German University in Cairo

Laplace Summary

Math301

Fall 2020

Contents

1 Basic Definition of Laplace Transform

$$\boxed{f(t)} \longrightarrow \boxed{\mathcal{L}f(t)} \longrightarrow \boxed{\text{F(S)}}$$

- Converts integral equation & DE's into algebric equations.
- COVERSION FROM TIME-DOMAIN TO FREQUENCY-DOMAIN

- Applications of Laplace Transform: in ODE,PDE, integral equations, etc ...

$$F(S) = \mathcal{L}f(t) = \int_0^\infty f(t).e^{-st}dt;\ for\ t \geq 0 \tag{1}$$

Improper Integral

$$\int_0^\infty f(t)e^{-st}dt = \lim_{b\to\infty}\int_0^b f(t).e^{-st}dt \tag{2}$$

Note

- Laplace Transform is applicable to continuous and discontinuous functions.
- The Laplace Transform is used if $f(t)$ is given without any integral, if given with interval use BASIC DEFINITION

2 Laplace Transform Properties

2.1 Lineraty of Laplace Transform

- $\mathcal{L}[f(t) \pm g(t)] = \mathcal{L}f(t) \pm \mathcal{L}g(t)$
- $\mathcal{L}[cf(t)] = c\mathcal{L}(f(t)) = cF(S)$

2.2 S- Shifiting property

$$\mathcal{L}(e^{at}f(t)) = F(S-a) \tag{3}$$

2.3 Laplace Transform of dervatives

- $\mathcal{L}[f'(t)] = S\mathcal{L}(f) - F(0)$
- $\mathcal{L}[f''(t)] = S^2\mathcal{L}[f] - Sf(0) - f'(0)$

- for n $\geq 1; \mathcal{L}[f^{(n)}] = S^{(n)}\mathcal{L}[f] - S^{(n-1)}f(0) - S^{(n-2)}f'(0) - \dots f^{(n-1)}(0)$

2.4 Laplace Transform of Integrals

$$\mathcal{L}[\int_0^t f(\tau)d\tau] = \frac{F(S)}{S} \tag{4}$$

2.5 Dervative of Laplace Transform

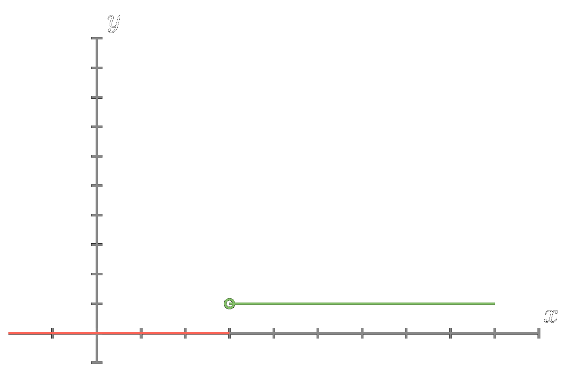
- $\mathcal{L}[tf(t)] = -F'(S)$
- $\mathcal{L}[t^2 f(t)] = F''(S)$
- $\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{dS^n} F(S)$

2.6 Integral of Laplace Transform

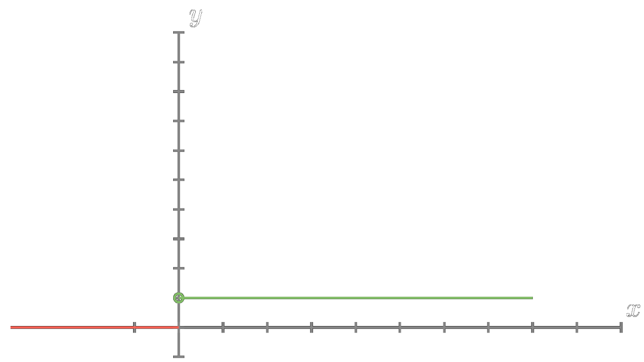
$$\mathcal{L}[\frac{f(t)}{t}] = \int_S^\infty F(U)dU \tag{5}$$

2.7 Heaviside function (Unit step Function)

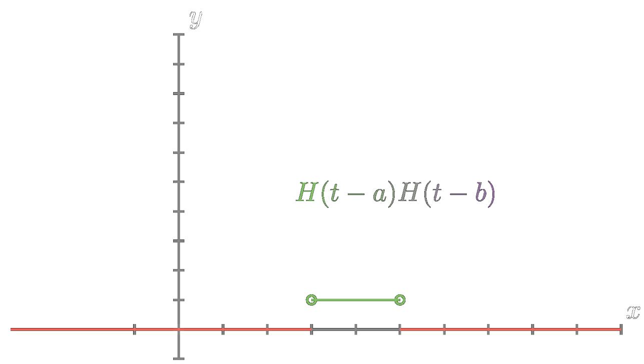
$$H(t-a) = \begin{cases} 0, & t < a \\ 1, & t > a \end{cases}$$



$$H(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$



$$H(t-b) = \begin{cases} 0, & t < b \\ 1, & t > b \end{cases}$$



2.7.1 Notes

- $H(f(t)) = \square; a < t < b$
  1. by definition $F(S) = \int_a^b \square e^{-st} dt$
  2. by unit step $f(t) = \square[H(t-a)H(t-b)]$
- $H(f(t)) = \square; t < b$
  1. by definition $F(S) = \int_b^\infty \square e^{-st} dt$
  2. by unit step $f(t) = \square[H(t-b)]$

2.8 t-shifiting property

- $\mathcal{L}[H(t-a)] = \frac{1}{s} e^{-as}$

- $\mathcal{L}[f(t-a)H(t-a)] = F(S)e^{-as}$

2.9 Dirac's Delta Function

$$\delta(t-a) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

$$\mathcal{L}[\delta(t-a)] = e^{-as}$$
$$\mathcal{L}[\delta(t)] = 1$$
$$\mathcal{L}[f(t)\delta(t-a)] = f(a)e^{-as}$$

2.10 Convolution Product

$$(f * g)(t) = \int_0^t f(\tau)g(t-\tau)d\tau;$$

2.11 Convolution Theorem

$$\mathcal{L}[f * g] = \mathcal{L}[f]\mathcal{L}[g]]$$