Steel Beam Hinge Modeling

Reference:

[1]: Hysteretic models that incorporate strength and stiffness deterioration

[2]: Deterioration modeling of steel components in support of collapse prediction of steel moment frames under earthquake loading

[3]: Global collapse of frame structures under seismic excitations

[4]: Sidesway collapse of deteriorating structural systems under seismic excitations

Modeling introduction

- No deterioration exits:

  Three parameters: initial stiffness K_e; yield strength F_y; strain-hardening stiffness K_s

  $$K_s = \alpha_s K_e$$

- Consider deterioration:

  Apart from above three parameters; cap deformation $delta_c$; peak strength F_c; Post-capping stiffness K_c; residual strength F_r

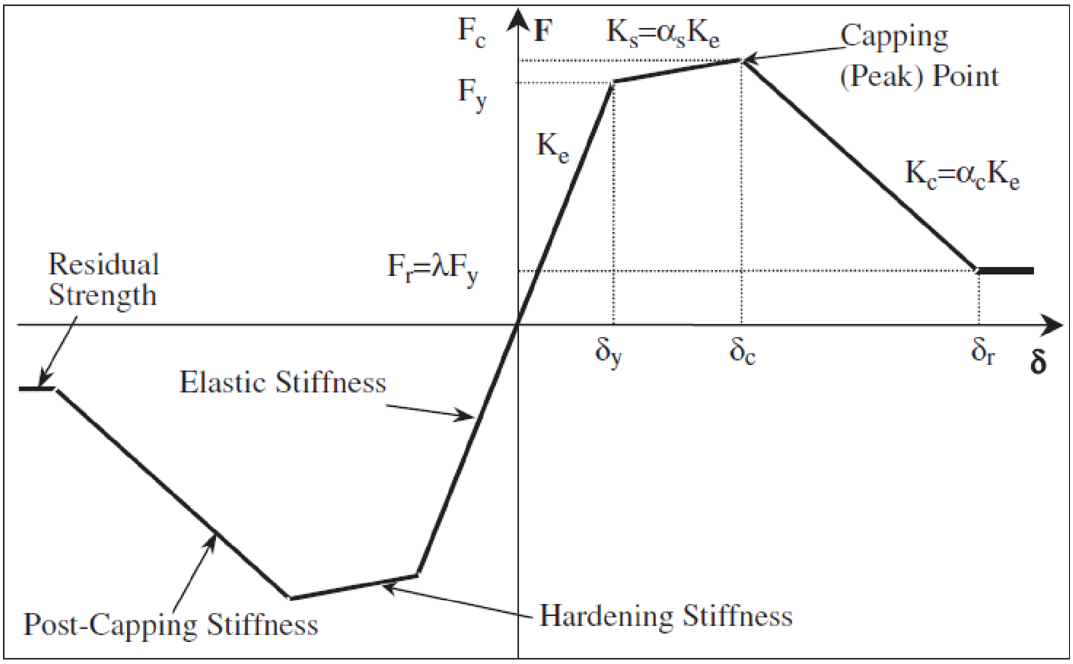

Figure 2. Backbone curve for hysteretic models.

$$K_s = \alpha_s K_e$$
$$K_c = \alpha_c K_c$$
$$F_r = \lambda F_y$$

Four types of deterioration involved in strength and stiffness deterioration: basic strength; post-capping strength, unloading stiffness, and accelerated reloading stiffness deteriorations.

