Lecture 3

Ibrahim Abou Elenein

May 16, 2021

Contents

1 Sample Spaces

The set of all possible outcomes of a random experiment is called a *sample space*

2 Events

An event E, is a set of some outcomes of a probability experiment, i.e. a subset of the sample

$$E \subseteq S$$

If an event E contains no outcomes, then E is an impossible event.

3 Probability

$$P(E) = \frac{N(E)}{N(S)}$$

$$0 \leq P(E) \leq 1; \; P(S) = 1$$

Example 3.1. A card is drawn from a standard deck. Find the probabilities of the following events.

- Getting a queen.
- Getting a club.
- Getting a number.

Solution 3.1.

$$|S| = 52$$

E1 is the event of getting a queen, $P(E1) = \dfrac{|E1|}{|S|} = \dfrac{4}{52} = \dfrac{1}{13}$

E2 is the event of getting a club, $P(E2) = \dfrac{|E2|}{|S|} = \dfrac{13}{52} = \dfrac{1}{4}$

E3 is the event of getting a number, $P(E3) = \dfrac{|E3|}{|S|} = \dfrac{40}{52} = \dfrac{10}{13}$

4 Counting Rules

4.1 Multiplication Rule

In a sequence of two experiments, if the first experiment can occur in m different ways and the second one can occur in n different ways, then the whole sequence can occur in $m \times n$ different ways.

Example 4.1. Consider the manufacturing of: number-plates consisting of two letters followed by four digits

1. How many plates are possible?

$$26 \times 26 \times 10 \times 10 \times 10 \times 10 = 6760000$$

2. How many plates are possible, if no letter or digit can be repeated?

$$26 \times 25 \times 10 \times 9 \times 8 \times 7 = 3276000$$

Example 4.2. In a class of 10 students, 6 are to be chosen and seated in a row for a picture. How many different pictures are possible?

$$10 \times 9 \times 8 \times 7 \times 6 \times 5 \ different\ pictures.$$

Tip what if they were to sit in a circle?

4.2 Permutaion

An arrangement of n objects A SPECIFIC ORDER using k objects at a time is called a permutation and it is denoted by ${}^{n}P_k$. The number of repetition-free permutations (linear arrangements) of size k from a set of n distinct objects is given by:

$${}^{n}P_k = \frac{n!}{(n-k)!}; \quad 0 \leq k \leq n$$

4.3 Combination

A selection of k distinct objects WITHOUT REGARD TO ORDER out of n objects is called a combination and it is denoted by ${}^{n}C_k$ The number of repetition-free combinations of size k from n distinct objects is given:

$${}^{n}C_k = \frac{{}^{n}P_k}{k!} = \frac{n!}{(n-k)!k!}; \quad 0 \leq k \leq n$$

5 Problems

Example 5.1.

Example 5.2. In a class of 10 students, three are to be chosen to represent the class in a competition. How many selections are possible?

Note that, here, the students are not selected in any specific order.
The number of selections is ${}^{10}C_3$

Example 5.3. A student is taking a Math-401 test in which 7 questions out of 10 are to be answered. In how many ways can the student answer the exam if :

1. Any 7 questions may be selected.

$$\text{The student can answer the exam in } {}^{10}C_7$$

2. The first 2 questions must be selected.

$$\text{The student can answer the exam in } {}^{8}C_5$$

3. The student must choose 3 questions from the first 5 and 4 questions from the

$$\text{The student can answer the exam in } {}^{5}C_3 \times {}^{5}C_4 = 50$$

Example 5.4. What is the number of (possibly meaningless) words that are made up of all the letters in the word **chemistry**

$$\text{The number of such words id } {}^{9}P_9$$

Example 5.5. What is the number of permutations of the letters in the word **ball**?

Note that there are repeated letter so it's not 4 distinct objects.

$$\text{The number of permutations is } \frac{{}^{4}P_4}{2!} = 12$$

Example 5.6. What is the number of permutations of the letters in the word **Pepper**?

$$\text{The number of permutations is } \frac{{}^{6}P_6}{2!3!}$$

Example 5.7. The owner of a pizzeria prepares every pizza by always combining 4 different ingredients. How many ingredients does he need, at least, if he would like to offer 30 different pizzas in the menu?

$$\text{we need to find } n \text{ such that } {}^{n}C_4 = 30$$

$$\frac{n!}{(n-4)!4!} = 30$$

$$\frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!} = 30 \times 4!$$

$$n(n-1)(n-2)(n-3) - 720 = 0$$

By solving the equation $n = 6.8 \Rightarrow$he needs at least 7 ingredients

Example 5.8. You play a simple card game. You draw 3 cards. If any of them is a King or the three cards are of the same suit, you win otherwise you lose. How many hands are you winning?

$$\text{A} = \text{draw at least one king in a hand of 3 cards}$$
$$\text{N(A)} = \text{One King or Two or Three} =$$
$$({}^{4}C_1 \times {}^{48}C_2) + ({}^{4}C_2 \times {}^{48}C_1) + ({}^{4}C_3 \times {}^{48}C_0)$$

$$\text{B} = \text{A hand draw 3 cards with same suit}$$
$$N(B) = {}^{4}P_1 \times {}^{13}P_3$$

$$A \cap B = \text{a hand with draw 3 cards with a king and the same suit.}$$
$$N(A \cap B) = {}^{4}C_1 \times {}^{1}C_1{}^{12}C_2$$

$$N(A \cup B) = N(A) + N(B) - N(A \cap B) = 5684.$$

Example 5.9. In how many ways you can arrange the word "ORANGE", if:

- 2 vowels and 2 consonants are used to make 4-letter words.

$$\text{No.of ways} = {}^{3}C_2 \times {}^{3}C_2 \times 4!$$

- 2 vowels and 3 consonants are used to make 5-letter words.

$$\text{No.of ways} = {}^{3}C_2 \times {}^{3}C_3 \times 5!$$

Example 5.10. There are 8 men and 9 women on a committee selection pool. A committee consisting of: a president, a vice-president, and 3 coordinators; is to be formed. In how many ways can exactly three women be on the committee?

$$N = {}^{9}C_3 \times {}^{8}C_2 \times [\frac{5!}{3!}]$$