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# CPS TRANSFORM

## CPS -- CONTINUATION PASSING STYLE

- ▶ continuation 指广义  $(:: a \rightarrow b)$  上的后续计算。
- ▶ continuation passing style 指使用 continuation  $(:: a \rightarrow b)$  和接受 continuation  $(:: (a \rightarrow b) \rightarrow c)$  的函数来构建计算的一种编程思路。

`"I'm a fox" :: String`



`\ c -> c "I'm a fox" :: (String -> r) -> r`

`(\ c -> c "I'm a fox") id -- "I'm a fox"`

CPS和代数类型一样，可以用来作为控制结构的基础。

```
data Bool = True | False
if :: Bool -> a -> a -> a
if True x _ = x
if False _ y = y
```

```
checkFoo :: Bool -> String
checkFoo b = if b "Foo" "Bar"
```

```
type Bool = forall a. a -> a -> a
true :: Bool
true = \ x -> \ _ -> x
false :: Bool
false = \ _ -> \ y -> y
```

```
checkFoo :: Bool -> String
checkFoo b = b "Foo" "Bar"
```

## 常见的CPS套路

```
data Position = Position Pico Pico
p = Position 3 4 :: Position
type PositionCPS = forall r. (Pico -> Pico -> r) -> r
pCPS k = k 3 4    :: Position
```

```
data Either a b = Left a | Right b
type EitherCPS a b = forall r. (a -> r) -> (b -> r) -> r
```

```
error = left "i'll be back"      :: Either String b
errorCPS k _ = k "i'll be back" :: EitherCPS String b
```

```
data Maybe a = Just a | Nothing
type MaybeCPS a = forall r. (a -> r) -> r -> r
JustCPSFoo sk _ = sk "Foo"
nothingCPS _ fk = fk
```

## CPS和MONAD

```
instance Monad Maybe where
    return x = Just x
    Just x  >>= f = f x
    Nothing >>= _ = Nothing

envPort = Just "8080"
readInt :: String -> Maybe Int
foo :: Int -> Maybe Foo

envPort >>= readInt >>= foo
case (case envPort of
        Nothing -> Nothing
        Just foo  -> readInt foo
    ) of
    Nothing -> Nothing
    Just i  -> foo i
```

## CPS和MONAD

```
newtype MaybeCPS r a = MaybeCPS
  { runMaybeCPS :: (a -> r) -> r -> r }
```

```
instance Monad (MaybeCPS r) where
  return x = MaybeCPS (\ks kf -> ks x)
  MaybeCPS c >>= f = MaybeCPS $
    \ ks kf -> (c $ \ a ->
      runMaybeCPS (f a) ks kf) kf
```

Diagram illustrating the CPS transformation of the monad bind operator (`>>=`).

The expression `runMaybeCPS (f a) ks kf` is the core of the transformation. The diagram shows the following components and their relationships:

- `\ ks kf ->` (lambda expression for `ks`)
- `(c $ \ a ->` (lambda expression for `a`)
- `runMaybeCPS (f a) ks kf` (the main function call)
- `kf` (the final lambda argument)

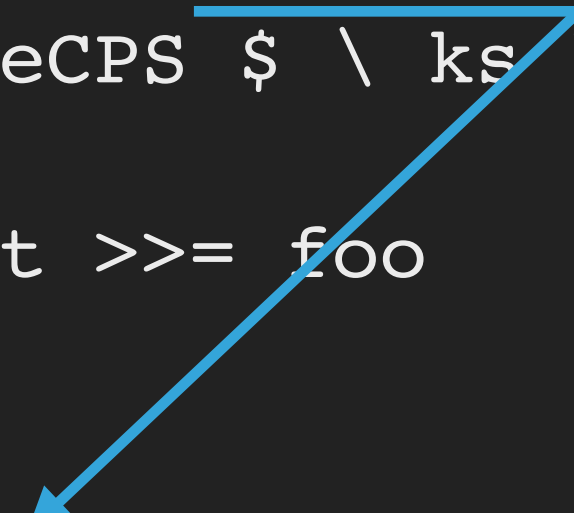
Arrows indicate the mapping from the lambda arguments to their corresponding lambda expressions:

