sz < n; sz *= 2){
            sort(sz, t), sort(0, t);
            t = nr[ sa[0] ] = 0;
            for(int i = 1; i < n; i++){
                a = sa[i]+sz < n ? r[ sa[i]+sz ] : -1;
                b = sa[i-1]+sz < n ? r[ sa[i-1]+sz ] : -1;
                nr[ sa[i] ] = r[ sa[i] ] == r[ sa[i-1] ] && a
                    == b ? t : ++t;
            }
            if(t == n-1) break;
            memcpy(r, nr, sizeof(int) * n);
        }

        void build_lcp(){ // lcp[i] = lcp(s[:i], s[:i+1])
            int k = 0;
            for(int i = 0; i < n; i++) r[ sa[i] ] = i;

            for(int i = 0; i < n; i++){
                if(r[i] == n-1) k = 0;
                else{
                    int j = sa[r[i]+1];
                    while(i+k < n && j+k < n && s[i+k] == s[j+k])
                        k++;
                }
                lcp[r[i]] = k;
                if(k) k--;
            }

            int comp_lcp(int i, int j){
                if(i == j) return n - i;
                if(r[i] > r[j]) swap(i, j);
                return min(lcp[k] for k in [r[i], r[j]-1]);
            }
        }
    }

    int comp_lcp(int i, int j){
        if(i == j) return n - i;
        if(r[i] > r[j]) swap(i, j);
        return min(lcp[k] for k in [r[i], r[j]-1]);
    }
}

```

Z Algorithm

```

vector<int> z_algo(const string &s) {
    int n = s.size(), L = 0, R = 0;
    vector<int> z(n, 0);
    for(int i = 1; i < n; i++){
        if(i <= R) z[i] = min(z[i-L], R - i + 1);
        while(z[i]+i < n && s[ z[i]+i ] == s[ z[i] ])
            z[i]++;
        if(i+z[i]-1 > R) L = i, R = i + z[i] - 1;
    }
    return z;
}

```

Prefix function/KMP

```

vector<int> prefix_function(const string &s){
    int n = s.size();
    vector<int> b(n+1);

```

```

    b[0] = -1;
    int i = 0, j = -1;
    while(i < n){
        while(j >= 0 && s[i] != s[j]) j = b[j];
        b[++i] = ++j;
    }
    return b;
}

```

```

void kmp(const string &t, const string &p){
    vector<int> b = prefix_function(p);
    int n = t.size(), m = p.size();
    int j = 0;
    for(int i = 0; i < n; i++){
        while(j >= 0 && t[i] != p[j]) j = b[j];
        j++;
        if(j == m){
            //patern of p found on t
            j = b[j];
        }
    }
}

```

Min rotation

```

int min_rotation(int *s, int N) {
    REP(i, N) s[N+i] = s[i];

    int a = 0;
    REP(b, N) REP(i, N) {
        if (a+i == b || s[a+i] < s[b+i]) { b += max(0, i-1);
            break; }
        if (s[a+i] > s[b+i]) { a = b; break; }
    }
    return a;
}

```

All palindrome

```

void manacher(char *s, int N, int *rad) {
    static char t[2*MAX];
    int m = 2*N - 1;

    REP(i, m) t[i] = -1;
    REP(i, N) t[2*i] = s[i];

    int x = 0;
    FOR(i, 1, m) {
        int &r = rad[i] = 0;
        if (i <= x+rad[x]) r = min(rad[x+x-i], x+rad[x]-i);
        while (i-r-1 >= 0 && i+r+1 < m && t[i-r-1] == t[i+r+1]) ++r;
        if (i+r >= x+rad[x]) x = i;
    }

    REP(i, m) if (i-rad[i] == 0 || i+rad[i] == m-1) ++rad[i];
    REP(i, m) rad[i] /= 2;
}

```

Palindromic Tree

```

const int MAXN = 105000;

struct node {
    int next[26];
    int len;
    int sufflink;
    int num;
};

```

