

Lecture 1 Summary

Ibrahim Abou Elenein

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1 Energy and Power

1.1 Energy

Continuous

$$E_{\infty} = \int_{-\infty}^{\infty} |x(t)|^2 dt \quad (1)$$

Discrete

$$E_{\infty} = \sum_{n=-\infty}^{\infty} |x[n]|^2 \quad (2)$$

1.2 Power

1.2.1 For non Periodic Signals

Continuous

$$P_{\infty} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt \quad (3)$$

Discrete

$$P_{\infty} = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x[n]|^2$$

1.2.2 For Periodic

$$P = \frac{1}{T_o} \int_{-\frac{T_o}{2}}^{\frac{T_o}{2}} |x_p(t)|^2 dt$$

where T_o is The Fundamental Period

2 Signal Decomposition

$$x(t) = Evx(t) + Odx(t)$$

With

$$Ev\{x(t)\} = \frac{1}{2}[x(t) + x(-t)]$$

$$Od\{x(t)\} = \frac{1}{2}[x(t) - x(-t)]$$

3 Notes

- A Power Signal is signal its power is finite
- An Energy Signal is signal its energy is finite

3.1 From Euler Formula

$$e^{jx} = \cos x + j \sin x$$
$$|e^{jx}| = |\cos x + j \sin x| = \sqrt{\cos^2 x + \sin^2 x} = 1$$

3.2 Unit step function

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$
$$|f(t)u(t)| = f(t) \quad 0 < t < \infty$$

3.3 Geometric Series

$$a + ar + ar^2 + \dots + ar^n = \sum_{k=0}^n ar^k = a \left(\frac{1 - r^{n+1}}{1 - r} \right)$$
$$a + ar + ar^2 + \dots = \sum_{k=0}^{\infty} ar^k = \frac{a}{1 - r} \quad \text{for } |r| < 1$$

3.4 Series

$$\sum_{i=1}^n c = n \cdot c$$
$$\sum_{n=0}^N c = (N + 1) \cdot c$$
$$\sum_{n=-N}^N c = \sum_{n=0}^{2N} c = c(2N + 1)$$

Shifting by adding the upper and lower bound by N

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$
$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$