How do we handle words?

Last time, we saw how to improve on BOW representations of documents

- TF-IDF
- PMI
- LSA
- NMF

Now, we ask how should individual words be represented?

How do we handle words?

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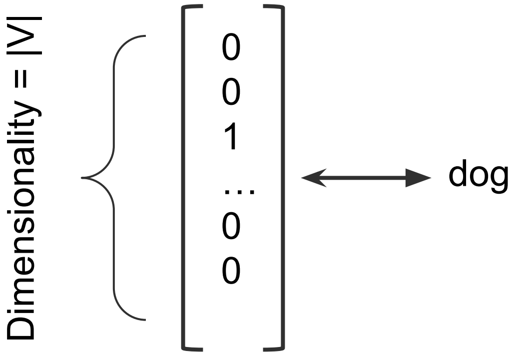
The simplest idea is to use a **one-hot encoding**



How do we handle words?

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$$\text{Dimensionality} = |V| \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \\ \dots \\ 0 \\ 0 \end{bmatrix} \right. \longleftrightarrow \text{dog} \qquad\qquad \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dots \\ 1 \\ \dots \\ 0 \end{bmatrix} \longleftrightarrow \text{pomeranian}$$

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What are some problems with this approach?

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Pixels have relative meaning

vector(dog) • vector(pomeranian) = 0 = vector(dog) • vector(bookshelf) = …

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omeranian

What approaches let us represent word **meanings**?

vector(dog) • vector(pomeranian) = 0 = vector(dog) • vector(bookshelf) = …

Distributional Hypothesis

Statistical ML models expect numeric inputs. What is the **meaning** of a word?

Word to search for: happy Search WordNet

Display Options: (Select option to change) Change

Key: "S:" = Show Synset (semantic) relations, "W:" = Show Word (lexical) relations
Display options for sense: (gloss) "an example sentence"

Adjective

- S: (adj) **happy** (enjoying or showing or marked by joy or pleasure) *"a happy smile"; "spent many happy days on the beach"; "a happy marriage"*
  - ***see also***
    - S: (adj) cheerful (being full of or promoting cheer; having or showing good spirits) *"her cheerful nature"; "a cheerful greeting"; "a cheerful room"; "as cheerful as anyone confined to a hospital bed could be"*
    - S: (adj) contented, content (satisfied or showing satisfaction with things as they are) *"a contented smile"*
    - S: (adj) glad (showing or causing joy and pleasure; especially made happy) *"glad you are here"; "glad that they succeeded"; "gave a glad shout"; "a glad smile"; "heard the glad news"; "a glad occasion"*
    - S: (adj) elated (exultantly proud and joyful; in high spirits) *"the elated winner"; "felt elated and excited"*
    - S: (adj) euphoric (exaggerated feeling of well-being or elation)
    - S: (adj) felicitous (exhibiting an agreeably appropriate manner or style) *"a felicitous speaker"*
    - S: (adj) joyful (full of or producing joy) *"make a joyful noise"; "a joyful occasion"*
    - S: (adj) joyous (full of or characterized by joy) *"felt a joyous abandon"; "joyous laughter"*

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Word to search for: inebriated Search WordNet

Display Options: (Select option to change) Change

Key: "S:" = Show Synset (semantic) relations, "W:" = Show Word (lexical) relations

Display options for sense: (gloss) "an example sentence"

Verb

- **S: (v) exhilarate, tickle pink, inebriate, thrill, exalt, beatify (fill with sublime emotion) *"The children were thrilled at the prospect of going to the movies"; "He was inebriated by his phenomenal success"***

Distributional Hypothesis

Statistical ML models expect numeric inputs. What is the **meaning** of the word *bardiwac*

John enjoys sipping bardiwac in the warm weather

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He drank too much bardiwac, so he can’t drive tonight

The bardiwac grapes didn’t fare well in this summer’s heat

Distributional Hypothesis

Statistical ML models expect numeric inputs. What is the **meaning** of the word *bardiwac*

John enjoys sipping bardiwac in the warm weather

She knocked over

n the carpet

He drank too much

The bardiwac grap

Distributional Hypothesis

"You shall know a word by the company it keeps."

