Lecture 1 Summary

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1 Energy and Power

1.1 Energy

Continuous

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt \qquad (1)$$

Discrete

$$E_\infty = \sum_{-\infty}^{\infty} |x[n[|^2 \qquad (2)$$

1.2 Power

1.2.1 For non Periodic Signals

Continuous

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \qquad (3)$$

Discrete

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} |x[n]|^2$$

1.2.2 For Periodic

$$P = \frac{1}{T_o} \int_{\frac{-T_o}{2}}^{\frac{T_o}{2}} |x_p(t)|^2 dt$$

where T_o is The Fundamental Period

2 Signal Decomposition

$$x(t) = Evx(t) + Odx(t)$$

With

$$Ev\{x(t)\} = \frac{1}{2}[x(t) + x(-t)] \qquad Od\{x(t)\} = \frac{1}{2}[x(t) - x(-t)]$$

3 Notes

- A Power Signal is signal its power is finite
- An Energy Signal is signal its energy is finite

3.1 From Euler Formula

$$e^{jx} = \cos x + j\sin x$$
$$|e^{jx}| = |\cos x + j\sin x| = \sqrt{\cos^2 x + \sin^2 x} = 1$$

3.2 Unit step function

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$
$$|f(t)u(t)| = f(t) \qquad 0 < t < \infty$$

3.3 Geometric Series

$$a + ar + ar^2 + \cdots + ar^n = \sum_{k=0}^{n} ar^k = a(\frac{1 - r^{n+1}}{1 - r})$$
$$a + ar + ar^2 + \cdots = \sum_{k=0}^{\infty} ar^k = \frac{a}{1 - r} \qquad for \quad |r| < 1$$

3.4 Series

$$\sum_{i=1}^{n} c = n.c$$
$$\sum_{n=0}^{N} c = (N + 1).c$$
$$\sum_{n=-N}^{N} c = \sum_{n=0}^{2N} c = c(2N + 1)$$

Shifting by adding the upper and lower bound by N

$$\sum_{i=1}^{n} i = \frac{n(n + 1)}{2}$$
$$\sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6}$$