

Lecture 2 Summary

Ibrahim Abou Elenein

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1 Periodicity

1.1 General

1.1.1 Continuous

if $x(t) = x(t + T) \implies x(t)$ is Periodic

1.1.2 Discrete

if $x[n] = x[n + N] \implies x[n]$ is Periodic

1.2 Special Cases

1.2.1 Continuous

Complex exponential is always periodic with period $T_o = \frac{2\pi}{\omega_o}$

Example 1.

$$e^{j\omega_o t}, \cos \omega_o t, \sin \omega_o t \quad T_o = \frac{2\pi}{\omega_o}$$

1.2.2 Discrete

Complex exponential may be periodic if $\frac{\omega_o}{2\pi} = \text{rational number} = \frac{m}{N}$ where N is the period

Example 2.

$$e^{j\omega_o n}, \cos(\omega_o n), \sin(\omega_o n)$$

if $\frac{\omega_o}{2\pi} = \frac{m}{N}$ where N = Period

1.3 Examples

- $x(t) = je^{j10t}$ Periodic with $T_o = \frac{2\pi}{\omega_o} = \frac{\pi}{5}$
- $x[n] = e^{j\pi n}$ $\frac{\omega_o}{2\pi} = \frac{1}{2}$ $N = 2$

- $x[n] = \cos(\frac{2\pi}{4}n) \quad \frac{\omega_o}{2\pi} = \frac{1}{4}$
- $x[n] = 3e^{j\frac{3\pi}{5}(n+\frac{1}{2})} = 3e^{j\frac{3\pi}{5}(0.5)}e^{j\frac{3\pi}{5}n} \quad \frac{\omega_o}{2\pi} = \frac{3}{10}$

1.4 Notes

- constant functions is periodic with arbitrary period
- Periodic \pm aperiodic = aperiodic
- periodic \times aperiodic = aperiodic
- periodic \pm periodic = periodic if $\frac{\omega_{o1}}{\omega_{o2}} = \text{rational}$

The Period of the sum of two periodic signals is the lowest common multiple LCM of the individual periods.

The frequency of the sum of two periodic signal is the greatest common divisor GCD of the individual frequencies.

1.4.1 Examples

1. $x(t) = e^{(j-1)t} = e^{jt}(\text{complex exp}) * e^{-t}(\text{real exp}) =$
periodic \times aperiodic $\longrightarrow x(t)$ is aperiodic
2. $x(t) = \cos(\pi t) + \sin(2t) \quad \frac{\omega_{o1}}{\omega_{o2}} = \frac{\pi}{2}$ is not rational s $x(t)$ is not periodic