J.R. Firth (1957)

Distributional Hypothesis

Our goal is to describe a word's meaning using the contexts it appears in

Put another way, can we find **representations** so that words that appear in similar contexts are represented similarly and words that do not appear in similar contexts are represented dissimilarly

Harry and Sally sat <u>on the river</u> bank <u>and had a</u> picnic

The robbers <u>struck the largest</u> bank <u>in all of</u> New York

It doesn't matter <u>if you always</u> bank <u>the shot you</u> just need to make it in

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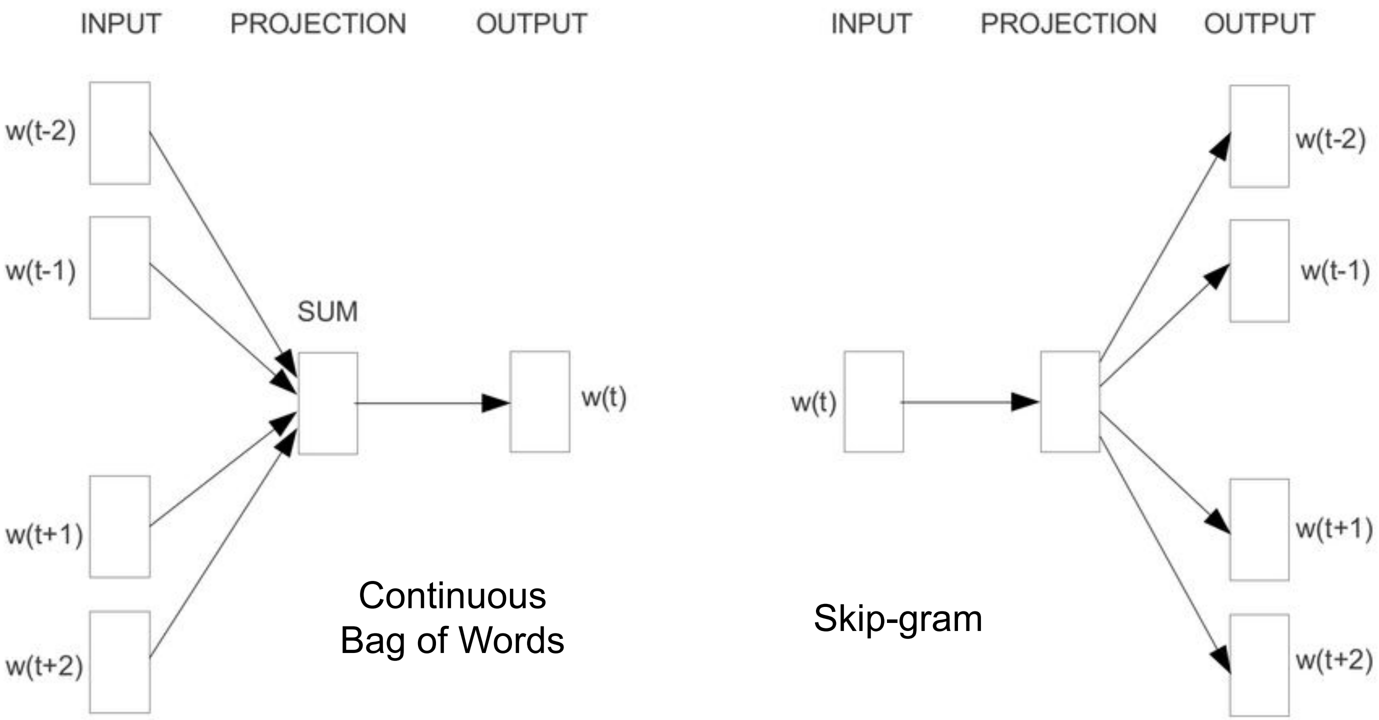
Tasks: (1) predict center word from context (2) predict context from center word

