Introduction to Computational Physics

PHYS 250 (Autumn 2018) – Lecture 4

David Miller

Department of Physics and the Enrico Fermi Institute
University of Chicago

October 11, 2018

Outline

1. *The physics of randomness and emergent properties*
   - Coin flips
   - Random walks
   - Central limit theorem and analytic descriptions of random behavior

Slow random diffusion

Last time, we determined that if the diffusion (i.e. walking speed) is also slow, such that the time derivative of the position probability distribution function, ρ, is approximately linear with respect to time and $\rho(x, t + \Delta t) - \rho(x, t) \approx (\frac{\partial \rho}{\partial t})\Delta t$, then

$$\frac{\partial \rho}{\partial t} = \frac{a^2}{2\Delta t}\frac{\partial^2 \rho}{\partial x^2}. \tag{1}$$

We observed that this **is** the diffusion equation Eq. 15 with $D = \frac{a^2}{2\Delta t}$.

The point is that we obtained an analytical description of a random walk via the diffusion equation under minimal assumptions: the probability distribution is broad and slowly varying compared to the size and time of the individual steps.