```

int len;
char s[MAXN];
node tree[MAXN];
int num; // node 1 - root with len -1, node 2 - root
        with len 0
int suff; // max suffix palindrome
long long ans;

bool addLetter(int pos) {
    int cur = suff, curlen = 0;
    int let = s[pos] - 'a';

    while(true){
        curlen = tree[cur].len;
        if (pos-1 - curlen >= 0 && s[pos-1 - curlen] == s
            [pos])
            break;
        cur = tree[cur].sufflink;
    }
    if (tree[cur].next[let]) {
        suff = tree[cur].next[let];
        return false;
    }

    num++;
    suff = num;
    tree[num].len = tree[cur].len + 2;
    tree[cur].next[let] = num;

    if (tree[num].len == 1){
        tree[num].sufflink = 2;
        tree[num].num = 1;
        return true;
    }

    while (true){
        cur = tree[cur].sufflink;
        curlen = tree[cur].len;
        if(pos-1 - curlen >= 0 && s[pos-1 - curlen] == s[
            pos]){
            tree[num].sufflink = tree[cur].next[let];
            break;
        }
    }

    tree[num].num = 1 + tree[tree[num].sufflink].num;

    return true;
}

void initTree() {
    num = 2; suff = 2;
    tree[1].len = -1; tree[1].sufflink = 1;
    tree[2].len = 0; tree[2].sufflink = 1;
}

int main() {

    initTree();

    for (int i = 0; i < len; i++) {
        addLetter(i);
    }

    return 0;
}

```

Suffix Automaton

```

map<char, int> to[2*N];
int link[2*N], len[2*N], last = 0, sz = 1;

void add_letter(char c){
    int p = last;
    last = sz++;
    len[last] = len[p] + 1;
    for(; !to[p][c]; p = link[p]) to[p][c] = last;
    if(to[p][c] == last){
        link[last] = 0;
        return;
    }
    int u = to[p][c];
    if(len[u] == len[p]+1){
        link[last] = u;
        return;
    }
    int c1 = sz++;
    to[c1] = to[u];
    link[c1] = link[u];
    len[c1] = len[p]+1;
    link[last] = link[u] = c1;
    for(; to[p][c] == u; p = link[p]) to[p][c] = c1;
}

```

Suffix Tree

```

namespace sf {
// const int NS = ; const int N = * 2;
int cn, cd, ns, en = 1, lst;
string S[NS]; int si = -1;
vector<int> sufn[N]; // sufn[si][i] no do sufixo S[si][i
...]
struct node {
    int l, r, si, p, suf;
    map<char, int> adj;
    node() : l(0), r(-1), suf(0), p(0) {}
    node(int L, int R, int S, int P) : l(L), r(R), si(S)
        , p(P) {}
    inline int len() { return r - l + 1; }
    inline int operator[](int i) { return S[si][l + i]; }
    inline int& operator()(char c) { return adj[c]; }
} t[N];
inline int new_node(int L, int R, int S, int P) { t[en]
    = node(L, R, S, P); return en++; }
void add_string(string s) {
    s += '$'; S[++si] = s; sufn[si].resize(s.size() + 1)
        ; cn = cd = 0;
    int i = 0; const int n = s.size();
    for(int j = 0; j < n; j++)
        for(; i <= j; i++) {
            if(cd == t[cn].len() && t[cn](s[j])) { cn = t
                [cn](s[j]); cd = 0; }
            if(cd < t[cn].len() && t[cn][cd] == s[j]) {
                cd++;
                if(j < s.size() - 1) break;
            }
            else {
                if(i) t[lst].suf = cn;
                for(; i <= j; i++) { sufn[si][i] = cn;
                    cn = t[cn].suf; }
            }
        }
    else if(cd == t[cn].len()) {
        sufn[si][i] = en;
        if(i) t[lst].suf = en; lst = en;
        t[cn](s[j]) = new_node(j, n - 1, si, cn);
    }
}

```

```

        cn = t[cn].suf; cd = t[cn].len();
    } else {
        int mid = new_node(t[cn].l, t[cn].l + cd -
            1, t[cn].si, t[cn].p);
        t[t[cn].p](t[cn][0]) = mid;
        if(ns) t[ns].suf = mid;
        if(i) t[lst].suf = en; lst = en;
        sufn[si][i] = en;
        t[mid](s[j]) = new_node(j, n - 1, si, mid);
        ;
        t[mid](t[cn][cd]) = cn;
        t[cn].p = mid; t[cn].l += cd; cn = t[mid].
            p;
        int g = cn? j - cd : i + 1; cn = t[cn].suf
            ;
        while(g < j && g + t[t[cn](S[si][g])].len
            () <= j) {
            cn = t[cn](S[si][g]); g += t[cn].len();
        }
        if(g == j) { ns = 0; t[mid].suf = cn; cd =
            t[cn].len(); }
        else { ns = mid; cn = t[cn](S[si][g]); cd
            = j - g; }
    }
}
}
};

```

Geometry

2D basics

```

typedef double cod;
double eps = 1e-7;
bool eq(cod a, cod b){ return abs(a - b) <= eps; }

```