Word2Vec

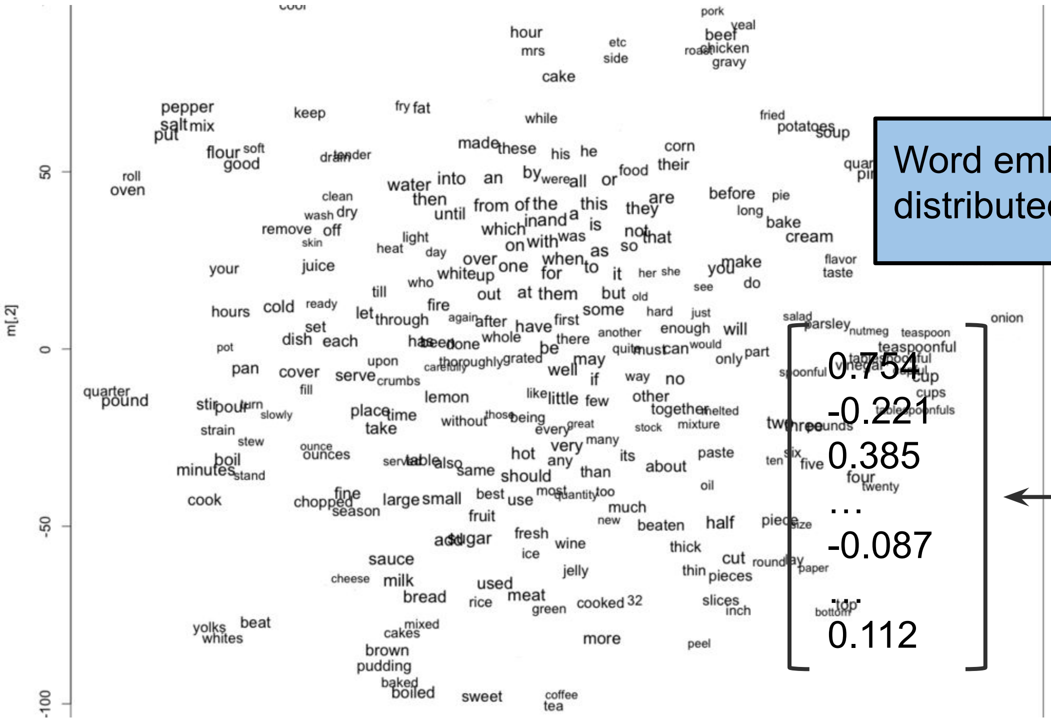


Continuous
Bag of Words

Skip-gram

Efficient Estimation of Word Representations in Vector Space, Mikolov et al., 2013a

Word2Vec



Word embeddings are distributed representations

$$\begin{bmatrix} 0.754 \\ -0.221 \\ 0.385 \\ \dots \\ -0.087 \\ \dots \\ 0.112 \end{bmatrix} \longleftrightarrow \text{pomeranian}$$

Also see: A Neural Probabilistic Language Model, Bengio, et al., 2003

Skip-gram model

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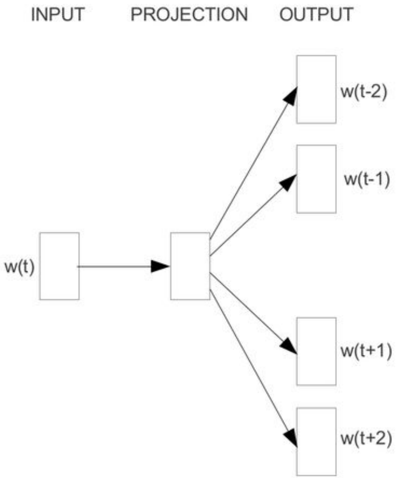
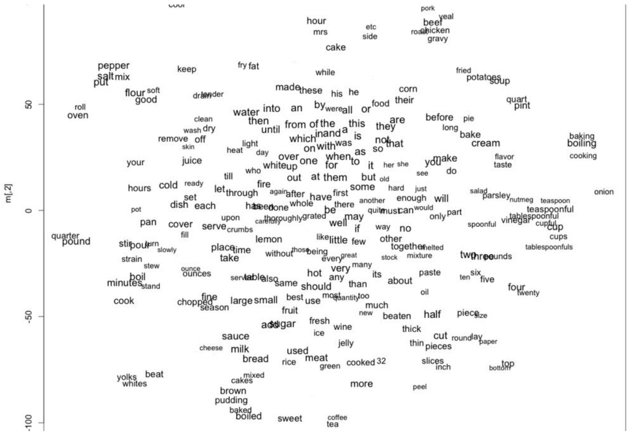




