

Haskell与代数数据类型

data关键字

```
data Point = MakePoint Double Double
```

↑ ↑
数据类型 构造函数

```
MakePoint :: Double -> Double -> Point
```

```
-- 构造数据
```

```
pointA = MakePoint 3.0 4.0
```

```
-- 解构数据
```

```
case pointA of MakePoint x y -> ... x ... y
```

```
getX :: Point -> Double
```

```
getX (MakePoint x y) = x
```



中缀构造函数

```
data Point = Double :: Double
```



数据类型

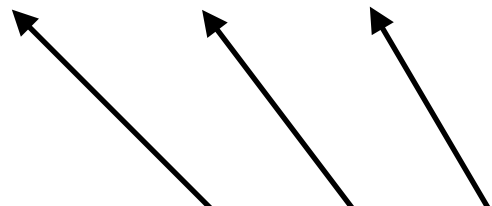


构造函数

```
(::) :: Double -> Double -> Point
```

-- 构造数据

```
pointA = 3.0 :: 4.0
```



-- 解构数据

```
case pointA of x :: y -> ... x ... y
```

```
case pointA of (::) x y -> ...
```



记录语法

-- 添加新的绑定的时候都可以对数据进行解构

```
let MakePoint x y = pointA in ... x ... y  
MakePoint x y = pointA
```

-- 使用记录语法，可以让编译器自动生成解构函数

```
data Point = MakePoint  
  { getX :: Double  
  , getY :: Double  
  }
```

-- `getX, getY :: Point -> Double`

-- 构造数据的时候可以通过标签函数指定顺序

```
pointB = MakePoint{getY = 4.0, getX = 3.0}
```



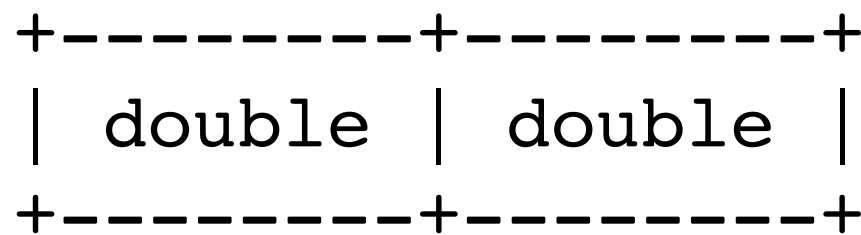
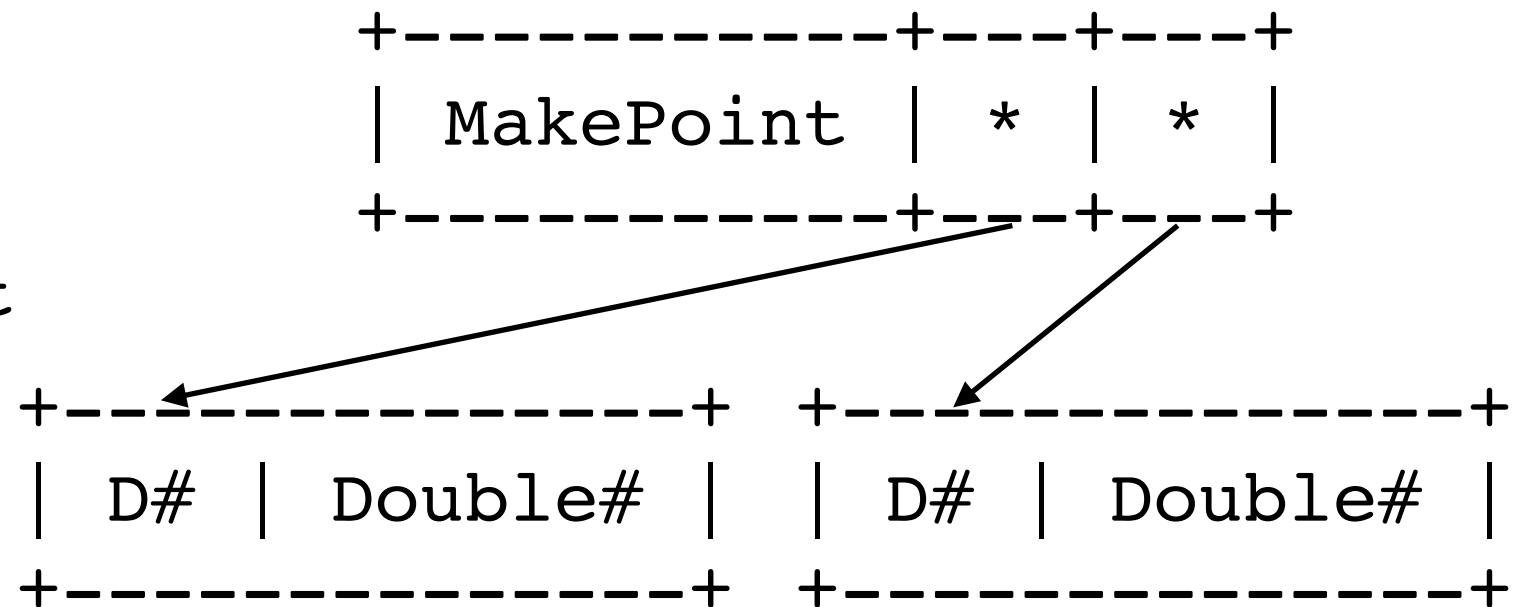
c struct?

```
-- Haskell
data Point = MakePoint
  { getX :: Double
  , getY :: Double
  }
```

```
data Double = D# Double#
```

```
// In C
```

```
typedef struct {
    double x;
    double y;
} point;
```