```

struct vec{
    cod x, y; int id;
    vec(cod a = 0, cod b = 0) : x(a), y(b) {}
    vec operator+(const vec &o) const{
        return {x + o.x, y + o.y};
    }
    vec operator-(const vec &o) const{
        return {x - o.x, y - o.y};
    }
    vec operator*(cod t) const{
        return {x * t, y * t};
    }
    vec operator/(cod t) const{
        return {x / t, y / t};
    }
    cod operator*(const vec &o) const{ // cos
        return x * o.x + y * o.y;
    }
    cod operator^(const vec &o) const{ // sin
        return x * o.y - y * o.x;
    }
    bool operator==(const vec &o) const{
        return eq(x, o.x) && eq(y, o.y);
    }
    bool operator<(const vec &o) const{
        if(!eq(x, o.x)) return x < o.x;
        return y < o.y;
    }
    cod cross(const vec &a, const vec &b) const{
        return (a-(*this)) ^ (b-(*this));
    }
    int ccw(const vec &a, const vec &b) const{

```

```

        cod tmp = cross(a, b);
        return (tmp > eps) - (tmp < -eps);
    }
    cod dot(const vec &a, const vec &b) const{
        return (a-(*this)) * (b-(*this));
    }
    cod len() const{
        return sqrt(x * x + y * y); // <
    }
    double angle(const vec &a, const vec &b) const{
        return atan2(cross(a, b), dot(a, b));
    }
    double tan(const vec &a, const vec &b) const{
        return cross(a, b) / dot(a, b);
    }
    vec unit() const{
        return operator/(len());
    }
    int quad() const{
        if(x > 0 && y >=0) return 0;
        if(x <=0 && y > 0) return 1;
        if(x < 0 && y <=0) return 2;
        return 3;
    }
    bool comp(const vec &a, const vec &b) const{
        return (a - *this).comp(b - *this);
    }
    bool comp(vec b){
        if(quad() != b.quad()) return quad() < b.quad();
        if(!eq(operator^(b), 0)) return operator^(b) > 0;
        return (*this) * (*this) < b * b;
    }
    template<class T>
    void sort_by_angle(T first, T last) const{
        std::sort(first, last, [=](const vec &a, const
            vec &b){
            return comp(a, b);
        });
    }
    vec rot90() const{ return {-y, x}; }
    vec rot(double a) const{
        return {cos(a)*x -sin(a)*y, sin(a)*x +cos(a)*y};
    }
};

struct line{
    cod a, b, c; vec n;
    line(vec q, vec w){ // q.cross(w, (x, y)) = 0
        a = -(w.y-q.y);
        b = w.x-q.x;
        c = -(a * q.x + b * q.y);
        n = {a, b};
    }
    cod dist(const vec &o) const{
        return abs(eval(o)) / n.len();
    }
    bool contains(const vec &o) const{
        return eq(a * o.x + b * o.y + c, 0);
    }
    cod dist(const line &o) const{
        if(!parallel(o)) return 0;
        if(!eq(o.a * b, o.b * a)) return 0;
        if(!eq(a, 0))
            return abs(c - o.c * a / o.a) / n.len();
        if(!eq(b, 0))
            return abs(c - o.c * b / o.b) / n.len();
        return abs(c - o.c);
    }

```

```

}
bool parallel(const line &o) const{
    return eq(n ^ o.n, 0);
}
bool operator==(const line &o) const{
    if(!eq(a*o.b, b*o.a)) return false;
    if(!eq(a*o.c, c*o.a)) return false;
    if(!eq(c*o.b, b*o.c)) return false;
    return true;
}
bool intersect(const line &o) const{
    return !parallel(o) || *this == o;
}
vec inter(const line &o) const{
    if(parallel(o)){
        if(*this == o){ }
        else{ /* dont intersect */ }
    }

    auto tmp = n ^ o.n;
    return {(o.c*b - c*o.b)/tmp, (o.a*c - a*o.c)/tmp};
}
vec at_x(cod x) const{
    return {x, (-c-a*x)/b};
}
vec at_y(cod y) const{
    return {(-c-b*y)/a, y};
}
cod eval(const vec &o) const{
    return a * o.x + b * o.y + c;
}
};

