

Sample Contest

Fractals

November 30, 2021

1. Define The operation $a@b$ to be $3+ab+a+2b$. There exists a number x such that $x@b = 1$ for all b . Find x .
2. Let $y = x^2 + bx + c$ be a quadratic function. it has only one real root. if b is postive, find $\frac{b+2}{\sqrt{c}+1}$.
3. A circle of nonzero radius r has a circumference numerically equal to $\frac{1}{3}$ of its area. What is its area?
4. Let set \mathcal{A} be a 90-element subset of $\{1, 2, 3, \dots, 100\}$, and let S be the sum of the elements of \mathcal{A} . Find the number of possible values of S .
5. A *gorgeous* sequence is a sequence of 1's and 0's such that there are no consecutive 1's. For instance, the set of all gorgeous sequences of length 3 is $[1, 0, 0]$, $[1, 0, 1]$, $[0, 1, 0]$, $[0, 0, 1]$, $[0, 0, 0]$. Determine the number of gorgeous sequences of length 7.
6. A 8×8 chessboard with the northeast and southwest corner unit squares removed is given. Is it possible to partition such a board into thirty-one unit dominoes(where a domino is a 1×2 rectangle)? Show your work.
7. The function f satifies

$$f(x) + f(2x + y) + 5xy = f(3x - y) + 2x^2 + 1$$

for all real numbers x, y . Determine the vlaue of $f(10)$

8. Let

$$a = \underbrace{19191919191 \dots 1919}_{19 \text{ is repeted } 3838 \text{ times}}$$

What is the reminder of a when divided by 13?