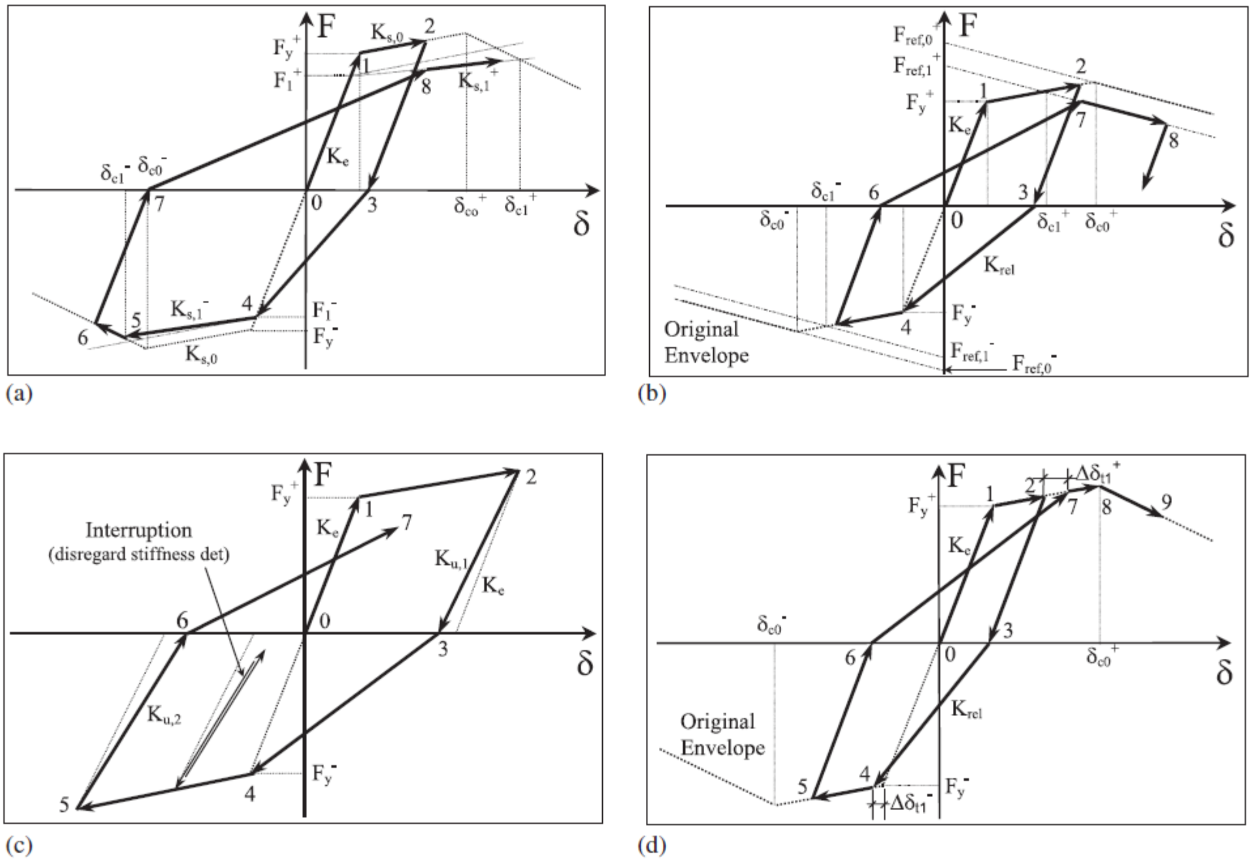

Figure 6. Individual deterioration modes, illustrated on a peak-oriented model: (a) basic strength deterioration; (b) post-capping strength deterioration; (c) unloading stiffness deterioration; and (d) accelerated reloading stiffness deterioration.

Modeling parameters

Reference 2 provides a set of equations to calculate modeling parameters

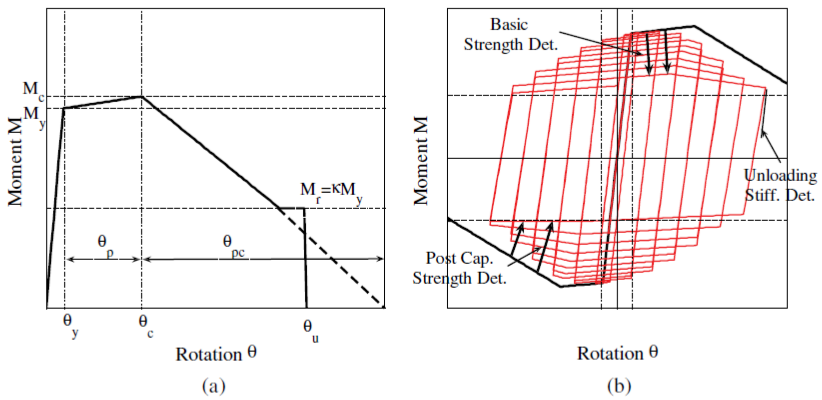

Fig. 1. Modified IK deterioration model: (a) monotonic curve; (b) basic modes of cyclic deterioration and associated definitions

- Trend of modeling parameters
  - Generally, modeling parameters for beams with other-than-RBS are smaller than that with RBS connection.
  - Dispersion of modeling parameters for beams with RBS connection are smaller.
  - Modeling parameters decrease as beam depth increases.
  - θ_p is linearly proportional to beam shear span L (distance from plastic hinge location to point of inflection)
  - Providing lateral bracing close to RBS portion of a beam decreases the rate of cyclic deterioration.

- For most deep beam, small $b_f/2t_f$ ratio has detrimental effect on theta_p, but benefits the parameters theta_pc and \LAMBDA.
- h/t_t is very important for all three modeling parameters (\theta_p, \theta_pc, \LAMBDA).