struct segment{
    vec p, q;
    segment(vec a = vec(), vec b = vec()): p(a), q(b) {}
    bool onstrip(const vec &o) const{ // onstrip strip
        return p.dot(o, q) >= -eps && q.dot(o, p) >= -eps;
    }
    cod len() const{
        return (p-q).len();
    }
    cod dist(const vec &o) const{
        if(onstrip(o)) return line(p, q).dist(o);
        return min((o-q).len(), (o-p).len());
    }
    bool contains(const vec &o) const{
        return eq(p.cross(q, o), 0) && onstrip(o);
    }
    bool intersect(const segment &o) const{
        if(contains(o.p)) return true;
        if(contains(o.q)) return true;
        if(o.contains(q)) return true;
        if(o.contains(p)) return true;
        return p.ccw(q, o.p) * p.ccw(q, o.q) == -1
            && o.p.ccw(o.q, q) * o.p.ccw(o.q, p) == -1;
    }
    bool intersect(const line &o) const{
        return o.eval(p) * o.eval(q) <= 0;
    }
    cod dist(const segment &o) const{
        if(line(p, q).parallel(line(o.p, o.q))){
            if(onstrip(o.p) || onstrip(o.q))
                || o.onstrip(p) || o.onstrip(q))
                return line(p, q).dist(line(o.p, o.q));
        }
    }
}

```

```

    else if(intersect(o)) return 0;
    return min(min(dist(o.p), dist(o.q)),
        min(o.dist(p), o.dist(q)));
}
cod dist(const line &o) const{
    if(line(p, q).parallel(o))
        return line(p, q).dist(o);
    else if(intersect(o)) return 0;
    return min(o.dist(p), o.dist(q));
}
};

struct hray{
    vec p, q;
    hray(vec a = vec(), vec b = vec()): p(a), q(b){}
    bool onstrip(const vec &o) const{ // onstrip strip
        return p.dot(q, o) >= -eps;
    }
    cod dist(const vec &o) const{
        if(onstrip(o)) return line(p, q).dist(o);
        return (o-p).len();
    }
    bool intersect(const segment &o) const{
        if(!o.intersect(line(p,q))) return false;
        if(line(o.p, o.q).parallel(line(p,q)))
            return contains(o.p) || contains(o.q);
        return contains(line(p,q).inter(line(o.p,o.q)));
    }
    bool contains(const vec &o) const{
        return eq(line(p, q).eval(o), 0) && onstrip(o);
    }
    cod dist(const segment &o) const{
        if(line(p, q).parallel(line(o.p, o.q))){
            if(onstrip(o.p) || onstrip(o.q))
                return line(p, q).dist(line(o.p, o.q));
            return o.dist(p);
        }
        else if(intersect(o)) return 0;
        return min(min(dist(o.p), dist(o.q)),
            o.dist(p));
    }
    bool intersect(const hray &o) const{
        if(!line(p, q).parallel(line(o.p, o.q)))
            return false;
        auto pt = line(p, q).inter(line(o.p, o.q));
        return contains(pt) && o.contains(pt); // <<
    }
    bool intersect(const line &o) const{
        if(line(p, q).parallel(o)) return line(p, q) == o;
        if(o.contains(p) || o.contains(q)) return true;
        return (o.eval(p) >= -eps)^(o.eval(p)<o.eval(q));
        return contains(o.inter(line(p, q)));
    }
    cod dist(const line &o) const{
        if(line(p,q).parallel(o))
            return line(p,q).dist(o);
        else if(intersect(o)) return 0;
        return o.dist(p);
    }
    cod dist(const hray &o) const{
        if(line(p, q).parallel(line(o.p, o.q))){
            if(onstrip(o.p) || o.onstrip(p))
                return line(p,q).dist(line(o.p, o.q));
            return (p-o.p).len();
        }
        else if(intersect(o)) return 0;
        return min(dist(o.p), o.dist(p));
    }
}

```

```

    }
};

double heron(cod a, cod b, cod c){
    cod s = (a + b + c) / 2;
    return sqrt(s * (s - a) * (s - b) * (s - c));
}

Nearest Points

struct pt {
    int x, y, id;
};

inline bool cmp_x (const pt & a, const pt & b) {
    return a.x < b.x || a.x == b.x && a.y < b.y;
}

inline bool cmp_y (const pt & a, const pt & b) {
    return a.y < b.y;
}

pt a[MAXN];

double mindist;
int ansa, ansb;

inline void upd_ans (const pt & a, const pt & b) {
    double dist = sqrt ((a.x-b.x)*(a.x-b.x) + (a.y-b.y)
        *(a.y-b.y) + .0);
    if (dist < mindist)
        mindist = dist, ansa = a.id, ansb = b.id;
}

void rec (int l, int r) {
    if (r - l <= 3) {
        for (int i=l; i<=r; ++i)
            for (int j=i+1; j<=r; ++j)
                upd_ans (a[i], a[j]);
        sort (a+l, a+r+1, &cmp_y);
        return;
    }