UNPACK to rescue!

```
-- Haskell
data Point = MakePoint
    { getX :: {-# UNPACK #-} !Double
    , getY :: {-# UNPACK #-} !Double
    }
data Double = D# Double#
```

```
// In C
typedef struct {
    double x;
    double y;
} point;
```

```
+-----+-----+-----+
| MakePoint | Double# | Double# |
+-----+-----+-----+
```

```
+-----+-----+
| double | double |
+-----+-----+
```



『更新』 数据

-- Haskell中数据默认是不可变的，只能根据原有的数据创建新的

```
pointB :: Point
```

```
pointB = MakePoint (getX pointA) 5.0
```

-- 记录语法提供了创建新数据的语法糖

```
pointB = pointA {getY = 5.0}
```

-- 等价于

```
pointB = MakePoint{ getX = getX pointA
```

```
      , getY = 5.0
```

```
      }
```

```
+-----+-----+-----+
```

```
| MakePoint | * | * |
```

```
+-----+-----+-----+
```

```
+-----+-----+-----+
```

```
| D# | 3.0# |
```

```
+-----+-----+-----+
```

```
+-----+-----+-----+
```

```
| D# | 4.0# |
```

```
+-----+-----+-----+
```

```
+-----+-----+-----+
```

```
| D# | 5.0# |
```

```
+-----+-----+-----+
```



抽象出类型?

-- 使用其他数值类型?

```
data FloatPoint = MakeFloatPoint Float Float
data IntPoint = MakeIntPoint Int Int
...
```

-- 使用类型变量抽象盒子的数据类型!

```
data Point a = MakePoint a a
```

```
type IntPoint = Point Int
```

```
type FloatPoint = Point Float
```

-- Point Int 和 Point Float 是不同的类型

-- Point Int :: Type

-- Point :: Type -> Type



抽象出类型?

- 使用 `Point a` 定义, 我们可以抽象出一些通用的操作
- `flipXY :: Point a -> Point a`
- `flipXY (MakePoint x y) = MakePoint y x`
- `flipXY` 可以工作在 `IntPoint`, `FloatPoint...` 之上
- 假如我们可以要求 `a` 类型的一些性质, 就可以实现更有趣的操作
- `moveX :: Num a => Point a -> a -> Point a`
- `moveX (MakePoint x y) dx = MakePoint (x + dx) y`
- 因为 `moveX` 使用到了 `(+) :: Num a => a -> a -> a`
- 即要求 `a` 类型是一个数字 (`Num a => ...`)
- 所以 `moveX` 也必须要求 `a` 类型是一个数字



面临选择?