Equations for parameters

- Database for the equations:

  Set 1: Beams with other-than-RBS connections and depth 18 in. <= d <= 36 in.

  Set 2: **Beams with RBS connections and depth 18 in <= d <= 36 in**

  Set 3: Beams with other-than-RBS connections and depth d >= 21 in.

  Set 4: Beams with RBS connections and depth >= 21 in.

  Sets 2 and 4 do not differ too much!!!

- Pre-capping plastic rotation:

  Data set 1:

$$\theta_p = 0.0865 \cdot (\frac{h}{t_w})^{-0.365} \cdot (\frac{b_f}{2 \cdot t_f})^{-0.140} \cdot (\frac{L}{d})^{0.340} \cdot (\frac{c_{unit}^1 \cdot d}{533})^{-0.721} \cdot (\frac{c_{unit}^2 \cdot F_y}{355})^{-0.230}$$

  If millimeters and megapascals are used:

$$c_{unit}^1 = c_{unit}^2 = 1.0$$

  If depth is in inches and F_y is in ksi:

$$c_{unit}^1 = 25.4$$
$$c_{unit}^2 = 6.895$$

  Data set 3:

$$\theta_p = 0.318 \cdot (\frac{h}{t_w})^{-0.550} \cdot (\frac{b_f}{2 \cdot t_f})^{-0.345} \cdot (\frac{L_b}{r_y})^{-0.0230} \cdot (\frac{L}{d})^{0.090} \cdot (\frac{c_{unit}^1 \cdot d}{533})^{-0.330} \cdot (\frac{c_{unit}^2 \cdot F_y}{355})^{-0.130}$$

  Data set 2:

$$\theta_p = 0.19 \cdot (\frac{h}{t_w})^{-0.314} \cdot (\frac{b_f}{2 \cdot t_f})^{-0.100} \cdot (\frac{L_b}{r_y})^{-0.185} \cdot (\frac{L}{d})^{0.113} \cdot (\frac{c_{unit}^1 \cdot d}{533})^{-0.760} \cdot (\frac{c_{unit}^2 \cdot F_y}{355})^{-0.070}$$

- Post-capping plastic rotation:

  Data set 1:

$$\theta_{pc} = 5.63 \cdot (\frac{h}{t_w})^{-0.565} \cdot (\frac{b_f}{2 \cdot t_f})^{-0.800} \cdot (\frac{c_{unit}^1 \cdot d}{533})^{-0.280} \cdot (\frac{c_{unit}^2 \cdot F_y}{355})^{-0.430}$$

  Data set 3:

$$\theta_{pc} = 7.50 \cdot (\frac{h}{t_w})^{-0.610} \cdot (\frac{b_f}{2 \cdot t_f})^{-0.710} \cdot (\frac{L_b}{r_y})^{-0.110} \cdot (\frac{c_{unit}^1 \cdot d}{533})^{-0.161} \cdot (\frac{c_{unit}^2 \cdot F_y}{355})^{-0.320}$$

  Data set 2:

$$\theta_{pc} = 9.52 \cdot (\frac{h}{t_w})^{-0.513} \cdot (\frac{b_f}{2 \cdot t_f})^{-0.863} \cdot (\frac{L_b}{r_y})^{-0.108} \cdot (\frac{c_{unit}^2 \cdot F_y}{355})^{-0.360}$$

- Reference cumulative plastic rotation:

  Data set 1:

$$\Lambda = \frac{E_t}{M_y} = 495 \cdot (\frac{h}{t_w})^{-1.34} \cdot (\frac{b_f}{2 \cdot t_f})^{-0.595} \cdot (\frac{c_{unit}^2 \cdot F_y}{355})^{-0.360}$$

  Data set 3:

$$\Lambda = \frac{E_t}{M_y} = 536 \cdot (\frac{h}{t_w})^{-1.26} \cdot (\frac{b_f}{2 \cdot t_f})^{-0.525} \cdot (\frac{L_b}{r_y})^{-0.130} \cdot (\frac{c_{unit}^2 \cdot F_y}{355})^{-0.291}$$

  Data set 2:

$$\Lambda = \frac{E_t}{M_y} = 585 \cdot (\frac{h}{t_w})^{-1.14} \cdot (\frac{b_f}{2 \cdot t_f})^{-0.632} \cdot (\frac{L_b}{r_y})^{-0.205} \cdot (\frac{c_{unit}^2 \cdot F_y}{355})^{-0.391}$$

- Remarks:

  The range of validity of these equations is only as good as the experimental data allows it to be.