    int m = (l + r) >> 1;
    int midx = a[m].x;
    rec (l, m), rec (m+1, r);
    static pt t[MAXN];
    merge (a+l, a+m+1, a+m+1, a+r+1, t, &cmp_y);
    copy (t, t+r-l+1, a+l);

    int tsz = 0;
    for (int i=l; i<=r; ++i)
        if (abs (a[i].x - midx) < mindist) {
            for (int j=tsz-1; j>=0 && a[i].y - t[j].y <
                mindist; --j)
                upd_ans (a[i], t[j]);
            t[tsz++] = a[i];
        }

    sort (a, a+n, &cmp_x);
    mindist = 1E20;
    rec (0, n-1);
}

```

Convex Hull

```

vector<vec> monotone_chain_ch(vector<vec> P){
    sort(P.begin(), P.end());

```

```

    vector<vec> L, U;
    for(auto p : P){
        while(L.size() >= 2 && L[L.size() - 2].cross(L.
            back(), p) < 0)
            L.pop_back();

        L.push_back(p);
    }

    reverse(P.begin(), P.end());
    for(auto p : P){
        while(U.size() >= 2 && U[U.size() - 2].cross(U.
            back(), p) < 0)
            U.pop_back();

        U.push_back(p);
    }

    L.pop_back(), U.pop_back();

    L.reserve(L.size() + U.size());
    L.insert(L.end(), U.begin(), U.end());

    return L;
}

```

Check point inside polygon

```

bool below(const vector<vec> &vet, vec p){
    auto it = lower_bound(vet.begin(), vet.end(), p);
    if(it == vet.end()) return false;
    if(it == vet.begin()) return *it == p;
    return prev(it)->cross(*it, p) <= 0;
}

bool above(const vector<vec> &vet, vec p){
    auto it = lower_bound(vet.begin(), vet.end(), p);
    if(it == vet.end()) return false;
    if(it == vet.begin()) return *it == p;
    return prev(it)->cross(*it, p) >= 0;
}

// lowerhull, upperhull and point, borders included
bool inside_poly(const vector<vec> &lo, const vector<vec>
    &hi, vec p){
    return below(hi, p) && above(lo, p);
}

```

Check point inside polygon without lower/upper hull

```

// borders included
// must not have 3 colinear consecutive points
bool inside_poly(const vector<vec> &v, vec p){
    if(v[0].ccw(v[1], p) < 0) return false;
    if(v[0].ccw(v.back(), p) > 0) return 0;
    if(v[0].ccw(v.back(), p) == 0)
        return v[0].dot(p, v.back()) >= 0
            && v.back().dot(p, v[0]) >= 0;

    int L = 1, R = (int)v.size() - 1, ans = 1;

    while(L <= R){
        int mid = (L+R)/2;
        if(v[0].ccw(v[mid], p) >= 0) ans = mid, L = mid
            +1;
        else R = mid-1;
    }
}

```



```
    return v[ans].ccw(v[(ans+1)%v.size()], p) >= 0;
}
```

Minkowski sum

```
vector<vec> mk(const vector<vec>&a, const vector<vec>&b){
    int i = 0, j = 0;
    for(int k = 0; k < (int)a.size(); k++){if(a[k] < a[i])
        i = k;
    for(int k = 0; k < (int)b.size(); k++){if(b[k] < b[j])
        j = k;

    vector<vec> c;
    c.reserve(a.size() + b.size());
    for(int k = 0; k < int(a.size()+b.size()); k++){
        vec pt{a[i] + b[j]};
        if((int)c.size() >= 2
            && c[c.size()-2].ccw(c.back(), pt) == 0)
            c.pop_back();
        c.push_back(pt);
        int q = i+1, w = j+1;
        if(q == int(a.size())) q = 0;
        if(w == int(b.size())) w = 0;
        if(c.back().ccw(a[i]+b[w], a[q]+b[j]) < 0) i = q;
        else j = w;
    }
    c.shrink_to_fit();

    return c;
}
```

Miscellaneous

LIS

```
multiset<int> S;
for(int i = 0; i < n; i++){
    auto it = S.upper_bound(a[i]); // low for inc
    if(it != S.end()) S.erase(it);
    S.insert(a[i]);
}
ans = S.size();
```

DSU rollback

```
#include <bits/stdc++.h>
```

```
using namespace std;
```

```
struct DSU{
    vector<int> sz, p, change;
    vector<tuple<int, int, int>> modifications;
    vector<size_t> saves;
    bool bipartite;

