

Unit Step and Unit Impulse

Ibrahim Abou Elenein

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1 Discrete Unit Impulse

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

2 Discrete Unit Step

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

3 Rectangler Signal

$$x(n) = \begin{cases} 1, & 0 \leq n \leq N \\ 0, & otherwise \end{cases}$$

4 Continuous Unit Step

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

5 Continuous Unit Impulse

$$\delta(t) = 0 \text{ for } t \neq 0$$

6 Relation between Unit Impulse and Unit Step

$$\delta(n) = u(n) - u(n-1)$$

$$\delta(t) = \frac{d}{dt}[u(t)]$$

$$u(n) = \sum_{k=0}^{\infty} \delta(n-k)$$

$$u(n) = \sum_{m=-\infty}^n \delta(m)$$

$$u(t) = \int_{-\infty}^t \delta(\tau) d\tau$$

7 Notes

$$\int_{-\infty}^{\infty} \delta(\tau) d\tau = 1$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k) \delta(n-k)$$

$$\int_{t_o-\epsilon}^{t_o+\epsilon} f(t) \delta(t-t_o) dt = f(t_o) \forall \epsilon > 0$$

Example

$$\int_0^2 \sin(\pi t) \delta(t-2.5) dt$$

at $t = 2.5$ this value are not in the interval $[0,2]$ so it's zero

Example

$$\int_{-4}^7 \sin\left(\frac{\pi t}{2}\right) \delta(t-1) dt$$

the value $t = 1$ is in the interval so

$$\int_{-4}^7 \sin\left(\frac{\pi t}{2}\right) \delta(t-1) dt = f(1) = \sin\left(\frac{\pi \times 1}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$$