Basic Equation Solving

Fractals

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1 Basic Equation Solving

1.1 Algebraic Manipulation

To introduce the topic of **algebraic manipulation**, let us start with a little known example: **egyptian fractions**

Theorem 1 (Egyptian Fractions). For all a, b where $ab \neq 1$

$$\frac{a}{ab-1} = \frac{1}{b(ab-1)} + \frac{1}{b}.$$

From here, we can see that putting things together (factoring) is just as important as taking them apart (distributing). Now, let us turn the power of products:

Example 1.1. For positive real numbers a, b,

$$a + \frac{1}{b} = 4,$$

$$b + \frac{1}{a} = 5,$$

Find $ab + \dfrac{1}{ab}$.

Solution. It is very easy to get lost in the problem if we directly try to solve for a and b. Instead, let us multiply the equations:

$$(a + \frac{1}{b})(b + \frac{1}{a}) = 4(5) = 20.$$

$$ab + \frac{a}{a} + \frac{b}{b} + \frac{1}{ab} = ab + \frac{1}{ab} + 2 = 20,$$

$$\boxed{ab + \frac{1}{ab} = 18.}$$

Theorem 2. Let x, y be nonzero real numbers such that $x + y = a$ and $xy = b$ Then,

$$x^2 + y^2 = a^2 - 2b,$$

$$(x + 1)(y + 1) = a + b + 1,$$

$$x^2 + xy^2 = ab,$$

$$|x - y| = \sqrt{a^2 - 4b},$$

$$x^3 + y^3 = a^3 - 3ab,$$

$$\frac{1}{x} + \frac{1}{y} = \frac{a}{b},$$

1.2 Quadratic Equations

A polynomial is an equation of the following form:

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

where

$$a_0, a_1, \ldots, a_n$$

are constants. A quadratic equation is a polynoimal with $n = 2$:

$$ax^2 + bx + c = 0,$$

A common way to solve a quadratic equation is to use the quadratic formula:

Theorem 3 (Quadratic Formula). For the equation $ax^2 + bx + c = 0$, the roots x_1, x_2 must be equal to

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a},$$

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a},$$