Corpus

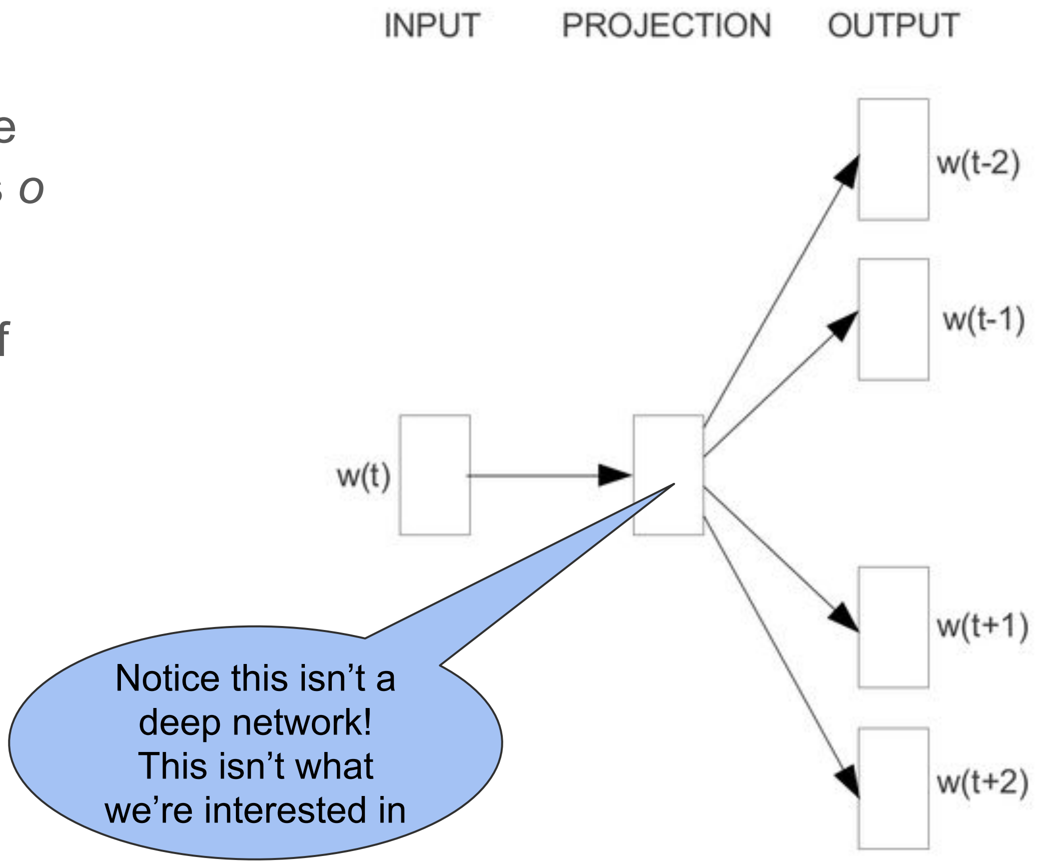
Model

2-d representation

Skip-gram model

The skip-gram model learns the probability of the context words *o* given the center word *c*

- Start with a large corpus of text
- Each word representation starts as a random vector
- Calculate p(o|c) using the word vectors
- Adjust word vectors to maximize the probabilities



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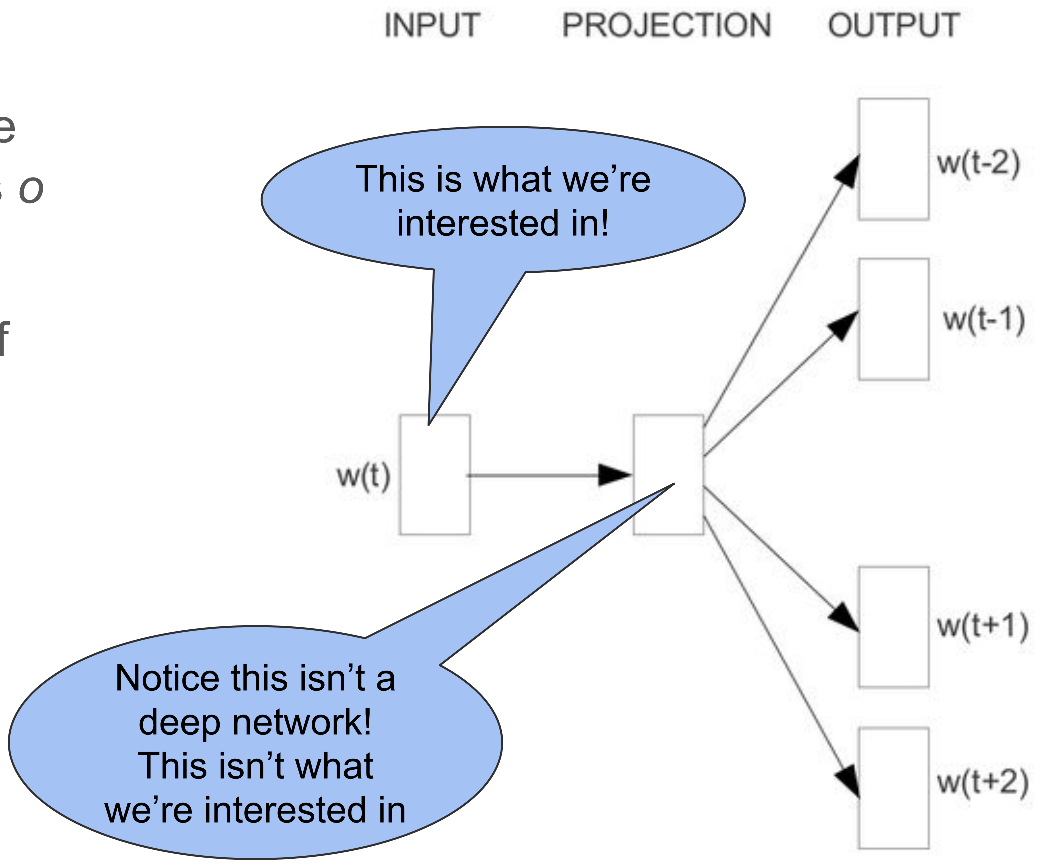

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$p(w_{t-3}|w_t)$ $p(w_{t-2}|w_t)$ $p(w_{t-1}|w_t)$ $p(w_{t+1}|w_t)$ $p(w_{t+2}|w_t)$ $p(w_{t+3}|w_t)$

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But how do we train the model?