    DSU(int n): sz(n+1, 1), p(n+1), change(n+1),
        bipartite(true){
        iota(p.begin(), p.end(), 0);
    }

    void add_edge(int u, int v){
        if(!bipartite) return;
        int must_change = get_colour(u) == get_colour(v);
        int a = rep(u), b = rep(v);
        if(sz[a] < sz[b]) swap(a, b);
        if(a != b){
            p[b] = a;
            modifications.emplace_back(b, change[b],
                bipartite);
```

```
            change[b] ^= must_change;
            sz[a] += sz[b];
        }
        else if(must_change){
            modifications.emplace_back(0, change[0],
                bipartite);
            bipartite = false;
        }
    }

    int rep(int u){
        return p[u] == u ? u : rep(p[u]);
    }

    int get_colour(int u){
        if(p[u] == u) return change[u];
        return change[u] ^ get_colour(p[u]);
    }

    void reset(){
        modifications.clear();
        saves.clear();
        iota(p.begin(), p.end(), 0);
        fill(sz.begin(), sz.end(), 1);
        fill(change.begin(), change.end(), 0);
        bipartite = true;
    }

    void rollback(){
        int u = get<0>(modifications.back());
        tie(ignore, change[u], bipartite) = modifications
            .back();
        sz[ p[u] ] -= sz[u];
        p[u] = u;
        modifications.pop_back();
    }

    void reload(){
        while(modifications.size() > saves.back())
            rollback();
        saves.pop_back();
    }

    void save(){
        saves.push_back(modifications.size());
    }
};

const int N = 100005;
const int B = 318;

int n, m, q;
int x[N], y[N], l[N], r[N], ans[N];

vector<int> qu[N];

int brute(int lef, int rig, DSU &s){
    s.save();
    for(int i = lef; i <= rig; i++)
        s.add_edge(x[i], y[i]);
    int ret = s.bipartite;
    s.reload();
    return ret;
}

int main(){
```

```

scanf("%d %d %d", &n, &m, &q);

for(int i = 1; i <= m; i++)
    scanf("%d %d", x+i, y+i);

DSU s(n);
for(int i = 0; i < q; i++){
    scanf("%d %d", l+i, r+i);
    if(r[i] - l[i] <= B + 10)
        ans[i] = brute(l[i], r[i], s);
    else qu[l[i] / B].push_back(i);
}

for(int i = 0; i <= m / B; i++){
    sort(qu[i].begin(), qu[i].end(), [](int a, int b){
        return r[a] < r[b];
    });
    s.reset();

    int R = (i+1)*B-1;

    for(int id : qu[i]){
        while(R < r[id]) ++R, s.add_edge(x[R], y[R]);
        s.save();
        for(int k = l[id]; k < (i+1)*B; k++)
            s.add_edge(x[k], y[k]);
        ans[id] = s.bipartite;
        s.reload();
    }
}

for(int i = 0; i < q; i++)
    printf("%s\n", ans[i] ? "Possible" : "Impossible");
}

```

Buildings

```

// count the number of circular arrays
// of size m, with elements on range
// [1, c**(x*x)]

```

```

#include<bits/stdc++.h>
using namespace std;

```

```

#define debug(x) cerr << fixed << #x << " = " << x << endl;
#define ll long long

```

```

const int MOD = 1e9 + 7;
const int MAX = 1e5 + 5;
int dp[MAX];

```

```

inline int add(int a, int b) {
    a += b;
    if(a >= MOD) {
        a -= MOD;
    }
    return a;
}

```

```

inline int sub(int a, int b) {
    a -= b;
    if(0 > a) {
        a += MOD;
    }
    return a;
}

```

```

inline int mult(int a, int b) {
    return (1LL * a * b) % MOD;
}

int f_exp(int x, int exp) {
    if(exp == 0) {
        return 1;
    }
    else if(exp & 1) {
        return mult(x, f_exp(x, exp - 1));
    }
    return f_exp(mult(x, x), exp / 2);
}

inline int inv(int x) {
    return f_exp(x, MOD - 2);
}

int main()
{
    ios::sync_with_stdio(false);
    cin.tie(NULL); cout.tie(NULL);

    int n, m, c;
    cin >> n >> m >> c;

    int x = f_exp(c, n * n);
    int ans = f_exp(x, m);
    for(int i = 1; i <= m; i++) {
        if(m % i == 0) {
            int y = f_exp(x, i);
            for(int j = 1; j < i; j++) {
                if(i % j == 0) {
                    y = sub(y, mult(j, dp[j]));
                }
            }
            dp[i] = mult(y, inv(i));
            ans = sub(ans, mult(i - 1, dp[i]));
        }
    }

    cout << ans << '\n';

    return 0;
}

```

Rand

```

cout << RAND_MAX << endl;
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
vector<int> permutation(N);

iota(permutation.begin(), permutation.end(), 0);

shuffle(permutation.begin(), permutation.end(), rng);

iota(permutation.begin(), permutation.end(), 0);

for(int i = 1; i < N; i++){
    swap(permutation[i], permutation[
        uniform_int_distribution<int>(0, i)(rng)]);
}

```

Klondike