  Though the data does not include heavy W14 sections (heavier than W14X370) and heavy (heavier than W36X150) and deep (deeper than W36) beam sections. Predictions from regression equations were compared with existing heavy W14 sections and found to provide reasonable close values of experimentally obtained parameters.

  Until more experiments are conducted, the preceding equations provide the best estimates that can be offered to columns.

Table 1. Modeling Parameters for Various Beam Sizes (Other-than-RBS) Based on Regression Equations

Section size	θ_p (rad)	θ_{pc} (rad)	Λ	h/t_w	$b_f/2t_f$	L_b/r_y	L/d	d (mm)
W21 × 62	0.031	0.14	0.90	46.90	6.70	50.00	7.14	533
W21 × 147	0.038	0.22	2.23	26.10	5.43	50.00	6.79	561
W24 × 84	0.028	0.15	1.00	45.90	5.86	50.00	6.22	612
W24 × 207	0.034	0.28	2.81	24.80	4.14	50.00	5.84	653
W27 × 94	0.024	0.13	0.83	49.50	6.70	50.00	5.58	683
W27 × 217	0.029	0.22	2.14	28.70	4.70	50.00	5.28	721
W30 × 108	0.021	0.12	0.82	49.60	6.91	50.00	5.03	757
W30 × 235	0.024	0.19	1.76	32.20	5.03	50.00	4.79	795
W33 × 130	0.019	0.11	0.79	51.70	6.73	50.00	4.53	841
W33 × 241	0.021	0.16	1.42	35.90	5.68	50.00	4.39	869
W36 × 150	0.017	0.12	0.81	51.90	6.38	50.00	4.18	912
W36 × 210	0.020	0.18	1.45	39.10	4.49	50.00	4.09	932

Note: Assumed beam shear span $L = 3{,}810$ mm (150 in.); $L_b/r_y = 50$; and expected yield strength $F_y = 379$ MPa (55 ksi).

Table 2. Modeling Parameters for Various Beam Sizes (Beams with RBS) Based on Regression Equations

Section size	θ_p (rad)	θ_{pc} (rad)	Λ	h/t_w	$b_f/2t_f$	L_b/r_y	L/d	d (mm)
W21 × 62	0.028	0.16	0.97	46.90	6.70	50.00	7.14	533
W21 × 147	0.033	0.27	2.15	26.10	5.43	50.00	6.79	561
W24 × 84	0.026	0.19	1.08	45.90	5.86	50.00	6.22	612
W24 × 207	0.030[a]	0.34[a]	2.71[a]	24.80	4.14	50.00	5.84	653
W27 × 94	0.022	0.16	0.91	49.50	6.70	50.00	5.58	683
W27 × 217	0.026[a]	0.29[a]	2.12[a]	28.70	4.70	50.00	5.28	721
W30 × 108	0.020	0.16	0.89	49.60	6.91	50.00	5.03	757
W30 × 235	0.023	0.25	1.78	32.20	5.03	50.00	4.79	795
W33 × 130	0.018	0.16	0.86	51.70	6.73	50.00	4.53	841
W33 × 241	0.020	0.22	1.46	35.90	5.68	50.00	4.39	869
W36 × 150	0.017	0.16	0.89	51.90	6.38	50.00	4.18	912
W36 × 210	0.019[a]	0.25[a]	1.53[a]	39.10	4.49	50.00	4.09	932

Note: Assumed beam shear span $L = 3{,}810$ mm (150 in.); $L_b/r_y = 50$; and expected yield strength $F_y = 379$ MPa (55 ksi).
[a]Values slightly outside the range of experimental data.

- Effective yield strength M_y:

  It is found to be slightly greater than predicted bending strength $M_{y,p}$, which is defined as plastic section modulus Z times the measured material yield strength. For RBS, the ratio of M_y to $M_{y,p}$ is 1.06, whereas for non-RBS is 1.17.

Table 3. Statistics of Ratios of Effective-to-Predicted Component Yield Strength and Capping Strength-to-Effective Yield Strength

Connection type	Mean $M_y/M_{y,p}$	$\sigma_{My/My,p}$	Mean M_c/M_y	$\sigma_{Mc/My}$
RBS	1.06	0.12	1.09	0.03
Other-than-RBS	1.17	0.21	1.11	0.05

- Residual strength ratio k:

  From the data sets for W-sections, a residual strength ratio $k = M_r/M_y$ of approximately 0.4 is suggested for sets 3 and 4.

  To assess it more reliably, more experiments with very large deformation cycles need to be conducted.