Skip-gram model

The skip-gram model learns the probability of the context words *o* given the center word *c*

For each position in the corpus t = 1, …, T, predict the context with window size *m* given the center word w_j. We train by maximizing a likelihood:

$$L(\theta) =$$

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What is the probability model?

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Skip-gram model

The skip-gram model learns the probability of the context words *o* given the center word *c*

In fact, we use two vectors per word—one for when the word is a center word and one for when the word is a context word—which makes optimization easier

vector representation of *o*

vector representation of *c*

θ are all the u, v vectors for every word in our vocabulary!

$$p(o|c) = \frac{\exp\left(u_o^T v_c\right)}{\sum_{w \in V} \exp\left(u_w^T v_c\right)}$$

Computes **similarity**

sum over vocabulary (softmax regression)

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Exercise: Write down the formula for the context word optimization: $\partial/\partial v_c \; p(o|c)$.

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Practical considerations:

- The denominator is **expensive to compute when V is large (|V| > 10^5)**. So, we modify the model to perform binary classification so we don't have to sum over all vocabulary words ("what is the probability that this word is a context word for *c*" vs. "what is the probability that this word is right as a context word")

$$p(o \text{ is right word} | c) = \sigma(u_o^T v_c)$$

Only using this term pushes similar words together but doesn't discourage dissimilar words from being far apart—add in examples of *o* words that are not the right word for *c*

$$p(o \text{ is wrong word} | c) = \sigma(-u_o^T v_c)$$

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Skip-gram model

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$$L(\theta) = -\frac{1}{T}\sum_{c,o}\left(\log\sigma(u_o^T v_c) + \sum_w \log\sigma(-u_w^T v_c)\right)$$

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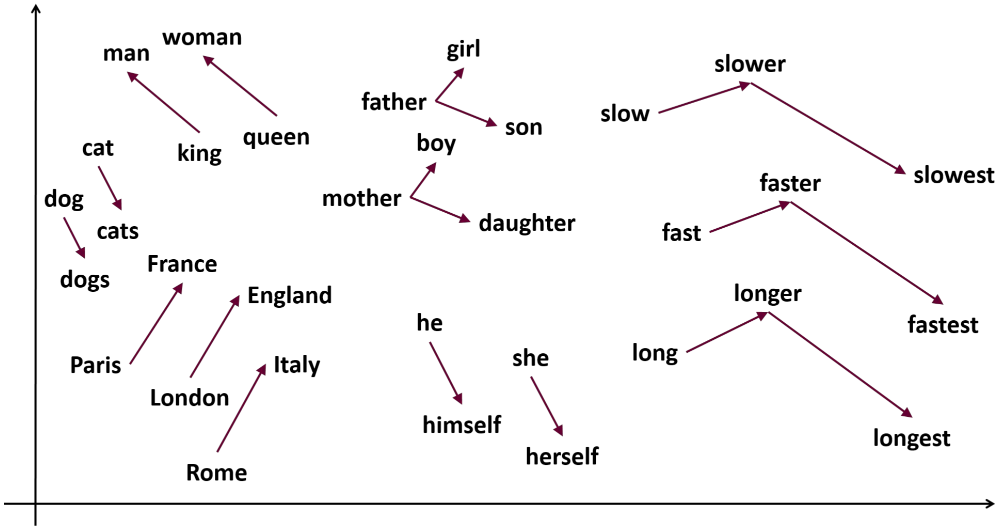
Word2vec embeddings

Advantages of word2vec:

- Word vectors staged in a semantically-meaningful space
- Low-dimensional embedding compared to one-hot
- Common-sense operations are interpretable
- Labels come for free (self-supervised)



Disadvantages of word2vec:

- Relative distances alone are meaningful
- Embeddings are static
- Requires lots of data

Take-home message

- Working with language data is challenging
- Embeddings are the most profitable approach to representing individual words as input to a neural model
- We saw static embedding approaches and will later see how embeddings can be determined on the fly