- `ks` maps to `b -> r`
- `kf` maps to `(a -> r) -> r -> r`
- `a` maps to `a -> MaybeCPS b`
- `kf` maps to `(b -> r) -> r -> r`

## CPS和MONAD

```
envPort = MaybeCPS $ \ ks kf -> kf
readInt str = MaybeCPS $ \ ks kf -> ...
```

```
envPort >>= readInt >>= foo
(MaybeCPS $
  \ ks' kf' ->
    ((\ks kf -> kf) $ \ a ->
      runMaybeCPS (readInt a) ks' kf') kf'
) >>= foo
```



```
MaybeCPS $ \ ks' ' kf' ' -> ((\ ks' kf' -> ... ) $ ...)
runMaybeCPS s f
```

```
-- MaybeCPS $ \ ks kf -> kf
```

## CONT MONAD

```
newtype Cont r a = Cont
  { runCont :: (a -> r) -> r }
```

```
instance Monad (Cont r) where
  return x = Cont $ \ k -> k x
  Cont c >>= f = Cont $ \ k ->
```

$c \ \$ \ \backslash \ a \ ->$

$\text{runCont } (f \ a) \ k$

$b \ -> r$

$(a \ -> r) \ -> r$

$a \ -> \text{Cont } r \ b$

$(b \ -> r) \ -> r$



## CONT MONAD

```
envPortCPS :: Cont r String
```

```
envPortCPS = Cont $ \ k -> k "8080"
```

```
readIntCPS :: String -> Cont r Int
```

```
readIntCPS str = Cont $ \ k -> k (readInt str)
```

```
fooCPS :: Cont r Int
```

```
fooCPS = do
```

```
    port <- envPortCPS
```

```
    readIntCPS port
```

```
runCont fooCPS foo
```

```
-- foo (readInt envPort)
```

## CALL-WITH-CURRENT-CONTINUATION

```
callCC :: ((a -> Cont r b) -> Cont r a)
        -> Cont r a
```

```
callCC f = Cont $ \ a ->
    runContT $ f (\ x -> Cont $ \ _ -> a x)
```

```
fooCPS :: Cont r Int
```

```
fooCPS = do
```

```
    port <- envPortCPS
```

```
    err <- callCC $ \ k -> do
```

```
        port_num <- readInt port
```

```
        if port_num < 80 then k "not available"
```

```
            else ...
```

```
    catchErr err
```

## REALWORLD EXAMPLE: ATTOPARSEC

```
type Failure i t r = t -> Pos
                    -> More
                    -> [String]
                    -> String
                    -> IResult i r
```

```
type Success i t a r = t -> Pos
                      -> More
                      -> a
                      -> IResult i r
```

```
newtype Parser = Parser { runParser :: forall r. Input
                          -> Pos
                          -> More
                          -> Failure i Input r
                          -> Success i Input a r
                          -> IResult i r
```

## REALWORLD EXAMPLE: ATTOPARSEC

```
instance Monad (Parser i) where
    return v = Parser $ \t pos more _lose succ ->
                        succ t pos more v

m >>= k = Parser $ \t !pos more lose succ ->
    let succ' t' !pos' more' a =
        runParser (k a) t' pos' more' lose succ
    in runParser m t pos more lose succ'
```

## COMPOSE CONTINUATION

```
type ShowS = String -> String
```

```
shows "hello" -- ("hello" ++)
```

```
shows 'X'      -- ('X':)
```

```
newtype DList a = DL { unDL :: [a] -> [a] }
```