-- 一个只有一个居民(`inhabitant`)的类型

```
data () = ()
```

-- GHC 会优化空盒子, 程序里所有的`()`都指向

-- 静态内存的一个地址

-- 一个拥有两个居民的类型

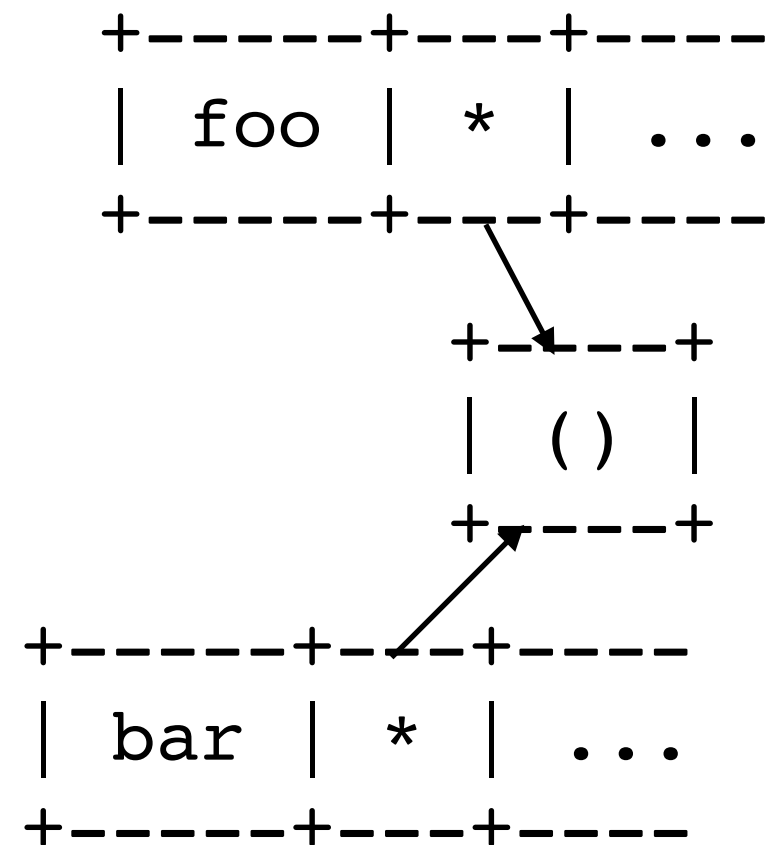
```
data Bool = True | False
```

```
case x of True  -> ...
```

```
        False -> ...
```

-- 上述 `Bool` 类型就是标准库里的逻辑值类型

-- 程序里所有的 `True, False` 都指向静态内存的两个地址



Sum&Product

-- 在类型之间选择的类型，又被称为和类型 (Sum Type)

```
data Either a b = Left a | Right b
```

-- `Either a b` 居民的数量，是 `a` 和 `b` 的居民数量之和

-- 同时包含若干类型的类型，又被称为积类型 (Product Type)

```
data (a, b) = (a, b)
```

-- `(a, b)` 居民的数量，是 `a` 和 `b` 的居民数量之积

-- 代数数据类型里的『代数』，指的就是和类型和积类型。

"sum" is alternation ($A \mid B$, meaning A or B but not both)

"product" is combination ($A \ B$, meaning A and B together)



重要的代数类型

-- 标准库里表示可能不存在的值的类型 Maybe a

```
data Maybe a = Just a | Nothing
```

```
divMaybe x y | y == 0 = Nothing  
              | otherwise = Just (x `div` y)
```

```
case x `divMaybe` y of Just ... -> ...  
                      _         -> ...
```

-- 标准库里的单链表

```
data [a] = a : [a] | []
```

```
-- [1,2,3,4]
```

```
1:2:3:4:[]
```



和类型的记录语法?

-- 和类型里也可以使用记录语法, 但非常不推荐

```
data Candidate = Fresh School
                | Experienced
                { getCompany :: Company
                , getPosition :: Position }
```

```
getCompany :: Candidate -> Company
getCompany (Experienced comp _) = comp
```

-- getCompany (Fresh _) = ???

-- 在和类型里使用记录语法, 会引入部分(Partial)函数!

