Math 501
Lecture 1

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1 Preposition

a preposition is a claim or a declerative statement which has a truth value , can be proven to be either $True$ or $False$

For Example:

- "Shimaa studies discrete mathimatics" is considered as $Preposition$
- "How was your day?" is considered as $not - Preposition$
- $x + 1 = 2$ is considered as *non declerative statement* because it is truth value depends on a variable so cannot be proven to be $True$ or $False$ without knowing the value of that variable

2 Logical Operators

Definition 2.1 (**Negation**). let p a preposition the negation of p , denoted by $\neg p$ .The truth value of the negation of p $\neg$ p is the opposite value of p , expressed in English as "It's not the case that. p"

For Example:

- the negation of "I have more than 5 friends" will become "I have at most 5 friends"

Definition 2.2 (**Conjunction**). let p and q be prepositions , the conjunction of p and q denoted by $p \wedge q$ is a preposition "p and q" that become true only if both p and q are both $True$

Definition 2.3 (**Disjunction**). let p and q be prepositions .The Disjunction of p and q denoted by $p \vee q$ is a preposition "p or q" which is $False$ only if both of p and q are $False$

Definition 2.4 (**Exclusive Disjunction**). let p and q be prepositions .The esclusive or denoted by $p \oplus q$ is a preposition that is $True$ if exactly one of p or q are $True$, and $False$ otherwise

3 Conditional Statements

Definition 3.1. let p and q be prepositions . The Conditional Statements $p \to q$, "if p then q" is false whenever p is $False$ or q is $True$

The meaning of $p \to q$ assert that q is true whenever p holds but not vise versa ,when p is $False$ it does not matter what the value of q for implication to be $True$,p is called (hypothesis or antecedent or premise) while q is called conclusion or consequence .

English Phrases to express conditional statements:

- "if p, then q"
- "p implies q"
- "p is sufficient of q"
- "p only if q"
- "q is necessary for p"
- "q unless $\neg$ p" **important**
- "p only if q"
- "q whenever p "

Converse, Contrapositive, and Inverse: for conditional statement $p \to q$

- $q \to p$ called the converse .
- $\neg q \to \neg p$ called the Contrapositive and has the same truth value as the original statement
- $\neg p \to \neg q$ called the inverse

Definition 3.2. Let p and q be prepositions , the biconditional statement $p \leftrightarrow q$ is a preposition "p if and only if q" which is $True$ when p and q have the same truth values and $False$ otherwise

4 Precedence of Logical Operators

Logical Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\to$	4
$\leftrightarrow$	5

5 Logical Equavilance

two preposition p, q are said to be equavilant if $p \leftrightarrow q$ is a *tautology*

- $p \to q \equiv \neg p \vee q$
- $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
- $p \wedge T_0 \equiv p$, $p \vee F_0 \equiv p$ (identity law)
- $p \wedge F_0 \equiv F_0$, $p \vee T_0 \equiv T_0$ (domination law)
- $p \wedge p \equiv p$, $p \vee p \equiv p$ (idempotent (okay to apply many times))

- $\neg(p \land q) \equiv \neg p \lor \neg q$, $\neg(p \lor q) \equiv \neg p \land \neg q$ (De morgans law)
- $p \land (p \lor q) \equiv p$, $p \lor (p \land q) \equiv p$ (apsorption law)