```

// minimum number of moves to make
// all elements equal
// move: change a segment of equal value

```

```
// elements to any value

int v[305];
int dp[305][305];
int rec[305][305];

int f(int l, int r){
    if(r == l) return 1;
    if(r < l) return 0;
    if(dp[l][r] != -1) return dp[l][r];
    int ans = f(l+1, r) + 1;
    for(int i = l+1; i <= r; i++){
        if(v[i] == v[l])
            ans = min(ans, f(l, i - 1) + f(i+1, r));
    }

    return dp[l][r] = ans;
}

int main() {
    int n, m;
    memset(dp, -1, sizeof dp);
    scanf("%d %d", &n, &m);
    for(int i = 0; i < n; i++){
        scanf("%d", &v[i]);
        if(i && v[i] == v[i-1]){
            i--;
            n--;
        }
    }
    printf("%d\n", f(0, n-1) - 1);
    // printf("%d\n", rec[0][n-1]);
    // printf("%d\n", rec[1][n-1]);
    // printf("%d\n", rec[2][n-3]);
}

```

Hilbert Order

```
// maybe use B = n / sqrt(q)
inline int64_t hilbertOrder(int x, int y, int pow = 21,
    int rotate = 0) {
    if(pow == 0) return 0;
    int hpow = 1 << (pow-1);
    int seg = (x < hpow) ? (
        (y < hpow) ? 0 : 3
    ) : (
        (y < hpow) ? 1 : 2
    );
    seg = (seg + rotate) & 3;
    const int rotateDelta[4] = {3, 0, 0, 1};
    int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
    int nrot = (rotate + rotateDelta[seg]) & 3;
    int64_t subSquareSize = int64_t(1) << (2*pow - 2);
    int64_t ans = seg * subSquareSize;
    int64_t add = hilbertOrder(nx, ny, pow-1, nrot);
    ans += (seg == 1 || seg == 2) ? add : (subSquareSize
        - add - 1);
    return ans;
}

```

Modular Factorial

```
// Compute (1*2*...*(p-1)*1*(p+1)*(p+2)*...*n) % p
// in O(p*log(n))

int factmod(int n, int p){
    int ans = 1;
    while(n > 1){
        for(int i = 2; i <= n % p; i++)

```

```
            ans = (ans * i) % p;
        n /= p;
        if(n % 2) ans = p - ans;
    }
    return ans % p;
}

int fac_pow(int n, int p){
    int ans = 0;
    while(n) n /= p, ans += n;
    return ans;
}

int C(int n, int k, int p){
    if(fac_pow(n, p) > fac_pow(n-k, p) + fac_pow(k, p))
        return 0;
    int tmp = factmod(k, p) * factmod(n-k, p);
    return (f_exp(tmp, p-2, p) * factmod(n, p)) % p;
}

```

Enumeration all submasks of a bitmask

```
// loop through all submask of a given bitmask
// it does not include mask 0
for(int sub = mask; sub; sub = (sub-1)&mask){
}

```

Slope Trick

///By woqjal25, contest: Codeforces Round #371 (Div. 1),
problem: (C) Sonya and Problem Wihtout a Legend,
Accepted, #

```
#include<stdio.h>
#include<queue>

int main()
{
    int n, t;
    long long ans = 0;
    std::priority_queue<int> Q;
    scanf("%d%d", &n, &t);
    Q.push(t);
    for(int i=1; i<n; i++){
        scanf("%d", &t); t-=i;
        Q.push(t);
        if(Q.top() > t)
        {
            ans += Q.top() - t;
            Q.pop();
            Q.push(t);
        }
    }
    printf("%lld", ans);
    return 0;
}

```

Fast IO

```
#define pc(x) putchar_unlocked(x)
#define gc(x) getchar_unlocked(x)

inline void scan_int(int &x){
    register int c = gc();
    x = 0;
    int neg = 0;
    for(;; ((c < '0' || c > '9') && c != '-'); c = gc());
    if(c == '-'){

```

