The **Stable Motion** pop-up menu

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This text presents a brief explanation of the functionalities the **Stable Motion** pop-up menu.

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1 Overview

This menu allows to perform various operations related to stable motions. A stable motion is a process with i.i.d increments which follow a stable law. Stable laws are a generalization of the Gaussian, in the sense that they are the only laws that are preserved under convolutions. Except for the Gaussian, stable laws do not have a variance, and there exists a real number alpha in (0,2) such that all moments of order equal to or greater than alpha do not exist. The most well known non Gaussian stable law is the Cauchy law.

Apart from the **Test of Stability** sub-menu, all others operations in this menu are also present in other **Fraclab** menu: The **Synthesis** sub-menu calls the same window as the corresponding sub-menu in the **Synthesis** menu. The **Mac Culloch Method for Parameters Estimation** and the **Koutrouvelis Method for Parameters Estimation** are the same as the ones in the **1D Exponents Estimation** sub-menu. The **Test of Stability** sub-menu is described below. Finally, both the **Spectral Measure Estimation** and the **Covariation Estimation** sub-menu are not implemented yet. The reason why there is a special menu for Stable motions although most operations are already scattered in other sections of

Fraclab is for visibility reasons: The operations here are strongly parametric, and rather different in nature from those in the remaining menus.

2 Synthesis

In this menu, you can synthesize a stable process by specifying the four parameters characterizing the stable law governing its increments. The **Characteristic Exponent**, alpha, between 0 and 2, controls the thickness of the tail of the process. alpha equal 1 gives the Cauchy law, and alpha lower than or equal to one yields a process without a mean. The **Skewness Parameter** is between -1 (distribution totally skewed to the left) and 1 (distribution totally skewed to the right), and controls the symmetry : the distribution is symmetric (around its location parameter) if and only if the skewness parameter is 0. The **Location Parameter**, which is simply the mean when alpha is greater than 1, is any real number. Finally, the **Scale Parameter**, often denoted gamma, is a positive number related to the "size" of the distribution : multiplying the process by a positive number w results in a multiplication of the scale parameter by w. When alpha equals 2 (Gaussian case), the square of the scale parameter is simply the variance divided by 2.

Once the computation is over, two signals are generated : the stable process itself, named *stable_process#* and its increments, which are iid realizations of a stable law, *stable_increments#*. Both are vectors.

3 Mac Culloch Method for Parameters Estimation

Check as usual your **Input** signal, and just hit **Compute**. The estimated values of the **Characteristic Exponent**, the **Skewness**, **Location** and **Scale** parameter will appear, along with estimates of their standard deviation (in the column **std**). Be careful that if you want to estimate the parameters of the stable motion X, you should input here the signal Y which contains the increments of X. In other words, this procedure estimates the parameters of the process, the increments of which are the input signal.

4 Koutrouvelis Method for Parameters Estimation

This is exactly the same as above, except this time no estimates are available for the standard deviations. The same remark about the increments applies.

5 Test of Stability

This menu allows you to test that a particular signal may be well modeled by a stable motion. In that view, one investigates whether its increments taken at different lags display some scale invariance. First check your **Input** signal (recall that you must input here the increments of the signal for which you want to test adequacy to a stable motion). Then decide how many time lags you want to investigate. In that view, set the **Maximum Resolution** parameter to the desired value: A value of n will mean that that you'll estimate the four parameters of the stable motion for the original signal and its versions sub-sampled by factors 2,..,n. If the signal is stable, the parameter alpha should be approximately the same for all sub-sampled versions, while the evolution of the scale parameters gamma should follow a power law with exponent related to alpha (see the references cited below for precise statements). Hit **Compute**. On the line **Characteristic parameter** will appear the exponent alpha estimated using the power law behaviour of the scale parameter alluded to above. To check stability, look at the outputs of this computation that have been displayed in the **Variables** list. There are four of them: *Estim_Param_M#*, *Estim_Sd_Dev_M#*, *plot_alpha#* and *plot_gamma#*. *Estim_Param_M#* is an n by 4 matrix (n is the chosen **Maximum Resolution**): Estim_Param_M#(i,j), for i=1, ...n, yields the characteristic exponent alpha for j=1, the skewness parameter for j=2, the location parameter (j=3) and the scale parameter gamma (j=4) all estimated at resolution i using the Mac Culloch method. *Estim_Sd_Dev_M#* is the matrix of the associated standard deviations. Instead of looking at the numerical values, it is often more illustrative to display the two graphs *plot_alpha#* and *plot_gamma#*. *plot_gamma#* displays a log-log plot of the evolution of the scale parameter with respect to resolution. Ideally, this should be a straight line, with slope related to alpha. In practice, one computes the least square regression line, the slope of which will be used to estimated the alpha value displayed on the line **Characteristic parameter**. It is always a good idea to check whether approximate linearity holds. As a second test, look at *plot_alpha#*, which displays the estimated values of alpha at the different scales: Stability entails that the estimated alpha should be roughly the same at all resolutions. Moreover, the estimated values should be in agreement with the one obtained by regressing on the gamma values. You should thus display *plot_alpha#* and verify that the graph is approximately horizontal (remember to check the ordinates before making a conclusion...), with common value close to the one displayed on the line **Characteristic parameter**.

6 Spectral Measure Estimation

This is not implemented in the current version of **Fraclab**.

7 Covariation Estimation

This is not implemented in the current version of **Fraclab**.