-- 在运行过程中会发生异常:

```
-- *** Exception: No match in record selector
getCompany
```

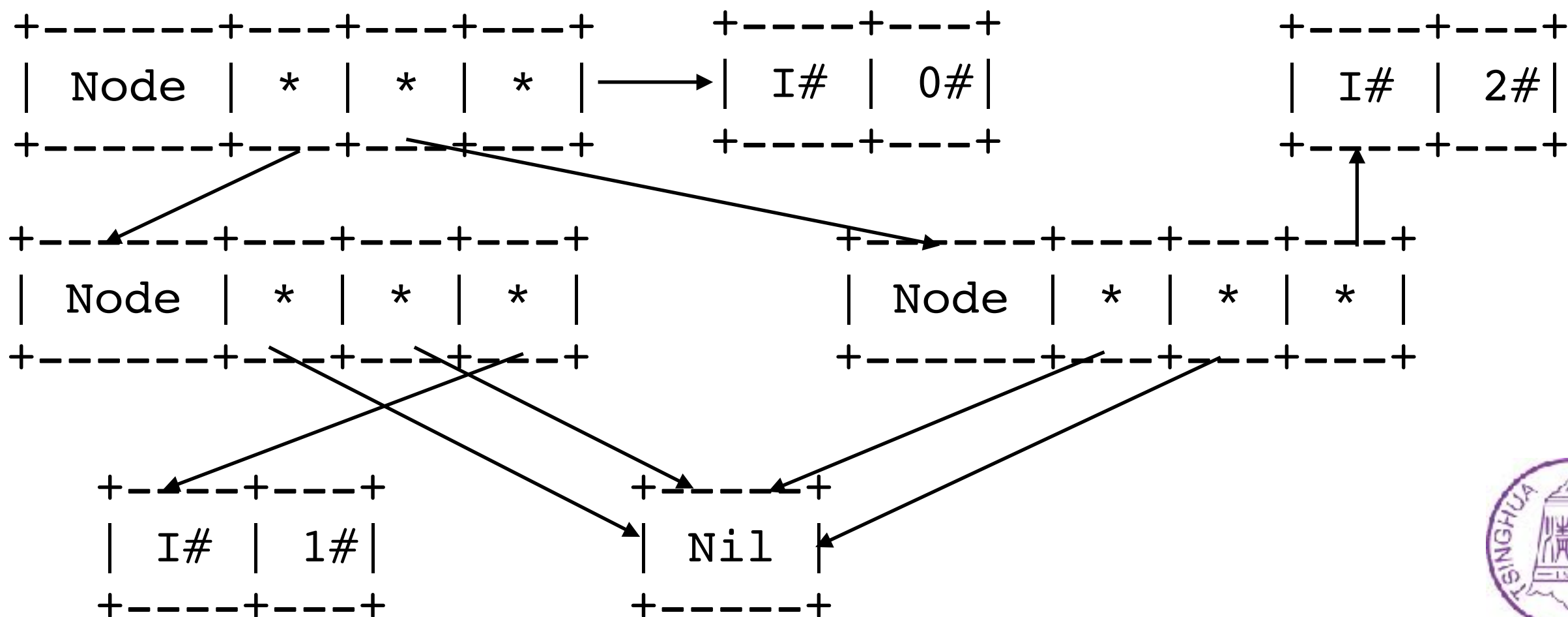


递归的数据定义

-- 二叉树

```
data BinTree a = Node (BinTree a) (BinTree a) a
                | Nil
```

```
Node (Node Nil Nil 1) (Node Nil Nil 2) 0
```



递归数据递归处理

```
data BinTree a = Node (BinTree a) (BinTree a) a
                | Nil
```