- Ultimate rotation capacity θ_u:

  This quantity is highly dependent on loading history and my be very large for cases in which only a few very large cycles are executed.

  Estimation regarding this parameter are made only for experiments with stepwise increasing cycles of the type required in AISC.

  For other than RBS, it is 0.05 to 0.06 rad.

  For RBS, it is 0.06 to 0.07 rad.

  For monotonic loading, \theta_u is found to be on the order of three times as large as the \theta_u reported in symmetric cyclic loading protocols.

OpenSees IMK material model

Reference: OpenSees IMK material model

- Elastic stiffness K_0:

$$K_0 = (n+1)\frac{6 \cdot E \cdot I_z}{L}$$

  where E, I and L are elastic modulus, moment of inertia, and length of beam. Typically, n is set as 10.

- Strain hardening ratio for positive (negative) directions:

  Will be computed at he very end of this note.

- Effective yield strength M_y:

  According to findings reported in Ref [2], for beams with RBS connection:

$$My = 1.06 \cdot M_{y,p} = 1.06 \cdot Z \cdot F_y$$

- Cyclic deterioration for *basic strength*, *post-capping strength*, *accelerated reloading stiffness*, and *unloading stiffness*.

$$\begin{aligned} &\Lambda_S : \text{basic strength} \\ &\Lambda_C : \text{post} - \text{capping strength} \\ &\Lambda_A : \text{accelerated reloading stiffness} \\ &\Lambda_K : \text{unloading stiffness} \end{aligned}$$

  A very large number means almost no cyclic deterioration.

  In my modeling

$$\Lambda_K = \Lambda_A = \Lambda_S = \Lambda_C = (n+1) \cdot \Lambda = 11 \cdot \Lambda$$

  Where Λ is calculated from regression equation in preceding section.

- Rate of deterioration for *basic strength*, *post-capping strength*, *accelerated reloading*, and *unloading stiffness*.

  By default: (**In my modeling**)

$$\begin{aligned} c_S &= 1.0 \\ c_C &= 1.0 \\ c_A &= 1.0 \\ c_K &= 1.0 \end{aligned}$$

- Plastic rotation capacity *theta_p*

  In my modeling, use the regression equation in the preceding section.

- Post-capping rotation capacity *theta_pc*:

  In my modeling, use the regression equation in the preceding section.

- Residual strength ratio *Res*:

  Based on Ref [2], the ratio is assumed to be 0.4.

- Ultimate rotation *theta_u*:

Based on preceding section and official OpenSees example, it is assumed to be 0.4. Ultimate rotation is associated with the failure of ductile tearing. Ref [4] reveals that ductile tearing will not be critical in most of cases.

In my modeling, use 0.4.

- Rate of cyclic deterioration *D* (for symmetric hysteretic response use 1.0):

  In my modeling, use 1.0.

- elastic stiffness amplification factor *nFactor*:

  This is optional, default value is 0.

  In my modeling, ignore this argument and the program would automatically use 0.

- **Strain hardening ratio from Official OpenSees example**

  For *elastic beam-column element*, modified moment of inertia is:

$$I_{mod} = \frac{n + 1.0}{n} \cdot I_z$$

  Rotational stiffness for *elastic beam-column element*:

$$K_{bc} = \frac{n+1}{n} \cdot \frac{6 \cdot E \cdot I_z}{L}$$

  Initial stiffness for rotational spring for beam:

$$K_s = n \cdot \frac{6 \cdot E \cdot I_{mod}}{L} = (n + 1.0) \cdot \frac{6 \cdot E \cdot I_z}{L}$$

where I_z is the moment of inertia for beam section.

Strain hardening ratio for spring:

$$a_{mem} = (n + 1.0) \cdot \frac{My \cdot (M_c/M_y - 1.0)}{\theta_p} \cdot \frac{1}{K_s}$$

Modified strain hardening ratio for spring (**used for modeling**):

$$b = \frac{a_{men}}{1 + n \cdot (1 - a_{men})}$$

- Strain hardening ratio from Prof. Burton's codes:

  Initial stiffness for rotational spring for beam:

$$K = n_1 \cdot K_0 = 11 \cdot \frac{6 \cdot E \cdot I_z}{L}$$

  Straining hardening ratio for spring:

$$asPosScaled = \frac{a_s}{1 + n_2 \cdot (1 - a_s)}$$

  Where $n_2 = 10$.

  a_s is calculated using the following equation:

$$a_s = 0.11 \cdot \frac{My}{\theta_p} \cdot \frac{1}{K_0}$$

$$K_0 = \frac{6 \cdot E \cdot I_z}{L}$$