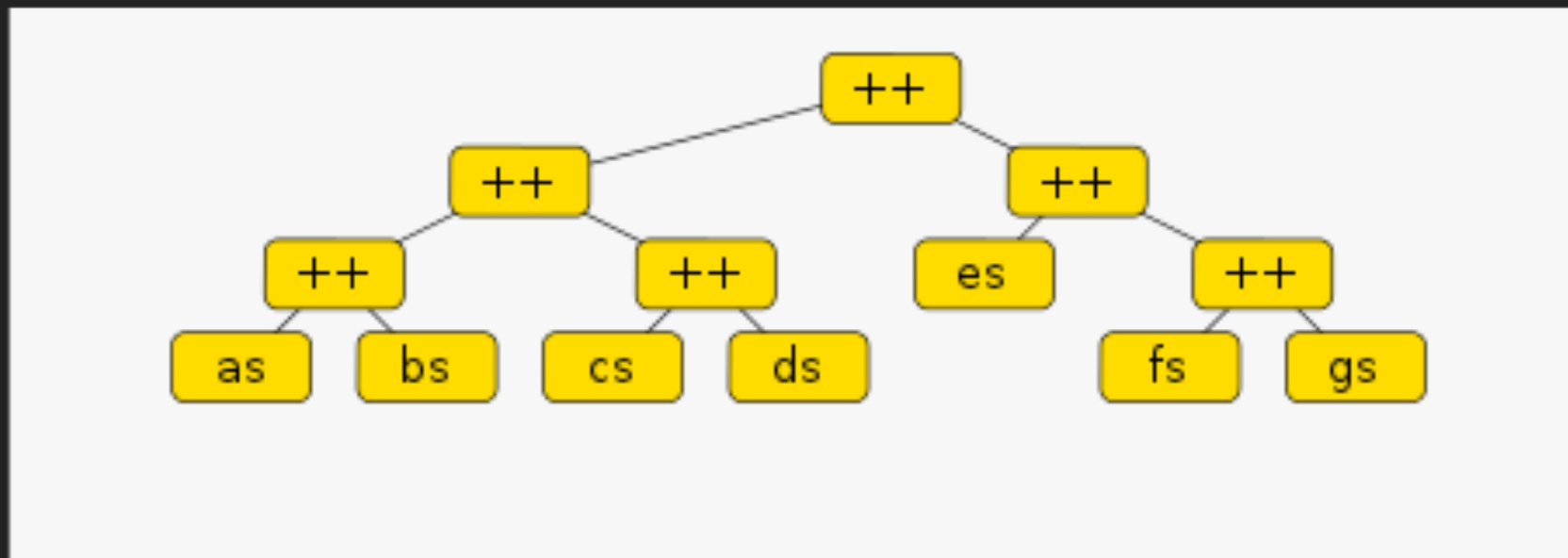
```
append :: DList a -> DList a -> DList a
```

```
append xs ys = DL (unDL xs . unDL ys)
```

## LIST

```
l = ((as ++ bs) ++ (cs ++ ds))  
    ++ (es ++ (fs ++ gs))
```

```
xs ++ ys = case xs of []      -> ys  
              (x:xs') -> x : (xs' ++ ys)
```

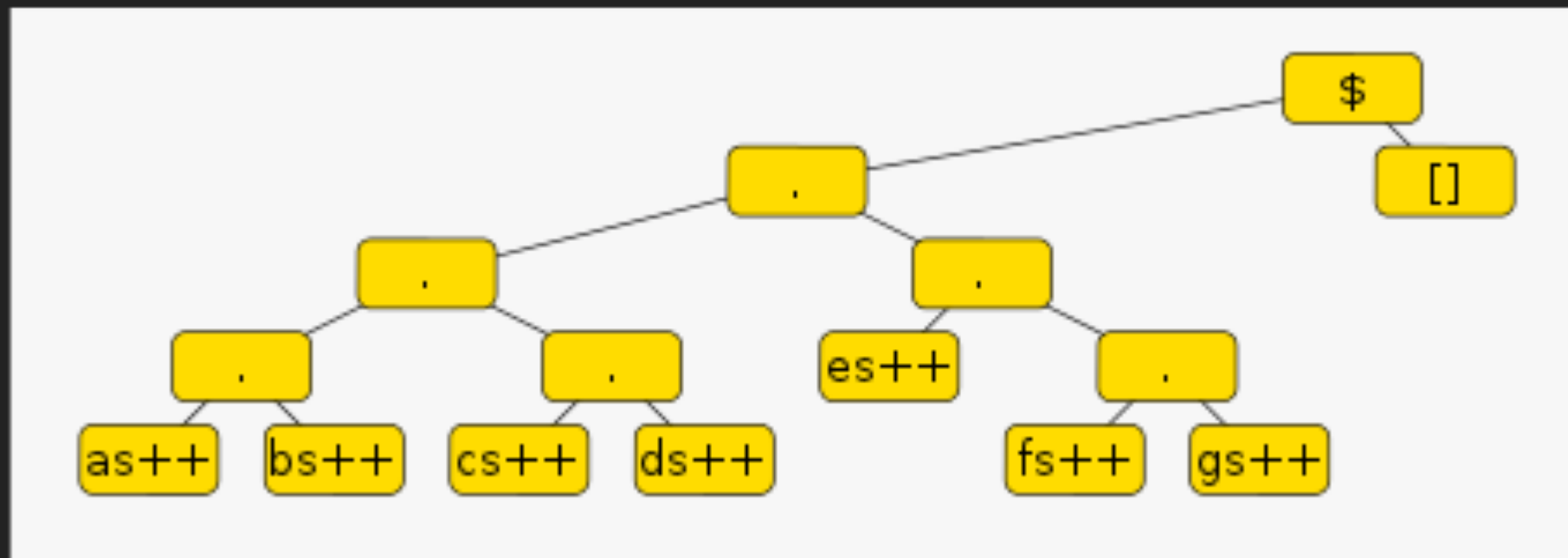


```
process (x:xs) = ...  
process l
```