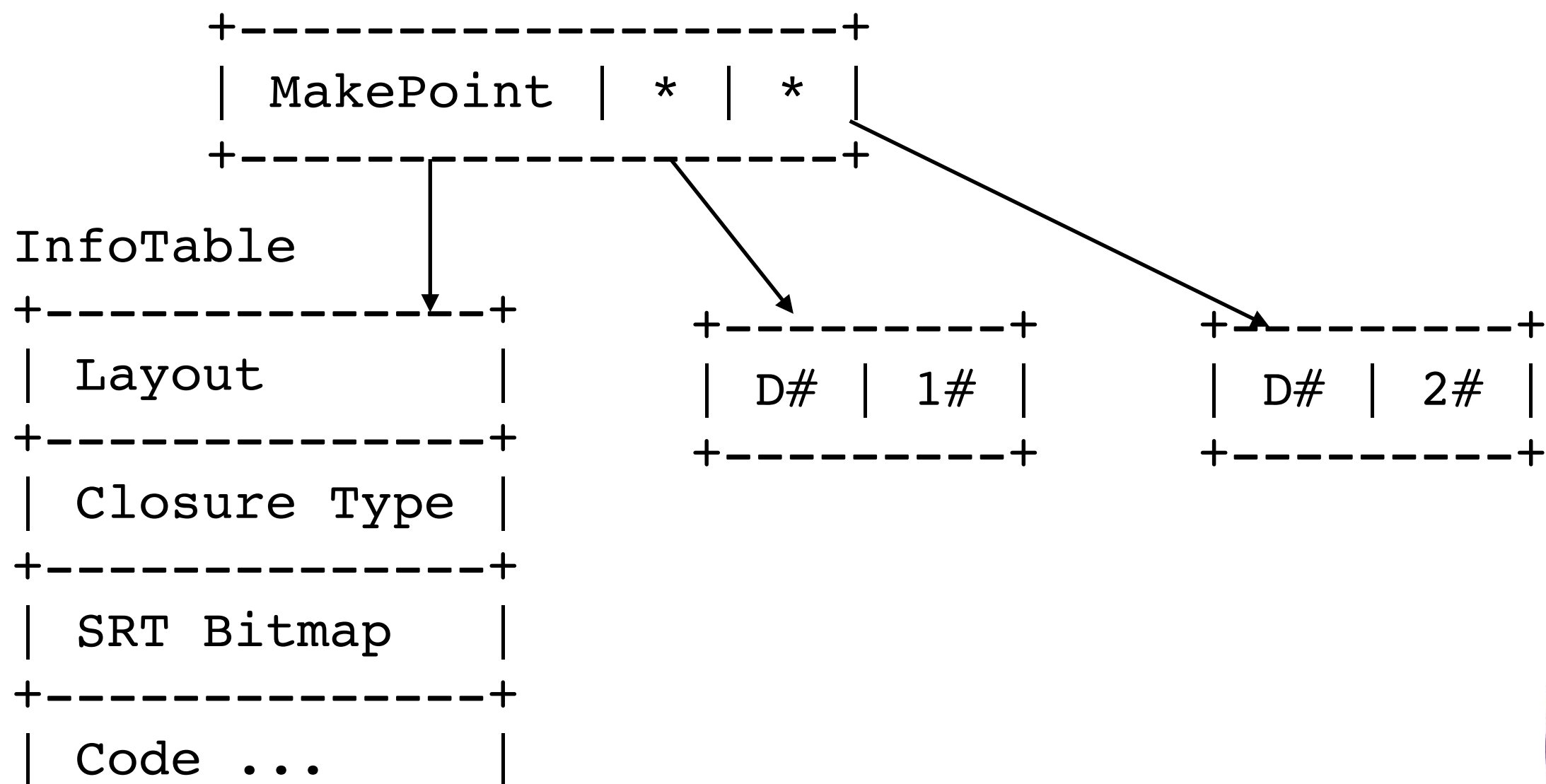
```
countNode :: BinTree a -> Int
countNode (Node left right _) =
    countNode left + countNode right + 1
countNode Nil = 0
```

```
countNode (Node (Node Nil Nil 1)
                (Node Nil Nil 2) 0)
-- countNode (Node Nil Nil 1)
--   + countNode (Node Nil Nil 2) + 1
-- (countNode Nil + countNode Nil + 1)
--   + (countNode Nil + countNode Nil + 1) + 1
-- (0 + 0 + 1) + (0 + 0 + 1) + 1
```