8 Homework

First synthesize some stable motions, and observe the effect of the various parameters, of which the most important is the characteristic exponent alpha. It should be particularly clear from the graphs of the process and of its increments that smaller alpha-s lead to processes with more discontinuities. You should also observe easily how the skewness parameter affects the output. For instance, with a skewness of 0.6 there will be mainly negative jumps, and positive jumps for a skewness of 0.6. You may also check that, in agreement with the theory, when alpha is smaller than 1 and for a skewness of 1 or -1, you only get positive or negative jumps. In other terms, the corresponding processes are monotonous although still very irregular.

You may then test the estimation methods. Open both the Mc Culloch and Koutrouvelis estimation menus. Choose an input signal in the **Variables** list and hit **Refresh** (recall that you must input the increments of the stable motion and not the process itself). You'll find that, as long as you have a reasonably long signal (e.g. 5000 points), both methods give approximately correct results concerning the two most interesting parameters, i.e. the characteristic exponent and the skewness. The location and scale parameters are sometimes strongly off, especially for low values of alpha and extreme skewness. You may finally try the **Test of Stability** facilities. You'll notice that, as long as you don't go to too large **Maximal Resolution**, the numerical result fit nicely the theory. For instance, if you synthesize a stable motion with the default values of 5000 points, alpha = 1.5 and gamma = 1, both the Mac Culloch and Koutrouvelis methods will yield estimates of alpha and gamma very close to their theoretical values. Now the **Test of Stability** with the default **Maximal Resolution** 12 will yield a **Characteristic parameter** of around 1.43 slightly below the true one. *plot_gamma#* is reasonably linear for small and medium lags, and becomes erratic for lags close to 12 (recall that the abscissa is the logarithm of the scale). Finally, *plot_alpha#* shows that the estimated values of alpha are between 1.48 and 1.63. Since the original signal was indeed a stable motion, you see that, even in the ideal case, there is some discrepancy between the estimated values of alpha using the two methods, i.e. the direct one and the regression on the values of gamma. In general, you'll find that the direct estimation yields a larger value than the one obtained by regression.

To make a test on a real signal, we'll consider the financial log already studied in the **Homework** section of the **Overview** of this help: This is a record of the Nikkei225 index during the period 01/01/80 to 05/11/2000. The log consists in 5313 daily values corresponding to that period. Load first these data into **Fraclab**: Press the **Load** button in the main window. A new window appears, showing the files of your current directory. Change directory to the **DATA** directory that comes with the **Fraclab** release. Choose the file called *nikkei225.txt* by clicking on it. Its name is then displayed at the top of the window, in the **Name:** box. Since this file is plain text, click on the button to the right of **Load as:**, and select the item **ASCII**. Then press **Load**, and **Close** the loading window. The *nikkei225* file should appear in your **Variables** list of the main window, under the name *fnikkei225*. View this signal: Open the **View** window by pressing on the **View** button. In the **View** window, click on **View in new**. This will open a window displaying the stock market log. Like most data of this type, this signal is quite erratic. Other obvious features include a steady increase at the beginning of the log, and strong discontinuities around the points 1780, 2040, 2650, 2760 or 3200.

Financial analysts do not work directly on the prices, but on their logarithms, so we'll first type *lnikkei = log(fnikkei225);* in the matlab window, and import *lnikkei* into **Fraclab**. To do this, press the **Scan Workspace** button in the main window. In the new windows that appears, titled **Import Data from MATLAB Workspace**, locate the signal *lnikkei*, select it by clicking on it, and hit **Import**, then **Close** this window. *lnikkei* will appear in the **Variables** list of the main window, under the same name.

Finally, because we want to check whether *lnikkei* can be well modeled by a stable motion, and estimate the corresponding parameters, we need to compute its increments. Type *incnikkei = diff(lnikkei);* in the matlab window, and import *incnikkei* into **Fraclab**. You may want to view this new signal, and check that it indeed displays a lot of spikes of wildly varying sizes, somewhat reminiscent of what happens for a "true" stable process.

Open the **Mac Culloch Estimation** and **Koutrouvelis Estimation** windows, verify that the **Input** signal is *incnikkei* or hit **Refresh**, and **Compute** the parameters using both methods. You'll find an relatively good agreement for the estimations of alpha (1.37 versus 1.49) and the skewness parameter (0 versus 0.18), and an excellent agreement for the location and scale parameters, which are both estimated as 0. Let us now test the stability. Open the **Test of Stability** window, choose a **Maximum Resolution** of 10 and hit **Compute**. You'll get an alpha of around 1.62, somewhat larger that the estimates above. The graph of *plot_gamma#* is nicely linear for small and medium lags, and becomes erratic only for the largest lags. In fact, the linearity in this graph is quite comparable to the one we had for a true stable process. However, the situation is not so good for *plot_alpha#* : this graph displays strong variations, with estimated values of alpha ranging in (1.36, 1.73). This casts a doubt of the stability of *lnikkei*. The problem most probably comes from the fact that the increments of *lnikkei* are certainly not independent but on the contrary are likely to exhibit strong dependence. Thus, although *lnikkei* may be a process without a variance, or even a stable process (i.e. a process with stable marginals), it is not a stable motion, i.e. its increments are not independent. We hope to include in future releases of **Fraclab** some tools that will allow to analyze general stable processes.

9 References

(1) S. Rachev, S. Mittnik, *Stable Paretian Models in Finance*, John Wiley & Sons, 2000.

(2) L. Belkacem, *alpha-SDE and Option Pricing Model*, Fractals in Engineering (J. Lévy Véhel, E. Lutton and C. Tricot Eds.), Springer Verlag, 1997.