## DLIST

```
dl = (((as++). (bs++)). ((cs++). (ds++))) .  
      ((es++). ((fs++). (gs++)))
```

```
dl []
```



## DLIST

```

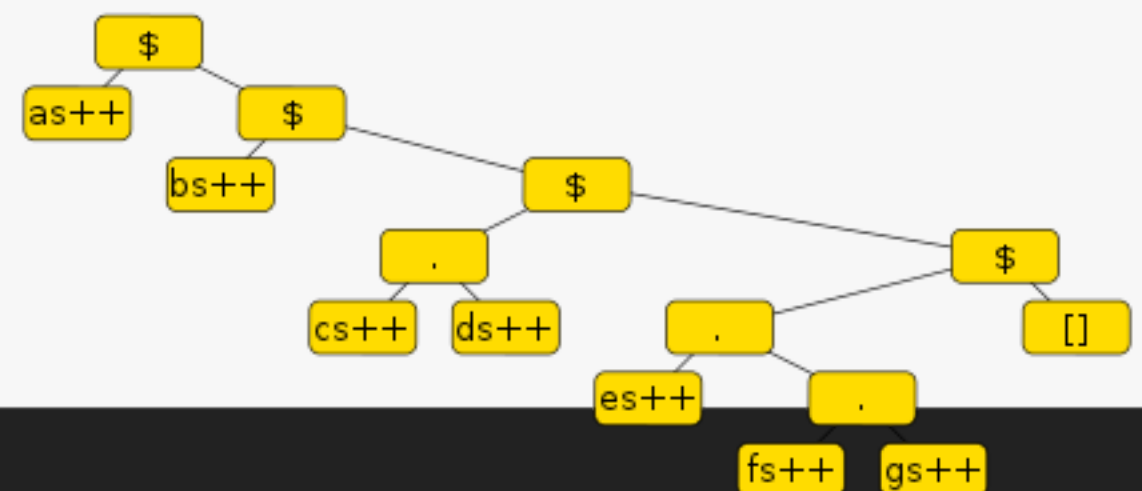
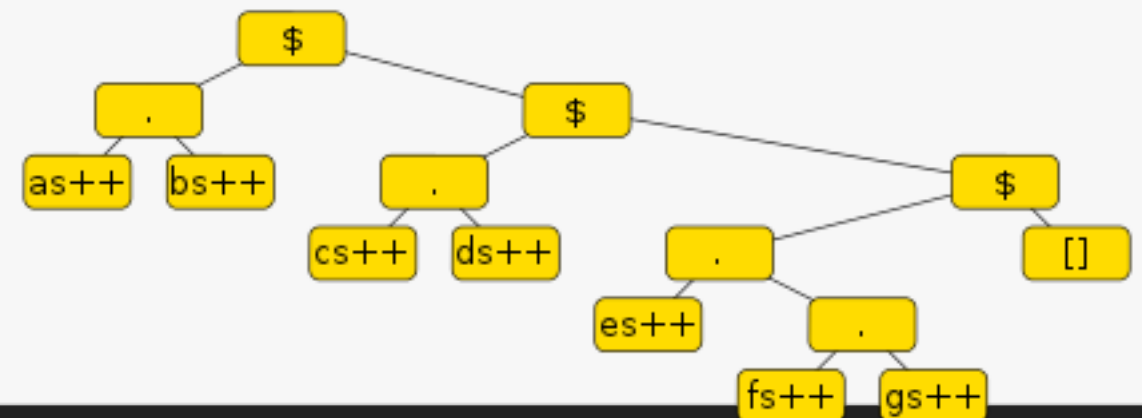
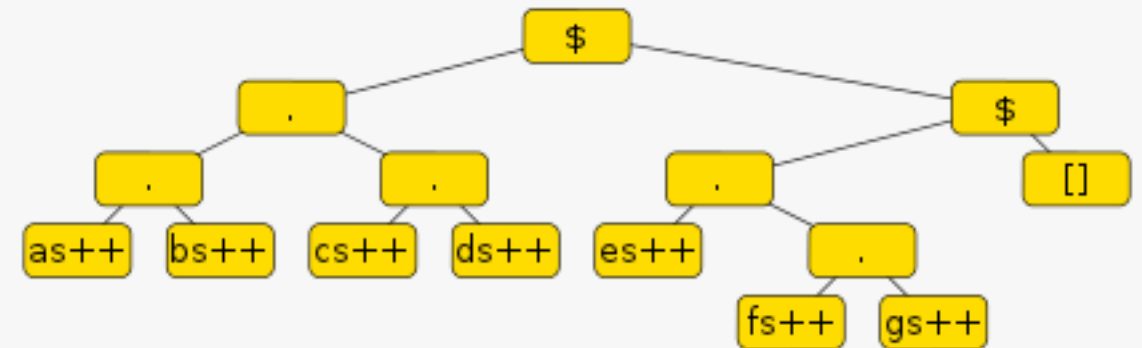
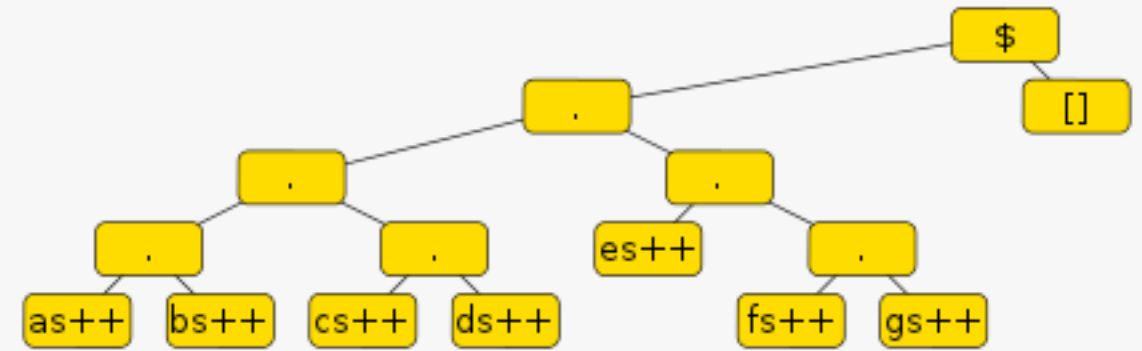
(as++
 (bs++
  (cs++
   (ds++
    (es++
     (fs++
      (gs++[])))))))

```

```

process (x:xs) = ...
process 1

```





## FORMAT TIME

```
data Part = Year | Month | Day | Hour | Min | Sec | Str String
          deriving (Show, Eq)
```

```
class FormatTime t where
    formatPart :: t -> [Part] -> [Part]
```

```
data Date = Date
    { year :: Int
    , month :: Int
    , day :: Int
    }
```

```
data Clock = Clock
    { hour :: Int
    , min :: Int
    , second :: Int
    }
```

## FORMAT TIME

```
instance FormatTime Date where
  -- formatPart :: Date -> [Part] -> [Part]
  formatPart (Date y m d) = map go
    where
      go p =
        case p of Year -> Str (show y)
                  Month -> Str (show m)
                  Day -> Str (show d)
                  _ -> p

formatPart (Date 2016 12 27)
  [ Month, Str "-", Day, Str " ",
    , Hour, Str ":", Min]
-- [ Str "12", Str "-", Str "27", Str " ",
--   , Hour, Str ":", Min]
```

