German University in Cairo

Lectures 16

Math301

Fall 2020

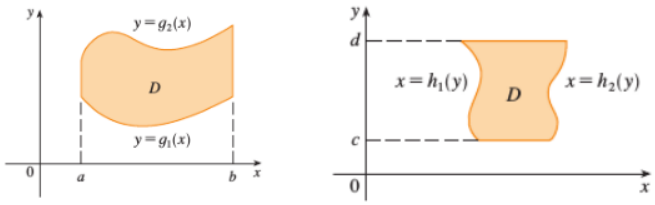
Contents

1 Lecture 6

Rule 1. Fubini's Thm.

says that double integrals are just iterated integrals. So you integrate wrt a variable while the other is const and then integrate again wrt too the other one.

Rule 2. How to solve double integrals of areas ?



Type I region Type II region

We have two cases I, II
So how to know which region we are talking about ?
simply type I as we see we draw two vertical lines making a recctangle (look at the roman I)
and type two is a the same rectangle but horizontal
So for type I regions:

$$\iint_D f(x,y)dA = \int_a^b [\int_{y=g_1(x)}^{y=g_2(x)} f(x,y)dy]dx \tag{1}$$

and for type II regions:

$$\iint_D f(x,y)dA = \int_c^d [\int_{x=u_1(x)}^{x=u_2(x)} f(x,y)dx]dy \tag{2}$$

Rule 3. Where to use double integrals ?

1. Volume Calculation
   from the double integral defention it's so clear that it's used to calculate volumes :)

$$Volume = Area * hight \tag{3}$$

$$V = \iint_D f(x,y)dA \tag{4}$$

we assume the function to be our hight (z-coordinate) (5)

2. Area Calculation
   if we thought of $f(x,y) = 1$ then the integral is just an area

$$V = \iint_D 1dA = A \tag{6}$$

3. Mass computation
   if we thought of $f(x,y) = \rho(x,y)$
   $TotalMass = massdensity * totalarea$

$$Mass = \iint_D \rho(x,y)dA \tag{7}$$

4. Charge computation
   if we thought of $f(x,y) = \sigma(x,y)$
   $TotalCharge = massdensity * totalarea$

$$Charge = \iint_D \sigma(x,y)dA \tag{8}$$

5. moment of inertia

6. center of gravity of flat regions

7. etc. you can find more in the textbook Stewart's Section 15.5