Lecture 2 Summary

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1 Periodicity

1.1 General

1.1.1 Continuous

if $x(t) = x(t+T) \Longrightarrow x(t)$ is Periodic

1.1.2 Discrete

if $x[n] = x[n+N] \Longrightarrow x[n]$ is Periodic

1.2 Special Cases

1.2.1 Continuous

Complex exponential is always periodic with period $T_o = \frac{2\pi}{\omega_o}$

Example 1.

$$e^{j\omega_o t}\ ,\ \cos\omega_o t\ ,\ \sin\omega_o t \qquad T_o = \frac{2\pi}{\omega_o}$$

1.2.2 Discrete

Complex exponential may be periodic if $\frac{\omega_o}{2\pi} = \text{rational number} = \frac{m}{N}$ where N is the period

Example 2.

$$e^{j\omega_o n}\ ,\ \cos(\omega_o n)\ ,\ \sin(\omega_o n)$$

if $\frac{\omega_o}{2\pi} = \frac{m}{N}$ where N = Period

1.3 Examples

- $x(t) = je^{j10t}$ Periodic with $T_o = \frac{2\pi}{\omega_o} = \frac{\pi}{5}$
- $x[n] = e^{j\pi n}$ $\frac{\omega_o}{2\pi} = \frac{1}{2}$ $N = 2$

- $x[n] = \cos(\frac{2\pi}{4}n) \quad \frac{\omega_o}{2\pi} = \frac{1}{4}$
- $x[n] = 3e^{j\frac{3\pi}{5}(n+\frac{1}{2})} = 3e^{j\frac{3\pi}{5}(0.5)}.e^{j\frac{3\pi}{5}n} \quad \frac{\omega_o}{2\pi} = \frac{3}{10}$

1.4 Notes

- constant functions is periodic with arbitrary period
- Periodic $\pm$ aperiodic = aperiodic
- periodic x aperiodic = aperiodic
- periodic $\pm$ periodic = periodic if $\frac{\omega_o 1}{\omega_o 2}$ = rational

  The Period of the sum of two periodic signals is the lowest common multiple LCM of the individual periods.

  The frequency of the sum of two periodic signal is the greatest common divisor GCD of the individual frequencies.

1.4.1 Examples

1. $x(t) = e^{(j-1)t} = e^{jt}(complex \;\; exp) * e^{-t}(real \;\; exp) =$
   periodic x aperiodic $\longrightarrow x(t)$ is aperiodic
2. $x(t) = \cos(\pi t) + \sin(2t) \quad \frac{\omega_{o1}}{\omega_{o2}} = \frac{\pi}{2}$ is not rational s $x(t)$ is not periodic