Lecture 2

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1 Predicates

predicates are statements involving variables such that " $x > 3$ " . These statements are neither tru or false where the value of the variable is not specified. The statement "x is greater than 3" involving two parts . the variable part x and the predicate(property) it self " great than 3" .its denoted by $P(x)$ once a value is assigned to x the $P(x)$ become a preposition

For example:
let $P(x)$ denote the statement "$x > 3$" what are truth values of $P(4)$ and $P(2)$

Solution: we obtain $P(4)$ by setting $x = 4$ in the statement, Hence $P(4)$ is the statement "$3 > 4$" which is $True$. $P(2)$ is the statement "$2 > 3$" which is $False$

2 Quantifiers

Definition 2.1 (**Universal Quantifier**)**.** the universal Quantifier of $P(x)$ is the statement "$P(X)$ is $True$ for all values of x in the domain". It is denoted by $\forall x P(x)$

Definition 2.2 (**Esistential Quantifier**)**.** the existential quantifier of $P(x)$ is the preposition " there exist an element x such that $P(x)$ is $True$" . It is denoted by $\exists x P(x)$

statement	when true	when false
$\forall x P(x)$	P(x) is $True$ for all values of x	there is an x for which $P(x)$ is $False$
$\exists x P(x)$	there is an x such that $P(x)$ is $True$	$P(x)$ is $False$ for all x

English phrases with quatifiers:

- "no one is P(x)" $\longleftrightarrow \forall x \neg P(x)$ or $\neg(\exists x P(x))$
- "not every one is P(x)"$\longleftrightarrow \neg(\forall x P(x))$ or $\exists x \neg P(x)$
- "exactly one is P(x) " $\longleftrightarrow \exists x(P(x) \land \forall y(P(y) \to x = y))$
- "all Q(x) is P(x) " $\longleftrightarrow \forall x(Q(x) \to P(x))$
- "all Q(x) isnot P(x)" $\longleftrightarrow \forall x(Q(x) \to \neg P(x))$
- "some Q(x) are P(x)" $\longleftrightarrow \exists x(Q(x) \land P(x))$
- "some Q(x) are not P(x)" $\longleftrightarrow \exists x(Q(x) \land \neg P(x))$

Nested Quantifiers

statement	when true?	when false?
$\forall\forall y P(x,y)$ $\forall y \forall x P(x,y)$	P(x,y) is true for all possible pairs	there is a pair (x,y) such that P(x,y) is false
$\forall x \exists y P(x,y)$	for every x there is y (not necessary same for different values of x) such that P(x,y) is true	there is an x such that P(x,y) is false for every y
$\exists y \forall x P(x,y)$	there is exist a specific (same) y that P(x,y) is true for all x	for every y there is at leasy one x that P(x,y) fails
$\exists x \exists y P(x,y)$ $\exists y \exists x P(x,y)$	therer is a pair (x,y) for which P(x,y) is true	for all pairs (x,y) P(x,y) is False