Unit Step and Unit Impulse

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1 Discrete Unit Impulse

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

2 Discrete Unit Step

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

3 Rectangler Signal

$$x(n) = \begin{cases} 1, & 0 \leq n \leq N \\ 0, & otherwise \end{cases}$$

4 Continuous Unit Step

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

5 Continuous Unit Impulse

$$\delta(t) = 0 \ for \ t \neq 0$$

6 Relation between Unit Impulse and Unit Step

$$\delta(n) = u(n) - u(n-1)$$

$$\delta(t) = \frac{d}{dt}[u(t)]$$

$$u(n) = \sum_{k=0}^{\infty} \delta(n-k)$$

$$u(n) = \sum_{m=-\infty}^{n} \delta(m)$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau$$

7   Notes

$$\int_{-\infty}^{\infty} \delta(\tau)d\tau = 1$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$$

$$\int_{t_o-\epsilon}^{t_o+\epsilon} f(t)\delta(t-t_o)dt = f(t_o)\forall \quad \epsilon > 0$$

Example

$$\int_{0}^{2} sin(\pi t)\delta(t-2.5)dt$$

at $t = 2.5$ this value are not in the interval [0,2] so it's zero

Example

$$\int_{-4}^{7} sin(\frac{\pi t}{2})\delta(t-1)dt$$

the value $t = 1$ is in the interval so

$$\int_{-4}^{7} sin(\frac{\pi t}{2})\delta(t-1)dt = f(1) = \sin(\frac{\pi \times 1}{2}) = \sin(\frac{\pi}{2}) = 1$$