## FORMAT TIME

```
instance FormatTime Clock where
  -- formatPart :: Clock -> [Part] -> [Part]
  formatPart (Clock h m s) = map go
    where
      go p =
        case p of Hour -> Str (show h)
                  Min -> Str (show m)
                  Sec -> Str (show s)
                  _   -> p

formatPart (Clock 20 43 00)
  [ Month, Str "-", Day, Str " ",
    , Hour, Str ":", Min]
-- [ Month, Str "-", Day, Str " "
--   , Str "20", Str ":", Str "43"]
```

## FORMAT TIME

```
data DateTime = DateTime Date Clock
```

```
instance FormatTime DateTime where
```

```
    -- formatPart :: DateTime -> [Part] -> [Part]
```

```
    formatPart (DateTime d c) =
```

```
        formatPart d . formatPart c
```

```
formatPart
```

```
    (DateTime (Date 2016 12 27) (Clock 20 43 00))
```

```
    [ Month, Str "-", Day, Str " "
```

```
    , Hour, Str ":", Min]
```

```
--    [ Str "12", Str "-", Str "27", Str " "
```

```
--    , Str "20", Str ":", Str "43"]
```

## FORMAT TIME

```
formatTime :: (FormatTime t) => t -> [Part] -> String
formatTime t ps = go (formatPart t ps)
```

```
  where
```

```
    go [] = ""
```

```
    go (p:ps) = case p of Str s -> s ++ go ps
                        _      -> go ps
```

```
formatTime
```

```
  (DateTime (Date 2016 12 27) (Clock 20 43 00))
```

```
  [ Month, Str "-", Day, Str " "
```

```
  , Hour, Str ":", Min]
```

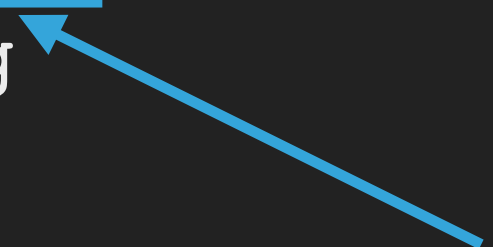
```
-- [ Str "12", Str "-", Str "27", Str " "
```

```
-- , Str "20", Str ":", Str "43"]
```

```
-- "12-27 20:43"
```

## FORMAT TIME CPS

```
class FormatTime t where
  formatPart :: t
               -> (Part -> String)
               -> Part -> String
```



**THE CONTINUATION K!**



```
formatTime :: (FormatTime t)
            => t -> [Part] -> String
formatTime t ps =
  concat $ map (formatPart t (const "")) ps
```

## FORMAT TIME CPS

```
instance FormatTime Date where
  -- formatPart :: Date
  --           -> (Part -> String)
  --           -> Part -> String

  formatPart (Date y m d) k p =
    case p of Year    -> show y
              Month   -> show m
              Day      -> show d
              Str x    -> x
              _        -> k p
```

**THE CONTINUATION K!**



## FORMAT TIME CPS

```
instance FormatTime Clock where
  -- formatPart :: Clock
  --           -> (Part -> String)
  --           -> Part -> String

  formatPart (Clock h m s) k p =
    case p of Hour -> show h
              Min  -> show m
              Sec  -> show s
              Str x -> x
              _    -> k p
```



## FORMAT TIME CPS

```
instance FormatTime DateTime where
  -- formatPart :: DateTime
  --           -> (Part -> String)
  --           -> Part -> String

  formatPart (DateTime d c) k =
    formatPart d (formatPart c k)

formatTime :: (FormatTime t)
           => t -> [Part] -> String
formatTime t ps =
  concat $ map (formatPart t (const "")) ps
```

## FIX POINT

```
fix :: (a -> a) -> a
```

```
fix f = f (fix f) -- f (f (f (f...)))
```

```
fix (const "x") -- "x"
```

```
f k = ... k ...
```

```
f rec x = if x == 0 then 1
           else x * rec (x - 1)
```

```
fix f 4 -- f (fix f) 4 -- f (f (f (f...))) 4
        -- 4 * f (f...)) 3
        -- 4 * 3 * 2 * f (f...)) 0
```