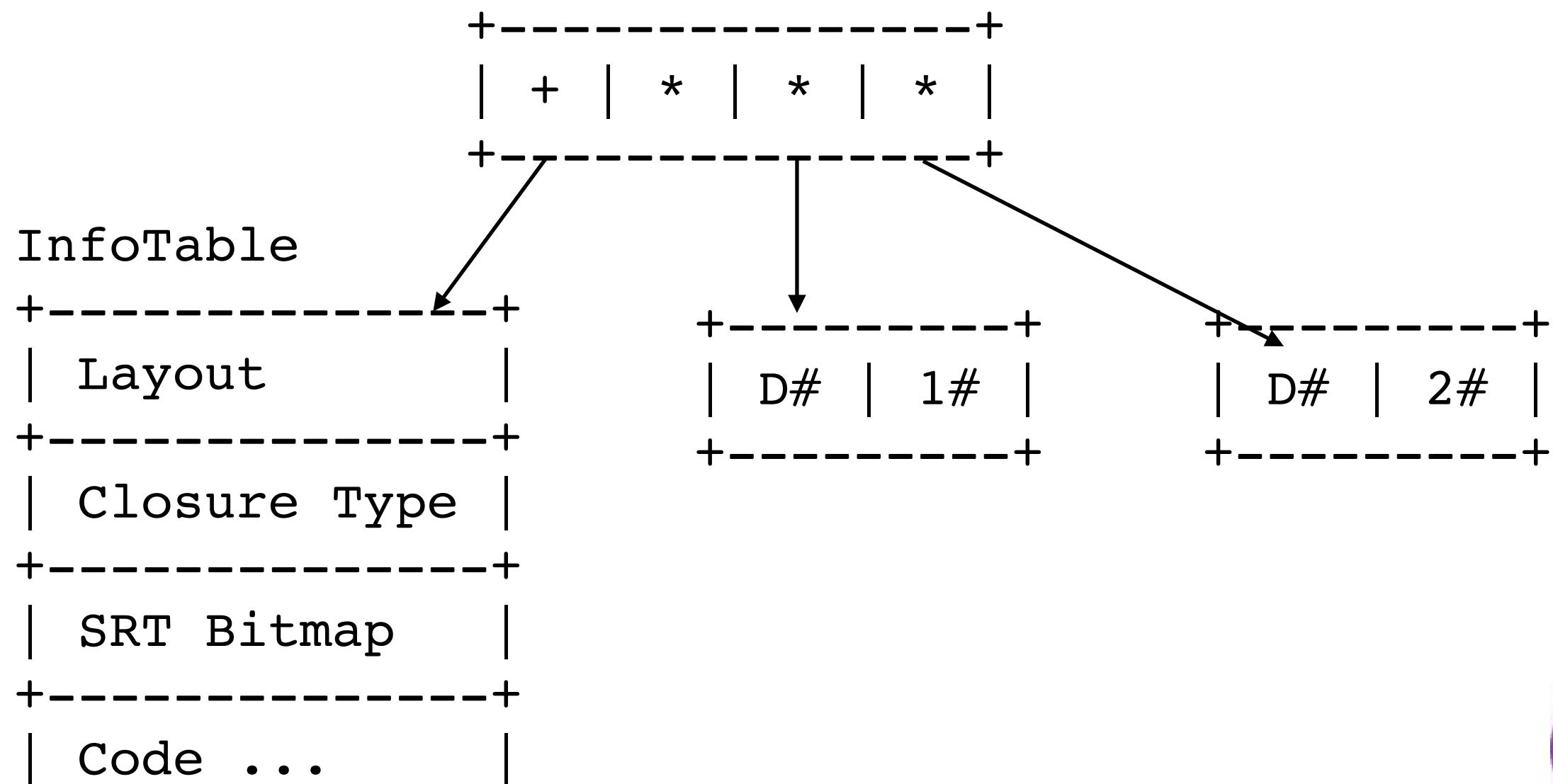
盒子比喻

-- 在 ghc 的堆(heap)上, 除了构造函数像一个盒子
pointA = MakePoint 1 2



任务盒?

-- 在 ghc 的堆(heap)上, 除了构造函数之外, 有一类特除的盒子
1 + 2 :: Double



递归求解任务盒

```
countNode (Node (Node Nil Nil 1)
                (Node Nil Nil 2) 0)
```

```
+-----+---+---+
| countNode | * | * |
+-----+---+---+
```

```
+-----+---+---+
| Node | * | * | * |
+-----+---+---+
```

```
+-----+---+---+
| Node | Nil | Nil | * |
+-----+---+---+
+-----+---+---+
| Node | Nil | Nil | * |
+-----+---+---+
```



递归求解任务盒

```
countNode (Node Nil Nil 1)
```

```
  + countNode (Node Nil Nil 2) + 1
```

```
  +---+---+---+---+
  | + | * | * | * |
  +---+---+---+---+
```

```
+---+---+---+---+
| + | * | * | * |
+---+---+---+---+
```

```
+-----+
| I# | 1# |
+-----+
```

```
+-----+---+---+---+
| countNode | * | * |
+-----+---+---+---+
```

```
+-----+---+---+---+
| countNode | * | * |
+-----+---+---+---+
```

```
+-----+---+---+---+
| Node | Nil | Nil | * |
+-----+---+---+---+
```

