

Functions

Fractals

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1 Introduction

1.1 Defintions

Defintion 1.1. A function f from a set X to a set Y is a relation that assigns to each element in set X exactly one element in set Y .

Defintion 1.2. The domain is the set of X (a.k.a. the input).

Defintion 1.3. The range is a subset of Y (a.k.a. the output).

1.2 Existence of a Function

Theorem 1 (Vertical Line Test). if you can draw a Vertical line that passes through more than one point of a relation on a grap, it's not a function, if you cannot, it's a function.

Example 1.1. what the domain and range of the function $f(x) = \sqrt{16 - x^2}$?

sloution Note that if $a < 0$, then \sqrt{a} is undefined for reals, Thus, $16 - x^2 \geq 0 \Rightarrow -4 \leq x \leq 4$
since $x^2 \geq 0$, we have that $0 \leq 16 - x^2 \leq 16$, so the range is $0 \leq y \leq 4$

2 Combinations of Functions

Theorem 2 (common function Combinations). The following are some common combinations of functions:

- **Sum** $(f + g)(x) = f(x) + g(x)$
- **Difference** $(f - g)(x) = f(x) - g(x)$
- **Product** $(fg)(x) = f(x)g(x)$
- **Quotient** $(\frac{f}{g})(x) = \frac{f(x)}{g(x)}$ where $g(x) \neq 0$
- **Composition** $(f \circ g)(x) = f(g(x))$

2.1 Domain and Range of a Composite Function

3 Types of Functions

3.1 Piecewise-Defined Function

4 Properties of Functions

4.1 Odd and Even Functions

4.2 Periodic Functions

5 Inverse Functions

5.1 Existence of an Inverse Function