```

    neg = 1;
    c = gc();
}
for(; c >= '0' && c <= '9'; c = gc())
    x = (x << 1) + (x << 3) + c - '0';
if(neg) x = -x;
}

inline void print_int(int n){
    int rev = 0, count = 0, neg;
    if(n == 0){
        pc('0');
        return;
    }
    if(n < 0) n = -n, neg = 1;
    while(n % 10 == 0) count++, n /= 10;
    for(rev = 0; n != 0; n /= 10)
        rev = (rev << 3) + (rev << 1) + n % 10;
    if(neg) pc('-');
    while(rev != 0) pc(rev % 10 + '0'), rev /= 10;
    while(count-->0) pc('0');
    pc('\n');
}

inline void print_string(char *str){
    while(*str) pc(*str++);
    pc('\n');
}

```

Knapsack Bounded with Cost

// menor custo para conseguir peso ate M usando N tipos diferentes de elementos, sendo que o i-esimo elemento pode ser usado b[i] vezes, tem peso w[i] e custo c[i]
// O(N * M)

```

int b[N], w[N], c[N];
MinQueue Q[M]
int d[M] //d[i] = custo minimo para conseguir peso i

for(int i = 0; i <= M; i++) d[i] = i ? oo : 0;
for(int i = 0; i < N; i++){
    for(int j = 0; j < w[i]; j++){
        Q[j].clear();
        for(int j = 0; j <= M; j++){
            q = Q[j % w[i]];
            if(q.size() >= b[i]) q.pop();
            q.add(c[i]);
            q.push(d[j]);
            d[j] = q.getmin();
        }
    }
}

```

LCA <O(nlgn), O(1)>

```

int start[N], dfs_time;
int tour[2*N], id[2*N];

void dfs(int u){
    start[u] = dfs_time;
    id[dfs_time] = u;
    tour[dfs_time++] = start[u];
    for(int v : g[u]){
        dfs(v);
        id[dfs_time] = u;
        tour[dfs_time++] = start[u];
    }
}

```

```

int LCA(int u, int v){
    if(start[u] > start[v]) swap(u, v);
    return id[min(tour[k] for k in [start[u], start[v]])];
}

```

Burnside's Lemma

Let (G, \oplus) be a finite group that acts on a set X . It should hold that $e_g * x = x$ and $g_1 * (g_2 * x) = (g_1 \oplus g_2) * x, \forall x \in X, g_1, g_2 \in G$. For each $g \in G$ let $X^g = \{x \in X \mid g * x = x\}$. The number of orbits its given by:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

Wilson's Theorem

$(n-1)! \equiv -1 \pmod n \iff n$ is prime

Fibonacci

- $F_{n-1}F_{n+1} - F_n^2 = (-1)^n$
- $F_{n+k} = F_kF_{n+1} + F_{k-1}F_n$
- $GCD(F_n, F_m) = F_{GCD(n,m)}$
- $F_n = \frac{(\frac{1+\sqrt{5}}{2})^n - (\frac{1-\sqrt{5}}{2})^n}{\sqrt{5}}$

Kirchhoff's Theorem

Laplacian matrix is $L = D - A$, where D is a diagonal matrix with vertex degrees on the diagonals and A is adjacency matrix.

The number of spanning trees is any cofactor of L . i -th cofactor is determinant of the matrix gotten by removing i -th row and column of L .

Multigraphs

In $D[i][i]$ all loops are excluded. $A[i][j]$ = number of edges from i to j .

Directed multigraphs

$D[i][i]$ = indegree of i minus the number of loops at i . $A[i][j]$ = number of edges from i to j .

The number of oriented spanning trees rooted at a vertex i is the determinant of the matrix gotten by removing the i th row and column of L .

Edge coloring

Data: A graph G

Result: A proper coloring c of the edges of G

Let $U := E(G)$;

while $U \neq \emptyset$ **do**

```

    Let  $(u,v)$  be any edge in  $U$ ;
    Let  $F[1:k]$  be a maximal fan of  $u$  starting at  $F[1]=v$ ;
    Let  $c$  be a color that is free on  $u$  and  $d$  be a color
        that is free on  $F[k]$ ;
    Invert the  $cd_u$  path;
    Let  $w \in V(G)$  be such that  $w \in F, F' = [F[1]...w]$  is a
        fan and  $d$  is free on  $w$ ;
    Rotate  $F'$  and set  $c(u,w)=d$ ;
     $U := U - (u,v)$ ;

```

end

Notes

When we repeat something and each time we have probability p to succeed then the expected number of tries is $\frac{1}{p}$, till we

succeed.