Fractulum Mathathon

Fractals Community

December 15, 2021

> God made the natural numbers; all else is the work of man.
>
> Leopold Kronecker

1. Given two integers, define an operation $*$ such that if a and b are integers, then $a * b$ is an integer. The operation $*$ has the following properties:

   - $a * a = 0$ for all integers a;
   - $(ka + b) * a = b * a$ for all integers a, b, and k;
   - $0 \leq b * a < a$.
   - $0 \leq b < a$ then $b * a = b$.

   Find $2021 * 16$

2. It's currently $6:00$ on a 12 hour clock. What time will be shown on the clock 100 hours from now? Express your answer in the form $hh:mm$.

3. We call a positive integer *binary-okay* if at least half of the digits in its binary (base 2) representation are 1's, but no two 1s are consecutive. For example, $10_{10} = 1010_2$ and $5_{10} = 101_2$ are both binary-okay, but $16_{10} = 10000_2$ and $11_{10} = 1011_2$ are not. Compute the number of binary-okay positive integers less than or equal to 2021 (in base 10).

4. Let n be an integer such that $n^4 - 2n^3 - n^2 + 2n + 2$ is a prime number. What's the sum of all possible n?

5. Find the number of subsets S of $\{1, 2, ..., 10\}$ such that no two of the elements in S are consecutive

6. Can an 8×8 board be coverd by 15 1×4 rectangles and only one 2×2 square without overlaping? prove your answer.

7. A 3×3 magic square is a grid of distinct numbers whose rows, columns, and diagonals all add to the same integer sum. Connie creates a magic square whose sum is N, but her keyboard is broken so that when she types a number, one of the digits $(0-9)$ always appears as a different digit (e.g. if the digit 8 always appears as 5, the number 18 will appear as 15). The altered square is shown below. Find N.

9	11	10
18	17	6
14	11	15

8. Compute

$$\sum_{k \geq 0} \binom{1000}{3k}$$

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>>> from engineering.math import pi, e
>>> pi == e
True
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