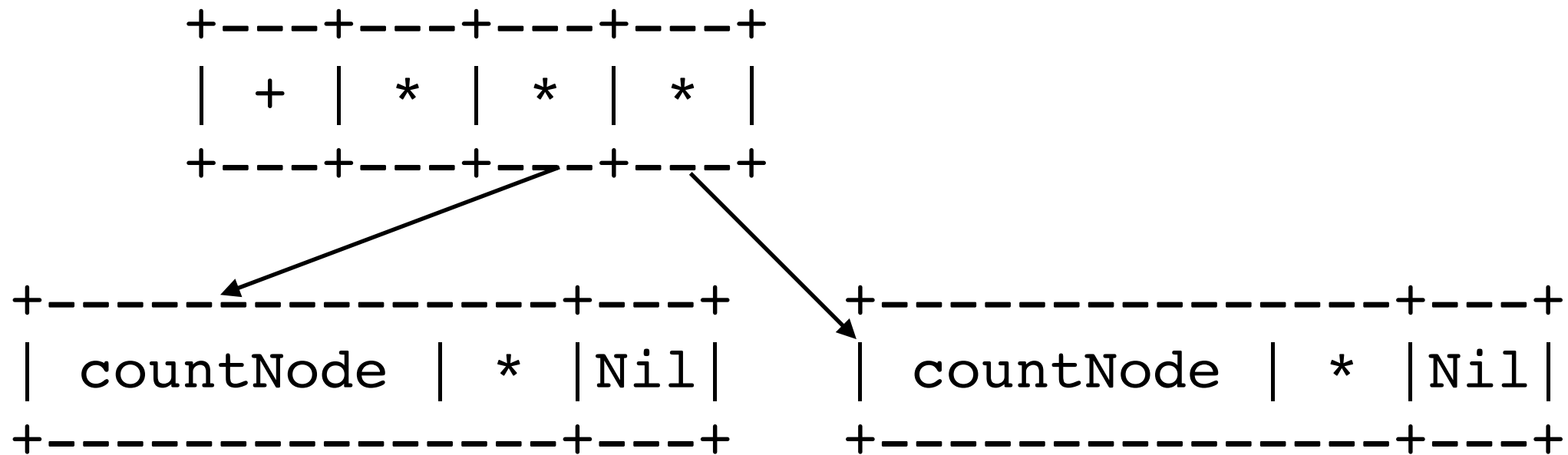
```
+-----+---+---+---+
| Node | Nil | Nil | * |
+-----+---+---+---+
```



