

Math 501

Lecture 1

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1 Proposition

a proposition is a claim or a declarative statement which has a truth value , can be proven to be either *True* or *False*

For Example:

- "Shimaa studies discrete mathematics" is considered as *Proposition*
- "How was your day?" is considered as *not – Proposition*
- $x + 1 = 2$ is considered as *non declarative statement* because its truth value depends on a variable so cannot be proven to be *True* or *False* without knowing the value of that variable

2 Logical Operators

Definition 2.1 (Negation). let p a proposition the negation of p , denoted by $\neg p$.The truth value of the negation of $p \neg p$ is the opposite value of p , expressed in English as "It's not the case that. p "

For Example:

- the negation of "I have more than 5 friends" will become "I have at most 5 friends"

Definition 2.2 (Conjunction). let p and q be propositions , the conjunction of p and q denoted by $p \wedge q$ is a proposition " p and q " that become true only if both p and q are both *True*

Definition 2.3 (Disjunction). let p and q be propositions .The Disjunction of p and q denoted by $p \vee q$ is a proposition " p or q " which is *False* only if both of p and q are *False*

Definition 2.4 (Exclusive Disjunction). let p and q be propositions .The exclusive or denoted by $p \oplus q$ is a proposition that is *True* if exactly one of p or q are *True* , and *False* otherwise

3 Conditional Statements

Definition 3.1. let p and q be prepositions . The Conditional Statements $p \rightarrow q$, "if p then q " is false whenever p is *False* or q is *True*

The meaning of $p \rightarrow q$ assert that q is true whenever p holds but not vise versa ,when p is *False* it does not matter what the value of q for implication to be *True* , p is called (hypothesis or antecedent or premise) while q is called conclusion or consequence .

English Phrases to express conditional statements:

- "if p , then q "
- " p implies q "
- " p is sufficient of q "
- " p only if q "
- " q is necessary for p "
- " q unless $\neg p$ " **important**
- " p only if q "
- " q whenever p "

Converse, Contrapositive, and Inverse: for conditional statement $p \rightarrow q$

- $q \rightarrow p$ called the converse .
- $\neg q \rightarrow \neg p$ called the Contrapositive and has the same truth value as the original statement
- $\neg p \rightarrow \neg q$ called the inverse

Definition 3.2. Let p and q be prepositions , the biconditional statement $p \leftrightarrow q$ is a preposition " p if and only if q " which is *True* when p and q have the same truth values and *False* otherwise

4 Precedence of Logical Operators

Logical Operator	Precedence
\neg	1
\wedge	2
\vee	3
\rightarrow	4
\leftrightarrow	5