递归求解任务盒

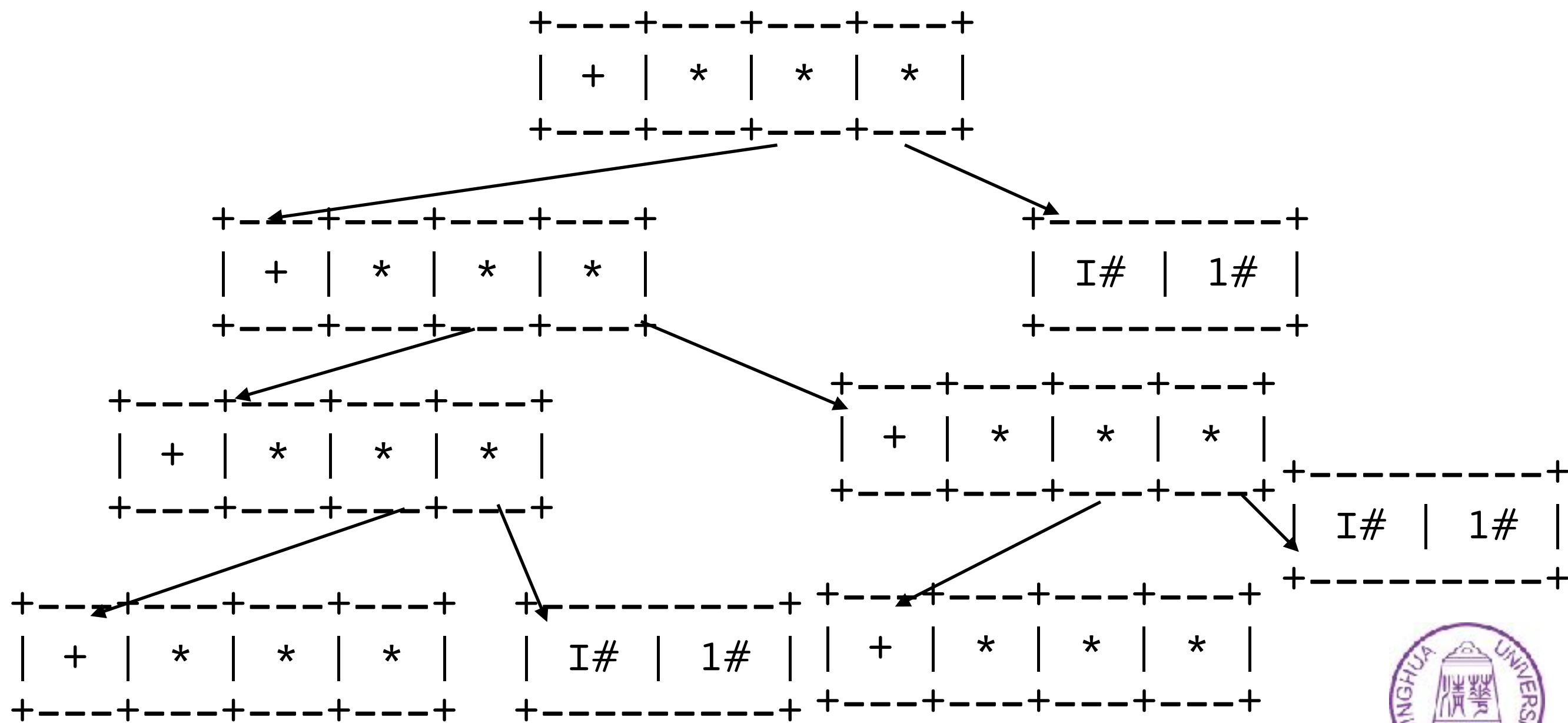
```
(countNode Nil + countNode Nil + 1)
+ (countNode Nil + countNode Nil + 1)
+ 1
```

```
countNode Nil + countNode Nil
```



统一的内存表示

$$(0 + 0 + 1) + (0 + 0 + 1) + 1$$



统一的内存表示

3

+-----+	
I# 3#	
+-----+	



Thinking recursively

```
data BinTree a = Node (BinTree a) (BinTree a) a
                | Nil
```

-- 对二叉树的结点求和

```
sumBinTree :: Num a => BinTree a -> a
sumBinTree = ?
```

-- 求二叉树的高度

```
binTreeHeight :: BinTree a -> Int
binTreeHeight = ?
```



Keep Invariant!

- 递归结合数据结构的性质可以解决一些有趣的问题
- 假如我们规定插入二叉树的节点保持比父节点 `key` 小的进左树
- 否则进右树的性质

```
insertNode :: Ord k
           => BinTree (k, v)
           -> (k, v) -> BinTree (k, v)

insertNode
  (BinTree left right kv'@(k', _)) kv@(k, _)
    | k < k'   = BinTree (insertNode left kv) right kv'
    | k == k'  = BinTree left right kv
    | k > k'   = BinTree left (insertNode right kv) kv'
insertNode Nil kv = BinTree Nil Nil kv
```



Keep Invariant!

-- 有了左右树和父节点大小的性质，递归搜索一个 k 就变得很简单

```
lookup :: Ord k => BinTree (k, v) -> k -> Maybe v
lookup k (Node left right (k', v))
    | k < k' = lookup k left
    | k == k' = Just v
    | k > k' = lookup k right
lookup _ Nil = Nothing
```



递归的两种形式

```
data BinTree a = Node (BinTree a) (BinTree a) a
                | Nil
```

```
-- direct style
```

```
countNode :: BinTree a -> Int
```

```
countNode (Node left right _) =
    countNode left + countNode right + 1
```

```
countNode Nil = 0
```

```
-- accumulator style
```

```
countNode' :: BinTree a -> Int -> Int
```

```
countNode' (Node left right _) acc =
    countNode' left (countNode' right acc) + 1
```

```
countNode' Nil acc = acc
```

